XXVII. COMMUNICATIONS BIOPHYSICS[*]

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RESEARCH OBJECTIVES AND SUMMARY OF RESEARCH

The major concern of the Communications Biophysics Group is the search for a better understanding of sensory communication, in particular, of hearing. As in the past, future research will tend to combine electrophysiological and behavioral experiments with machine data processing and analytical methods from communication theory. We continue to maintain a close friendship with the Eaton-Peabody Laboratory, at the

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Massachusetts Eye and Ear Infirmary, in Boston. The cooperative arrangements include the sharing of facilities and joint appointments for staff members. Our principal recent accomplishments and plans for the future can be summarized as follows.

1. Related Behavioral and Electrophysiological Studies

Electrophysiological studies of conditioning in the rat have been concerned with changes in sensory evoked potentials correlated with conditioned changes in behavior. Experimental work has been restricted for the most part to the primary projection pathways of the auditory system. Increases in the amplitudes of click-evoked potentials recorded from the auditory cortex and medial geniculate body have been demonstrated when the click stimulus is established as a conditioned one under several aversive conditioning procedures. Changes in auditory evoked potentials have not been found when the click is established as a discriminative stimulus under operant conditioning procedures employing positive reinforcement. It has also been shown that the changes in auditory potentials found with aversive conditioning procedures are not dependent upon the establishment of the click as the conditioned stimulus. Rather, they reflect a more general change that is not modally specific. Confounding effects related largely to movement variables have been experimentally eliminated. The increases in amplitudes of auditory evoked potentials have been attributed, therefore, to a radical change in state which the conditioned fear represents in aversive conditioning paradigms. Electrophysiological studies of conditioning will continue.

2. Physiological Studies

a. Sensory Systems

(i) The relationship between physiological properties and anatomical location of single units in the cochlear nucleus has been studied in more detail than previously. In the coming year our electrophysiological studies of the cochlear nucleus will continue with emphasis on the response characteristics of single units in the oral pole (anterior portion) of the ventral cochlear nucleus.

This research will be carried out in cooperation with the Eaton-Peabody Laboratory.

(ii) Recent experimental work on the efferent innervation of the cochlea has indicated that stimulation of the crossed olivo-cochlear bundle has a much greater effect on auditory nerve response to high-frequency than to low-frequency stimuli. This work will be extended to include the effect of the efferents on single VIIIth-nerve fibers.

(iii) Extensive study of the statistical characteristics of the electrical responses evoked by clicks, flashes, and somatic stimuli in sleeping and awake human subjects will continue.

(iv) Studies of inhibition phenomena in single primary auditory nerve fibers will continue.

(v) In order to study cochlear electrophysiology in more detail, techniques have been developed for introducing micropipettes into the cochlea, recording DC and microphonic potentials, and determining the position of the electrode. We hope that the continuation of this work will lead to better understanding of the mechanisms of transduction from mechanical signals into patterns of neural activity.

(vi) Some completed studies in this area include: measurements of the sound pressure transformation by the head and auditory meatus of the cat, and experimental work on the middle ear of anesthetized cats, which has led to a description of the effects of opening the bulla, and some indication of the role of the joints between middle-ear bones.

b. Other Systems

(i) Instruments and biological techniques have been developed to allow the insertion of microelectrodes into explants of brain tissue in vitro under direct microscopic vision. Computer programs have been worked out for the processing of the spontaneous potentials that arise in such explants. The coming year will be devoted to the application of these techniques and instruments.

(ii) Research will continue on correlation studies of normal and pathological tremors in relation to the EEG, intrinsic and induced EEG rhythms in metabolic disorder in relation to higher mental functions, and study of evoked responses in relation to temporal conditioning to paired stimuli.

(iii) Studies are in progress on the electrical activity of respiratory neurons of the medulla (especially in the region of the solitary tract) corresponding to artificially induced changes in respiration.

(iv) Apparatus has been constructed for intracellular recording from the nervous system of Aplysia californica, and a preliminary survey of the sensory characteristics of the periesophageal ganglia has been accomplished. In the coming year, work will proceed on a more detailed analysis of pathways involved in touch, with special attention to an interesting group of tactile cells in the cerebral ganglion.

(v) Response patterns of Purkinje cells in the cerebellum of the cat to sensory stimuli are being studied in relation to a possible stochastic model.

3. Other Achievements and Proposed Studies

a. New data processing techniques, based in part on the concept of a "recovered firing probability," have been developed and applied to electrophysiological data on the spike activity of single auditory nerve fibers in response to various acoustic stimuli. The results of such processing have yielded several important constraints on possible random-process models which will be further explored in the coming year.

b. Studies have been completed on the effects of noise bandwidth on binaural unmasking, and on the analogous phenomenon of bilateral unmasking on the skin. Also, efforts have been made to extend the equalization and cancellation model to phenomena other than unmasking. Next year, the work on unmasking will continue and projects are contemplated in areas related to speech, perception, and auditory memory.

c. Various models for the convergence of auditory-nerve input upon cells of the cochlear nucleus are being studied analytically; the present phase of this work should be completed during the coming year.

d. The PDP-4 computer has been equipped with a special interface that allows extensive control of stimuli and on-line data processing at any of several experimental stations in the laboratory. This facility has already proved its value and should be running at peak capacity during the coming year.

W. M. Siebert, W. A. Rosenblith

Selected Publications

1. J. S. Barlow, "Inertial Navigation in Relation to Animal Navigation," J. Institute of Navigation (in press).

2. J. S. Barlow, "Correlation Analysis of EEG-Tremor Relationships in Man," EEG Clin. Neurophysiol. Suppl. (in press).

3. J. S. Barlow, "Some Observations on Evoked Responses in Relation to Temporal Conditioning to Paired Stimuli in Man," Proc. International Colloquium on Mechanisms of Orienting Reaction in Man (in press).

4. S. K. Burns and R. Melzack, "A Method of Analyzing Variations of Evoked Responses," EEG Clin. Neurophysiol. (in press).

5. R. R. Capranica, The Evoked Vocal Response of the Bullfrog: A Study of Communication by Sound, Research Monograph No. 33 (The M.I.T. Press, Cambridge, Mass., 1965).

6. A. Cavaggioni and M. H. Goldstein, Jr., "Facilitation and Inhibition in the Visual System after Photic Stimulation," Arch. ital. Biol. (Pisa) 103, 397-420 (1965).

7. H. S. Colburn and N. I. Durlach, "Time-Intensity Relations in Binaural Unmasking," J. Acoust. Soc. Am. 38, 93-103 (1965).

8. G. Guilbaud, W. Rosenblith, S. Burns, et D. Albe-Fessard, "Évolution chez l'Homme au cours des différents stades du sommeil des réponses électrocorticales évoquée au vertex par deux modes de stimulation," C. R. Acad. Sci., Paris, t. 260, p. 5366-5369 (17 mai 1965).

9. J. L. Hall II, "Binaural Interaction in the Accessory Superior-Olivary Nucleus of the Cat," J. Acoust. Soc. Am. 37, 814-822 (1965).

10. R. D. Hall, R. J. Clayton, and R. Mark, "A Device for the Partial Restraint of Rats in Operant Conditioning Studies," J. Exptl. Analysis Behav. (in press).

11. N. Y. S. Kiang, "Stimulus Coding in the Auditory Nerve and Cochlear Nucleus," Acta Oto-Laryngol. 59, 186-200 (1965).

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13. N. Y. S. Kiang, T. Watanabe, E. C. Thomas, and L. F. Clark, Discharge Patterns of Single Fibers in the Cat's Auditory Nerve (The M.I.T. Press, in press).

14. R. Melzack and S. K. Burns, "Neurophysiological Effects of Early Sensory Restriction," Exptl. Neurol. 13, 163-175 (1965).

15. B. J. Murawski and S. K. Burns, "Day-to-Day Correlation of Urinary Adrenal Steroids and Alpha Frequency in the EEG," J. Exptl. Physiol. (in press).

16. R. R. Pfeiffer, "Classification of Response Patterns of Spike Discharges for Units in the Cochlear Nucleus: Tone-Burst Stimulation," Exptl. Brain Research (in press).

17. R. R. Pfeiffer and N. Y-s. Kiang, "Spike-Discharge Patterns of Spontaneous and Continuously Stimulated Activity in the Cochlear Nucleus of Anesthetized Cats," Biophys. J. 5, 301-316 (1965).

18. W. Simon, "The Real-Time Sorting of Neuro-electric Action Potentials in Multiple Unit Studies," EEG Clin. Neurophysiol. 18, 192-195 (1965).

19. W. Simon, "Microphotography of Deep Fields," Rev. Sci. Instr. (in press).

20. W. Simon, "The Use of a Small Computer in the Physiology Laboratory," NEREM Record (in press).

21. W. A. Rosenblith, "Engineering in the Sciences of Life and Man," Listen to Leaders in Engineering, A. Love and J. S. Childers (eds.) (Tupper and Love, Atlanta, 1965), pp. 275-288.

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24. W. A. Rosenblith and J. B. Wiesner, "Norbert Wiener: From Philosophy to Mathematics to Biology; the Life Sciences and Cybernetics," J. Nerv. Ment. Disease 140, 3-8 (January 1965).

25. W. M. Siebert, "Some Implications of the Stochastic Behavior of Primary Auditory Neurons," Kybernetik 2, 206-215 (1965).

26. F. M. Wiener, R. R. Pfeiffer, and A. S. N. Backus, "On the Sound Pressure Transformation by the Head and Auditory Meatus of the Cat," Acta Oto-Laryngol. (in press).

Theses

1. Lowell Ray Anderson, "Nervous System Activity — Is It Dependent upon the Breathing Cycle," S.B. Thesis, Department of Electrical Engineering, M.I.T.

2. Barry A. Blesser, "Radar Video Data Processor for Air Traffic Control," S.M. Thesis, Department of Electrical Engineering, M.I.T.

3. William H. Broadley, "Design and Operation of the CBL Variable-Rate Digital Clock," S.B. Thesis, Department of Electrical Engineering, M.I.T.

4. Joseph F. Dyro, "Cerebral Dominance in Perceiving and Learning Visual and Auditory Stimuli," S.B. Thesis, Department of Electrical Engineering, M.I.T.

5. Roger E. Graves, Jr., "The Effects of Temperature on the Lobster Cardiac Ganglion," S.B. Thesis, Department of Electrical Engineering, M.I.T.

6. Peter R. Gray, "A Statistical Analysis of Electrophysiological Data from Auditory Nerve Fibers in Cat," Ph.D. Thesis, Department of Electrical Engineering, M.I.T.

7. Philip L. Marcus, "Analysis of Spontaneous Electrical Activity in the Embryonic Brain Explants," S.M. Thesis, Department of Electrical Engineering, M.I.T.

8. Reed W. Markley, "Transmitter Noise and Clutter Affecting CW Radar Systems," S.M. Thesis, Department of Electrical Engineering, M.I.T.

9. Mark F. Medress, "An Analog to Digital Encoder Using Pulse Code Modulation," S.B. Thesis, Department of Electrical Engineering, M.I.T.

10. Wilbur E. Mosier, Jr., "The Acoustical Impedance of the Middle Ear of Anesthetized Cats," S.B. Thesis, Department of Electrical Engineering, M.I.T.

11. Edwin C. Moxon, "Triggered Laboratory Pulse Generator," S.B. Thesis, Department of Electrical Engineering, M.I.T.

12. Leonardo Peusner, "Complex Reaction Times for Pattern Recognition," S.B. Thesis, Department of Electrical Engineering, M.I.T.

13. Denis Poussart, "Measurements of Latency Distributions in Peripheral Nerve Fibers," S.M. Thesis, Department of Electrical Engineering, M.I.T.

14. Frederick C. Prout, "Electrical Communication in the Gymnarchus Niloticus, A Weakly Electric Fish," S.B. Thesis, Department of Electrical Engineering, M.I.T.

15. Michael J. Riezenman, "Binaural Unmasking as a Function of Bandwidth of Masking Noise," S.B. Thesis, Department of Electrical Engineering, M.I.T.

16. Jon A. Schafer, "Programming the PDP-4 for Use in Current Psychoacoustical Experiments," S.B. Thesis, Department of Electrical Engineering, M.I.T.

17. Philip E. Strause, "Effect of Subthreshold Stimuli on Frog Sciatic Nerve Refractory Periods," S.B. Thesis, Department of Electrical Engineering, M.I.T.

A. METHOD FOR STUDYING RESPONSE AND INHIBITION AREAS OF SOME SINGLE UNITS IN THE AUDITORY SYSTEM

Response and inhibition areas of single neural units have been of continual interest to investigators of the auditory system (for example, Galambos and Davis,[1] Rose, Galambos, and Hughes,[2] and Greenwood and Maruyama[3]). The response area of a single unit represents the region or range of signal-level and signal-frequency parameter settings within which a sinusoidal stimulus will elicit spike discharges. Conversely, an inhibition area represents that range of parameter settings within which the sinusoidal stimulus will reduce the number of spike discharges (in general, differences in inhibition areas depend on whether the discharges are spontaneous or in response to other stimuli).

One of two methods for determining these areas has been used by most investigators. In the first, the frequency range of response or inhibition for any given signal level is measured — with visual (oscilloscope) and auditory (loudspeaker) cues — by detecting the presence or absence of spike discharges synchronized to the stimulus. In the second, the frequency range of response or inhibition for any given signal level is measured by counting the number of spike discharges in a fixed time interval and either applying an arbitrary definition, for example, the frequency range within which the spike discharge rate is, say, 20 per cent greater (or less, for inhibition) than the spontaneous spike-discharge rate, or plotting average discharge rates for many points, and constructing rate contours from which the appropriate areas can be determined.

The first method is essentially a signal (response) in noise (spontaneous activity) detection problem. The subjective evaluations of whether or not there is a response are easily made by the experimenter for units that have little or no spontaneous activity, but are usually more difficult to make for units with moderate or high rates of spontaneous activity. The second method, while yielding more uniform results from unit to unit than the first, is usually more time-consuming because several counts must be made in order to determine a single data point for the response area. Neither of these methods yields much information concerning the response activity within the boundaries of the response area.

Figure XXVII-1 illustrates the details of a method of stimulation and data processing for obtaining response and inhibition (of spontaneous activity) areas, as well as iso-discharge rate contours, discharge rate versus signal-level functions, and tone-on-tone inhibition regions. The stimulus (A) is a fixed-amplitude sinusoid with a frequency varying logarithmically with time and cycling through a fixed range (one octave, two octaves or one decade, etc.). The sync signal (B) is used to initiate the computation of a histogram (D) (identical to the poststimulus time histograms[4] that are often computed for responses to periodic stimuli) from the action potentials (C) of a single unit. The abscissa of the histogram is calibrated in frequency, and, of course, can represent the

A

f_1

f_2

f_1

t

REPEAT

STIMULUS

B

SYNCHRONOUS SIGNAL

C

ACTION POTENTIALS (SCHEMATIC)

Σ OF N SWEEPS

D

No. of Events

$$\text{Average Firing Rate (Sp/sec)} = \frac{\text{No. of Events}}{\text{Bin width (sec)} \times \text{No. of Sweeps}}$$

f_1

f_2

f_1

RESPONSE PROFILE HISTOGRAM

E CENTER FREQUENCY OF BIN n

$$f_{c_n} = f_1 \, 10 \exp\left[\frac{\left(n - \frac{1}{2}\right)}{N} \cdot (\text{number of decades}) \right] = f_{c_{2N-n}} \ \text{cps}$$

Low to High
(1st half of histogram)

High to Low
(2nd half of histogram)

n = bin numbers ; 1,2,3, ···· N.

f_1 = low - frequency limit

N = Total number of bins / 2.

F BAND SWEEP OF STIMULUS PER BIN n

$$f_{B_n} = \left\{ f_1 \, 10 \exp\left[\frac{n}{N} \cdot \text{number of decades} \right] \right\} \left\{ 1 - 10^{-\frac{1}{N} \cdot \text{number of decades}} \right\} \text{cycles}$$

To calculate E and F for octave instead of decade sweeps ,
2 instead of 10 is raised to the powers indicated.

Fig. XXVII-1. Details of stimulus, data processing, and numerical calculations
for simple method of measuring response and inhibition areas.

frequency sweep from f_1 to f_2 or from f_2 to f_1 or both (as shown), as determined by the polarity of the sync signal used and the time resolution of the computation. Obvious numbers of interest, such as shown in D, E, and F, are easily computed.

Each histogram obtained in this way approximately represents a constant signal-level contour of the response region of a unit. Figure XXVII-2 is an isometric plot of a response region reconstructed from the rising frequency portion of histograms obtained for 6 different signal levels from a single unit located in the cochlear nucleus of a cat.
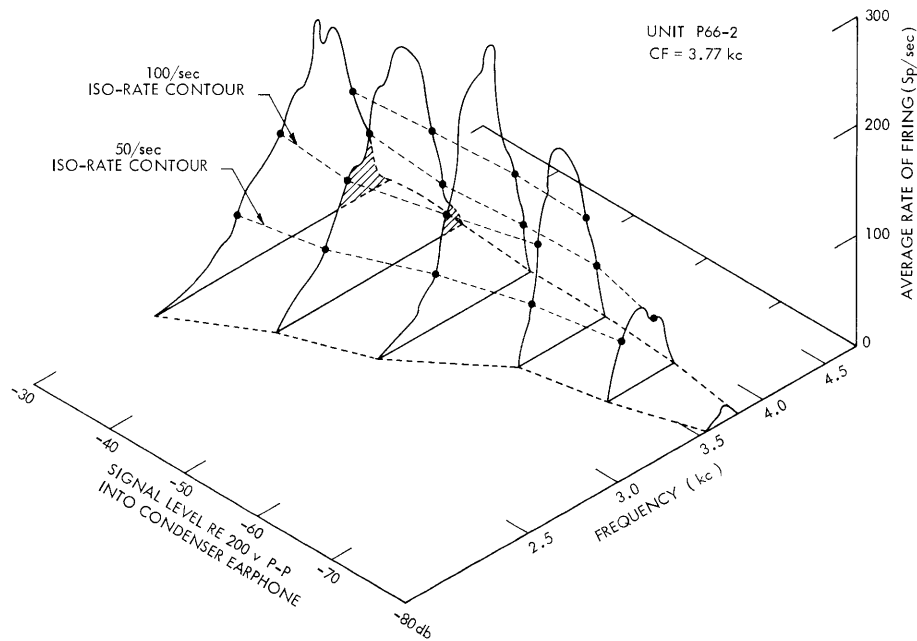

Fig. XXVII-2. Isometric plot of constant signal-level contours obtained from portions of 6 different histograms. Sweep rate, 1 cycle per 2 seconds. Each contour is based on results from 60 sweeps.

This particular unit had no spontaneous activity. From histograms or plots such as these, iso-discharge rate contours, discharge rate functions at fixed frequencies, and the usual response areas can be obtained. The data necessary to construct the plot in Fig. XXVII-2 were obtained in approximately 15 minutes.

As anticipated, the results obtained by an increasing frequency (f_1 to f_2) are often different from those obtained by a decreasing frequency (f_2 to f_1) and result in asymmetric histograms of the complete cycle. The degree of asymmetry has been found to vary from unit to unit, and is presumably a result of adaptation and fatigue effects. These differences, however, can be reduced for many units by using slow sweep speeds (tens of seconds per sweep). Nevertheless, it must be pointed out that this asymmetry
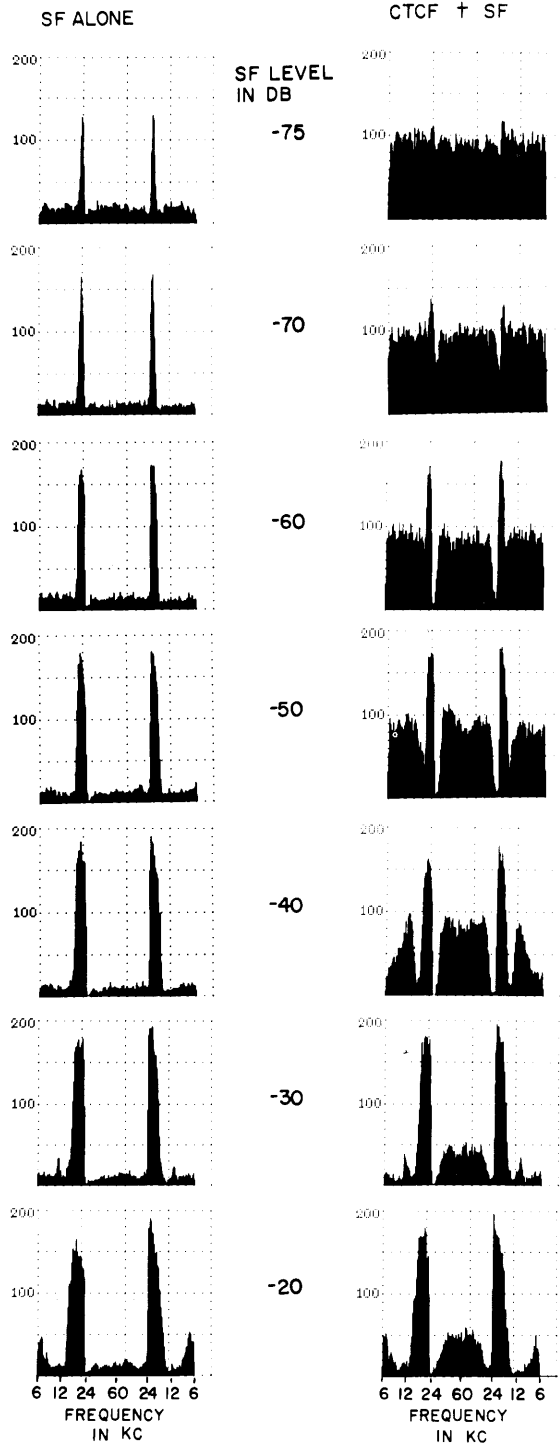
Fig. XXVII-3.

Application of sweep frequency stimulation to study of single auditory-nerve fibers as presented by Kiang and Sachs.[5] The left-hand column shows histograms of responses to sweep frequency alone at different signal levels; the right-hand column shows results of sweep frequency applied while a continuous tone was also present. Each of these histograms was obtained from responses to a single sweep of 22-sec duration.

effect does limit the use of this method, especially for measurements of iso-discharge rate contours.

Useful application of the method has been made by others in our laboratory.[5] Figure XXVII-3 illustrates the results obtained from a single auditory nerve fiber when the animal was stimulated by the sweep frequency alone (left column), and the sweep frequency and a continuous tone together (right column). The absence of inhibition of the spontaneous activity and the presence and extent of inhibition of the responses to the continuous tone for this unit are evident in the left- and right-hand columns, respectively.

The device used to generate the fixed-amplitude, logarithmically varying sinusoid is a modified Brüel and Kjaer 3306 Automatic Frequency Response Recorder. Briefly, the major requirements of the system are:

1. A range of available sweep rates,

2. Fast, minimum backlash, driver reversal,

3. Linear motion (logarithmic change in frequency) of the oscillator control throughout the sweep range of frequencies, and

4. Stability of the frequency limits, f_1 and f_2.

A block diagram of the system is shown in Fig. XXVII-4. Desired sweep rates are obtained by driving a synchronous motor with a variable frequency source. Linear
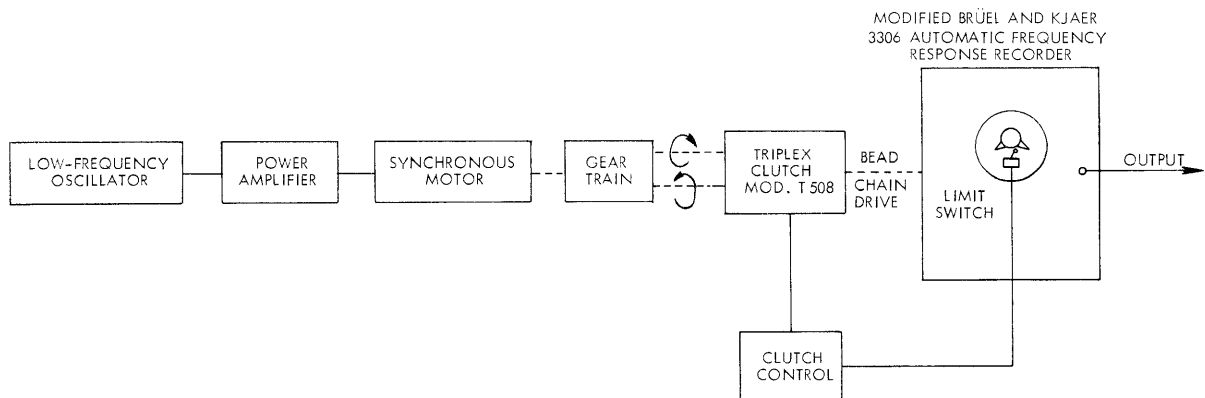

Fig. XXVII-4. Block diagram of the Triplex clutch-controlled
sweep frequency oscillator.

motion throughout the sweep range, fast reversal, and minimum backlash are obtained by simultaneously driving two inputs to a Triplex[6] magnetic clutch at the same, constant speed, but in opposite directions. The sweep direction of the oscillator is reversed in approximately 5 msec by coupling the Triplex clutch output to the alternate gear-train input. Backlash is minimized because the drive motor and the entire gear train always
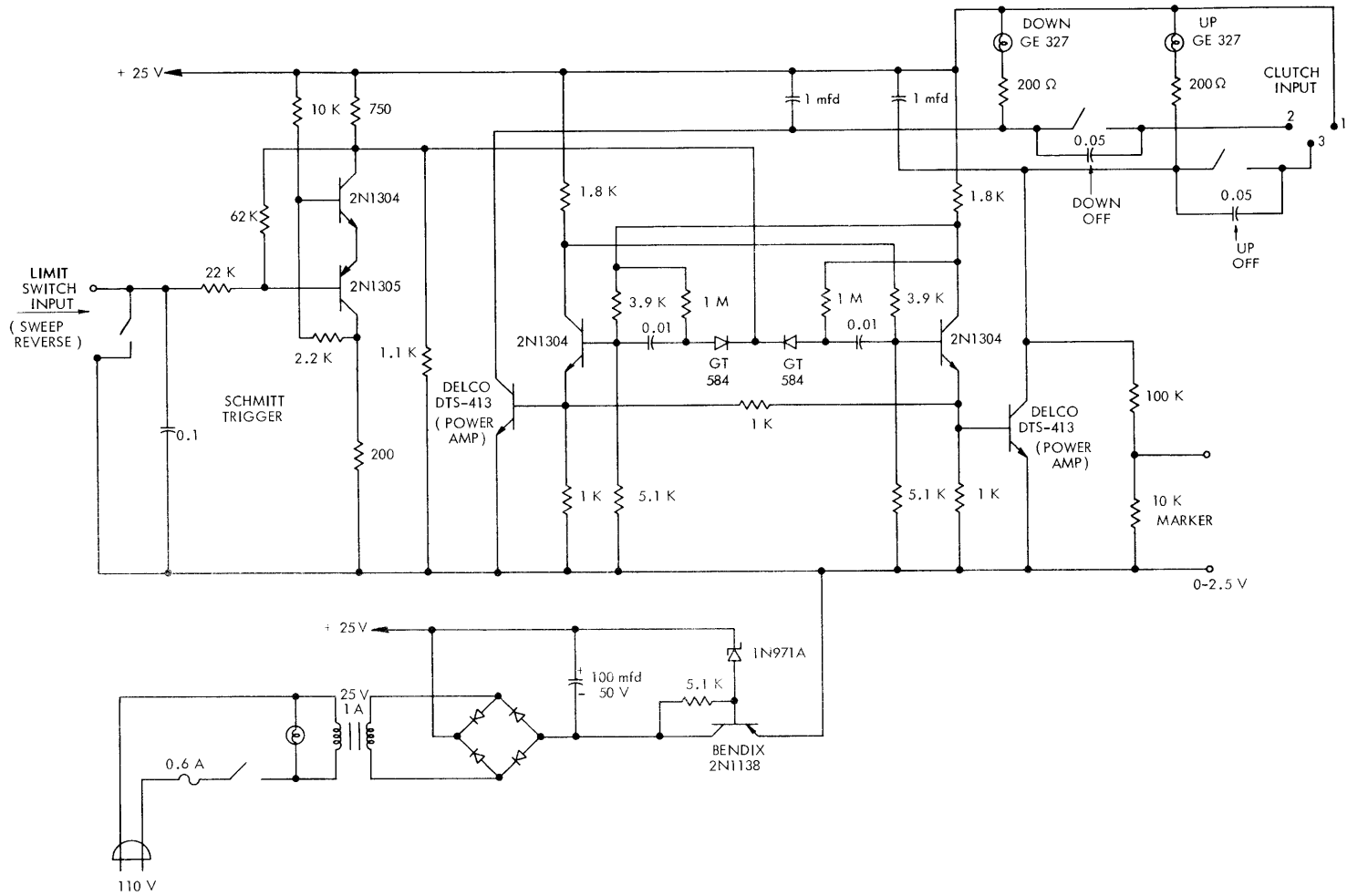
Fig. XXVII-5. Schematic diagram of Triplex clutch control (see Fig. XXVII-4).

turn in the same direction. The clutch control (Fig. XXVII-5) is a transistorized clutch coil energizer that energizes alternate clutch coils each time the limit switch is closed by a cam device mounted on the oscillator dial. The design of the cam device permits the operator to select a sweep range of 1 or 2 octaves or decades.

R. R. Pfeiffer, A. H. Crist

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2. J. E. Rose, R. Galambos, and J. R. Hughes, "Microelectrode Studies of the Cochlear Nuclei of the Cat," Bull. Johns Hopkins Hosp. 104, 211-251 (1959).

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6. Manufactured by Sterling Instruments, Mineola, New York.

B. STATISTICAL ANALYSIS OF ELECTROPHYSIOLOGICAL DATA FROM
AUDITORY NERVE FIBERS IN CAT

[This report is a brief summary of a Ph. D. thesis of the same title, submitted to
the Department of Electrical Engineering, M. I. T., August 1965. Further details
and results may be found in the thesis itself.]

In the last few years, a considerable amount of electrophysiological data collected
from single afferent auditory nerve fibers has become available. Many aspects of these
data appear to be rather simply related to the details of the anatomy, physiology, and
mechanical properties of the peripheral auditory system as they are, at present, under-
stood. This situation has encouraged efforts to develop some sort of abstract model to
describe the transformation from acoustic stimulus to firing patterns of the auditory-
nerve fibers. At least two attempts at such a model have recently been made, and this
study represents the beginning of still another attempt. The distinguishing characteristic
of this study is that the data have been processed in new ways in order to test more directly and refine the assumptions relating to models. The long-term objective in trying to develop this model is twofold. A manageable description of the peripheral auditory system would be valuable, if not essential, in considering what kinds of processing of auditory information could be performed by the brain. And, a model such as this could also provide new insight into the detailed processes within the peripheral auditory system which result in these firing patterns.

The initial portion of this study was based on electrophysiological recordings collected by Kiang and his co-workers.[3,4] As the study progressed, the acquisition of further data became necessary, and these were obtained in the same laboratory and with the same techniques as those used by Kiang and his co-workers. As in previous studies based on Kiang's data,[6,7] the principal statistics employed

UNIT 356-25
SPONTANEOUS ACTIVITY
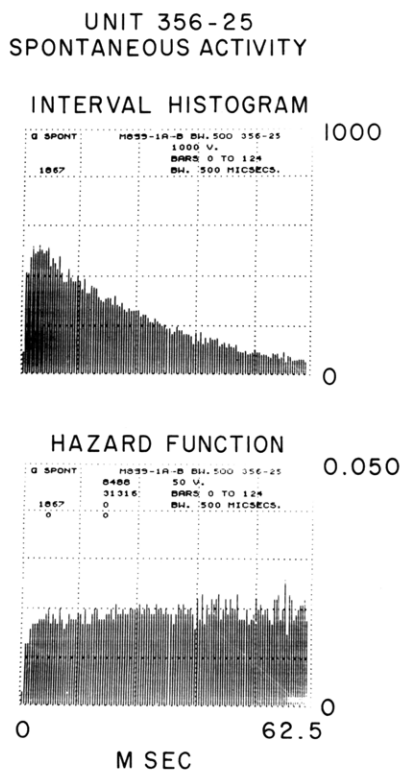
INTERVAL HISTOGRAM

HAZARD FUNCTION

Fig. XXVII-6. Unit 356-25. Interval his-
togram and hazard func-
tion (bin width 0.5 msec,
124 bars) calculated from
a long (≈14 minutes) re-
cording of spontaneous
activity.

have been poststimulus time (PST) and interval histograms.[2] For the present study, however, various conditional probability analyses have also been performed. Two examples of these analyses are given below.

The histograms presented in Fig. XXVII-6 are based on a long recording of the spontaneous activity of an auditory-nerve fiber. A conventional interval histogram is presented in the upper part of the figure. In this histogram, x_k, the height of the kth bar, represents the number of interspike intervals with duration between $k/2$ and $(k+1)/2$ msec. Previous investigations have indicated that the times between spontaneous spikes (or "events" or "firings") in auditory-nerve fibers may be regarded as essentially independent identically distributed random variables,[3] i. e., the event times may be considered as generated by a "renewal process."[1,5] In the analysis of such processes, a function called the "age specific failure rate" or "hazard function" is of some importance. By using the formula $\phi(k) = \dfrac{x_k}{\sum\limits_{n=k}^{\infty} x_n}$, an estimate of the hazard function may be calculated from the interval histogram of Fig. XXVII-6; the result of this calculation appears in the lower part of the figure. Except for statistical fluctuations caused by the finite sample size, this histogram suggests that the hazard function is approximately constant for times greater than 20-25 msec. Since this histogram is effectively an estimate of the conditional probability of an event during the next 1/2 msec as a function of the time since the last event, it may be concluded that (for spontaneous events at least) the unit "recovers" from the effects of the previous firing in approximately 20-25 msec.

The data of Fig. XXVII-6 may be regarded as typical of auditory-nerve fibers. Although spontaneous rates vary among fibers from less than 1/sec to more than 100/sec, it appears that all interval histograms have a mode at less than 10 msec and decay exponentially thereafter.[3] It is easy to show that an exponential decay of the interval histogram implies a constant value of the hazard function. This suggests that all auditory-nerve fibers "recover" from spontaneous events within approximately 20 msec.

That auditory-nerve fibers also "recover" in the presence of acoustic stimuli may be demonstrated by appropriate calculations. Specifically, it is necessary to estimate the conditional probability of an event in a particular interval of time, as a function of the time since the last event. These calculations have been performed, and they indicate that the probability of an event does not vary with the time-since-the-last-event, provided it has been at least 20 msec since the last event. Probabilities that are conditional on this minimum time since the last event are referred to as "recovered probabilities." In an operational sense, the calculation of recovered probabilities presumably removes the complications in the data resulting from the refractory properties of the fiber. For this reason, these calculations are particularly valuable in comparing some
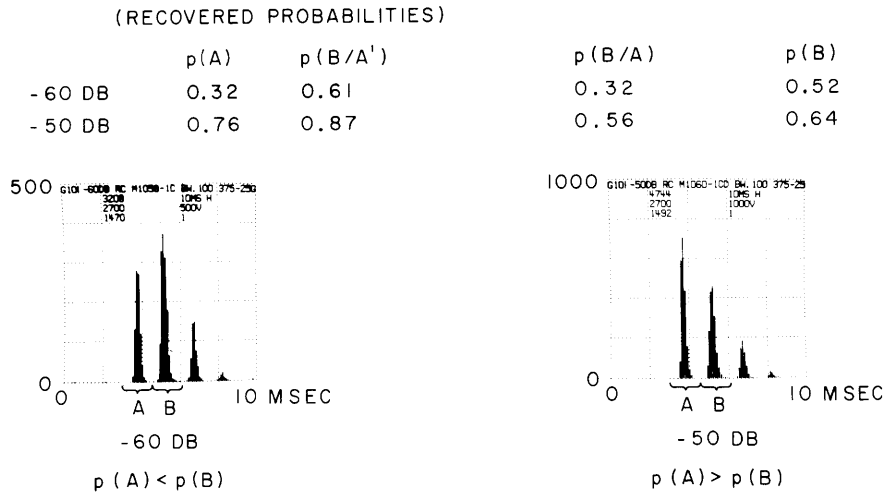
(RECOVERED PROBABILITIES)

	p(A)	p(B/A')	p(B/A)	p(B)
-60 DB	0.32	0.61	0.32	0.52
-50 DB	0.76	0.87	0.56	0.64

Fig. XXVII-7. Unit 375-25. PST histograms (bin width 0. 1 msec, 100 bars) and various conditional probabilities. Stimuli: 10/sec, -50 db and -60 db rarefaction clicks, 4. 5 minutes. All probability estimates are conditional on no spike for 30 msec before the interval. A [B] is the event: a spike during interval A [B]. p(B) is calculated from p(B) = p(A) p(B/A) + [1-p(A)] p(B/A').

aspects of possible models with the electrophysiological data.

The calculations presented in Fig. XXVII-7 are based on data obtained from one fiber with click runs at two different intensities. The PST histograms are given for each run. The height of the kth bar in the PST histogram represents the number of spikes that occurred between k/10 and (k+1)/10 msec after a click presentation, in 2700 repetitions of the click presentation. The time structure that is evident in these histograms corresponds, at least qualitatively, to what is known of the mechanical motion of the cochlear partition, but this is not important in this brief discussion.

Several conditional probabilities corresponding to intervals A and B were estimated from these same data. All probabilities given in this figure are conditional on no event for 30 msec before A. Thus p(A) is a recovered probability, as is p(B/A'), the conditional probability of an event in B, given none in A (and none for 30 msec before A). p(B/A) is the conditional probability of an event in B, given that there was an event in A (and none for 30 msec before A). p(B) is calculated from p(B) = p(A) p(B/A) + p(B/A') [1-p(A)], and is the conditional probability of an event in B, given that there is no event for 30 msec before A (with no conditions on whether or not there was an event in A).

As well as presenting some typical results of the conditional probability calculations performed in this study, Fig. XXVII-7 illustrates some inadequacies of the conventional PST histograms The fact that peak A is larger than peak B in the -50 db run could be misleading if the PST histogram were interpreted as a direct measure of the strength of

an effective stimulus. In terms of recovered probabilities, peak B is larger than A, but if there has been an event during A, the unit is "refractory" during interval B and a smaller probability (p(B/A)) applies. Since, at the -50 db intensity, the unit is likely to fire during interval A and hence be "refractory" during interval B, the size of peak B in the histogram is smaller than would be expected from the recovered probability alone. As the recovered probabilities are presumably free of any refractory complications, it seems reasonable to regard them as a measure of an effective stimulus. In these terms, the PST histogram reflects a somewhat complicated combination of both the character-istics of the effective stimulus and the refractory characteristics of the neural unit.

The data presented above are intended to illustrate the concept of a recovered firing probability and the value of this concept in analyzing data from auditory-nerve fibers. These and other related calculations have been applied to a great deal of data obtained with short acoustic clicks as stimuli, and to a lesser amount of data obtained with other stimuli. The use of these calculations to study the nonrefractory aspects of the data has provided a somewhat simpler picture of the relation between these firing patterns and the mechanical motion of cochlear structures, but it has also exposed some unexpected phenomena. These results and the implied constraints on possible models are discussed in the author's thesis.

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C. STATISTICAL MECHANICS AND FIELD THEORY

It has been exhibited in many excellent works[1] that, at present, the density matrix provides the most general description of a quantized system. This formalism embodies simultaneously the quantum-mechanical and statistical-mechanical aspects of the description and reduces to the usual quantum theory when the knowledge about the system is complete.

The general method of finding the density operator, ρ, of incompletely specified systems[2] is essentially an information theory formalism that takes advantage of the fact that the entropy $\overline{S} = -\text{tr}\,\rho\ln\rho$ is an additive invariant measure of ignorance about the state of the system. To find ρ one maximizes the entropy, subject to whatever features of the system are known. Let the average values $\overline{\Omega}_a = \langle \Omega_a \rangle = \text{tr}\,\rho\Omega_a$ corresponding to observables Ω_a be known by observation. Then subject to these and the ever-present constraint of normalization $\text{tr}\,\rho = 1$ (which implies $\text{tr}\,\rho\Omega_o = \Omega_o$, where Ω_o is a c-number), one obtains the desired operator by the method of Lagrangian multipliers:

$$\rho = e^{-(\lambda_o+1) - \sum_a \lambda_a \Omega_a}. \tag{1}$$

The multiplier λ_o is determined from the condition of normalization, and $Z = \exp(\lambda_o+1) = \text{tr}\,(\exp(-\sum_a\lambda_a\Omega_a))$ is the partition function of the distribution. To give a concrete example, consider the equilibrium problem of a cavity radiation. Since radiation is equivalent to a collection of oscillators, we have the Hamilitonian operator, $H_k = \omega_k a_k^* a_k$. Furthermore, at thermal equilibrium the only known feature of the system is the (average) energy, $\overline{\mathscr{E}}_k = \text{tr}\,\rho_k H_k$. The density operator is then $\rho_k = \left(1-e^{-\omega_k\beta}\right)\exp(-\beta\omega_k a_k^* a_k)$. The energy may be evaluated in any representation because of the invariant character of the trace. Evaluating in a representation with $a_k^* a_k$ diagonal and summing over all k, we have

$$\overline{\mathscr{E}} = \sum_k \overline{\mathscr{E}}_k = \frac{V}{(2\pi)^3}\int d^3k\,\text{tr}\,\rho_k H_k = \frac{V}{(2\pi)^2}\int \frac{\omega^3\,d\omega}{e^{\beta\omega}-1}. \tag{2}$$

Here, as usual, $V^{-1}(2\pi)^3 \sum_k \longrightarrow \int d^3k$ is used. We recognize Eq. 2 as the Planck expression for the total energy of the cavity.

Although the density matrix formalism is of utmost power and simplicity, it suffers from certain deficiencies which, in this report, we shall try to exhibit and correct. We shall see that this method, when properly modified and extended, offers even greater power and generality for dealing with the problems of quantized systems and their statistical treatments. For reasons that will become clear in the discussion, we propose

two related modifications of the density matrix formalism.

(i) First, we split ρ into a product $\rho = \sigma^{*}\sigma$ and express statistical averages as $\overline{\Omega}_{a} = \mathrm{tr}\,\sigma^{*}\Omega_{a}\sigma$. This allows us to write $\sigma = \exp(-(\lambda_{0}+1)/2 - \Sigma_{a}\mu_{a}\Omega_{a})$, where $\mu_{a} = \zeta_{a} + i\ell_{a}$ are complex numbers. The real parts are $\zeta_{a} = \lambda_{a}/2$, but the imaginary parts will also be considered. For example, in the case of the Hamiltonian operator, H, the real and imaginary parts are $\zeta = \beta/2$ and time $\ell = t$, respectively. The motivation behind this way of writing the density matrix is to extend the parameters to complex planes. For example, by writing $\tau = \zeta + it$, one deals with inverse temperature and time as a single complex parameter.[3] Another point is that an extension to relativistic and field theory situations is strongly suggested by this form. Thus from relativistic symmetry we may associate with the momentum operators $\vec{P}$ the variables $\vec{\tau} = \vec{\zeta} + i\vec{r}$. With this generalization it is possible to treat statistically the virtual quanta surrounding charged particles and find a finite value for the self-energy.

(ii) The second modification of the density matrix method is concerned with the possibility of improving the formalism and extending its applicability by considering the entropy principle and the action principle as a single dynamical postulate. Indeed, action and entropy have many similarities. They are related to real and imaginary parts of our complex numbers, and both lead to extremum problems. Also, both are additive and invariant quantities of the system. The usual treatments of the two principles, however, differ procedurally: the action principle provides infinitesimal characterization, whereas the entropy formalism yields finite Lagrangian multipliers. Since we are dealing with operators, the finite parameters are, in general, not desirable because finite transformations are not always commutative and infinitesimal ones always are. What we need is a general postulate allowing us to construct finite parameters through a succession of an infinite number of infinitesimal transformations.

Consider the operator $\Omega = -\ln \sigma = \Gamma + iW$. Here, $\Gamma = -\mathrm{Re}\,\ln \sigma = -\frac{1}{2}\ln \sigma^{*}\sigma$ is half of the entropy operator, and W is the action operator. Both Γ and W are invariant additive quantities. We propose a direct generalization of the quantum action principle in which the extended complex quantity Ω is used instead of W. A suitable formulation of this idea may be obtained by using the complex four-dimensional variables $\tau_{\mu} = \zeta_{\mu} + ix_{\mu}$, instead of ordinary space-time coordinates. Also, we must use the more general statistical (mixed) states, $|a\rangle = \Sigma_{\eta} \sigma_{a\eta}|\eta\rangle$, in place of pure states $|1\rangle$. Subject to the known values, $\overline{\Omega}_{a} = \mathrm{tr}\,\sigma^{*}\Omega_{a}\sigma$, and the normalization conditions, $\mathrm{tr}\,\sigma^{*}\sigma = 1$, we postulate

$$\delta(2|1) = -(2|\delta\Omega|1). \qquad (3)$$

For $\Gamma = 0$, the mixed states become pure states and Eq. 3 reduces to the usual action principle. Then, as is well known, the Schrödinger equation can be obtained by fixing $\langle 2|$ and applying the variation to $|1\rangle$, $\delta| \rangle = -\delta W| \rangle$. For variations arising from time

we have $\delta_t W = H\delta t$, and $i\frac{\partial}{\partial t}\Big|\Big\rangle = H\Big|\Big\rangle$. Starting from an initial state $\Big|_{t=0}\Big\rangle$, this gives the solution $\Big|t\Big\rangle = e^{-i\int H\,dt}\Big|_{t=0}\Big\rangle = e^{-iWt}\Big|_{t=0}\Big\rangle$. Here the normalization condition is automatically satisfied. For $\Gamma = 0$ we thus have the usual theory. In the corresponding statistical case we have $\delta\Big|\Big) = -\{\delta\mu_o + H\delta(\zeta+it)\}\Big|\Big)$. Thus we shall have a formal solution $\Big|\mu_o, \tau\Big) = \exp(-\int d\mu_o - \int H\,d\tau)\Big|\mu_o = 0, \tau = 0\Big)$, where $\tau = \zeta + it$. Since in this simple case H and $\int H\,d\tau$ commute, we may write $\sigma = \exp(-\mu_o - H\tau)$, reproducing essentially the earlier result with μ_o and τ finite. Of course, such simplification is not always possible. In general, the solution must be constructed as a perturbation series through a proper iteration process. From our action principle the general result of this iteration process may be represented formally by the exponential expression

$$\sigma = e^{-\int d\mu_o - \int \sum_\alpha \Omega_\alpha d\mu_\alpha}, \tag{4}$$

where $\Omega_\alpha = \partial\sigma/\partial\mu_\alpha\Big|_{\mu_\alpha \to 0}$ are observables. To secure uniqueness of the observables, we must also impose an analyticity condition $\partial\sigma/\partial\mu_\alpha^*\Big|_{\mu_\alpha \to 0} = 0$. In our opinion this generalized formulation embodied in Eqs. 3 and 4 is superior to the usual density matrix method, since it extends to the complex domain, is applicable to relativistic and field theory problems, unifies action and entropy into a single principle, and provides a general integration procedure instead of the Lagrangian multipliers which are of limited applicability.

We may now indicate the extended usefulness of our new formalism by applying it to the self-energy problem. For simplicity, we consider a soluble neutral scalar theory. The self-energy divergence arises because the contributions from virtual quanta become arbitrarily large in the limit of high momenta. The usual quantum field theory has no way of reducing the efficiency of these high-momentum virtual (spacelike) quanta. In our theory we have an analogy to cavity radiation in which the infrared divergence is removed by reducing the efficiency of real (timelike) quanta by the statistical weight leading to the finite result of Eq. 2. From relativistic symmetry we consider, then, the momentum operator $\vec{P}_k = \vec{k}a_k^*a_k$ and the associated parameter $\vec{\mu}_r = \vec{\zeta}_r + i\vec{r}$. By maximizing the entropy for fixed $\overline{P}_k$, we have $\sigma_k = \left(1 - e^{-2k\cdot\zeta_r}\right)^{1/2}\exp\left(-(\zeta_r + ir)\cdot ka_k^*a_k\right)$. On the other hand, the contribution to self-energy of a single virtual particle is known to be[4] $\mathscr{E}_k = -g_o^2(2\omega_k^2 r)^{-1}$. We thus have the operator $H_k = \mathscr{E}_k N_k$, where $N_k = a_k^*a_k$ is the number operator for the virtual quanta. The total energy of the virtual cloud, therefore, is given by

$$\overline{\mathscr{E}}_o = \sum_k \text{tr}\left(\sigma_k^* H_k \sigma_k\right) = \frac{-1}{2(2\pi)^3}\int \frac{d^3k}{k^2 + m^2}\text{tr}\left(\sigma_k^* g_o^2 N_k \sigma_k\right). \tag{5}$$

Since

$$tr\left(\sigma_k^* N_k \sigma_k\right) = \left(e^{2k \cdot \zeta_r} - 1\right)^{-1} \xrightarrow[k \to \infty]{} e^{-2k \cdot \zeta_r},$$

this integral is absolutely convergent for large momenta.[5] For $k \to 0$, we must require Eq. 5 to behave as the Yukawa case, which "renormalizes" the coupling constant as $g_o \to g^2 2k \cdot \zeta_r$ (implying that the charge g corresponds to low-energy observations such as Thomson scattering). This analogous to the cavity situation where in the limit $\omega \to 0$ we obtain the law of equipartitions of energy.

If this interpretation of the self-energy problem is correct, we have no difficulty in computing the interaction energy, $\overline{\mathscr{E}}_{12}$, between two sources separated by a distance $r = |x|$.

$$\overline{\mathscr{E}}_{12} + \sum_k \sum_{n=1} \sum_{n'=1} tr\left(\sigma_k^* H_k \sigma\right) - 2\overline{\mathscr{E}}_o = \frac{g^2}{(2\pi)^3} \int \frac{d^3 k}{k^2 + m^2} 2k \cdot \zeta_r \frac{e^{ik|x|} + e^{-ik|x|}}{e^{2k \cdot \zeta_r} - 1}.$$

(6)

For large $|x|$, we have the well-known Yukawa interaction, whereas, for small $|x| = r$, that is, for large k, the statistical weight acts as a cutoff. Note, however, that our method is completely idfferent from the usual (extended-body) cutoff procedures which violate the relativity and locality postulates of the theory. Here, there is no such mutilation of the basic framework. What we are doing is simply to extend the long-respected quantum statistical mechanics relativistically without changing the fundamental tenets of the whole theory. If valid, the new approach seems to imply a greater unity and symmetry between statistical mechanics and quantum theory. For example, it seems necessary to consider the fluctuation relations $\Delta P_\mu \cdot \Delta \zeta_\mu \geq 1/2$ that are due to the statistical part, together with and on the same footing with the Heisenberg uncertainty relations $\leq P_\mu \cdot \Delta x_\mu \geq 1/2$. A method for expanding the formal solution (4) and applications to scattering, production, and polarization problems will not be reported here.

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5. The first attempt to solve the self-energy problem through information theory ideas was made by L. Brillouin, Ann. Phys. 5, 243 (1958). Although his reasoning was completely classical, Brillouin obtained a similar statistical (for large k) weight, purely on the basis of his theory of information and measurement.