ESSAYS ON ECONOMIC THEORY

AND APPLICATIONS

by

Mario Draghi

Laurea, University of Rome

1970

SUBMITTED IN PARTIAL FULFILLMENT

OF THE REQUIREMENTS FOR THE

DEGREE OF DOCTOR OF

PHILOSOPHY

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

November 1976

Signature of Author ____________________

Certified by ____________________

Accepted by ____________________

NOTE:

The Table of Contents does not coincide with the pagination of this thesis.
In Section II, pages 8, 9, 10 & 11 are missing.

Institute Archives
6-23-77

ESSAYS ON ECONOMIC THEORY AND APPLICATIONS

by

MARIO DRAGHI

Submitted to the Department of Economics on November 30, 1976
in partial fulfillment of the requirements for the degree of
Doctor of Philosophy

Abstract

The thesis is formed of three chapters on quite unrelated problems. The first essay is an empirical investigation of what has been termed the short-run productivity puzzle namely, the procyclical movement of labor productivity with respect to output. The U.S. economy is analyzed both in the aggregate and at a sectoral level. It is shown that the common finding that relates the paradoxical behavior of labor productivity to the presence of short run increasing returns is due to incorrect estimation techniques. When care is taken of this econometric shortcoming, increasing returns to labor are not any more detected. This chapter contains an appendix investigating the relation between real wages, employment and output. The common finding is a positive relation between real wages and employment. This appendix shows how such an outcome may be due to the presence of specification error bias and to the use of an incorrect price deflator.

The second chapter considers various aspects of the theory of devaluation. It contains a discussion of the monetarist model where it is shown that the conditions under which a devaluation improves the balance of payments of the devaluing country are more stringent than what used to be thought. It shows that the differences between the absorption, the elasticities and the monetarist models lie not in the more or less explicit consideration given to the demand and supply elasticities on the goods markets but in the different underlying description of the assets markets, namely the monetarist and the absorption models rely on flow asset equilibrium, while the classical elasticities model can be reinterpreted and justified when the assets markets are assumed to be in stock equilibrium. This chapter finally contains a discussion of a two countries, two goods, two assets model when interest payments on foreign debt are explicitly taken into account in the definition of disposable income.

The last chapter is concerned with the tradeoff between short run stabilization policies and long run plans. A planner tries to control a short run economy characterized by variable unemployment rates. The benchmark path is an optimal full employment growth path reflecting the planner's preferences. It is shown that if the planner implements the policies suggested by short run optimization, the long run path will never be reached. On the other hand, if policies that are optimal from a long run point of view are actually enforced on the short run economy, this will not be destabilized in a finite time and the optimal growth path will be achieved.

Thesis Supervisor: Franco Modigliani

Title: Professor of Economics & Finance

Acknowledgements

I am deeply indebted to the members of my committee for their advice at different stages of writing this dissertation. Professor Stanley Fischer offered constructive criticisms and substantial suggestions in the writing of Chapter II. When he went on leave, Professor Jerry Hausman agreed to join my thesis committee. His always available assistance was essential and his influence is clearly visible throughout the first essay. Professors Franco Modigliani and Robert Solow supervised this dissertation all along its variegated development. The numerous and lengthly conversations I had with them were a constant source of inspiration and wide perspectives. My debt to them goes beyond this dissertation.

I wish to thank my friends Pentti Kouri of Stanford University and Glenn Loury of Northwestern University for the valuable suggestions they offered on various occasions.

I wish to acknowledge a special debt I owe to my teachers in Rome who helped the formation of my ideas on economic theory and policy matters. Only the author of this dissertation knows how important is their contribution.

Ms. Vicki Elms not only did a splendid job in typing, as everybody can see, but fixed a last-minute editorial disaster, making possible a timely completion.

This dissertation is dedicated to my wife Serena.

Table of Contents

Abstract ii

Acknowledgements iv

Table of Contents v

Chapter I. 1

Introduction 1

Survey of the Literature 4

The Model to Be Reestimated. 11

Conclusions. 30

Appendix A .33

Notes. 35

References .38

Real Wages, Employment, & Output 41

Chapter II 57

Introduction .57

Section 2. 61

Section 3. 70

Section 4. 72

Section 5. 80

Conclusions. 86

Appendix 88

Footnotes. 99

References 101

Chapter III. 102

Introduction 102

The Short Run Economy. 106

An Alternative Formulation 110

Endogenous Choice of Consumption
and Investment. 113

The Tying Up 117

Conclusions. 126

Appendix 129

References 132

Chapter 1

This paper investigates the relation between output and employment in the context of a short run production function. This is certainly not a new problem so that interest in it can be motivated only by the quite different conclusions reached in this instance. With respect to the question of what is the value of the elasticity of output with respect to employment, there seems to be a striking unanimity in previous empirical results, in the sense of pointing to the existence of generalized marginal increasing returns to labor. As Solow observed: "When output stagnates or falls away from a peak, productivity (output per man-hour) tends to fall, or to rise slower than trend; when output recovers, productivity rises faster than trend.... The crude observations seem, therefore, to contradict diminishing returns to labor in the short run. When increased employment works on a given stock of capital goods, output per man-hour "ought" to fall."[1] It should be added that some evidence contradicts the existence of diminishing returns to labor in the long run also.

This puzzling evidence was quite welcome by those who never trusted the realism of the neoclassical apparatus. So that it is probably not an understanding to say that this type of result inspired both theoretical considerations on "The Irrelevance of Equilibrium Economics"[2] and empirical statements, generally expressed

---

This paper owes much to the helpful comments made by J. Hausman and R. M. Solow. As the reader will easily realize by himself, a good deal of the contentions that will follow are based on previous work by the mentioned people. So that little is left to the author but the recognition of the full responsibility for the errors.

The CENTRO ALBERTO BENEDUCE is gratefully acknowledged for financial assistance.

in form of laws, namely the Kaldor-Berdoorm Law[3] asserting the existence of increasing returns to labor in the short run.

In what follows it will be argued that such estimates of the labor productivity coefficient are biased in the upward direction, the source of the bias being a specification error.

The next section will be a brief survey of the empirical results obtained by other researchers. The third section will be devoted to the exposition of an alternative set of results based on a model recently explored by Nordhaus.[4] The relation of real wages output and employment will then be investigated.

2. Survey of the Literature

The relation between employment and output has been studied from two apparently different standpoints. When it is interpreted as a structural relation involving the long-run determinants of labor productivity, explanations usually center on facts like changes in personal efficiency of workers, capital-labor substitution, technical progress accruing through gross investment, economies of scale and increasing returns depending on the extent of the market.[5] On the other hand, when what is looked for are explanations of the cyclical behavior of labor productivity, the emphasis goes to short-run phenomena such as adjustment costs, capital utilization rates and overhead labor.

Unfortunately, given the lack of data on most of these variables such a variety of interpretations is not accompanied by an equal variety of estimated regressions, so that it is quite hard to distinguish short from long run.[6] For someone who claims that the previous estimates of labor productivity are biased, a good question to be asked is then, "biased with respect to what?" The answer is that at least with respect to the direction of the bias, it does not make any difference whether we are dealing with short or long run, the estimate of the output-labor input elasticity is biased upward. Consider the estimation of

$$e_t = \alpha_0 + \alpha_1 y_t + \varepsilon_t \tag{1}$$

where the "true" relation is

$$e_t = \alpha_0 + \alpha_1 y_t + \alpha_2 k_t + \varepsilon_t$$

it is clear that the estimated $\hat{\alpha}_1$ will be

$$\hat{\alpha}_1 = \alpha_1 + \alpha_2 \frac{\text{Cov } (y_t, k_t)}{v(y_t)}$$

where y and e are the log of output and the log of labor and cov(·), v(·) are respectively covariance and variance. It is apparent now that whether we interpret k as the actual capital stock at some point in time, reflecting capital-labor substitution, or the capital stock corresponding to a certain age distribution of different machines as long run considerations would suggest, or should k be the short run utilization rate of existing machines, to estimate (1) with OLS would yield estimates inconsistent and biased towards increasing returns to labor.

As Table I shows whether we are dealing with short or long run the estimated forms look quite similar, the main difference being the presence of one or more lagged terms for employment and a first or second degree polynomial in time accounting for capital stock and technical progress.

Table I[a),b)]

KALDOR[c)] [ ] (U.K., 1954-1964)	$E_t = -1.028 + 0.516X_t$
NEILD [ ] (U.K., 1949-1961)	$E_t = 0.260 + 0.723 \cdot E_{t-1} + 0.158 \cdot X_t + 0.0080 \cdot X_{t-1} - 0.000235 \cdot t$
	$M_t = 0.092 + 0.838M_{t-1} + 0.252 \cdot X_t - 0.133X_{t-1} - 0.000283 \cdot t$
BRECHLING [ ] (U.K.	$\Delta E_t = 1.096 + .182X_t + .00013 \cdot t + .0000087t^2 - .319H_t - .408E_{t-1}$
	$\Delta M_t = 1.455 + .341X_t + .0061t - .000024t^2 - .347H_t - .717E_{t-1}$
KUH [ ][8] (U.S., 1948-1960)	$X_t = ? + 1.426M_t - 1.380M_{t-1} + .257M_{0t} + .019K_t + .690X_{t-1}$
	$X_t^D = ? + 1.506M_t - 1.322M_{t-1} + .179M_{0t} - .117K_t + .627X_{t-1}$
	$X_t^{ND} = ? + 1.250M_t - 1.428M_{t-1} + .326M_{0t} - .130K_t + .814X_{t-1}$

a) This summary table does not give full justice to the authors mentioned in the sense that it arbitrarily reports the parts of their empirical results that seemed more relevant in this context.

b) E_t, M_t, M_{0t}, X_t, X_t^D, X_t^{ND} are respectively employment, manhours, overhead manhours, aggregate output, output in the durables and non-durables sector, all in log terms. Data are quarterly in all cases with the exception of Kaldor who uses log changes in annual data. All the U.K. regressions are relative to the manufacturing sector.

c) This regression is relative to the manufacturing sector of the U.K. economy. Kaldor actually produces some other cross sectional (across countries)-time series evidence for other sectors as well. However, the sample period is different for different countries, so that the results do not seem to have an immediate interpretation.

		$\hat{\alpha}_1^{-1}$
Ball and St. Cyr [ ] (U.K., 1955I to 1964II)		1.16
	Austria	1.23
	Belgium	1.12
	Canada	1.07
	France	3.19
	Germany	1.02
	Ireland	1.47
Brechling and O'Brien [ ] (Various periods)	Italy	1.46
	Netherlands	1.80
	Norway	1.49
	Sweden	1.38
	U.K.	1.79
	U.S.A.	1.39
Smyth and Ireland [ ]	Australia	1.42

Most of the mentioned authors implicitly assumed that the capital stock is continuously operated at full capacity, no matter the level of output, and that the capital labor ratio is fully flexible. In this case cov $(y_t, k_t) = 0$, and therefore a regression of the logarithm of output versus the logarithm of employment would give unbiased estimates of the coefficient, although the estimates of the variance-covariance matrix would still be biased. However, it is generally believed that this is not true: when employment falls so do capital utilization rates. Furthermore, it should be pointed out that also in a model with machines of different vintages and with ex post fixed coefficients the capital labor ratio may well be flexible, capital may well be continuously utilized and we should not observe[8] such high values of output-employment elasticities. When output falls, newer types of machines with lower labor input requirement will be utilized. And in fact it has been shown[9] that to ignore the age distribution of the machines defined as the age of marginal machines, when this is positively correlated with K causes an upward bias in the output-labor elasticity estimate.

To reconcile the evidence with the prediction of the theory two routes have been followed. The first consists in the explicit introduction of changes in the capital utilization rates. This can be done either approximating them by changes in variables like "electricity consumption relative to the installed horsepower of electric motors"[10] or assuming the existence of a user cost of capital so that sometimes it may be optimal to keep some capital idle.[11] In the first case the limit of the attempt is obviously in the quality of the proxy chosen.

In the second, the authors, in order to get a testable specification, make the unsatisfactory assumption that the ratio of the user cost of capital to the cost of labor is a constant, so that when output falls, labor and capital utilization fall in the same proportion, and what appeared to be returns to labor are really returns to scale.

Before proceeding with the exposition of the empirical results, it may be worthwhile to outline the general idea of what follows:

(1) Estimation of production functions involves use of variables that are often unobservable. If one just estimates by OLS what is observable is very likely to get inconsistent estimates.

(2) The choice is then between trying to approximate the unobservables by a known series, or using an instrumental variables estimator, or both. While the consistency and the unbiasedness of the explicit results one gets with the first procedure are conditional on the "goodness" of the proxy chosen, a legitimate instrumental variables estimator would yield consistent estimates in all cases. The "legitimacy" of an instrument is conditional on the absence of correlation between the instrument and the variables left out because unobservable, but correlated with included variables. In our case, for instance, the selected instruments should be uncorrelated with capital utilization rates but correlated with output.

(3) The confidence intervals of the instrumental variables estimates are, however, fairly large, so that often their difference from the OLS estimates is statistically insignificant on the basis of conventional but theoretically unjustified criteria of significance -- two standard

error intervals. Then what is needed is a criterion of comparison between the two estimates that, loosely speaking, weights the difference in the expected value of the coefficients with the difference in the expected value of their standard errors. Such a criterion is provided by the Hausmann Specification Error Text.[12]

(4) Finally, a fairly interesting conjecture can be stated as a consequence of not having specified the unobserved variables -- i.e., to use proxies for capital utilization rates or series for the capital stock where, however, the benchmark value and the depreciation rate are unobservable anyway. Provided that, (i) the functional form relating employment and output is of the type of (1); (ii) the selected set of instruments is orthogonal both to the long run and to the short run unobservables, the estimates of the effect of changes in current output on current man-hours are unbiased whatever is the story told about the underlying technology. More specifically, there should be no difference between the long and the short run current labor input-output elasticity estimate.

3. The Model to Be Reestimated

Most of the authors just mentioned analyze the relation between labor input and output either at an aggregate level or at a two sector level of disaggregation. One of the most careful and detailed investigations at a more disaggregated level is that by Nordhaus.[13] His paper is partly aimed at different goals, but the starting point is the estimation of an equation relation man-hours to output and time for twelve industries of the U.S. economy. The main features and results of this section of his paper are:

(i) The specification of a cyclical correction of the productivity measure based on the fact that cyclical movements in output influence productivity. From this follows the use of the concept of normal output as the "level that GNP would attain if the unemployment rate were at its normal level defined as its postwar average of 4.7%."[14] Roughly the same concept has then been used at industry level.

(ii) The results, all based on OLS estimation procedure, were that "Six of the twelve industries show significant increasing returns (to labor): agriculture, durable manufacturing, transportation, public utilities, trade and services. The remainder display no significant departure from constant returns; no industry has significant decreasing returns."[15]

His model starts with the estimation of

$$\log(GNP)_t = \alpha_0 + x_1 t + \alpha_2(U_t - 4.7\%) + \alpha_3(U_{t-1} - 4.7\%) + \varepsilon_t$$

then the definition of normal output, x_n for the aggregate is derived

$$xn = \log(XN) = \log(GNP)_t - \hat{\alpha}_2(U_t - 4.7\%) - \hat{\alpha}_3(U_{t-1} - 4.7\%) \qquad (2)$$

obviously equivalent to

$$xn = \hat{\alpha}_0 + \hat{\alpha}_1 t + u_0 \qquad (2')$$

where the estimate of the inclusion of the residual is justified because "Any changes in the underlying growth rate of output, such as the acceleration in the late 1960s due to the more rapid growth of the labor force, should also appear in the estimated growth of normal output."[16]

An obvious remark would be that these effects should either be estimated separately or care should be used in the subsequent estimates to avoid bias and inconcistency. A relationship for the aggregate is then derived

$$e - xn = \delta_{00} + \delta_{10}(x - xn) + \delta_{20}t + v_0 \qquad (3)$$

where: e is the log of man hours.

TABLE II

-- Regression Estimates for Aggregate Labor Productivity

-- Samples 48-73 and 48-71

-- (Standard Errors)

Independent Variables	OLS Sample:48-71	OLS Sample:48-73	TSLS(1) Sample:48-73	TSLS(2) Sample:51-73
δ_{00} (constant)	- 1.0746 (0.0094)	- 1.0799 (0.0094)	- 1.086 (0.015)	- 1.104 (0.0091)
δ_{10} (x-xn)	0.8981 (0.1437)	0.8816 (0.150)	1.783 (0.500)	1.229 (0.139)
δ_{20} (time)	- 0.02413 (0.0006)	- 0.0235 (0.0005)	- 0.0231 (0.0009)	- 0.022 (0.0005)
S.E.	0.02110	0.0219	0.0351	0.0123
D.W.	0.511	0.43	0.78	1.87
$\bar{R}^2$	0.98	0.98	0.96	0.994

Nordhaus' results are replicated, abstracting from negligible differences, in the first column.[17] The second contains the same results over the extended sample 48-73. Col. 3 reports the result of a TSLS procedure where the log of total money supply, the New York Fed discount rate and the lagged value of exports have been used as instruments. Since the null hypothesis of significant serial correlation could not be rejected than the OLS estimates were corrected in col. 4 and an instrumental variables procedure correcting for it was performed using Fair's method.[18]

The results of it are reported under the heading of I.V.(2). Furthermore, given the quite substantial difference between the OLS estimates and the ones reported in Column 3, it was considered appropriate to use a different set of instruments just in order to see how much the estimates of δ_{10} were effected by a different selection. So that besides the instruments suggested by Fair in order to obtain a consistent estimate when the errors are serially correlated, the following instruments were used in Column 4: the same set as in Column 3, an age variable describing the ratio of people of age between 16 and 19 out of the total population, female labor force and short term capital flows. There is no specific explanation of why these were the instruments used besides the fact that they were considered _a priori_ uncorrelated with the variables left out, but correlated with the included ones. Results did not change substantially so that it then becomes clear that for the aggregate, the two contentions advanced in the first paragraph, namely that (i) OLS estimates of equations like (3) are biased and inconsistent and that (ii) if account is taken of this, diminishing marginal returns to labor could be detected, are verified. To see this more clearly consider the following test for specification error, recently discovered by J. Hausman. If $\hat{\delta}_{10}$ and $\hat{\hat{\delta}}_{10}$ are respectively the estimates of Columns 2 and 4,[19] let

$$\hat{q} = \hat{\delta}_{10} - \hat{\hat{\delta}}_{10}$$

It will then be true that

$$m = \frac{\hat{q}^2}{1 \cdot V(\hat{q})} \sim F(1, 21)$$

$\hat{q}$	m	m^*
0.315	9	7.95

As it can be seen, m exceeds its critical value m^* at 1% confidence level, therefore suggesting evidence of serious misspecification in production functions of the form of (3).

Finally it may be comforting to know that the I.V. estimate of $\frac{1}{\delta_{10}} = 0.813$ obtained in the last column of Table 2 coincides almost perfectly with the one obtained by Solow (0.815), [20] where he chooses the route of approximating capital utilization rates by electricity consumption relative to the installed horsepower of electric motors.

Since at the disaggregate level there is no concept immediately equivalent to normal output such that cyclical elements could be eliminated, Nordhaus found necessary to introduce the concept of "normal industrial demand defined as "...that level of industrial demand that would be forthcoming if aggregate demand were at its normal level."[21] More specifically, demand for output in each sector i is

$$x_i = \alpha_{0i} + \alpha_{1i}(p_i - \overline{p}) + \alpha_{2i}(U_t - 4.7\%) + \alpha_{3i}\overline{xn} + \varepsilon_{1i} \qquad (4)$$

where

x_i = log of gross output originating by industry i in 1958 prices

p_i = log of deflator for x_i

$\overline{p}$ = log of a geometric index of prices using 1958 output weights

Normal output is then:

$$xn_i = \hat{\alpha}_{0i} + \hat{\alpha}_{1i}(p_i - \overline{p}) + \alpha_{3i}\ xn \qquad (5)$$

This time residuals are excluded from the definition of xn_i, but in this case OLS is a textbook example of estimation biased from simultaneity. Normal output is produced by a log linear production function

$$qn_i = \quad _{0i} + \beta_{1i}en_i + \beta_{2i}k_i + \beta_{3i}t + \beta_{2i} \tag{6}$$

where

en_i = log of normal labor input

k_i = log of net capital stock

qn_i = log of normal output

Assuming finally: that (1) normal output is always equal to normal demand, (2) CRTS are prevailing and (3) the capital-labor ratio grows at an exponential rate, $_{ui}$, we have that:

$$en_i = xn_i - \beta_{0i} - (\beta_{3i} + \beta_{ui}\beta_{2i})t - \varepsilon_{2i} \tag{7}$$

For each industry i, it is assumed that manhours adjust to short-run demand according to

$$e_i - en_i = \delta_{1i}(x_i - xn_i) + \varepsilon_{3i} \tag{8}$$

So that substituting the form to be estimated is

$$e_i - xn_i = \delta_{0i} + \delta_{1i}(x_i - xn_i) + \delta_{2i}t + \eta_i \tag{9}$$

Estimates of equation (9) for the twelve sectors of the U.S. economy are produced in Table III.

TABLE III

Coefficients for Labor Productivity Equations[23]

Industry	Independ. Variable	OLS Sample: 48-71	OLS Sample: 48-73	I.V. Sample 48-73	OLS Sample: 51-73	I.V.(2) Sample: 51-73	3SLS(2)
1 Agriculture	δ_{01}	9.48	0.0046	0.0058	0.0105	0.173	
	δ_{11}	0.476 (0.136)	0.533 (0.229)	1.164 (0.489)	0.116 (0.191)	1.669 (0.467)	
	δ_{12}	-0.053 (0.0003)	-0.0614 (0.0007)	-0.065 (0.00083)	-0.061 (0.0019)	-0.0621 (0.00079)	
	S.E.	0.0132	0.0276	0.0319	0.0243	0.0229	
	D.W.	1.28	1.19	2.09	1.57	1.63	
	$\bar{R}^2$	0.999	0.996	0.995	0.997	0.997	
2 Mining	δ_{02}	7.82	-1.66	-1.67	-4.93	4 .53	
	δ_{12}	1.080 (0.136)	0.786 (0.202)	1.182 (0.414)	0.969 (0.104)	1.112 (0.168)	
	δ_{22}	-0.0375	-0.0357	-0.0354	0.036 (0.310)	0.0243	
	S.E.	0.0317	0.0474	0.0512)	0.0224	0.0288	
	D.W.	0.47	0.33	0.35	1.20	1.59	
	$\bar{R}^2$	0.988	0.971	0.967	0.992	0.986	

Industry	Independ. Variable	OLS Sample: 48-71	OLS Sample: 48-73	I.V. Sample: 48-73	OLS Sample: 51-73	I.V.(2) Sample: 51-73	3SLS(2)
3 Construction	δ_{03}	8.83	-1.31	01.30	-1.47	-1.42	
	δ_{13}	1.480 (0.374)	1.510 (0.251)	1.216 (0.368)	1.462 (0.150)	1.406 (0.218)	
	δ_{23}	-0.0090 (0.0024)	0.000561 (0.00147)	0.00035 (0.0015)	0.009 (0.0046)	0.00644 (0.00499)	
	S.E.	0.0800	0.0561	0.0577	0.0254	0.0350	
	D.W.	0.23	0.30	0.29	2.69	2.31	
	$\bar{R}^2$	0.608	0.56	0.54	0.911	0.831	
4 Manufacturing Non-durable	δ_{04}	8.55	5.98	5.98	6.03	6.195	
	δ_{14}	0.923 (0.0749)	0.969 (0.136)	1.06 (0.178)	0.845 (0.0717)	1.221 (.191)	
	δ_{24}	-0.0314 (0.00039)	-0.0326 (0.00060)	-0.0326 (0.00061)	-0.0128 (0.0466)	-0.0422 (0.0171)	
	S.E.	0.0130	0.0228	0.0231	0.0123	0.0188	
	D.W.	1.05	0.43	0.50	1.56	1.66	
	$\bar{R}^2$	0.997	0.992	0.99	0.997	0.993	

Industry	Independ. Variable	OLS Sample: 48-71	OLS Sample: 48-73	I.V. Sample: 48-73	OLS Sample: 51-73	I.V.(2) Sample: 51-73	3SLS(2)
	δ_{05}	8.49	5.84	5.83	5.88	5.83	
5 Manufacturing Durables	δ_{15}	0.830 (0.058)	0.825 (0.066)	1.030 (0.127)	0.822 (0.0744)	1.191 (0.235)	
	δ_{25}	-0.025 (0.00072)	-0.026 (0.00071)	-0.025 (0.00086)	-0.0281 (0.00153)	-0.025 (0.00154)	
	S.E.	0.0242	0.0270	0.0321	-.0200	0.0528	
	D.W.	1.06	0.87	0.77	1.53	2.10	
	$\bar{R}^2$	0.986	0.985	0.978	0.991	0.937	
	δ_{06}	8.53	-0.116	-0.11	-0.059	-0.06	
6 Transportation	δ_{16}	0.575 (0.145)	1.911 (0.198)	2.21 (0.479)	1.71 (0.219)	1.71 (0.236)	
	δ_{26}	-0.0332 (0.00.0)	-0.0441 (0.0012)	-0.0439 (0.0013)	-0.047 (0.0035)	-0.0467 (0.0031)	
	S.E.	0.0337	0.0486	0.0511	0.0390	0.0428	
	D.W.	0.85	0.78	0.93	1.36	1.47	
	$\bar{R}^2$	0.982	0.982	0.980	0.986	0.983	

Industry	Independ. Variable	OLS Sample: 48-71	OLS Sample: 48-73	I.V. Sample: 48-73	OLS Sample: 51-73	I.V.(2) Sample: 51-73	3SLS(2)
7 Communication	δ_{07}	8.27	-0.698	-0.698	-0.85	-0.72	
	δ_{17}	1.242 (0.351)	0.756 (0.306)	1.295 (0.559)	0.91 (0.176)	1.418 (0.575)	
	δ_{27}	-0.0565 (0.0011)	-0.0551 (0.00101)	-0.00551 (0.0010)	-0.0474 (0.0147)	-9.0534 (0.0036)	
	S.E.	0.037	0.038	(0.041	0.0252	0.0320	
	D.W.	0.54	0.42	0.46	1.72	1.60	
	$\bar{R}^2$	0.992	0.991	0.99	0.995	0.993	
8 Public/ Utilities	δ_{08}	7.79	-1.12	4.43			
	δ_{18}	0.413 (0.149)	0.159 (0.067)	0.474 (0.343)			
	δ_{28}	-0.0547 (0.0094)	-0.0523 (0.00053)	-0.0169 (0.0233)			
	S.E.	0.0320	0.0192	0.0323			
	D.W.	0.52	0.98	0.18			
	$\bar{R}^2$	0.994	0.99	0.91			

Industry	Independ. Variable	OLS Sample: 48-71	OLS Sample: 48-73	I.V. Sample: 48-73	OLS Sample: 51-73	I.V.(2) Sample: 51-73	3SLS(2)
9 Trade	δ_{09}	8.80	0.52	-0.98	-0.97	-0.97	
	δ_{19}	0.433 (0.150)	0.520 (0.131)	1.071 (0.99)	0.575 (0.138)	1.041 (0.169)	
	δ_{29}	-0.027 (0.00047)	-0.0232 (0.00039)	-0.0231 (0.00058)	-0.0234 (0.0006)	-0.0238 (0.00048)	
	S.E.	0.0160	0.0151	0.0201	0.0125	0.0102	
	D.W.	1.42	1.29	1.31	1.44	2.08	
	$\bar{R}^2$	0.99	0.99	0.98	0.993	0.995	
10 Fire	δ_{10}	7.47					
	δ_{110}	0.717 (0.270)					
	δ_{210}	-0.0167 (0.00098)					
	S.E.	0.0331					
	D.W.	0.46					
	$\bar{R}^2$						

Industry	Independ. Variable	OLS Sample: 48-71	OLS Sample: 48-73	I.V. Sample: 48-73	OLS Sample: 51-73	I.V.(2) Sample: 51-73	3SLS(2)
	δ_{011}	0.09	-1.10	-1.10	-1.12	-1.11	
11	δ_{111}	0.548	0.635	1.044	0.690	1.343	
		(0.123)	(0.115)	(0.227)	(0.146)	(0.368)	
Services	δ_{211}	-0.0095	-0.0047	-0.0047	-0.0035	-0.0045	
		(0.00055)	(0.00049)	(0.00062)	(0.0019)	(0.00051)	
	S.E.	0.0187	0.0190	0.0237	0.0130	0.0176	
	D.W.	0.82	0.50	0.38	1.66	1.92	
	$\bar{R}^2$	0.938	0.826	0.73	0.801	.757	
	δ_{012}	9.05					
12	δ_{112}	0.974					
		(0.0309)					
Government	δ_{212}	0.0014					
		(0.00023)					
	S.E.	0.0076					
	D.W.	1.32					
	$\bar{R}^2$	0.980					

The first and the second Column of the Table contain, respectively, Nordhaus' estimates and the ones over the larger sample 48-73.

The third column shows an instrumental variables estimate without correction for serial correlation. Since we could not reject the null hypothesis of zero first order serial correlation, unbiased estimates of the residuals to be used in subsequent testing, were derived correcting both the OLS and the instrumental variables estimates, these last using Fair's method. The fourth and the fifth columns display these results. As in the case of Table 2, different sets of instruments were used in deriving the results of Column 3 and 5.

There are not that many comments to be made: (i) Table III more dramatically confirms the aggregate results: diminishing marginal returns to labor are present and significant in all nine sectors of the U.S. economy; that is to say, in the sectors where the available data were such that some hypothesis could be tested. In the case of Public Utilities, Finance Insurance and Real Estate and Government, no meaningful tests were possible given the poor quality of the available data.

In particular, in the case of four sectors, agriculture, construction, transportation, and communication, the output-labor input elasticity is below 0.71. However, it should be mentioned that the hypothesis of constant or increasing returns to labor can be rejected at 5% level only in the case of construction and transportation. If full account were taken of a cross equations correlation, it would have been possible to obtain more precise estimates in the direction of diminishing returns. This suggests the use of a full information instrumental variables estimator having the same expected value as the ones in Columns 3 and 5, but a lower standard error. Unfortunately, computations seemed too cumbersome to be completed at the present moment.

A more interesting question appears to be the one concerning the bias of OLS estimates caused by the neglect of changes in utilization of other factors of production. Table 3 shows that in the cases where diminis-ing returns were detected starting with, instrumental variables estimates did not contradict the OLS results. The following Table 4 shows the outcome of the specification error test described above.

TABLE IV

Specification Error Test

Industry	$\hat{q}$	m
Agriculture	1.553	12.99
Mining	0.143	1.97
Construction	-0.062	0.80
Manufacturing Non-Durables	0.381	6.04
Manufacturing Durables	0.368	8.10
Transportation	0	0
Communication	0.50	0.88
Trade	0.466	13.64
Services	0.653	4.42

Since m has a critical value $m^* = 4.30$ at 5% level, the null hypothesis of absence of misspecification can be decisely rejected in the case of Agriculture, Manufacturing Durables and Non-Durables, Trade, and Services. Communication is the only industry where, although the use of instrumental variables resulted in a quite different point estimates, it was not possible to reject the hypothesis of misspecification, given the high standard error of the consistent estimate.

$$\hat{\delta}_{(I)} = \delta + (1 - \delta)\,\frac{\text{cov}(x,\ xn)}{V(x)} \tag{10}$$

Then suppose that $\hat{\delta}_{(I)}$ were an unbiased estimate if it were not for the cyclical correction. We would then rewrite (10) as

$$\hat{\delta}_{(I)} = \hat{\delta}_{(III)} + (1 - \hat{\delta}_{(I)})\,\frac{\text{cov}(x,\ xn)}{V(x)}$$

According to Table I results, $\delta_{(I)} < 1$. This together with the fact that cov(x, xn) > 0, under any reasonable assumption, should result in $\hat{\delta}_{(I)} > \hat{\delta}_{(III)}$, the corresponding estimate in Tables II and III. In fact the opposite is true. This shows that the presence of the cyclical correction does, if anything, strengthen the comparison in favor of Tables II and III results.

4. Conclusions

Instead of summarizing the techniques and results discussed in the previous sections I will try to sketch some implications and some very tentative conclusions of these findings.

1- Behavior of unobservables

As we saw in section 2, the upward bias in the estimate of labor productivity comes from having neglected some variables in the estimated form. By inspection of equations (1) and (2) one can give a deterministic interpretation of this bias. Suppose the function one estimates is $y = \beta_1 e$, while the true form is $y = \beta_1 e + \beta_2 k$. The bias would then be

$$(\hat{\beta}_1 - \beta_1) = \beta_2 \frac{dk}{de}$$

where $\hat{\beta}_1$ would be equal to $\frac{dy}{de}$ from the first specification. So that for given β_2, the extent of the bias depends on the degree of substitutability between labor and the unobserved variables along an output path. In particular for $\frac{dk}{de} = 0$, we would have the estimates of Table (I), for $\frac{dk}{de} = 1$, $\hat{\beta}_1$ would be a scale elasticity along the Ireland and Smith's interpretation. Therefore, for any plausible value of $0 < \beta_2 < 1$, $\frac{dk}{de}$ must be quite above 1 to yield the differences between biased and unbiased estimates reported in Tables (II) and (III). In particular, from the presence or absence of bias one can derive the behavior of $\frac{dk}{de}$. As we saw in the case of two industries, Construction and Transportation, the instrumental variable estimate did not yield results dignificantly different from the OLS estimates suggesting therefore that $\frac{dk}{de} = 0$. Then, if we interpret k as the amount of utilized capital, it turns out that capital utilization rates change more than proportionately than employment, as output changes, and this happens in the case of all sectors, with the exception of Construction and Transportation. The overall picture is then of labor being much more of a fixed factor relative to capital.

(ii) How can the above results be reconciled with the fact that observed labor productivity, i.e., output per man-hour, moves procyclically? The answer depends on the interpretation one gives to the unobserved variables and rests on the fact that observed data attribute changes in output to changes in man-hours only, while other factors of production are changing as well. A possible interpretation is the one outlined above suggesting that when output falls, man-hours decrease less than in proportion (labor productivity rises) but capital utilization rates fall more than proportionately with respect to the decline in man-hours. Another suggestive interpretation would identify the unobserved variables with other factors of production like raw materials. This is particularly interesting because while we may think of capital utilization rates as being essentially determined by demand factors, we can easily imagine exogenous factors affecting the quantity of raw materials used by different industries.[24]

(iii) Distribution Theory

In the past twenty years, one of the criticisms at the empirical level raised against the neoclassical theory of distribution, was the impossibility of measuring the share of capital defined as aggregate capital times its marginal product. If this is the issue, the method outlined in the previous sections seems to show, in principle at least, that we do not need what Hahn and Matthews call "our armchair omniscience (that) can take account of each machine separately."[25]

If some production function is assumed where the only factors are capital and labor, it is enough to have a consistent estimate of the labor share in order to be able to derive the share of capital, conditionally on the form of the production function assumed. Obviously all this may have nothing to do with distribution in reality, but for reasons other than the impossibility of measuring aggregate capital.[26]

(iv) Okun's Law:

It might be tempting to relate the above results to Okun's Law and to consider them as contradicting the fact that when unemployment fall by x per cent, output rises by more than x per cent. I think this is a mistake: findings of this nature can contradict only explanations of the Law and not the Law itself. More specifically, they seem to show that only one of these explanations is not verified.[27]

APPENDIX A

The data are substantially the same as those used by Nordhaus and the reader is referred to his careful discussion. There is only one notable exception: in the cases of total man-hours of production workers, in the Transportation and Communication industries, the published data are available only since 1964. For the previous years Nordhaus relied on the crude technique of using a fixed-weight index of hours for the industrial sector (mining and manufacturing) as a substitute up to 1964. Instead, we have used the concept of "Number of full-time equivalent employees" measuring man-years of full-time employment of wage and salary workers and its equivalent in work performed by part-time workers. The definition of full-time employment is the number of hours that are customary at a particular time and place. For a description of the concept, see "Survey of Current Business," June 1945, pp. 17-18.

The following Table describes the instruments used in obtaining the instrumental variables estimates of δ_{1i} without and with correction for serial correlation, respecively, shown in Columns 3 and 5 of Table III.

Industry	Equation	Age	Subal	IA	RMC	GF	Money	Ex	RMF	JMVMF	LCF	BOP35
Agriculture	(3)	+	+	+	+							
	(5)	+	+	+								
Mining	(3)					+	+	+				
	(5)					+			+	+		+
Construction	(3)	+				+			+			
	(5)					+			+		+	
Mfg.non-durables	(3)	+				+	+		+			
	(5)	+				+	+		+			+
Mfg. durables	(3)					+	+	+	+			
	(5)	+				+	+		+		+	+
Transportation	(3)	-			+	+						
	(5)	+			+	+					+	
Communication	(3)				+	+						
	(5)				+	+						
Trade	(3)					+	+					
	(5)					+	+		+		+	+
Services	(3)					+			+			+
	(5)											

where: Age is the age composition variable previously mentioned; Subal are subsidies to agriculture lagged two or more periods; IA is investment in farm residential structures lagged two or more periods; RMC is the money rate on prime commercial paper from 4 to 6 months; GF is Federal Government Purchases of goods and services; Money is maximal money supply total; Ex are exports of goods and services; RMF is the discount rate of N.Y.Fed.; JMVMF is the value index of U.S. domestic imports of finished manufactures; LCF is civilian labor force female; BOP35 are short term capital flows.

These instruments have indifferently been used as they appear, and/or lagged one or more periods, and/or in log form, or in other functional forms. It should finally be mentioned that in the case of each of the column 5 estimates the set of additional instruments suggested by Fair's method should be added to ones reported in previous table.

Notes

1. Solow, R.M. [18], p.316 , who is, as far as the nature of the results he gets in this paper, one notable exception.
2. Kaldor, N. [12].
3. Kaldor, N. [11]. For recent controversies on the Kaldor-Verdoorn Law, see Rowthorn, R.E. [16] and Cripps, T.F. and Tarling, R.J. [5].
4. Nordhaus, W.D. [15].
5. Salter [17], Kaldor, op. cit.
6. Distributed log extimation does not help in this sense. For an example consider the <u>three</u> years distributed lag of [9] necessary to have a meaningful estimate of such elasticity.
7. It should be pointed out that Kuh is the only exception who finds in one the many estimated forms a long run labor-output elasticity of 1.22, pp. 9-10 [13].
8. However, with respect to this, Solow [18] says, "firms own capacity of various vintages and efficiencies, and the age distribution varies among firms. When industry output falls, reduction in sales are distributed unevenly among firms because of geographical advantages, customer-relationships, and other imperfections of the market." But then it should be explained why in the early part of the cycle we generally do not observe significant shifts in the productivity weights of different industries. See Kuh, op. cit.
9. Solow [19].
10. Solow [18] p. 319, Whitaker [21]

11. Ireland, N.J. and Smyth, D.F. op. cit. The last three quotations of Table I are taken from [10].

12. Hausman, J. [8].

13. Nordhaus, W. D., op. cit.

14. Nordhaus, W. D., op. cit., p. 495.

15. Nordhaus, W. D., op. cit., p. 502, where the significance criterion is given by a 2σ confidence interval.

16. Nordhaus, W. D., op. cit., p. 495.

17. The δ_{10} should not be compared with the sum of the corresponding coefficients in Nordhaus' Table 1, but with the δ_{10} obtained using only the current value of (x-xn). See Nordhaus, op. cit., p. 504, note 17.

18. Fair [6].

19. $\hat{q}$ was obviously computed based on an OLS regression performed over the same sample and corrected for first order serial correlation as the I.V.(2) estimates. The results are in Col. 4.

20. Solow, R. M. [18].

21. Nordhaus, W. D. [15], p. 493.

23. The data used in the cases of Transportation, Communication and Public Utilities are different from the ones used by Nordhaus. For a description of the difference and of the instruments used in Cols. 3 and 5, see the Appendix.

24. For example one may be tempted to explain the fall in labor produ-tivity that happened when the U.S. 1974 recession was at its bottom, with the increase in the price of oil and the consequent reduction in energy utilization, while unemployment did not yet significantly change.

25. Quotation taken from Atkinson, A.B. [1], Hahn, F. H. and Matthews, R. C. O. [7].

26. However, consider the estimate of 1.22 obtained in the last column of Table III. The labor share is $e \cdot \frac{\partial y}{\partial e} = y \frac{1}{1.22}$. Assuming a Cobb-Doublas production function, this would yield an average estimate of the ratio of total profits to total wages, defined as total proprietors income and private wages and salaries, of approximately 0.234, where the actual mean ratio is 0.205.

27. A conceivable implication in this respect might only regard the welfare prescriptions derived from estimates of labor productivity that are upward biased. In particular, suppose that utilization rates of existing machines can be changed independently from employment, say through a change in the user cost of capital, we would then observe increases in output that do not follow corresponding changes in the employed labor force. For those who think that the main reason for lower unemployment rates lies in the great increases in output, evidence and hypothesis as the ones suggested above should lead to a downward revision of the weight attached to unemployment in a hypothetical welfare function.

REFERENCES

[1] Atkinson, A.B., The Economics of Inequality, Clarendon Press, Oxford, 1975.

[2] Brechling, F.P.R., "The Relationship between Output and Employment in British Manufacturing Industries," RES, July 1965, pp. 187-216.

[3] Cripps, T.F. and Tarling, R.J., Growth in Advanced Capitalist Economies 1950-1970, Cambridge University Press, 1973.

[4] Fair, R.C.,

Econometrica, Vol.38, May 1970, pp. 507-516.

[5] Hahn, F.H. and Matthews, R.C.O., "The Theory of Economic Growth: a Survey," in Surveys of Economic Theory (Vol. 2), Macmillan, 1965.

[6] Hausman, J.A., "Specification Tests in Econometrics," MIT Working Paper #185, August 1976.

[7] H.M. Treasury Macroeconomic Model (Technical Manual), H.M. Treasury, February 1976.

[8] Ireland, N.J. and Smith, D.J., "The Specification of Short-run Employment Models," RES, April 1970, pp. 281-285.

[9] Kaldor, N., Causes of the Flow Rate of Economic Growth in U.K., Cambridge University Press, 1966.

[10] Kaldor, N.,"The Irrelevance of Equilibrium Economics" (The Goodricke Lecture, University of York, May 10, 1972), EJ, Vol. 82, December 1972, pp. 1237-1255.

[11] Kuh, E., "Cyclical and Secular Labor Productivity in United States Manufacturing," R. E. and Stat., Vol. 47, February 1965.

[12] Nordhaus, W.D., "The Recent Productivity Slowdown," Brookings Papers on Economic Activity 3, 1972.

[13] Rowthorn, R.E., "What Remains of Kaldor Law," EJ, Vol. 85, March 1975, pp. 10-19.

[14] Salter, W.E.G., Productivity and Technical Change, Cambridge University Press, 1969.

[15] Solow, R.M., "Some Evidence on the Short-Run Productivity Puzzle," in Essays in Honor of Paul Rosenstein Rodan, ed. by J. Bhagwati and R. Eckaus, Allen and Unwin, London 1973.

[16] Solow, R.M., "Substitution and Fixed Proportions in the Theory of Capital," RES, June 1962, pp. 207-218.

[17] Verdoorn, P.J., "Fattori Che Regolano lo Sviluppo della Produttività," L'Industria, 1949.

[18] Whitaker, J.K., "Vintage Capital Models and Econometric Production Functions," RES, 1965, pp. 1-16.

Real Wages, Employment and Output

The last empirical point of this chapter concerns the relation between the demand for labor and real wages. As in the case of the output-labor input elasticity estimates, this is an open question in the sense that while the evidence[24] uniformly points in the direction of procyclical movements of real wages with respect to output and employment, neoclassical theory, whether its conclusions are derived from a fixed coefficients production function or from a Cobb-Douglas, predicts anticyclical movements. As historical account of the controversy that started immediately after the publication of the General Theory with an exchange of dissenting views, on the E.J. of 1938 between Keynes and Dunlop, can be found in Bodkin. Such a diversity of opinions is still actual since some recent contributions try to explain among other things, the dynamic behavior of real wages, employment and output such as the one by Barro and Grossman predicts procyclical movements while Solow and Stiglitz hold an opposite point of view.[25]
In this paragraph such relation will be explored at both aggregate and disaggregate level.

The first question to be asked is what is exactly the relation that someone wants to estimate. There is no unique answer to this question since it depends on whether the production function is supposed to describe a short or a long run technological constraint and on whether we assume that firms price their output at marginal cost or on a mark up based on a minimum average cost of which unit labor cost is the main component. In this case, the amount of the mark up would be determined by entry-preventing or other oligopolistic considerations. Therefore, while changes in output would call for changes in employment and money wages, prices would be insensitive (if we make abstraction from changes in the size of the market) in the short run and therefore real wages, should be positively correlated with changes in employment and output. Here are three possible specifications:

$$\text{(I)} \qquad \frac{w}{P} = F_e(e)e^{\gamma t} = \alpha(\frac{y}{e}) \quad , \qquad 0 < \alpha < 1$$

output in the short run can change only if labor changes because capital is always fully utilized, its costs have been completely repaid, and there is no user cost of capital; α and γ are respectively the labor coefficient and the rate of technological progress.

If capital is not fully utilized or if its costs are not completely sunk or if there is some kind of user cost, we would have

$$\text{(II)} \qquad \frac{w}{P} = \frac{\alpha}{\beta}\ \frac{r}{P}\ (\frac{y}{e})^{\alpha/\beta} e^{-\gamma t} \qquad \alpha + \beta = 1, \qquad 0 < \beta < 1$$

where β and r are respectively the capital coefficient in the production function and r/P can be interpreted either as the real cost of capital or as a user cost. It is worth being noticed that if we estimate (I) while the true model in log linear form is

$$\ln(w/P) = \alpha_0 + \ln\ (r/P) + \frac{\alpha}{\beta} \ln\ (\frac{y}{e}) - \frac{\gamma^t}{\beta}$$

the OLS estimate $\hat{\alpha}_{(I)}$ from (I) will be

$$\hat{\alpha}_{(I)} = \frac{\alpha}{\beta} + \frac{\text{Cov }[\ln(r/P),\ \ln(y/e)]}{V[\ln(y/e)]} - \frac{\gamma}{\beta}\frac{\text{Cov }[\ln(t),\ \ln(y/e)]}{V[\ln(y/e)]}$$

where it is assumed that the R.H.V. of (II) are uncorrelated with the error term, which says that:

(i) $\hat{\alpha}_I$ has to be reinterpreted as α/β, so that values of $\hat{\alpha}_I$ bigger than one should not be a surprise and do not contradict diminishing marginal returns to labor;

(ii) If there is any substitution in the short run also, Cov $(r/P,\ y/e) < 0$, at most it will be equal to zero in the case of no substitution at all. Further, we should expect Cov $(y/e,\ t) > 0$. From all this we should expect $\hat{\alpha}_{(I)}$ to be biased towards zero.

The third specification would be

$$\text{(III)} \qquad \frac{w}{P} = \frac{y^*}{e^*}\,(1 + m)^{-1}$$

where m is the mark up over the competive price p_c set equal to the minimum average cost

$$p = p_c(1 + m) = w\,\frac{1^*}{y^*}\,(1 + m)$$

where y^* and e^* are the cost minimizing level of output and labor input. This theory, while still holding that real wages move with long run labor productivity, will predict that in the short run output and employment will be largely uncorrelated with price changes and positively correlated with changes in nominal wages because of changes in the number of vacancies. So that a suitable test of this theory would be to estimate a relation of the form

$$\text{(IIIa)} \quad \ln\left(\frac{w}{p}\right) = \quad + \quad e + \quad \left(\frac{y^*}{e^*}\right) +$$

where $\frac{y^*}{e^*}$ can either be approximated by a time trend and/or by actual productivity. On the basis of this theory, we should expect to be positive and significant. There is one major qualification to be made with respect to this: "With much idle capacity, the temptation for individual members of the oligopolistic group to secure a larger share of the shurnken business is very strong. Thus.... a resultant fall in the effective price if not in the quoted one."[26] then may well turn up to be negative and the resultant behavior virtually undistinguishable from the competitive one. The main difference among these specifications is really between II and III(a). While the first says that real wages can change only if labor marginal productivity changes, the second contains additional elements of a behavior of the labor market that is in the short run largely independnet of the marginal productivity of labor. Therefore if we estimate III(a) while the true model is II, the estimated should be either negative and significant or positive and non-significantly different from 0. Whether positive or negative, an OLS estimate of III(a) when the true model is II should yield estimates biased toward zero.

The second question concerns the estimation of relations like (I)-(III) at the aggregate level. While at the sectorial level there is no doubt about which price deflator to use, i.e., the output price deflator of the sector, at aggregate level there is no clear correspondent between the wage, employment concepts and the price deflator. So that at the economy level, it is not clear whether we are testing for the equilibrium proposition that firms do by-and-large price at marginal cost or for some empirical regularity between the purchase in power of wages and output and employment. This explains why five different deflators have been used: P_1, the implicit price deflator (i/p.d.), net domestic product, non-farm business; P_2, i.p.d. of personal consumption expenditures; P_3, the wholesale price index of all commodities less farm products; P_4, the implicit price deflator of gross domestic product; P_5, the consumer price index of all items. The employment and the wage concepts are respectively total employment in non-agricultural establishments and hourly earnings of production workers, private non-agricultural. Tables IV and VI respectively show the OLS and the 2SLS estimates of equations II (where r/P has been assumed unobservable) and III, for different definitions of real wages.

TABLE IV

-- Estimates of real wages, employment and time at the aggregate level;

-- $\ln(\frac{w}{P_i})_t = \alpha_0 + \alpha_1 E_t + \alpha_2 t + \varepsilon_t$;

-- Observations = 26;

-- (Standard errors).

Real Wages	Independent Variable	OLS	TSLS
$\ln(\frac{w}{P_1})$	α_0	0.70	2.15
	α_1	0.0232	-0.366
		(0.0710)	(0.198)
	α_2	0.0188	0.0272
		(0.0015)	(0.0042)
	S.E.	0.0112	0.0171
	D.W.	0.48	0.50
	$\bar{R}^2$	0.994	0.98
$\ln(\frac{w}{P_2})$	α_0	1.13	2.87
	α_1	-0.0930	-0.560
		(0.0914)	(0.2451)
	α_2	0.0216	0.0316
		(0.0019)	(0.0052)
	S.E.	0.0144	0.0211
	D.W.	0.37	0.55
	$\bar{R}^2$	0.990	0.980
$\ln(\frac{w}{P_3})$	α_0	0.73	2.21
	α_1	-0.0861	-0.484
		(0.1144)	(0.2222)
	α_2	0.0290	0.0374
		(0.0024)	(0.0047)
	S.E.	0.0181	0.0223
	D.W.	1.40	0.84
	$\bar{R}^2$	0.992	0.988

Real Wages	Independent Variable	OLS	TSLS
	α_0	1.63	3.31
	α_1	-0.208	-0.658
$\ln(\frac{w}{P_4})$		(0.1114)	(0.2214)
	α_2	0.0217	0.0313
		(0.0024)	(0.0048)
	S.E.	0.0176	0.0230
	D.W.	0.36	0.53
	$\bar{R}^2$	0.982	0.97
	α_0	1.89	3.57
	α_1	-0.363	-0.811
$\ln(\frac{w}{P_5})$		(0.1296)	(0.2494)
	α_2	0.0268	0.0382
		(0.0028)	(0.0053)
	S.E.	0.0205	0.0253
	D.W.	0.59	0.83
	$\bar{R}^2$	0.983	0.975

TABLE V

Aggregate Relation: $\ln(\frac{w}{P_i}) = \beta_0 + \beta_1 \ln E + \beta_2 \ln y + \beta_3 T + y$

Observations = 26

Real Wages	Independent Variable	OLS	TSLS
$\ln(\frac{w}{P_1})$	α_0	-0.0221	-0.371
	α_1	-0.1052 (0.0950)	-0.3087 (0.1535)
	α_2	0.1963 (0.1026)	0.3770 (0.1681)
	3	0.0145 (0.0026)	0.0124 (0.0048)
	S.E.	0.0106	0.0117
	D.W.	0.50	0.47
	$\bar{R}^2$	0.994	0.993
$\ln(\frac{w}{P_2})$	α_0	0.50	-0.449
	α_1	-0.204 (0.1277)	-0.3028 (0.1422)
	α_2	0.1700 (0.1379)	0.3855 (0.1694)
	α_3	0.0179 (0.0036)	0.0123 (0.0045)
	S.E.	0.0143	0.0151
	D.W.	0.37	0.40
	$\bar{R}^2$	0.991	0.990

Real Wages	Independent Variable	OLS	TSLS
	α_0	0.90	-2.81
	α_1	-0.446 (0.2271)	-1.073 (0.412)
$\ln(\frac{w}{P_3})$	α_2	0.1856 (0.2451)	1.175 (0.5781)
	3	0.0313 (0.0064)	0.0092 (0.0143)
	S.E.	0.0254	0.0335
	D.W.	1.22	1.08
	$\bar{R}^2$	0.986	0.976
	α_0	0.002	-0.23
	α_1	-0.497 (0.1352)	-0.515 (0.147)
$\ln(\frac{w}{P_4})$	α_2	0.442 (0.1460)	0.491 (0.2274)
	3	0.0120 (0.0038)	0.0106 (0.0065)
	S.E.	0.0151	0.0152
	D.W.	0.48	0.50
	$\bar{R}^2$	0.987	0.987
	α_0	0.24	-1.80
	α_1	-0.6563 (0.1650)	-1.039 (0.274)
$\ln(\frac{w}{P_5})$	α_2	0.447 (0.1781)	1.014 (0.3848)
	α_3	0.0188 (0.0046)	0.0068 (0.0095)
	S.E.	0.0185	0.0223
	D.W.	0.61	0.59
	$\bar{R}^2$	0.986	0.980

The following remarks are worth being made: (i) Both Tables IV and V show that aggregate real wage is significantly and negatively correlated with employment, no matter which is the price index chosen to deflate aggregate nominal wages; (ii) This is true both of the OLS and the TSLS estimates, with the only exception of the first OLS estimate of Table IV showing that when the i/p.d. of the private business non-farm output is chosen, a statistically insignificant but positive relation can be detected. However, the OLS estimates of Table IV are inconsistent since form III(a) is subject to serious misspecification of the following table shows. Furthermore these estimates are biased towards zero as we expected, for the reasons just mentioned.

Test for Specification Error in III(a)

Equations	q	m
I		
II		
III		
IV		
V		

$m^* =$

(iii) It is interesting to notice that the TSLS of Table IV do by and large (within a two standard erros confidence interval) coincide with the OLS estimates of Table V which seems to say that II is a specification closer to the true one; (iv) In Table V labor productivity has been estimated unconstrained just in order to see whether the employment term can effect real wages independently from productivity:

neither OLS nor TSLS estimates show significant difference in the magnitude of the coefficients, leading therefore to the rejection of the hypothesis maintaining that employment has such independent effect on real wages in the aggregate; (v) The estimates of Table V and the TSLS estimates of Table IV square quite consistently with those of Table II, where the consistent estimates yield values between 0.58 and 0.81.

Similar tests have finally been performed at a disaggregate level for four sectors [28]: Mining, Construction, Manufacturing Durables and Non-Durables. They are reported in Tables VI and VII. The aggregate picture is confirmed quite extensively.

1) In all the estimates of Table VI the employment coefficient is negative and significantly different from zero, with the exceptions of the Mining and the Manufacturing Durables industries, where it is positive but not significantly different from zero. The corresponding TSLS estimates are always negative and significant. Furthermore it can be easily verified, applying Hausmann Specification Error test, that in the case of the above two industries the null hypothesis of absence of misspecification can be rejected respectively at the 1% and 5% confidence level.[29]

2) Although in the case of Table VII it has not been possible to find Instrumental variables predicting with the same efficiency as at the aggregate level, the general picture is that the relation between employment alone and real wages is statistically insignificant.

TABLE VI

Relation: $\ln(\frac{w}{P_i}) = \beta_0 + \beta_1 \ln E + \beta_2 \ln y + \beta_3 T + \varepsilon$

Observations = 26

(Standard Errors)

Real Wages	Independent Variable	OLS	TSLS
	β_0	- 0.086	-0.70
Mining $\ln(\frac{w}{P_{mi}})$	β_1	0.324 (0.2338)	-0.717 (0.2025)
	β_2	0.196 (0.3370	0.569 (0.2025)
	β_3	0.0329 (0.0090)	-0.717 (0.3028)
	S.E.	0.0435	0.0301
	D.W.	0.72	1.74
	$\bar{R}^2$	0.962	0.981
	β_0	-0.338	-1.012
Construction $\ln(\frac{w}{P_c})$	β_1	-0.738 (0.1367)	-0.774 (0.1567)
	β_2	0.857 (0.1637)	1.104 (0.1145)
	β_3	0.0008 (0.0028)	-0.0013 (0.0034)
	S.E.	0.0179	0.0265
	D.W.	1.57	0.58
	$\bar{R}^2$	0.921	0.879

Real Wages	Independent Variable	OLS	TSLS
	β_0	-0.46	-0.76
Aggregate Manufacturing	β_1	-0.714	-0.754
		(0.2604)	(0.2838)
$\ln(\frac{w}{P_m})$	β_2	0.758	0.855
		(0.1981)	(0.220)
	β_3	0.0024	0.0009
		(0.0054)	(0.0061)
	S.E.	0.0240	0.0244
	D.W.	0.42	0.49
	$\bar{R}^2$	0.981	0.980
	β_0	-1.25	-1.48
Manufacturing Nondurables	β_1	-1.05	-1.390
		(0.368)	(0.4321)
$\ln(\frac{w}{P_{mn}})$	β_2	0.999	1.243
		(0.239)	(0.2767)
	β_3	-0.0040	-0.011
		(0.0076)	(0.0086)
	S.E.	0.0277	0.0284
	D.W.	0.65	0.80
	$\bar{R}^2$	0.981	0.98
	β_0	-0.40	-0.64
Manufacturing Durables	β_1	0.300	-0.524
		(0.1780)	(0.2720)
$\ln(\frac{w}{P_{md}})$	β_2	0.4119	0.591
		(0.1275)	(0.1901)
	β_3	0.0099	0.0058
		(0.0032)	(0.0046)
	S.E.	0.0253	0.0265
	D.W.	0.38	0.51
	$\bar{R}^2$	0.97	0.97

TABLE VII

Relation: $\ln(\frac{w}{P_i}) = \alpha_0 + \alpha_1 \ln(E) + \alpha_2 t + \varepsilon$

Observations =

Real Wages	Independent Variable	OLS	TSLS
Mining $\ln(\frac{w}{P_{mi}})$	α_0		
	α_1		
	α_2		
	S.E.		
	D.W.		
	$\bar{R}^2$		
Construction $\ln(\frac{w}{P_c})$	α_0	1.62	2.72
	α_1	-0.438 (0.2471)	-1.22 (0.6027)
	α_2	0.0097 (0.0046)	0.0280 (0.0117)
	S.E.	0.0549	0.0935
	D.W.	0.44	0.42
	$\bar{R}^2$	0.11	-0.50

Chapter 2

The analysis of open economies has followed, roughly speaking, three main lines of thought, labelled by the literature as the elasticities, absorption and monetarist approaches. Although a number of related issues have been discussed along these lines, the first question, even chronologically, to which a literature of impressive dimensions was addressed, is "Under what circumstances will a devaluation improve the balance of payments of the devaluing country?" The debate was carried out sometimes assuming Keynesian underemployment, sometimes full employment. In what follows full employment will always be assumed.[1]

The main differences between the first approach and the other two is in the attention devoted to the traded goods market: in particular the eleasticities approach relies on the Marshall-Lerner condition, as a criterion for assessing the effect of a devaluation. It is now well understood that, with respect to this problem, the answer provided by this condition is unsatisfactory for the following reasons: (a) The world excess demand for one of the two goods is mistakenly defined to be equal to the balance of payments of one of the two countries; (b) The terms of trade are confused with the exchange rate; (c) The

---

Thanks are due to S.Fischer, F.Modigliani, P.Kouri and to all participants to the M.I.T. Monetary Workshop

change in excess demand for exportables, following a change in the terms of trade, is confused with a change in actual exports; but this is so only if markets for both goods clear before and after the devaluation.

However, all these misconceptions derive from the fact that the traditional model is a barter model. If the presence of an asset like money is not considered, the concepts of exchange rate and of balance of payments are not well defined and changes in the excess demand for exportables or importables must imply opposite changes in domestic expenditure, on the same good requiring therefore the assumption of continuous equilibrium, in order for the model to be consistent. But this is so only because savings were ruled out by assumption.

The absorption and the monetarist analysis on the other hand focus on the balance of payment as a whole neglecting its composition between trade balance, interest receipts from capital owned abroad and capital flows.[2] In the one good version of the monetarist model, the consequences of changes in the exchange rate on the goods market are then more or less neglected, relative prices changes and values of the elasticities are substituted by absolute price changes--being these by definition the only terms of trade concept we have to look at in a one good model. In line with the monetarist tradition it is assumed that the economy is in full employment and that changes in the interest rate do not affect the demand for money. This last assumption is usually implicit in the small country--with perfect capital mobility assumption or in the two countries model capital movements just do not appear explicitly in a bonds market clearing equation. Real hoarding or real savings is a concept used for three purposes: it is flow demand for real balances, since other assets are not explicitly considered, flow excess demand for real balances, since the nominal money stock is considered as given at each point in time--being denied to the government the possibility of running a positive deficit or of making stabilization operations--and it is identically equal to the balance of payments surplus--this last equality following from the flow budget constraint for the whole country. In the two goods version--

traded and non-traded--consumption of both goods is defined to be a function of expenditure and relative prices. Since expenditure is equal to real income minus real savings, i.e., balance of payments surplus, every time that something increases real savings, for example a devaluation through the real balance effect, consumption of both goods decreases. But since the non-traded goods market is assumed to clear all the times at full employment, a balance of payments surplus will always be accompanied by a decline in the price of home goods in terms of the traded ones. In this way the switching and the reducing expenditure effect of a devaluation described by H. G. Johnson [10] are well captured by this model.

The purposes of this paper are: (i) A reexamination of the conclusions of this model under more general assumptions concerning the specification of the consumption function and the flow demand for assets. With regard to the first, it is worth noting that the substitution effect of savings with respect to both goods is due to the assumption of separability--the marginal rate of substitution between the two goods is independent on the total quantity of money[3]--that will be dropped in the following discussion. With respect to the second, a formulation that more closely reflects the derivation of the flow demand from underlying stock demand functions, will be adopted. (ii) Explicit inclusion of government transfers in the definition of disposable income will permit a discussion, although from a different standpoint, of the other channel mentioned by Alexander [1, 2] through which a devaluation may affect the balance of payments, namely through changes in the distribution of real income.

(iii) An exposition and possibly a clarification of what seems to have been a long lively debate among the different approaches on the channels through which a devaluation is supposed to work.[4] It will be seen that there is nothing to debate--at least at the theoretical level--if a flow model is assumed as a description of the economy. On the other hand if demand for assets are depicted as demand for stocks, the choice of which market to look at becomes important. In other words, is the type of description of the assets markets what should have been debated.

These issues will be discussed with the help of a two goods--traded and non-traded--model. The second section is devoted to the presentation of a quite general macromodel. Then the demand for assets will be given the specification that we think proper for a flow model and a discussion of the effects of a devaluation on relative prices, absolute price level and interest rate follows in two stages: in the first the analysis will be a partial equilibrium one in which it will be assumed the existence of only one country; in the second it will be extended to the consideration of two countries. In the third section an alternative approach--the one we believe an elasticity theorist would prefer--will be suggested and the results will be compared.

2

Two goods are supplied according to

$$x_i^s = x_i^s(q) \qquad i=1,2 \qquad (1)$$

where x_1 is the traded good and x_2 is the non-traded, q is the relative price of the non-traded in terms of the traded one

$$q \equiv \frac{p_2}{p_1}$$

The reward to factors in real terms is

$$Y = x_1^{s}(q) + qx_2^{s}(q) \tag{2}$$

The government deficit in real terms is

$$d = G - T + i\,\frac{B^g}{P_1} + z \tag{3}$$

where G and T are government expenditure and tax collection in real terms, B^g is the outstanding stock of government debt, z are government transfers and i is the interest rate. Government debt in real terms is

$$g = \frac{D^s + B^g}{P_1} \tag{4}$$

where D^s is the outstanding domestic money stock. Government deficit is equal to the change per unit of time of government debt in real terms

$$d = \frac{\dot{D}^s + \dot{B}^g}{P_1} \tag{5}$$

(3) and (5) imply the government budget constraint. The total monetary base is made up of two components: the domestic one created by the central bank and that of foreign origin derived from past accumulation of balance of payments supluses.

$$M^s = D^s + F \tag{6}$$

Purchasing power parity is assumed to hold

$$e = \frac{P_1}{P_1{}^*} \tag{7}$$

where e is the exchange rate and the symbol * denotes the other country money price of the traded good. From equations (1) - (5) and (7), disposable income in real terms is

$$y^d \equiv x_1{}^s(q) + qx_2{}^s(q) + \frac{\dot{D}^s + \dot{B}^g}{P_1} - (G) + \frac{i}{P_1}(e\beta B^{*g} - (1-\alpha)B^g) \tag{8}$$

where α and β are the shares of the stock of domestic and foreign bonds initially held by domestic citizens, so that the last term of the right hand side of (8) represents flow of the net interest payments between the two countries. Domestically owned wealth in real terms is

$$a \equiv (M + \alpha B^g + e\beta B^{*g}) \frac{1}{P_1} \tag{9}$$

where it is assumed that households perceive government debt as wealth. The demand functions for the two goods are

$$x_i{}^d = x_i{}^d(q, y^d, a) \qquad i=1,2 \tag{10}$$

There are only two financial assets: outside money and government bonds. Their demand functions are

$$\begin{aligned} M^d &= p_1 L(y, a, i, \pi^e) \\ B^d &= p_1 J(y, a, i, \pi^e) \end{aligned} \tag{11}$$

Equations and definitions (1) - (11) are quite common to any macroeconomic model.[5] The original formulations of the different ways of analyzing the balance of payments may easily be respecified in these terms. If the only difference between absorption and monetarist approaches is that in the first explicit disaggregation of savings among different assets was not given explicit consideration, having taken care of this "shortcoming" will clearly eliminate any distinction.

Then the flow aspects of the monetarist model are introduced. Desired demand for money and bonds as flows are

$$\begin{aligned} \dot{M}^d &= p_1 DL(y,a,i,\pi^e) + \pi^e p_1 L(y,a,i,\pi^e) \\ \dot{B}^d &= p_1 DJ(y,a,i,\pi^e) + \pi^e p_1 J(y,a,i,\pi^e) \end{aligned} \qquad (12)$$

where $D \equiv \frac{d}{dt}$ and π^e is the expected rate of inflation. As it can be seen they are derived from (11) where we made the assumption of initial stock equilibrium. [6]

The flow budget constraint for the whole country in real terms is from eqs. (8) - (12)

$$x_1^{\,d} + qx_2^{\,d} + DL(\cdot) + DJ(\cdot) + \pi^e[L(\cdot) + J(\cdot)] =$$

$$= x_1^{\,s} + qx_2^{\,s} + \frac{\dot{D}^s + \dot{B}^g}{P_1} - (G) + \frac{i}{P_1}[e\beta B^{*g} - (1-\alpha)B^g] \qquad (13)$$

Equations (12) and (13) are the central point of the monetarist and absorption models where excess demands for assets and goods do add up in the flow budget constraint making possible the use of the Walras Law, [8].

The next step is to show under what conditions the money market contains all the information necessary to judge the effect of changes in the exogenous variables on the balance of payments. In other words what we want to derive are the necessary conditions under which the monetarist absoprtion is "correct." One small country with perfect capital mobility and fixed exchange rates is assumed, therefore p_1^*, i, e are predetermined. Then the only endogenous variable is q. In order to determine it, following [ ], we assume that the market for non-traded goods always clears

$$G + x_2^d(q, y^d, a) - x_2^s(q) = 0 \tag{14}$$

where mostly for sake of simplicity it is assumed that government buys only home goods. Capital flows are defined as flow excess demand for domestic bonds

$$K^f \equiv DJ(\cdot) + \pi^e J(\cdot) - \frac{\dot{B}^g}{P_1} \tag{15}$$

Balance of payments definition is

$$\tilde{BP} \equiv (x_1^s - x_1^d) + K^f + \frac{i}{P_1}[(1-\alpha)B^g - e\beta B^{*g}] \tag{16}$$

which by (13) is equal to the flow excess demand of real cash balances

$$\tilde{BP} = DL(y,a,i,\pi^e) + \pi^e L(y,a,i,\pi^e) - \frac{\dot{D}^s}{P_1}$$

Then, following the literature,[7] it is assumed that the adjustment of real balances to the desired level is costly, such that it cannot be done instantaneously. The flow demand for real cash balances is

$$\gamma[L(y,a,i,\pi^e) - \frac{M^s}{P_1}] + \pi^e L(\cdot) \tag{17}$$

from (16) and (17) the expression for the balance of payments surplus in nominal terms is

$$BP = \gamma[p_1 L(y,a,i,\pi^e) - M^s] + \pi^e p_1 L(y,a,i,\pi^e) - \dot{D}^s \tag{18}$$

which is basically the monetarist conclusion: everything that positively affects demand for money, automatically improves the balance of payments; increases in the rate of growth of the domestic money stock are "bad" in the sense that they automatically worsen the balance of payments position. In order for these conclusions to hold it has been necessary to assume: (i) Continuous clearing of the market for non-traded goods; (ii) Flow equilibrium in the money and bonds markets. Therefore, to have balance of payments disequilibrium we need to have stock disequilibrium either in the money market or in the bonds market. These conclusions have already been stated, although in a different form, by H. Johnson[10]. This description implies that persistent disequilibrium in the balance of payments is caused by persistent stock disequilibrium in the assets markets which instead are in general considered to adjust in short periods of time. Then it would seem that this model provides an explanation of short run disequilibria, but this does not agree very much with the assumption of continuous full employment clearing in the non-traded sector. It would seem, just to summarize the situation that we are in the short run when we talk about balance of payments disequilibria being at the same time in the long run on the real side of the economy. What is not satisfactory in this model is the treatment of the assets markets. In fact there does not seem to be any theoretical justification to equation (17) whose specification is on the other hand necessary to avoid substantial analytical difficulties. The presence of costs of adjusting an actual stock to a desired one is justified in the case of the theory of investment by the fact that firms' marginal costs rise with the rise of investment and this limits in the short run the firm's capability of achieving a certain desired stock of capital. But when similar reasoning is applied to the demand for money it is not clear what is meant.

In order to assess the effect of a devaluation in this model we first notice that the assumptions of one country, stock disequilibrium in the assets markets and free trade, imply that the domestic price level, or in our units, the price of the traded good becomes a control variable. Differentiating (7)

$$\frac{de}{e} = \frac{dp_1}{p_1}$$

being $\frac{dp_1^*}{p_1^*} = 0$ by assumption.

As a second step totally differentiate the non-traded goods market equilibrium condition (14)

$$\frac{dq}{q} = \frac{dG + m_2(d\tilde{Z} + d\tilde{I} - dT) + \delta_2\frac{dA}{A} - (\delta_2 + m_2(Z + I))\frac{dp_1}{p_1}}{(\varepsilon_2 + \eta_2)x_2} \tag{19}$$

where the symbol ~ denotes quantities in nominal level and where

$$\varepsilon_2 \equiv \frac{\partial x_2^s}{\partial q}\frac{q}{x_2} > 0$$

$$\eta_2 \equiv -\frac{\partial x_2^d}{\partial q}\frac{q}{x_2^d} > 0$$

$$m_2 \equiv \frac{\partial x_2^d}{\partial y^d} > 0$$

$$\delta_2 \equiv \frac{\partial x_2^d}{\partial a}\frac{a}{x_2^d} \cdot x_2^d > 0$$

$$I \equiv \frac{\alpha B^g}{P_1} i$$

denote respectively the elasticity of supply, the uncompensated elasticity of substitution, the marginal propensity to spend on traded goods, the wealth elasticity of consumption, interest payments on the shares of domestic and foreign government debt owned by domestic citizens.

Expression (19) deserves the following remarks:

1. One of the mentioned criticisms to the traditional model concerned the equilibrium assumptions underlying the Marshall-Lerner condition. The same criticism holds with respect to this model. In fact to derive expression (19) it was necessary to assume (a) equilibrium in the non-traded goods sector before and after the devaluation; (b) production always takes place on the production possibility frontier which implies continuous full employment or instantaneous shift of resources from the traded to the non-traded goods sector.

2. The relative price of home goods can change in consequence of a devaluation only insofar as it acts as a capital levy on real wealth, on the non-indexed part of the government deficit. This would create an excess supply of the non-traded good and would require a decrease in its relative price. Notice that if the excess demand were function only of relative prices, the effect of a devaluation on the non-traded goods market would be nil.

The third step is to differentiate totally equation (18) and to substitute (19) into it.

$$dBP = (\gamma+\pi^{e})\ [(1-\eta)L(\cdot) - \frac{\partial L}{\partial y}\frac{[\delta_2 + m_2(Z + I)]}{(\varepsilon_2 + \eta_2)}\ ]\ \frac{dp_1}{p_1} +$$

$$+ \frac{\partial L}{\partial y}(dG + m_2(d\tilde{Z} + d\tilde{I} - dT) + \delta_2\frac{dA}{A}) \cdot$$

$$\cdot\ (\varepsilon_2 + \eta_2)^{-1} + \frac{\partial L}{\partial a}\frac{dA}{P_1} \quad - \gamma dM^{s} - d\dot{D}^{s} \qquad (20)$$

where

$\eta \equiv \frac{\partial L}{\partial a}\frac{a}{L(\cdot)}$ and where we considered the variables $\underset{\sim}{i}$ and π^{e} as constant

denotes the real wealth elasticity of demand for real cash balances. The condition for a devaluation to improve the balance of payments of the devaluing countrv is then

$$\eta + \hat{\theta}\ [\ \frac{\delta_2 + m_2(Z + I)}{(\varepsilon_2 + \eta_2)}\ ] < 1 \qquad (21)$$

where

$$\hat{\theta} \equiv \frac{\partial L}{\partial y}\frac{y}{L}\frac{1}{y} > 0$$

The following remarks are worth making:

1. Even leaving aside the second member of the L.H.S. of (21), it is not clear that a devaluation should cause a positive real balance effect on the flow demand for money. Clearly once demand for money is specified as function of real wealth, among other things, the result will depend on the value of the elasticity with respect to this argument. The specification of demand for money as a stock adjustment does not help in this sense: a change in price changes both the actual and the desired real cash balances.

2. The distributional effects mentioned by Alexander[1] are shown to work

in a perverse sense. A devaluation decreases real disposable income of owners of assets that are fixed in nominal value. As such it decreases consumption _and_ savings. This is perhaps the main distinction with the monetarist model: once the separability assumption is dropped, savings, demand for money and balance of payments surpluses can change not only because of the real balance effect--which is transient--but even because real disposable income changes.

3. Movements in the relative price of home goods in terms of the traded ones are positively associated with movements in the balance of payments: in fact, being full employment real income deflated by the traded goods price, it will increase whenever q increases. This in turn will increase the flow demand for money and therefore the balance of payments surplus.

3

The two countries model

Before dealing with an alternative formulation of the balance of payments problem it is worth extending the previous discussion to the consideration of two countries. This will make it clear that there is no difference between the elasticities, monetarist, absorption approaches, at least with respect to the choice of the market relevant for assessing the effects of a devaluation.

The flow budget constraint of the other country is

$$x_1^{d*} + q^* x_2^{*d} + (DL^* + DJ^*) + \pi^{e*}[L^*(\cdot) + J^*(\cdot)] =$$

$$= x_1^{s*} + q^* x_2^{s*} + \frac{i}{p_1}[(1-\alpha)B^g - e\beta B^{*g}] +$$

$$+ \frac{e}{p_1}(\dot{D}^{*s} + \dot{B}^{*g}) - q^* \tag{22}$$

The excess demand functions for the second country are

$$Ex_1^* \equiv x_1^{d*}(q^*, y^{d*}, a^*) - x_1^{s*}(q^*)$$

$$Ex_2^* \equiv x_2^{d*}(q^*, y^{d*}, a^*) - x_2^{s*}(q^*)$$

$$E_M^* \equiv DL(\overset{*}{\cdot}) + {}^{e*}L(\cdot) - \tilde{I}^* - \frac{\dot{D}^*}{p_1}e$$

$$E_B^* \equiv DJ^*(\cdot) + \pi^{e*}J(\cdot) - \frac{\dot{B}^{g*}}{P_1}e$$

Similar relationships hold for the home country; where

$$\tilde{I}^* \equiv \frac{i}{p_1}[(1-\alpha)B^g - e\beta B^{*q}]$$

$$\tilde{I} \equiv \frac{i}{p_1}[e\beta B^{*q} - (1-\alpha)B^q]$$

The world market equilibrium conditions are

$$E_{x_1} + Ex_1^* = 0 \tag{23}$$

$$E_{x_2} = 0 \tag{24}$$

$$E_{x_2}^* = 0 \tag{25}$$

$$E_M + E_M^* = 0 \tag{26}$$

$$E_B + E_B^* = 0 \tag{27}$$

where we retained the assumption of continuous equilibrium in the non-traded goods markets in each country.

Due to the two budget constraints for the two countries one of the above five equations is redundant. Thus we have four independent equations in four variables q, q^*, i, p_1 or p_1^* --being one of the two determined by (7). Which is the equation to be chosen as redundant? This can be considered the trivial side of the debate elasticity vs. monetarist approach: an elasticity theorist would choose the first four eqs. emphasizing in such a way the traded goods market; a monetarust would probably choose the last four, putting more emphasis on the assets markets. Clearly it does not make any difference which set of equations we choose--provided we always maintain eqs. (24) and (25)--since any equilibrium set of solutions satisfying the first four eqs. will satisfy the last four.

4

Making use of the model expounded in the previous two sections we now want to discuss some of the major results obtained by the monetary analysis of the balance of payments. Unfortunately the greater complication of our model does not allow us to produce conclusions having the same degree of unambiguity as those presented by previous authors. The first step is to derive reduced forms of expressions for the endogenous variables of interest in assessing the effect of a devaluation on the balance of payments of the devaluing country. The tedious algebraic part of the discussion is relegated to the Appendix. However, it is worth pointing out that at least two alternative sets of assumptions guarantee the results stated in Propositions (1) - (2). When differences will arise in Proposition (3), they will be object of explicit discussion. We have chosen to work with

A.1 $\quad \Theta \equiv [(\gamma + a^{e})(1-\eta) + (\gamma^{*} + a^{e*})(1-\eta^{*})] \leqq 0$

A.2 $\quad (\eta_1 - \varepsilon_1 - m_1^{*})x_1 + m_1 x_2 \equiv a_{11} > 0$

A.3 $\quad (\eta_1^{*} - \varepsilon_1^{*} - m_1^{*})x_1^{*} + m_1^{*} x_2^{*} \equiv a_{12} > 0$

A.4 $\quad \left| \dfrac{\varepsilon_2^{*} - \eta_2^{*}}{m_2^{*}} \right| \geqq \left| \dfrac{L_y^{*}}{\lambda} \right|$

A.5 $\quad \dfrac{a_{11}}{a_{12}} \geqq \dfrac{L_y}{L_y^{*}}$

A.6 $\quad \dfrac{\phi_2}{m_2} \geqq \dfrac{\Psi_1}{\mu}$

The first assumption says that the effect of a change in the price of traded goods expressed in terms of the currency of the home country either reduces the world flow excess demand for money, expressed in terms of the same currency or leaves it unchanged. For positive expected rates of inflation this may imply values of the elasticities of demand for money with respect to real wealth equal or bigger than one. A.2 and A.3 are not very strong in the sense that neglecting initial quantities as it is often done in comparative statics they are automatically satisfied. They say that the sum of the uncompensated elasticity of demand for traded goods minus the elasticity of supply and the marginal propensity to import traded goods--weighted by the initial quantities of traded and non-traded goods--should be positive in both countries. In this way the traded goods market appears explicitly in this monetary model since we have chosen to work with the first four equations of the system (23) - (27). A.4 relates substitution and income effects in the non-traded goods market to substitution and income effects in the money market. To understand A.4 imagine--everything else constant--an increase in the relative price of non-traded goods in terms of traded ones in the other country. This has a substitution effect creating an excess supply of non-traded and an excess demand of traded goods. But it has a positive income effect working in the opposite way in the non-traded goods market and causing an excess demand for money. However since the non-traded goods market has to clear all the times, what is required is an increase in interest rate crowding out the excess demand for money and diverting this into expenditure on non-traded goods. A.4 says that substitution effects in the non-traded goods market relative to income effects should be bigger than income effects relative to substitution effects in the money market. A.5 tells a similar story for the traded goods market. Imagine a decline in the relative price of non-traded in the home country and an increase in the same price in the foreign country. What A.5 says is that the home country should reduce expenditure on traded goods relative to the reduction in demand for money more than the foreign country's increase in expenditure on traded relative to the increase in demand for money. As a result demand for non-traded goods should raise more in the home country than in the other country, given that demand for bonds would decline in the home country, as a consequence of the negative income effect caused by the decrease in the relative price of non-traded in terms of traded goods, and would increase in the other country on the same account. This would cause an excess flow supply of bonds in the home country and a corresponding flow excess demand in the other country, which would create a capital flow into the home country. Finally A.6 relates income and wealth effects in both commodity markets but does not seem to have any intuitive interpretation. As a last point it is worth noting that A.1 - A.6 are necessary although not sufficient to guarantee the local stability of the system (23) - (27).

<u>Proposition 1</u>

Given A.1 - A.6, a devaluation lowers the relative price of non-traded goods in terms of traded ones in the home country if

A.7 $$(\gamma^{*} + \pi^{e*})(1 - \eta^{*}) \leqq 0$$

A devaluation--everything else constant--causes a positive wealth effect on the other country's demand for goods and money--if A.7 is verified. It creates then an excess demand for money and traded goods by the other country. These markets--everything else constant--can be cleared only at a lower domestic relative price of non-traded goods. This in fact will induce a negative income effect that will reduce demand for money in the home country and a substitution effect that will reduce demand for traded and increase demand for non-traded goods. As it had to be expected, having considered real income among the arguments of the demand for money, makes much more ambiguous the "reducing expenditure effect" of the devaluation, placing much more emphasis on the substitution effect, along the classical lines.

Proposition 2

Given A.1 - A.7, a devaluation increases the price of traded goods measured in terms of currency units of the devaluing country. This obviously descends from the fact that the positive wealth effects originated by the devaluation and increasing the other country's demand for traded goods and money have to be offset by negative wealth effects--in order to clear those markets after the devaluation-- such as those caused by a higher domestic price level.

Proposition 1 and 2 have been derived assuming $\Theta \leqq 0$ and $(\gamma^* + \pi^{e*})\cdot(1-\eta^*) \leqq 0$. The same propositions would have been valid--although under an alternative set of assumptions--if we supposed $\Theta \geqq 0$ and $(\gamma^* + \pi^{e*})(1 - \eta^*) \geqq 0$. However, the two possibilities show an interesting difference.

Proposition 3

If a devaluation raises the price level with

$$\Theta < 0 \text{ and } (\gamma^* + \pi^{e*})(1 - \eta^*) < 0$$

or with

$$\Theta = 0 = (\gamma^* + \pi^{e*})(1 - \eta^*)$$

a devaluation will raise the domestic price level of traded goods measured in terms of currency units of the devaluing country less than proportionately to the amount of the devaluation if

$$(\gamma + \pi^e)(1 - \eta) \leqq 0$$

If a devaluation raises the domestic price level with

$$\Theta > 0 \text{ and } (\gamma^* + \pi^{e*})(1 - \eta^*) > 0$$

then it will have the same effect--a less than proportional increase--only if

$$(\gamma + \pi^e)(1 - \eta) > 0$$

However if a devaluation raises the price level in the intermediate cases when

$$\Theta > 0 \text{ and } (\gamma^* + \pi^{e*})(1 - \eta^*) < 0$$

$$\Rightarrow (\gamma + \pi^e)(1 - \eta) > 0$$

or when

$$\Theta < 0 \text{ and } (\gamma^* + \pi^{e*})(1 - \eta^*) > 0$$

$$\Rightarrow (\gamma + \pi^e)(1 - \eta) < 0$$

implying in both of these cases

$$|(\gamma + \pi^e)(1 - \eta)| > |(\gamma^* + \pi^{e*})(1 - \eta^*)|$$

then a devaluation will increase more than proportionately the domestic price level.

If we restrict our attention to the effect of the devaluation on the equilibrium conditions in the money market, the interpretation of the first two cases becomes rather easy.

In the first case the home country reduces its demand for money in proportion $(\gamma + \pi^e)(1 - \eta)$ of the positive change in the domestic price level. The only way the money market can be in equilibrium after the devaluation is if the other country increases demand for money. This--everything else constant--is true only if the price level of the other country falls after the devaluation. But this by the definition of the exchange rate implies that the price level of the devaluing country increases less than proportionately to the amount of the devaluation. The same reasoning holds in the second case. The other two cases are such that both countries increase or reduce their demand for money at the same time. In this situation what is required to clear the money market is an increase of the other country's prices level too. But this by the definition of the exchange rate implies a more than proportionate increase in the domestic price level of the devaluing country.

Proposition 4

A devaluation may raise or lower the world interest rate. The effect is uncertain under any set of assumptions about the form of the demand for money. This should be expected since, having considered a perfectly integrated capital market, movements in the world interest rate consequent to the devaluation would depend on the relative size of the two countries' wealth elasticity of consumption of both goods and demand for money.

The reduced forms of expression for the endogenous variables are

$$p_1 = p_1(e\ ;\ \xi)\ ;\ \frac{\partial p_1}{\partial e} > 0$$

$$q = q(e\ ;\ \xi) \qquad \frac{\partial q}{\partial e} < 0$$

$$i = i(e\ ;\ \xi) \qquad \frac{\partial i}{\partial e} \gtrless 0$$

where ξ is a vector of exogenous variables.

The balance of payments of the home country is

$$BP = \gamma\{p_1(e;\xi)L[y[q(e;\xi)],\frac{A}{p_1(e;\xi)},\ i(e;\xi),\pi^e -$$

$$- M^s\} + \pi^e p_1(e;\xi)L[\cdot] - \dot{D}^s \qquad (18')$$

differentiating it with respect to $\underset{\sim}{e}$

$$\frac{\partial BP}{\partial e/e} = (\gamma + \pi^e)\{L(\cdot)(1-\eta)\frac{\partial p_1/p_1}{\partial e/e} + \frac{\partial L}{\partial y}\frac{\partial y}{\partial q}\frac{\partial q}{\partial e/e} + \frac{\partial L}{\partial i}\frac{\partial i}{\partial e/e}\} \qquad (28)$$

<u>Proposition 5</u>

A devaluation does not necessarily improve the balance of payments of the devaluing country. Even neglecting the effect of changes in interest rate that are of ambiguous sign, it can be seen from (28) that the conditions for an improvement in the balance of payments are much more stringent than those usually derived by the monetarist model. In particular they amount to say that the wealth elasticity of demand for money should be close to zero, and that the positive stock-adjustment effect of the devaluation should be bigger than the negative income effect. Although there is nothing that can a priori exclude this possibility, it must be remembered that the stock-adjustment effect is transitory by definition while the income effect, in absence of other changes, is permanent. So that even if the premises of the monetarist model were verified, we should expect an initial improvement in the balance of payments in the period immediately after the devaluation followed by a permanent worsening due to the negative income effect on the demand for money.

5

In this section it is presented a different way of looking at the balance of payments: not anymore as flow excess demand for money, but simply as a definition. The two characteristics of the monetarist model, disequilibrium in the assets markets and the flow budget constraint are now replaced by the assumption of stock equilibrium and by a well defined wealth constraint.

The assets markets are described by the following equilibrium conditions

$$B^d + \tilde{B}^{d*} = B^g \qquad (28a)$$

$$B^{d*} + \tilde{B}^d = B^{*g} \qquad (28b)$$

$$M^d = D^s \qquad (28c)$$

$$M^{*d} = D^{*s} \qquad (28d)$$

and by the wealth constraints

$$B^d + \tilde{B}^d + M^d = W \qquad (29a)$$

$$B^{d*} + \tilde{B}^{*d} + M^{d*} = W^* \qquad (29b)$$

where $\tilde{B}^d$ is domestic demand for foreign bonds, $\tilde{B}^{*d}$ is foreign demand for domestic bonds and all the other variables are defined as before. Only two of the equations (28) are independent. However--having considered a single integrated capital market we now have five independent equations (23) - (24), (25) and any two of (28) in only four unknowns. The last assumption needed by this model is that one of the two independent equilibrium conditions be satisfied for whatever values of q^*, q, p_1, i. The main difference with respect to the previous model is that now in absence of the flow budget constraint we must retain all three equations (23) - (25) describing the commodity markets because there is no guarantee that solutions satisfying eqs. (28) will satisfy (23) - (25) being the last specified in terms of flows. In this way the traded goods market is given explicit consideration and the various elasticities on this market become of interest in assessing the effect of a devaluation.

The balance of payments is just the definition (16) not being directly related in this case to the flow excess demand for money. Capital flows are defined as

$$K_F \equiv \dot{\tilde{B}}^{d*} - \dot{\tilde{B}}^{d}$$

where

$$\dot{\tilde{B}}^{d*} = \gamma^*[p_1^* J^*(y^*, a^*, i, \pi^{e*}) - B^{g*}]$$

$$\dot{\tilde{B}}^{d} = \gamma[p_1 J(y, a, i, \pi^e) - B^g]$$

Trade deficit is in real terms

$$T \equiv x_1^d(y^d, a, q) - qx_1^s(q)$$

and net interest payments flows are

$$\tilde{I} \equiv \frac{i}{p_1}(e\beta B^{*g} - (1-\alpha)B^g)$$

(28d) is chosen as the independent equation.

<u>Proposition 6</u>

If $(1-\eta^*) \leqq 0$, and given A.2 - A.4 a devaluation lowers the relative price of the traded good in the devaluing country and it raises it in the other country. Furthermore it raises the domestic price level less than proportionately to the amount of the devaluation and it increases the world interest rate.

Therefore

$$\frac{\partial T}{\partial e} = \left(\frac{\partial T}{\partial q}\frac{\partial q}{\partial e} + \frac{\partial T}{\partial p_1}\frac{\partial p_1}{\partial e} + \frac{\partial T}{\partial i}\frac{\partial i}{\partial e}\right) \tag{30}$$

$$\frac{\partial T}{\partial q} = ((\eta_1-\varepsilon_1-m_1)x_1 + m_1x_2) > 0$$

$$\frac{\partial T}{\partial p_1} = -(m_1(Z + I) + \delta_1) < 0$$

$$\frac{\partial T}{\partial i} = m_1(\alpha B^g + \beta B^{*g}) > 0$$

Then if we exclude interest receipts a devaluation would unambiguously decrease the trade deficit. However, a devaluation creates positive excess demand for money in the world market, that, everything else constant, can be cleared only at a higher interest rate. This creates a positive income effect that may worsen the trade balance. Differentiating the capital account, measured in terms of the domestic price of traded goods, with respect to the exchange rate

$$\frac{\partial K_F}{\partial e/e} = [B^{*g}\gamma^*(1 - \eta_B^*) - B^g\gamma\,(1 - \eta_B)]\frac{\partial p_1/p_1}{\partial e/e} - B^{*g}\gamma^*[1 - \eta_B^*] +$$

$$+ \left(\gamma^*\frac{\partial J^*}{\partial i} - \gamma\frac{\partial J}{\partial i}\right)\frac{\partial i}{\partial e/e} + \gamma^*\frac{\partial J^*}{\partial y^*}\frac{\partial y^*}{\partial q^*}\frac{\partial q^*}{\partial e/e} - \gamma\frac{\partial J}{\partial y}\frac{\partial y}{\partial q}\frac{\partial q}{\partial e/e} \tag{31}$$

where

$$\eta_B^* \equiv \frac{\partial J^*}{\partial a^*}\frac{a^*}{B^{*g}/p_1^*}$$ and similarly for the other country,

are the wealth elasticities of demand for bonds. A devaluation affects the capital account through three channels: a wealth effect, an interest rate or substitution effect and an income effect. It unambiguously improves the capital account balance through the last channel: the decrease in real income in the devaluing country and the increase in real income in the other country respectively reduce the domestic flow demand for foreign bonds and increase the foreign flow demand for domestic bonds. A devaluation would improve the capital account balance through the other two channels if

$$\eta_B > 1 \quad , \quad \eta_B^* > 1 \tag{32a}$$

$$\gamma^* \frac{\partial J^*}{\partial i} \gtreqless \gamma \frac{\partial J}{\partial i} \tag{32b}$$

The interpretation of (32a) descends from the definition of capital flows. They arise from the existing gap between desired demand and available stock of domestic bonds. Desired demand is among other things a function of real wealth. Following a devaluation, wealth and the available stock of domestic bonds in real terms, decrease in the same proportion as the increase in the price of traded goods in the devaluing country and conversely in the other country. Only if desired demand for domestic bonds decreases more than proportionately to the change in real wealth, i.e., $\eta_B > 1$, and conversely in the other country, domestic flow demand for foreign bonds would decrease and foreign flow demand for domestic bonds would increase. (32b) is just a condition relative to the size of the two countries and it is of obvious interpretation.

The effect of a devaluation on net interest flows is not of particular interest as it depends on the initial holdings of domestic and foreign bonds.

Conclusion

The monetarist analysis of the balance of payments rests on the assumptions of : (i) an underlying utility function which is separable between commodities and money; (ii) the existence of an unambiguously positive real balance effect on the flow demand for money; (iii) the existence of a flow budget constraint for the whole economy such that an excess demand for traded goods is equal to a flow excess supply of money. The main implication of these assumptions is what used to be called "classical dichotomy," although in a particular form: demands for goods can change if their relative prices change or if out of a given real income savings increase with respect to expenditure.

In the first three sections of the paper the flow structure of the monetarist model has been retained but the other two assumptions have been dropped. It has been shown taht a devaluation affects the domestic price level and the relative price of non-traded goods in terms of the traded ones in the same way as it does in the monetarist model. However given the different specifications of the demand functions for goods and for assets, the channels through which a devaluation works are different from the monetarist model. In particular it has emphasized the fact that since demand for money is a positive function of real income and real wealth, changes in relative prices or in the absolute price level--such as those caused by a devaluation--that decrease real income and real wealth may decrease the flow demand for money and therefore may not improve the balance of payments of the devaluing country.

The second question to which we tried to answer was: Does the flow excess demand for money contain all the information necessary to judge the effect of a devaluation?" or, in other words, "Was the emphasis placed by classical authors on the traded goods market a misplaced one?" The answer provided was in two parts :

(i) In a general equilibrium model the equilibrium values of the endogenous variables are jointly determined by all the markets. We could have indifferently excluded by the Walras-Law the world money market or the traded goods market without this affecting in any substantial way the nature of the equilibrium solutions. It is true that we have chosen the flow demand for money as the expression of the balance of payments into which to substitute the reduced form expressions for the endogenous variables, but we could have done the same with the other side of the country's budget constraint which is by definition the balance of payments. Again there is no difference.

(ii) However, the difference arises where the assets markets are described as being in continuous stock equilibrium. In this case--closer to the IS-LM type of framework--we can still neglect the money market in virtue of the wealth constraint, but we must retain all the markets for commodities traded and non-traded.

Appendix

Part I

Eqn. (23) is

$$E_{x_1} + E_{x_1}^* \equiv x_1^d(q,y^d,a) - x_1^s(q) + x_1^{d*}(q^*,y^{d*},a^*) - x_1^{s*}(q^*) = 0$$

where $y^{d*} \equiv x_1^{s*}(q^*) + q^* x_1^{s*}(q^*) + \frac{Z}{p_1^*} + \frac{i}{p_1^*}(\frac{(1-\alpha)B^g}{e} + (1-\beta)B^{g*})$

$$a^* \equiv (M^* + \frac{(1-\alpha)B^g}{e} + (1-\beta)B^{g*}) \frac{1}{p_1^*}$$

Totally differentiating eqn. (23)

$$[(\eta_1 - \varepsilon_1 - m_1)x_1 + m_1x_2]\frac{dq}{q} + [(\eta_1^* - \varepsilon_1^* - m_1^*)x_1^* + m_2^*x_2^*]\frac{dq^*}{q^*} +$$

$$+ (m_1B + m_1^*B^*)di - (m_1(Z + B) + m_1^*(Z^* + B^*) + \delta + \delta^*)\frac{dp_1}{p_1} +$$

$$+ (m_1^*(Z^* + B^*) + \delta^*)\frac{de}{e} = 0$$

where all the other exogenous variables have been assumed constant

where

$$\eta_1 \equiv \frac{\partial s_1^d}{\partial q}\frac{q}{x_1} > 0$$

$$\varepsilon_1 \equiv \frac{\partial x_1^s}{\partial q}\frac{q}{x_1} < 0$$

$$m_1 \equiv \frac{\partial x_1^d}{y^d} > 0$$

$$B \equiv (\alpha B^g + \beta B^{g*}) > 0$$

$$\delta \equiv x_1 \cdot \frac{\partial x_1^d}{\partial a}\frac{a}{x_1} > 0$$

and similarly for the other country.

Since now on the following definitions will be introduced

$$[(\eta_1 - \varepsilon_1 - m_1)x_1 + m_1 x_2] \equiv a_{11}$$

$$[(\eta_1^* - \varepsilon_1^* - m_1^*)x_1^* + m_1^* x_2^*] \equiv a_{12}$$

$$-[m_1(Z + B) + m_1^*(Z^* + B^*) + \delta_1 + \delta_1^*] \equiv \Psi_1 < 0$$

$$(m_1 B + m_1^* B^*) = \mu < - \Psi_1$$

and it will be assumed that

$$a_{11} > 0 \ , \ a_{12} > 0$$

which correspond to A.2 and A.3 in the text.

Eqn. (23) is

$$E_{x_2} \equiv q + x_2^d(q, y^d, a) - x_2^s(q) = 0$$

totally differentiating it, assuming all the exogenous variables, but the exchange rate, to be constant

$$(\eta_2 - \varepsilon_2)x_2 \frac{dq}{q} - \phi_2 \cdot \frac{dp_1}{p_1} + m_2 B di = 0$$

where

$$\eta \equiv \frac{\partial x_2^d}{\partial q} \frac{q}{x_2} < 0$$

$$\varepsilon_2 \equiv \frac{\partial x_2^s}{\partial q} \frac{q}{x_2} < 0$$

$$m_2 \equiv \frac{\partial x_2^d}{\partial y^d}$$

$$\phi_2 \equiv \delta_2 + m_2(Z + B)$$

Similarly for the other country, from eqn. (24)

$$(\eta_2^* - \varepsilon_2^*)x_2^* \frac{dq^*}{q^*} - \phi_2^* \frac{dp_1}{p_1} + \phi_2^* \frac{de}{e} + m_2^* Bdi = 0$$

Eqn.(25) in real terms is

$$\gamma[L(y,a,i,\pi^e) - \frac{M^s}{P_1} + \pi^e L(\cdot) - \dot{D}^s + \gamma^*[L^*(y^*,a^*,i,\pi^e) - \frac{M^{*s}}{P_1}e] +$$

$$+ \pi^{e*} eL^*(\cdot) - \dot{D}^{*s} = 0$$

We totally differentiate this equation, assuming all the exogenous variables other than the exchange rate constant

$$(\gamma + \pi^e)\{[q\frac{\partial L}{\partial y}x_2]\frac{dq}{q} + \frac{\partial L}{\partial i}di + (-\frac{\partial L}{\partial a}a + \frac{M^s}{P_1})\frac{dp_1}{p_1}\} +$$

$$+ (\gamma^* + \pi^{e*})\{q^* \frac{\partial L^*}{\partial y^*}\frac{dq^*}{q^*} + \frac{\partial L^*}{\partial i}di + (-\frac{\partial L^*}{\partial a^*}a^* + \frac{M^{*s}}{P_1})\frac{dp_1}{p_1} +$$

$$+ (\frac{\partial L}{\partial a^*}a^* - \frac{M^{*s}}{P_1})\frac{de}{e}\} =$$

$$= (\gamma + \pi^e)L_y\frac{dq}{q} + (\gamma^* + \pi^{e*})L_y^* \frac{dq^*}{q^*} + [(\gamma + \pi^e)\frac{\partial L}{\partial i} + (\gamma^* + \pi^{e*})\frac{\partial L^*}{\partial i}]di +$$

$$+ [(\gamma + \pi^e)(1 - \eta) + (\gamma^* + \pi^{e*})(1 - \eta^*)]\frac{dp_1}{p_1} -$$

$$- (\gamma^* + \pi^{e*})(1 - \eta^*)\frac{de}{e} = 0$$

where

$$L_y \equiv q \frac{\partial L}{\partial y} x_2(\gamma + \pi^e) > 0$$

$$\eta \equiv \frac{\partial L}{\partial a} \frac{a}{M^s/P_1}(\gamma^* + \pi^{e*}) > 0$$

and we define

$$\theta \equiv [(\gamma + \pi^e)(1 - \eta) + (\gamma^* + \pi^{e*})(1 - \eta^*)] \gtrless 0$$

$$\lambda \equiv ((\gamma + \pi^e) \frac{\partial L}{\partial i} + (\gamma^* + \pi^{e*}) \frac{\partial L^*}{\partial i}] < 0$$

The matrix of the coefficients of endogenous variables is

$$\begin{bmatrix} a_{11} & a_{12} & \Psi_1 & \mu \\ (\varepsilon_2-\eta_2) & 0 & q_2 & -m_2B \\ 0 & (\varepsilon_2^*-\eta_2^*) & q_2^* & -m_2^*B^* \\ L_y & L_y^* & \theta & \lambda \end{bmatrix} \begin{bmatrix} dq/q \\ dq^*/q^* \\ dp_1/p_1 \\ di \end{bmatrix} = \begin{bmatrix} -\phi_1^* \\ 0 \\ \phi_2^* \\ (\gamma^*+\pi^{e*})(1-\eta^*) \end{bmatrix} \frac{de}{e}$$

The minors of the corresponding determinant are

$$A_{11} = \{-\phi_2 m_2^* B^* L_y^* - \theta(\varepsilon_2^* - \eta_2^*)m_2B + m_2B\phi_2^* L_y^* - (\varepsilon_2^* - \eta_2^*)\phi_2\lambda\} \gtrless 0$$

$$A_{12} = \{a_{12}\phi_2^*\lambda - m_2^* B^* \Psi_1 L_y^* + \theta(\varepsilon_2^* - \eta_2^*)\mu -$$

$$- L_y^* \phi_2^* \mu + m_2^* B^* \theta a_{12} - \Psi_1(\varepsilon_2^* - \eta_2^*)\lambda\} \gtrless 0$$

$$A_{13} = \{a_{12}\ {}_2\lambda - \Psi_1 m_2 B L_y^* - L_y^*\ {}_2\mu + m_2 B\theta a_{12}\} \gtrless 0$$

$$A_{14} = \{-a_{12}\ {}_2 m_2^* B^* - \Psi_1 m_2 B(\varepsilon_2^* - \eta_2^*) - (\varepsilon_2^* - \eta_2^*)\ {}_2\mu +$$

$$+ m_2 B g_2^* a_{12}\} \gtrless 0$$

The Jacobian of the partial derivatives of the endogenous variables is

$$\Delta = [a_{11}A_{11} - (\varepsilon_2 - \eta_2)A_{12} - L_y A_{14}] \qquad (1)$$

<u>1st Case</u>: $\theta \leqq 0$

it can be seen that $\Delta>0$ if the following conditions are satisfies

A.2 $a_{11} \geqq 0$

A.3 $a_{12} \geqq 0$

A.4 $\dfrac{\varepsilon_2^* - \eta_2^*}{m_2^*} \geqq \dfrac{L_y^*}{\lambda}$

A.5 $\dfrac{a_{11}}{a_{12}} \geqq \dfrac{L_y}{L_y^*}$

A.6 $\dfrac{\phi_2}{m_2} \geqq \dfrac{\Psi_1}{\mu}$

where we set the initial holdings of bonds equal to 1.

<u>2nd Case</u>: $\theta>0$

$\Delta<0$ if (A.2) - (A.6) are not true, their opposite holds and if

A'.6 $$\theta \geqq \frac{\phi_2}{(\varepsilon_2-\eta_2)}$$

<u>Proof of Proposition 1</u>

$$\frac{dq/q}{de/e} = \frac{1}{\Delta}\{-\phi_1^* A_{11} + \phi_2^* A_{13} - (\gamma^* + \pi^{e*})(1 - \eta^*)A_{14} \qquad (2)$$

<u>1st Case</u>: $\theta\leqq 0$, $(\gamma^* + \pi^{e*})(1 - \eta^*) \leqq 0$

(A.2) - (A.6) guarantee that $A_{11}>0$, $A_{13}<0$, $A_{14}<0$, $\Delta>0$

so that

$$\frac{dq/q}{de/e} < 0$$

<u>2nd Case</u>: $\theta>0$, $(\delta^* + \pi^{e*})(1 - \eta^*) > 0$

the opposite of (A.2) - (A.6) and A'.6 guarantee that $A_{11}<0$, $A_{14}>0$ and

A.8 $$\frac{\theta}{\lambda} \geqq \Psi_2$$

guarantees that $A_{13}<0$, so that $\frac{dq/q}{de/e} < 0$

Proof of Proposition 2

$$\frac{dp_1/p_1}{de/e} = \frac{1}{\Delta}\{a_{11}A'_{11} - (\varepsilon_2 - \eta_2)A'_{12} - L_yA'_{14}\} \tag{3}$$

$$A'_{11} = \{-(\gamma^* + \pi^{e*})(1 - \eta^*)(\varepsilon_2^* - \eta_2^*)m_2B + \phi_2^*m_2BL_y^*\}$$

$$A'_{12} = \{a_{12}\phi_2^*\lambda + \phi_1^*m_2^*B^*L_y^* + (\gamma^* + \pi^{e*})(1 - \eta^*)(\varepsilon_2^* - \eta_2^*)\mu -$$

$$- L_y^*\phi_2^*\mu + (\gamma^* + \pi^{e*})(1 - \eta^*)m_2^*Ba_{12} + \phi_1^*\lambda(\varepsilon_1^* - \eta_2^*)\}$$

$$A'_{14} = \{\phi_1^*m_2B(\varepsilon_2^* - \eta_2^*) + \phi_2^*m_2Ba_{12}\}$$

1st Case: $\theta \leqq 0$, $(\gamma^* + \pi^{e*})(1 - \eta^*) \leqq 0$

(A.2) - (A.6) allows to say $\frac{dp_1/p_1}{de/e} > 0$

2nd Case: $\Theta > 0$, $(\gamma^* + \pi^{e*})(1 - \eta^*) > 0$

the opposite of (A.2) - (A.6) guarantee the same result.

Proof of Proposition 3 to see if a devaluation increases the domestic price level more or less than proportionately it is necessary to compare the numerator and the denominator of (3). This proposition follows then by inspection of

$$[a_{11}(A_{11} - A'_{11}) - (\varepsilon_2 - \eta_2)(A_{12} - A'_{12}) - L_y(A_{14} - A'_{14})]$$

under alternative assumptions about values of θ and $(\gamma^* + \pi^{e*})(1 - \eta^*)$.

Proof of Proposition 4

$$\frac{di}{de/e} = \frac{1}{\Delta}\{a_{11}A''_1 - (\varepsilon_2 - \eta_2)A''_{12} - L_y A''_{14}\} \gtrless 0$$

$$A''_{11} = \{L_y^* \phi_2^* \phi_2 - (1 - \eta^*)(\gamma^* + \pi^{e*})\phi_2(\varepsilon_2^* - \eta_2^*)\}$$

$$A''_{12} = \{a_{12}\phi_2^*(\gamma^* + \pi^{e*})(1 - \eta^*) + \Psi_1\phi_2^* L_y^* - \theta(\varepsilon_2^* - \eta_2^*)\phi_1^* +$$
$$+ \phi_1^*\phi_2^* L_y^* - \phi_2^*\theta a_{12} - \Psi_1\lambda(\varepsilon_2^* - \eta_2^*)\}$$

$$A''_{14} = \{a_{12}\phi_2 \cdot \phi_2^* + \phi_1^*\phi_2(\varepsilon_2^* - \eta_2^*)\}$$

Part II

The matrix of coefficients of endogenous variables is now

$$\begin{bmatrix} a_{11} & a_{12} & \Psi_1 & \mu \\ \varepsilon_2 - \eta_2 & 0 & \phi_2 & -m_2 B \\ 0 & (\varepsilon_2^* - \eta_2^*) & \phi_2^* & -m_2^* B^* \\ 0 & L_y^* & (1-\eta^*) & L_i^* \end{bmatrix} \begin{bmatrix} dq/q \\ dq^*/q^* \\ dp_1/p_1 \\ di \end{bmatrix} = \begin{bmatrix} -1^* \\ 0 \\ \phi_2^* \\ (1-\eta^*) \end{bmatrix} \frac{de}{e}$$

The minors of the correspondent Jacobian are

$$A_{11} = \{-m_2^* B^* \phi_2 L_y^* - (1 - \eta^*)(\varepsilon_2^* - \eta_2^*) m_2 B +$$

$$+ m_2 B \phi_2^* L_y^* - \phi_2(\varepsilon_2^* - \eta_2^*) L_i^*\}$$

$$A_{12} = \{a_{12}\phi_2^* L_i^* - m_2^* B^* \Psi_1 L_y^* + (1 - \eta^*)(\varepsilon_2^* - \eta_2^*)\mu -$$

$$- \mu L_y^* \phi_2^* + (1 - \eta^*) m_2^* B^* a_{12} - \Psi_1(\varepsilon_2^* - \eta_2^* L_i^*\}$$

In this part of the appendix we will always assume: $(1 - \eta^*) \leqq 0$

it can be seen that an assumption parallel to (A.4) in the previous model

$$\text{A.4'} \qquad \frac{\varepsilon_2^* - \eta_2^*}{m_2^*} \geqq \frac{L_y^*}{L_i}$$

is enough to give a positive sign to the Jacobian

$$J = [a_{11}A_{11} - (\varepsilon_2 - \eta_2)A_{12}] > 0 \qquad \text{(II.1)}$$

<u>Proof of Proposition 6</u>

$$\frac{dq/q}{de/e} = \{-\phi_1^* A_{11} + \phi_2^* A_{13} - (1 - \eta^*) A_{14}\}\frac{1}{\Delta}$$

$$A_{13} = \{a_{12}\phi_2 L_i^* - \Psi_1 m_2 B L_y^* - L_y^* \phi_2 \mu + m_2 B(1 - \eta^*) a_{12}\}$$

$$A_{14} = \{-a_{12}\phi_2 m_2^* B^* - m_2 B \Psi_1(\varepsilon_2^* - \eta_2^*) - (\varepsilon_2^* - \eta_2^*)\phi_2 \mu + m_2 B \phi_2^* a_{12}\}$$

$A_{13}<0$, by (A.6) and $A_{14}<0$ by (A.6) and if

$$\frac{\phi_2}{m_2} > \frac{\phi_2^*}{m_2^*}$$

$$\frac{dq/q}{de/e} < 0$$

$$\frac{dq^*/q^*}{de/e} = \frac{1}{\Delta}\{a_{11}A'_{11} - (\varepsilon_2 - \eta_2)A'_{12}\}$$

$$A'_{11} = \{-\phi_2 m_2^* B^*(1 - \eta^*) - \phi_2^* - \phi_2\phi_2^* L_i\}$$

$$A'_{12} = \{-\phi_1^*\phi_2^* L_i - m_2^* B^* \Psi_1(1 - \eta^*) - \phi_1^* m_2^* B^*(1 - \eta^*) - \Psi_1 L_i \phi_2^*\}$$

In the case $(1 - \eta^*) \leqq 0$, $\frac{dq^*/q^*}{de/e} > 0$ unambiguously; in the other case the sign is uncertain.

$$\frac{dp_1/p_1}{de/e} = \frac{1}{\Delta}\{+a_{11}[-(1 - \eta^*)(\varepsilon_2^* - \eta_2^*)m_2 B + m_2 B\phi_2^* L_y^*]\} -$$

$$- (\varepsilon_2 - \eta_2)[L_i\phi_2^* a_{12} + \Psi_1^* m_2^* B^* L_y^* + (1 - \eta^*)(\varepsilon_2^* - \eta_2^*)\mu -$$

$$- \mu L_y^* \phi_2^* + m_2^* B^*(1 - \eta^*)a_{12} + \phi_1^*(\varepsilon_2^* - \eta_2^*)L_i]$$

if $\eta^* \geqq 1$, $\frac{dp_1/p_1}{de/e} > 0$ unambiguously.

if $\eta^* < 1$, sign is uncertain.

$$\frac{di}{de/e} = \frac{1}{\Delta}\{a_{11}(L_y\phi_2\phi_2^* - (\varepsilon_2^* - \eta_2^*)\phi_2(1 - \eta^*)) -$$

$$- (\varepsilon_2 - \eta_2)[a_{12} + \phi_2^*\Psi_1 L_y^* - \phi_1^*(\varepsilon_2^* - \eta_2^*)(1 - \eta^*) +$$

$$+ \phi_1^*\varphi_2^* L_y^* - \phi_2^* a_{12} - (\varepsilon_2^* - \eta_2^*)\Psi_1(1 - \eta^*)]$$

if $\eta^* > 1$, $\frac{di}{de/e} > 0$ unambiguously; otherwise the sign is uncertain.

by inspection of the unmerator and the denominator of $\frac{dp_1/p_1}{de/e}$, it can be seen that if (A.4') is true a devaluation will raise the domestic price level less than proportionately to the amount of the devaluation.

Footnotes

1. To exclude the possibility of unemployment or of trading out of equilibrium is indeed a major limitation of this type of analysis. Unfortunately as it is well known the consideration of more than one good in the presence of unempoloyment makes the analysis extremely difficult. For instance, within the present context of an open economy the key question would be "which would be the new allocation of excesses supply of goods and labor among different industries, once the devaluation is enacted?" Unless arbitrary allocation rules are assumed as it is done for example in Ch. XIV of [3] or in the Keynes-Wicksell monetary growth models [6], to answer this question seems at the present stage hopeless.

2. For "the monetarist model of the balance of payments" it is meant that body of economic literature which has its foundations in the works of Mundell [12, 13], Johnson [9], Dornbusch [4, 5], Negishi [14].

3. See Morishima [11] for a discussion of the concenpt of separability between consumption and savings decisions.

4.

5. A price index would have certainly been more appropriate than just choosing the price of one of the two goods as unit of measure. However, the degree of arbitrariness would have been reduced only insofar as there exists an unambiguous way of assessing the relative weight of the two goods in the representative basket. The choice of the price of the traded good has the satisfactory implication of assuming absence of money illusion following a devaluation and therefore seems appropriate for an "open" economy.

6. For the derivation of the flow demand for assets (12) see Foley [7].

7. See Dornbusch op. cit.

References

1. Alexander, S.S. "Effects of a Devaluation on a Trade Balance", International Monetary Fund, Staff Papers (April 1952), 263-278.

2. Alexander, S.S. "Effects of a Devaluation: A Simplified Synthesis of Elasticities and Absorption Approaches", American Economic Review (March 1959).

3. Arrow, K.J. and Hahn, F.H. General Competitive Analysis, San Francisco, Holden Day, 1971.

4. Dornbusch, R. "Currency Depreciation, Hoarding and Relative Prices", Journal of Political Economy (July 1973), 110-125.

5. Dornbusch, R. "Devaluation, Money, and Non-Traded Goods", American Economic Review (December 1973), 871-880.

6. Fischer, S. "Keynes-Wicksell and Neoclassical Models of Money and Growth", American Economic Review (December 1972), 880-890.

7. Foley, D.K. "On Two Specifications of Asset Equilibrium in Macro-economic Models", Journal of Political Economy (April 1975), 303-324.

8. Hahn, F.H. "The Balance of Payments in a Monetary Economy", Review of Economic Studies (February 1959), 110-125.

9. Johnson, H.G. Further Essays in Monetary Economics , Cambridge, Mass., Harvard University Press, 1973.

10. Johnson, H.G. "Towards a General Theory of a Balance of Payments", in International Finance ed. by R.N. Cooper, Baltimore, Maryland, Penquin Books, Inc., 1969.

11. Morishima, M. "Consumer Behavior and Liquidity Preference," Econometrica (April 1952), 223-246.

12. Mundell, R.A. International Economics, New York, MacMillan, 1968.

13. Mundell, R.A. Monetary Theory, Pacific Palisades, 1971.

14. Negishi, T. General Equilibrium Theory and International Trade, Amsterdam, North-Holland, 1972.

Short run Stabilization Policy and Long run Economic Plans*

by Mario Draghi
Massachusetts Institute of Technology
February 1975

The optimal growth literature is primarily concerned [5,6][1] with a description of an economy in which the planner chooses certain levels of consumption per man or certain capital-labor ratios, or a certain distribution in the ownership of private and public capital,[2] so as to maximize a utility function defined in the above arguments or in any of their combinations. When an optimal plan exists, it can be achieved either by brute force, choosing exogenously one of the control variables [1,10] or through the use of some policy instruments under the planner's control [2].

The underlying models are more or less elaborate versions of the Solow 1956 growth model [12]. As such, all the results derived by this type of model hold in the long run, regardless of the horizon chosen by

---

[1] These references obviously do not intend to be exhaustive of the whole literature.

[2] All the following arguments will be based on the assumption of a single existing capital good. The results obtained in such a way do not depend at all on the theory of distribution that may be derived from models making this assumption.

---

* I am grateful to Professor R.M.Solow for lenghty discussions and helpful comments. I also wish to thank P.A.Samuelson, Glenn Loury and other members of the M.I.T. Advanced Economic Theory Workshop for their suggestions. I will obviously retain resposibility for the remaining errors.

the planner. The so-called "short run" aspect of the economy characterized by non market clearing conditions and by variable unemployment rates is not discussed by this literature. As a consequence policies that are optimal in the long run may not be optimal for short run stabilization purposes. Two explanations may tentatively be offered for this lack of practical usefulness of timal growth models. One is that there may be an implicit assumption that the political environment where the planner acts resembles more closely a stalinist economy where a private sector following a keynesian type of short run behavior just does not exist. An alternative implicit assumption might be that the solutions originated by these models should serve as "benchmarks" for the short run stabilization policy models.

But if this were the case they should be used as such in this last type of model. Instead policy simulations in the largest econometric models [ ] are generally studied under the assumption that the full employment-full capacity utilization--however defined--level of GNP is either a constant or a known exponential function of time, in both cases exogenously determined without any consideration of the preferences of the policy maker.

If this suggestion were followed we would discover that there are constraints on the policy variables such that either the long run or the short run problem as they have been traditionally formulated may not have a solution. For example, the tax rate necessary to achieve a certain level of consumption per man at a given number of years from now may prove to be completely destabilizing in the next six months.

The purpose of this paper is to offer a qualitative description of the trade off ---when it exists--between policies that are optimal in the short run but not in the long run and vice versa. The reason for this is both practical and theoretical. What is called in common language the trade-off between reforms and stabilization policies has always been an open problem especially in dual economies. What usually happens in this type of economies is that policies aimed at growth of the backward sector are sacrificed--for instance because of a negative reaction of private investments--in favor of day-to-day stabilization of total aggregate demand. The usual argument being that growth in the poor part of the country cannot be promoted starting from an unemployment situation in the developed part of the country. The theoretical reason refers to a remark by Koopmans [6] noting how some assumptions crucial for the existence of a solution in the optimal growth problem--the choice of the planner's preference ordering--were unverifyable and somehow empty of an intuitive meaning. The simultaneous consideration of the short and the long run problem will show that the conditions usually assumed on the Central Planning Board's (C.P.B.) utility function, in order to guarantee existence and uniqueness of the solution, have indeed economic meaning.

The outline of the paper is the following: in the first section an example considered representative of the current literature on optimal stabilization policies will be stated; in the second and the third section an alternative formulation will be suggested. The two final sections are devoted to a discussion of what a reformist planner could do when facing a short run stabilization problem.

1. The short run economy

The optimal stabilization policy literature is usually concerned [11,13] with the design of an optimal control of the feedback type, keeping actual aggregate demand "close enough" to some predetermined level of full employment GNP.

Consider the classical Phillips stabilization policy problem [7,8]:

(1) $D(t) = cy(t) + I(t) + g(t)$

(2) $I(t) = \alpha\dot{y}(t) - k\dot{I}(t)$

(3) $\dot{y}(t) = r[D(t) - y(t)]$

Eliminating D(t) and I(t)

(4) $\ddot{y}(t) + b_1\dot{y}(t) + b_2 y(t) = r\dot{g}(t) + \frac{r}{k}g(t)$

$$b_1 = r(1-c) + \frac{1}{k} - \frac{\alpha r}{k}$$

$$b_2 = \frac{r(1-c)}{k}$$

where D(t) is aggregate demand, I investment, g government expenditure, y GNP. All magnitudes are in real terms, price level will be considered since now on a constant.[4]

The state representation of (4) is

---

[4] Money supply will therefore not be explicitly considered. There is only one type of ~~long~~ short run interest bearing bond B^g describing government debt. A tax rate can easily be included--and this will be done at a later stage--in (1) through (3), provided it is by now treated as a constant.

$$x_1(t) = y(t)$$

$$x_2(t) = \dot{y} - rg(t)$$

$$\dot{x}(t) = Ax(t) + Bu(t) \qquad (5)$$

$$x(t) = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} ; \quad A = \begin{bmatrix} 0 & 1 \\ -b_2 & -b_1 \end{bmatrix} ; \quad B = \begin{bmatrix} r \\ r(\frac{1}{k} - b_1) \end{bmatrix}$$

The following assumptions will be made:

A.1 A and B are time invariant

A.2 The representation (5) is completely controllable, i.e.,

$$\text{rank } [B \vdots AB] = 2$$

A.3 The representation (5) is completely observable

A.4 (5) is zero state bounded input, bounded output stable.[5]

What is called the "output function" of this system is not of particular interest from an economic point of view, because no attempt is made in these models of distinguishing between changes in aggregate demand and changes in supply. Anyway, in view of A.3 and A.4, substituting (2) into (1)

$$D(t) = cy(t) + \alpha\dot{y}(t) = k\dot{I}(t) + y(t) \qquad (1')$$

$$\dot{y}(t) = r[cy(t) + \alpha\dot{y}(t) - k\dot{I}(t) + y(t) - y(t)] \qquad (3')$$

$$D(t) = \dot{y}(t)\,\frac{1}{r} + y(t)$$

---

[5] For a discussion of existence, uniqueness and stability of the optimal policies in this type of problem see [3].

is the output function or in state form

$$D(t) = x_2(t)\,\frac{1}{r} + u(t) + x_1(t) \tag{6}$$

when the state is driven to zero by the control g(t), $D(t) = x_2(t)$.

Following the usual practice, let's redefine all the variables of (5) as deviations from some predetermined equilibrium values, considered constant through time.

$$\tilde{x}_i(t) = x_i(t) - \bar{x}_i$$

$$\tilde{g}(t) = g(t) - \bar{g} \tag{7}$$

The planner in the short run, minimizes

$$J = \frac{1}{2}\,[x'(T)Fx(T)] + \frac{1}{2}\int_{t_0}^{T} [x'(t)Qx(t) + g(t)'Rg(t)]\,dt \tag{8}$$

Q and F are 2 x 2 positive semidefinite matrices.

R is a strictly positive scalar matrix.

The optimal control is

$$g(t) = -R^{-1}B'K(t)x(t) \tag{9}$$

The matrix Riccati equation is

$$\dot{K}(t) = -K(t)A - A'K(t) + K(t)BR^{-1}B'K(t) - Q \tag{10}$$

with boundary condition

$$K(T) = F$$

The optimal trajectory of the state is

$$\dot{x}(t) = [A - BR^{-1}B'K(t)]\, x(t) \tag{11}$$

with the boundary condition

$$x(t_o) = \xi_o$$

Solutions of (11) will give the optimal trajectory of D(t) in (6).

What makes this problem, the way it has been discussed, not terribly interesting from an economic point of view, are the following facts:

(a) The business cycle described in equations (1) through (3) is of a linear fashion and is deterministic: once the state is driven to zero there is nothing that creates new cycles such as those we observe in reality;

(b) The control g(t) is treated as if it were unconstrained. In fact it obeys the government budget constraint.

$$g(t) = T(t) + \dot{B}^g(t) - rB^g \tag{5'}$$

where the notation is the usual one. If T, $\dot{B}^g$ are already determined by outside considerations it is quite possible that an optimal control does not exist.

(c) The method of redefining the state variables as deviations from some constant <u>a priori</u> determined desired value can be justified only if those equilibrium levels do not change through time, are not affected by the control and do not affect the present value of the control. This is in a certain sense the essence of what is called in the control literature "the state regulator problem." As we shall see if we allow those desired values to change through time, the structure of the problem becomes sensibly

$$\tilde{Z}^d = (1-x)(\tilde{Z} + iB^g) \tag{25}$$

$$x^* = x(\tilde{Z} + iB^g) \tag{26}$$

$$c_p = (1-s)Z^d \tag{27}$$

$$c_g = g \tag{28}$$

$$x^* + \dot{B}^g + nB^g = g + iB^g \tag{29}$$

$$\tilde{Z}^d - c_p - c_g = \dot{k} + nk - x^* + \underline{r}B^g \tag{30}$$

where $\tilde{Z}^d$ is disposable income, x the income tax rate, x^* total tax collection, c_p private consumption, s the constant marginal propensity to save, c_g is social consumption, $\dot{B}^g$ is the change per unit of time of the stock of government bonds. Time subscripts have been suppressed for the sake of clarity and all magnitudes are measured in per capita terms. Finally notice that (29) and (30) together imply portfolio equilibrium for the private sector and that this with the assumption of perfect capital markets and of perfect substitutability of bonds with capital implies

$$f_k(\cdot) = i \tag{31}$$

The planner is assumed to

$$\max_{\{c_g, x\}} \int_0^\infty e^{-\lambda t} u[c_p(t), c_g(t)]dt \tag{32}$$

u(·) strictly concave

subject to (29), (30) and to the transversality conditions

$$\lim_{t\to\infty} e^{-\lambda t} p(t) \geqq 0$$

$$\lim_{t\to\infty} e^{-\lambda t} g(t) \geqq 0 \qquad (32')$$

$$\lim_{t\to\infty} e^{-\lambda t} p(t)k(t) = 0$$

$$\lim_{t\to\infty} e^{-\lambda t} g(t)B^g(t) = 0$$

The current value Hamiltonian is

$$H(\cdot) = u(c_p, c_g) + p[s\tilde{Z}^d + x(\tilde{Z} + iB^g) - iB^g - g - nk] +$$

$$q[g + iB^g - nB^g - x(\tilde{Z} + iB^g)] \qquad (33)$$

where p and g are the auxiliary variables corresponding to the transition equations (29) and (30).

The optimal tax rate and government expenditure are determined by[8]

$$\frac{\partial H}{\partial x} = 0 = -(1-s)u_1(\ ) + p(1-s) - q$$

$$u_1[(1-s)(1-x)(\tilde{Z} + iB^g)g] = p - \frac{q}{(1-s)} \qquad (34)$$

$$\frac{\partial H}{\partial c_g} = 0 = u_2 - p + q$$

$$u_2[(1-s)(1-x)(\tilde{Z} + iB^g), c_g] = p - q \qquad (35)$$

---

[8]Concavity of $H^*(\cdot) = \max H(\cdot)$ in the state variables with the tranversality conditions (32') have to be satisfied in order for an optimal to exist. Concavity of $u(\cdot)$ and $f(\cdot)$ implies concavity of $H(\cdot)$ and it can be seen that this implies concavity of $H^*(\cdot)$ in the state variables.

The movement along the optimal path is governed by

$$\dot{p} = (\lambda+n - f'(k))p + q(f'(k) - f''(k)B^g) \tag{36}$$

$$\dot{g} = (\lambda+n)g \tag{37}$$

and by the differential equations (29) and (30).

The optimal tax rate and the optimal amount of government expenditure are obtained solving (34) and (35) in x and c_g. Uniqueness of the solutions in x and g is warranted by the strict concavity assumption. So that we have a simultaneous system of 2 static equations (34) and (35), and 4 autonomous differential equations (29), (30), (36), and (37) in 6 variables $x(t)$, $c_g(t)$, $p(t)$, $g(t)$, $k(t)$, $B^g(t)$. Solving this system the planner will now obtain the optimal time paths of the instruments as functions at each instant of time of the state and costate variables and he will derive the optimal time path of the GNP, to be used in (15) for short run stabilization purposes.

Before proceeding with the analysis of the optimal growth policy it is necessary to establish two properties of the nonlinear system (25) through (30):

p.1 The system (25) through (30) is controllable. Since the planner has three instruments, equations (29) through (30) can be solved for each pair of the three. In particular once the optimal income tax rate and the desired amount of social consumption are chosen, they will determine the change per unit of time in the stock of government bonds. Therefore, the optimal policy is always controllable.

p.2 The system (25) through (30) is controllable with stable instruments deriving from (29) and (30) the portfolio equilibrium condition for the whole economy and solving for B^g

$$\dot{B}^g = \frac{s}{1-s} c_p - (\dot{k} + nk) - nB^g \tag{38}$$

If the optimal policy is such that c_p and k converge to some finite limits and $\dot{k}$ converges to zero--and the optimal policy satisfies this condition [9] the first two terms of the right-hand side converge to a finite limit and since $n > 0$ any solution is stable from the instruments point of view.

4. The tying up

The two problems--of motion along the optimal path, and recovery from unemployment-- have been treated quite separately until now, their only link being the tracking equation (15). If this were really the case the C.P.B. could choose at time 0 a series of values of g(t) that at each point in time minimize (13) and stick with this policy for whatever is the horizon taken into consideration. In this section it is shown that this is not the case.

At time 0 the C.P.B. faces the following situation: there is a certain ratio of utilized capacity to employed labor force $\tilde{k}_0$; then there is a certain potential full employment capital-labor ratio k_0. The C.P.B. on the basis of k_0 computes the optimal long run path that maximizes (32), the optimal state at time 0 Z(0), and the policies that would be optimal if the system were at full employment. Then solving the Riccati equation and the tracking equation the C.P.B. finds the optimal g(0). As a by-product of this computation, from the two budget constraints (5') and (29) the C.P.B. obtains the additional information of how much the rate of change of B^g differs from the rate of change that there would be along the optimal path.

Now comes the question of actual implementation: if the C.P.B. implements the policy optimal for problem (13), it will bring the system close to some full employment-full capacity utilization state at that point in time, but it will never achieve the combination of social and private consumption prevailing on the optimal path. In fact, implementation of g(t) will, among other things, stimulate net private investments and will produce a new initial condition, say $k^*(1)$, that will differ from the k(1) belonging to the previous optimal path because this was achievable only through implementation of the policies optimal for problem (32).[9] This implies a new round of computations of the new desired state $Z^*(t+1)$ and through (15) of the new value of the optimal control $g^*(t+1)$. But as a practical implication this means that there will always be a problem of stabilization and there will never be room for actually achieving the desired combination of private and social consumption.

On the other hand, if instead the C.P.B. implements the policies optimal from a long run point of view in the sense that their computation assumes an economy at full employment and full capacity utilization, two possible outcomes are possible. The first is that this will destabilize the economy described by equation (5), producing lower and lower levels of utilization of capital and labor and therefore departing further and further from the optimal path. The other is the opposite outcome: at some

---

[9] Obviously k(1) cannot be reached by any other policy by definition of optimality.

point in time--obviously after T, the terminal time of problem (13)--the time path (5) is not only stable but it will converge to the optimal path. In fact, if the C.P.B. implements the policies optimal for (32), a situation in which the economy is in full employment but not on the turnpike is impossible.

To answer this question, first the optimal policies for (32) have to be computed.

Totally differentiating equations (34) and (35)

$$\begin{bmatrix} -u_{11}Z_p\cdot\beta & u_{12} \\ -u_{21}Z_p\cdot\beta & u_{22} \end{bmatrix} \begin{bmatrix} dx \\ dg \end{bmatrix} = \begin{bmatrix} -u_{11}\alpha\beta & -u_{11}\beta f'(k) & 1 & -\frac{1}{1-s} \\ -u_{21}\alpha\beta & -u_{21}\beta f'(k) & 1 & -1 \end{bmatrix} \begin{bmatrix} dk \\ dB^g \\ dp \\ dg \end{bmatrix}$$

$$\Delta = (-u_{11}\cdot u_{22} + u_{12}u_{21})Z_p\beta < 0$$

because by assumption $u(c_p, c_g)$ is strictly concave

$$\beta \equiv (1-x)(1-x) > 0$$

$$\alpha \equiv f'(k) + f''(k)B^g \gtrless 0$$

and solving for the optimal instruments

$$\frac{dx}{dk} = \frac{(-u_{11}u_{22} + u_{12}u_{21})\alpha\cdot\beta}{\Delta} = \alpha\cdot\beta \qquad (39)$$

$$\frac{dx}{dB^g} = \frac{(\qquad\cdot\qquad)}{\Delta} = \beta f'(k) > 0 \qquad (40)$$

$$\frac{dx}{dp} = \frac{u_{22} - u_{12}}{\Delta} \tag{41}$$

$$\frac{dx}{dq} = \frac{-u_{22}/(1-s) + u_{12}}{\Delta} \tag{42}$$

$$\frac{dg}{dk} = \frac{(u_{11}u_{21} - u_{11}u_{21})Z_p \cdot \beta^2 \alpha}{\Delta} \tag{43}$$

$$\frac{dg}{dB^g} = 0 \tag{44}$$

$$\frac{dg}{dp} = \frac{(-u_{11} + u_{21})Z_p \cdot \beta}{\Delta} \tag{45}$$

$$\frac{dg}{dq} = \frac{(u_{11} - u_{21}/(1-s))Z_p \cdot \beta}{\Delta} \tag{46}$$

If,(i), Public and private consumption are complementary everywhere and, (ii), increases in the capital labor ratio do not increase interest payments on the oustanding per capita stock of government debt more than they increase per capita output, i.e., $\alpha > 0$, then

$$\begin{aligned} & x^*(t) = x^*(k(t),\ p(t),\ q(t),\ B^g(t)) \\ & g^*(t) = g^*(p(t),\ q(t)) \\ & x^*_1 > 0,\ x^*_2 > 0,\ x^*_3 < 0,\ x^*_4 > 0 \\ & q^*_1 < 0,\ g^*_2 > 0 \end{aligned} \tag{47}$$

The sign of these derivatives--under the assumption of sufficiently accurate local approximation--is of importance for asserting the stability of the short run economy when the optimal policies are actually implemented.

The next step is to modify the short run model in order to take explicitly into account a tax rate and a government expenditure varying through time. Furthermore we have to allow for the introduction of interest bearing debt. Consequently

$$y_p = y + iB^g$$

$$y_d = (1-x)y_p$$

where y_p and y_d are the actual personal and disposable income. Then, since the optimal policies (47) are expressed in per capita terms, it is necessary to normalize by the same terms the short run model

$$\bar{y}_p = y_p \cdot e^{-nt}, \quad \bar{B}^g = B^g \cdot e^{-nt}, \quad \bar{I} = e^{-nt} I$$

Consequently we have the following definitions

$$\dot{\bar{y}}^d = (1-x)(\dot{\bar{y}}_p + n\bar{y}_p) - \dot{x}\bar{y}_p$$

The short run model can be rewritten as

$$\overline{D} = c\overline{y}^{d} + \overline{I} + \overline{g} \qquad (1')$$

$$\overline{I} = \alpha(\dot{\overline{y}}^{d} + n\overline{y}^{d}) - \beta(\dot{\overline{I}} + n\overline{I}) \qquad (2')$$

$$\dot{\overline{y}} = n\overline{y} = r(\overline{D} - \overline{y}^{d}) \qquad (3')$$

$$x^{*}\overline{y}_p + \dot{\overline{B}}^{g} + n\overline{B}^{g} = g^{*} + i\overline{B}^{g} \qquad (5'')$$

where x^* and g^* are now the optimal policies as derived in (47). The question we want to ask is: What happens to the system (1') through (5") when policies $\{x^*, g^*\}$ are actually implemented? Equations (1') through (3') can be reduced to

$$\ddot{\overline{y}} + \overline{b}_1\dot{\overline{y}} + \overline{b}_2(t)\dot{\overline{y}}_p + (\dot{\overline{b}}_3(t) + \overline{b}_4(t))\overline{y}_p = \frac{r}{\beta}\, g(p(t),q(t)) + \dot{g}(p(t),q(t)) \qquad (6')$$

$$\overline{b}_1 \equiv r(2n + \frac{1}{r\beta})$$

$$\overline{b}_2(t) \equiv (1 - c - \frac{\alpha}{\beta})(1-x^{*}(k(t),p(t),q(t),B^{g}(t))$$

$$\dot{\overline{b}}_3(t) \equiv -(1 - c - \frac{\alpha}{\beta})\dot{x}(\cdot)$$

$$\overline{b}_4(t) \equiv r[n(1-x)(1 - c - \frac{\alpha}{\beta}) + \frac{n}{r\beta} + \frac{(1-c)}{\beta}(1-x)]$$

The following features of eqn.(6') are worth being noticed:

a) Since the optimal policies have the characteristic that as time goes to infinity they converge to finite limits, $\dot{g}(\cdot)$, $\dot{x}^{*}(\cdot)$, and therefore $\dot{\overline{b}}$, will converge to 0--being eqns. (29), (30), (36), (37) and (6') a system of autonomous differential equations [4] --- and $x^*(\cdot)$ and $g(\cdot)$ will converge to finite limits. The other coefficients are still positive. Therefore introduction of policies $\{x^*(\cdot), g^*(\cdot)\}$ optimal for problem (32) but sub-optimal for problem (13) did not affect the stability of the time path of actual per capita output y(t).

b) However, just because $\{x^*(\cdot) g^*(\cdot)\}$ are non-optimal from the stabilization point of view, it is not obvious that they should eliminate even after T--the short run planning horizon--the original decrease in aggregate demand that started the business cycle described by (6"). If this happens at the steady state[10], not only would savings equal investments at a level below full employment, but the desired composition of social and private consumption would not be achieved, being policies $\{x^*(\cdot), g^*(\cdot)\}$ optimal only conditionally to a state of full employment. Then the stationary solution of (6") becomes of interest

$$[n^2 \bar{y}^{\infty} + n\bar{y}^{d\infty}(1 - c - \frac{\alpha}{\beta}) + \frac{n\bar{y}^{\infty}}{r\beta} + \frac{(1-c)\bar{y}^{d\infty}}{\beta}] = \frac{r}{\beta} g^{\infty} + ng^{\infty} \qquad (48)$$

where the symbol ∞ denotes the limit value. Are there conditions that make (48) equal to 0 and such that they are automatically satisfied on the long run optimal path? It can be seen[11] that (48) can be rewritten as

---

[10] It is clear that (6") cannot have a stationary state solution at a date other than that at which the system (29) through (37) has a steady state solution too.

[11] See Appendix Note II.

$$s\overline{y}^{d\infty} = n(\overline{k}^{\infty} + \overline{B}^{g\infty}) \qquad (48')$$

The pair of policies $\{x^*(\cdot), g^*(\cdot)\}$ optimal for problem (32) is not destabilizing in the short run and fully eliminates the initial decrease in aggregate demand. Moreover, differently from the alternative strategy described in section (2) , the economy once stabilized is automatically on the turnpike. In fact (48') satisfies the necessary and sufficient conditions for this to happen:

1) It is always verified at the steady state so that (48) is satisfied at $\overline{y}^{d\infty} = 0$, which implies full employment-full capacity utilization;

2) This is the result not of any policy but of the implementation of the pair $\{x^*(\cdot), g^*(\cdot)\}$ optimal for (32);

3) (48') is the condition of portfolio equilibrium and as such is always satisfied on the optimal path.

How will private investment and government debt behave on the transition path from unemployment to the full employment turnpike? Comparing the government budget constraints of the two economies (5") and (30) we see that since the planner adopts policies $\{x^*(\cdot), g^*(\cdot)\}$ on the short un path, the tax rate and government expenditure will be the same; on the other hand, $\overline{y} < z$ by definition, therefore

$$\overline{\dot{B}}^{g} + (n-i)\overline{B}^{g} > \dot{B}^{g} + (n-i)B^{g} \qquad (49)$$

on the transition path government debt will grow at a rate faster than that on the optimal path for any given initial level of per capita stock of government debt.

Then writing an expression similar to (38') for the short run economy and subtracting it from (38')[12]

$$(\dot{B}^g - \dot{\overline{B}}^g) + n(B^g - \overline{B}^g) + (\dot{k} - \dot{\overline{k}}) + n(k - \overline{k}) = \frac{s}{1-s}(c_p - \overline{c}_p) + \overline{u} \tag{50}$$

Since private consumption is an increasing function of disposable income it will be smaller on the transition path than on the optimal path. This together with (49) and (50) will imply that for any given initial level of the capital stock and stock of government bonds, private investments will be lower on the path approaching the turnpike than on the turnpike itself.

---

[12]See Appendix Note III.

5. Conclusions

Stabilization policies are generally defined as a set of actions taken by the government in order to achieve certain desired values. In a one period world the description of how these policies work is accomplished by the static IS-LM model. But when this description is abandoned and the dynamic behavior of the economy becomes of interest, the horizon chosen, the optimality criteria need to be redefined and consequently the problem has to be reformulated. As far as concerns the horizon to choose, a short run economy is defined for descriptive purposes: where the short run as distinct from the static momentary equilibrium, is characterized by variable unemployment and capacity utilization rates, and by adjustment lags, such that they would produce dynamic behavior of a business cycle type.

As an introductory exercise the problem of finding optimal stabilization policies in this linear model is discussed. This formulation is found unsatisfactory for several reasons that can be summarized by saying that it is not justified to specify some kind of dynamic behavior for one variable of the model without doing the same for the other variables. In particular, failure to recognize that government deficits and net private investment are identically equal to changes in the stock of government bonds and capital is important because neglecting all types of links with what happens after the "short run" horizon of T years, any stabilization policy is given a patent of myopia.

Then in order to take care of this deficiency a "long run" economy characterized by continuous full employment, perfectly competitive markets, instantaneous adjustments, is defined so that the target of the short run stabilization policy is the output that would be produced if the economy were in this ideal situation. But there are many full employment-full capacity utilization paths that the planner can choose and around which stabilizes the short run economy. Each of these paths, among other things, describes different combinations of investment and consumption, and within the second of social and private consumption. A path optimal in the sense of maximizing (32) is chosen; in order to be able to drive the economy to this optimal path the C.P.B. has to implement certain policies that are assumed to be different from those required to stabilize aggregate demand in the short run economy. The C.P.B. then faces a choice: either to use an undistinguished aggregate defined government expenditure to stabilize the economy and delaying desired "reforms" when full employment is achieved, or to adopt immediately a set of policies having the characteristic of being more selective in the sense of producing the desired combination of private and social consumption--what has been loosely defined as "reforms."

It is shown that if the first possibility is followed, the problem of short run stabilization will always be present and "reforms" will never be accomplished. On the other hand if the second alternative is followed, implementation of the long run policies will not result in a short run explosive behavior and will actually produce a long run state optimal not only in the sense of being at full employment, but of being there having accomplished the necessary "reforms."

The final question concerns the plausibility of our previous assumptions concerning the planner's behavior. Why should short run and long run policies be different? Why was the C.P.B. constrained to choose between short run stabilization and long run reforms? Why not achieve them at the same time?

To achieve simultaneously both targets the C.P.B. has to know present and future response of private investments to policy actions; in particular, since the situation considered is one of unemployment, at the initial time he has to know how current and future excess supplies will be allocated between private consumption and private net capital formation. On the other hand, if the C.P.B. follows the procedure outlined in section (4) of the paper, the only necessary information is the initial full employment, full capacity utilization capital-labor ratio, on whose basis optimal policies are computed. Whatever are the decisions taken by the private sector they will be constrained by these policies to be consistent with both full employment and desired reforms. Surely there will be no guarantee that these policies will minimize the frequency and/or the amplitude of short run oscillations, but they will always be successful in driving the system to full employment on the optimal path.

These assumptions reflect some rational behavior in the sense of minimizing the informational requirement needed by the planner but of further importance they seem to describe current behavior in many countries.

Appendix

Note I:

$$Z_p = \tilde{Z} + iB^g \qquad (I.1)$$

$$\tilde{Z} = c_p + c_g + \dot{k} + nk \qquad (I.2)$$

$$\tilde{Z} = Z_p - iB^g = Z^d + xZ_p - iB^g = Z^d + x^* - iB^g \qquad (I.3)$$

where Z_p is per capita full employment personal GNP. Combination of (I.2) and (I.3) gives the flow budget constraint (30).

Note II:

Consider the term $\frac{ny^{\infty}}{\beta}$ in (48). Making use of the same transformation as in Note I and evaluating (1') and (5") at the stationary state

$$\frac{ny^{\infty}}{\beta} = r\,\frac{(\bar{D}^{\infty} - \bar{y}^{d\infty})}{\beta} = r\,\frac{(-s\bar{y}^{d\infty} + \bar{I} + g^{\infty})}{\beta} = r\,\frac{(g^{\infty} - n\bar{B}^{g\infty})}{\beta}$$

Consider the other terms in the square brackets of (48'), they can be shown to be equal to

$$n^2 y^{\infty} = -n^2 B^{g\infty} + ng^{\infty}$$

$$(\bar{I}^{\infty} + \beta n^{\infty}\bar{I}^{\infty}) = \alpha n y^{d\infty}$$

$$\frac{y^{d\infty}}{\beta} - \frac{cy^{d\infty}}{\beta} = \frac{s}{\beta} y^{d\infty}$$

$$ny^{d\infty} - cny^{d\infty} = nsy^{d\infty}$$

Then recognizing the fact that investment is identically equal to changes in the capital stock and evaluating these changes at the steady state

$$\alpha ny^{d\infty} = n\bar{k}^{\infty} + \beta n^{2}\bar{k}^{\infty}$$

Then multiplying (48') by β, dividing by r and eliminating offsetting terms

$$s\bar{y}^{d\infty} + \beta ns\bar{y}^{d\infty} = n(\bar{k}^{\infty} + B^{g\infty}) + \beta n^{2}(\bar{k}^{\infty} + \bar{B}^{g\infty}) \qquad \text{(II.1)}$$

But differentiating twice with respect to time the portfolio equilibrium condition III.3 and evaluating it at the steady state, it can be seen that

$$\beta nsy^{d\infty} = \beta n^{2}(\bar{k}^{\infty} + B^{g\infty})$$

and expression (48") is obtained.

Note III:

$$\bar{D} - \bar{u} = \bar{y} \qquad \text{(III.1)}$$

where $\bar{u}$ is an exogenously given constant. Then using I.3

$$\bar{y} = \bar{y}^{d} + x^{*} - iB^{g}$$

$$D = y^{d} + x^{*} - iB^{g} - \bar{u}$$

Substituting this expression into (1') and remembering that investment is identically equal to changes in the capital stock

$$y^d - c\overline{y}^d - g^* = \dot{\overline{k}} + nk - x^* + i\overline{B}^g + u \qquad \text{(III.2)}$$

III.2 with the government budget constraint (5") and the definition of consumption will give

$$\dot{\overline{B}}^g + n\overline{B}^g = \frac{s}{(1-s)}\overline{c}_p - (\dot{\overline{k}} + nk) - u \qquad \text{(III.3)}$$

then subtracting this from (38')

$$(\dot{B}^g - \dot{\overline{B}}^g) + n(B^g - \overline{B}^g) + (\dot{k} - \dot{\overline{k}}) + n(k - \overline{k}) = \frac{s}{1-s}(c_p - \overline{c}_p) + u$$

which is expression (50).

Bibliography

Arrow, K.J., "Applications of Control Theory to Economic Growth," in Mathematics of the Decision Sciences, Part II, (American Mathematical Society, Providence, R.I., 1969)

Arrow, K.J. and Kuth, M., "Optimal Public Investment Policy and Controllability with Fixed Private Savings Ratio," Journal of Economic Theory 1 (1969)

Athans, M. and Falb, P.L., Optimal Control: An Introduction to the Theory and its Applications, (McGraw-Hill, New York, 1966)

Bellman, R., Stability Theory of Differential Equations, (McGraw-Hill, New York, Toronto, and London, 1953)

Chakravarty, S., Capital and Development Planning (M.I.T. Press, Cambridge, 1969)

Koopmans, T.C., "Objectives, Constraints, and Outcomes in Optimal Growth Models," Econometrica 35 (January, 1967)

Phillips, A.W., "Stabilization Policy in a Closed Economy," Economic Journal 64 (June, 1957)

Phillips, A.W., "Stabilization Policy and the Time Form of Lagged Responses," Economic Journal 67 (June, 1957)

Pontryagin, L.S., et al., The Mathematical Theory of Optimal Processes, (Interscience, New York, 1962)

Ramsey, F.P., "A Mathematical Theory of Savings," Economic Journal 38 (December, 1928)

Sengupta, J.K., "Optimal Stabilization Policy with a Quadratic Criterion Function," Review of Economic Studies 37 (January, 1970)

Solow, R.M., "A Contribution to the Theory of Economic Growth," Quarterly Journal of Economics 70 (1956)

Turnovsky, S.J., "Optimal Stabilization Policies for Deterministic and Stochastic Linear Economic Systems," Review of Economic Studies 40 (January, 1973)

Econometric models