Design and Fabrication of Active Tilt-Stage for Laser Rangefinder

by

Hector C. Vargas

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ABSTRACT

A design for an active tilt-stage that will carry the UTM-30LX, a laser rangefinder sensor (LRF) that is fabricated to scan in a 2-D plane, was determined by analysis. The LRF will be mounted with the tilt-stage on the PIONEER 3-AT (P3-AT), a robot designed to be mobile in rough-terrains, such that autonomous navigation will be made possible. In particular, the purpose of the active tilt-stage is to have the laser rangefinder’s scan function expand from 2-D to 3-D by rotating it on the tilt-stage with a servomotor. The main functional requirement for the LRF’s new function is for the new 3-D scan to be able to provide good enough resolution such that the failure to detect crucial obstacles that can hinder the movement of the robot is avoided.

The analysis was done for worst scenarios in order to determine a good design for the active tilt-stage system that will allow the LRF to perform viably even under these conditions. It turns out that the higher the angular velocity at which the LRF is rotated at by the servomotor is, the worse the resolution becomes. Also, the higher the velocity at which the P3-AT moves, the worse the resolution becomes. The tilt-stage system was designed such that to allow a large potential range of angular velocities that the LRF could be rotated at. Ultimately, this gives the programmer of the LRF/servomotor/P3-AT many options in how the robot should behave under different situations. The main constraints on the physical design of the active-tilt stage are that it should not block the view of the LRF and that a motor that can rotate the LRF at desired angular velocities be selected. Dynamixel’s DX-117 motor has been chosen for its sturdiness and for the option to actively control its position and angular velocity. The software portion of the project will be left for a future student who can hopefully find the enclosed analysis useful.

Thesis Supervisor: Karl Iagnemma
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Acknowledgements

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1. Introduction and Motivation

1.1 Introduction

Robotic systems that can be set to function autonomously have come to have many uses today with the many developments in technology in the 20th and 21st century (Shelton). Nowadays, robots are being developed to help in areas like surgery, space exploration, manufacturing, military, and rescue missions. In particular, robots with mobility system are usually driven by onboard computers that allow them to have a sense of autonomy (Asada). Moreover, these autonomous robots usually mount sensors to perceive the environment around the robot. This interaction with the environment is necessary in order for the robot to determine its next appropriate action. One such sensor, called the laser rangefinder (LRF) and illustrated on a robot in figure 1-1, emits laser beams and scans for reflected beams in order to measure the distance to objects within its planar field of view. These sensors are often installed in robots used to navigate through rubble in search for victims in disaster areas (Awesom-o). Research continues across the globe to develop the use of robots and also of sensors.

![Figure 1-1. Shows generally how LRF functions. LRF is illustrated mounted on a robot, very much like in the project.](image)

1.2 Objective and Motivation

Robotic Mobility Group, a research team at M.I.T., is developing a method of having laser rangefinder Hokuyo model UTM-30LX that normally scans in 2D to scan in 3D. The PIONEER 3-AT (P3-AT), an intelligent four-wheeled robot, will use the LRF in this method to ultimately navigate through unknown terrains autonomously.

One possible solution to expand the vision capability of the LRF to 3D is to mount the LRF on an active tilt-stage that is rotatable in the pitch direction at controlled speeds by a
servomotor attached to the robotic platform. A tilt-stage is just a platform on which the LRF will be installed and that can be tilted at different angles by a servomotor. Then, by actively knowing the LRF’s angular position through the servomotor, the P3-AT can be designed to reconstruct a 3-D view of the environment and to respond appropriately. The main scope of this thesis is to design and fabricate an active tilt-stage system for the LRF with regard to necessary constraints, and then to validate the developed stage. The constraints specifically consist of the necessity for the LRF’s scan to be able to detect crucial obstacles for the P3-AT such that the P3-AT won’t get stuck. Analysis on how the constraints should be accounted for in the tilt-stage design was done on MATLAB and modeling of the tilt-stage was done on SOLIDWORKS.

1.3 Laser RangeFinder: Design and Function

Because of the good resolution, laser rangefinders are being used more commonly as the quasi-visual sensor of robots (Price). Compared to other rangefinder sensors like sonar sensors, it is cheap and small but it can still perform almost as well as the other types of sensors. As a matter of fact, it works similarly to a sonar sensor only that it uses light instead of sound. Figure 1-2 shows the UTM-30LX developed by HOKUYO corp. (ROBOTIS).

![Laser rangefinder](image)

**Figure 1-2. Laser rangefinder developed by HOKUYO (UTM-30LX) © HOKUYO corp.**

A laser ray is emitted from a laser source (\( \lambda = 785 \) nm) inside the UTM-30LX LRF and the corresponding laser ray reflected from an object in the environment is sensed and processed in order to find the distance \( D \) between the LRF and the object. After counting the time the ray takes to go the object and be reflected back, the LRF is designed to use this information and the fact that the ray travels at the speed of light \( c \) in order to calculate the distance by equation 1-1:

\[
c(t_{\text{return}} - t_{\text{exit}}) = 2D
\]

(1-1).

For the project, an individual reading of the distance is henceforth referred to as a ray scan.
Actually, the LRF is designed to successively take several of these individual ray scans in a 25 ms time frame in order to build a 2-D planar view of the environment. Near the top of the UTM-30LX LRF, there is a window screen extending 270° around the LRF as indicated in figure 1-3.

Figure 1-3. The 270° planar scan range in +\( \beta \) direction It cannot see through the other 90°. It has an angular resolution of 0.25° which means that there are 1,440 ray scans in the plane scan.

It is through this screen that the LRF repeatedly takes ray scans in the counterclockwise +\( \beta \) direction (viewed from the top) that are separated by an angular resolution size characteristic to the LRF brand/type. In other words, a planar sweep scan consists of several individual ray scans separated by the angular resolution size. The mechanism by which the emission of the laser ray is rotated along the +\( \beta \) direction consists of reflection by a rotating mirror inside the LRF. The sensing of the returning laser ray works in a similar manner. Moreover, after having completed one 270° planar sweep, the LRF begins another sweep after waiting 1 ms as indicated by Figure 1-4.

Figure 1-4 It takes the LRF 25 ms to completely sweep through one scan: 24 ms for scanning and the other 1 ms for waiting interval

Of course, the working of the laser rangefinder is complicated in many ways: the LRF needs to be designed to distinguish one ray scan from another; the LRF needs to be able to detect
if the incoming ray scan has been only reflected once or many times; the LRF also needs to respond quickly upon detection for even the imprecision of 1μs can lead to a misreading of 30 m and so on. Specifically, when a laser ray is emitted from the LRF and reflected off of a surface in the environment, it turns out that some of the ray’s energy is absorbed by the surface and some of it is reflected according to the law of reflection which is illustrated in figure 1-5.

![Figure 1-5. theoretical law of reflection is illustrated. The parts of the ray that are absorbed into the surface and that are reflected in other directions are not shown here.](image)

However, due to imperfections, a small component of the ray’s energy is reflected straight back to the LRF and it is this component that the LRF detects to calculate the distance. This phenomena illustrated in figure 1-6 is crucial since this is what the LRF depends on to even be able to detect the surface.

![Figure 1-6. Shows how the emitted laser ray has to be partially reflected back to LRF for detection to even occur](image)

However, the UTM-30LX is equipped with the technology to take care of these issues and the details of it are not discussed here. In other words, in the project it is assumed that a single laser ray is emitted, reflected, and detected without consideration to the complex optical activities involved. As far as the project is concerned, it will be assumed that the LRF always works this way without complications due to optics.
Nonetheless, for the project, another feature will be developed for the LRF: a way of improving the vision capability of the LRF from 2-D to 3-D will be investigated in terms of the physical function of the LRF. In addition to scanning in the $\beta$ direction, scanning in the new $\hat{\theta}$ direction for the LRF will be enabled by a servomotor to make the scan 3-D. The development of the software counterpart will be continued through the Robotic Mobility Group. In particular, in order to accurately reconstruct the 3-D view, the software will not only need information from the LRF, but it will need to also actively know about the LRF’s current angular position/angular velocity and the mobile robot’s position/velocity.

1.4 PIONEER 3-AT: Design and Function

The PIONEER 3-AT, shown in Figure 1-7, has been designed for mobility on rough-terrain areas (MobileRobots Inc.).

![Figure 1-7. Shows the PIONEER 3-AT with some customizations (MobileRobots Inc.)](image)

It can be driven with a joystick and it is also capable of autonomous navigation by using a sonar sensor. Its feature of versatile customization is what sets the P3-AT apart from other rough-terrain robots. This feature will be used as the LRF/tilt-stage system will be, after design and fabrication, installed onto it in an effort to improve its navigating capabilities. The P3-AT itself is about 26 cm tall and the plan is to have the LRF installed on top of it. Thus, considering the physical dimensions of the LRF and the tilt-stage, the point of scanning will be about $h = 30$ cm off of a flat ground. More information about its physical design and functions is provided in the appendix.

2. Functional Requirements and Analysis

2.1 Objective of Analysis

Before beginning the design of the active tilt-stage system in which a servomotor will be allowed to rotate the LRF, it is first necessary to determine any requirements that the LRF is supposed to meet in order to function properly with its 3-D scan feature. In particular, the necessity of the detection by the LRF of crucial obstacles that have the potential of hampering the motion of the P3-AT at the wheels imposes key functional requirements for the 3-D scan feature.
of the LRF to meet. Moreover, even in the worst case scenario, the function of the LRF, which, as far is the project is concerned, basically consists of the angular velocity at which it is rotated at, should still produce a good enough resolution at point of contact with the environment such as to be able to detect the crucial obstacles. Before delving into detail about how the function of the LRF can meet the functional requirements, an analysis of the geometric motion of the LRF’s scan of the environment is necessary.

2.2 Analysis of Geometric Motion of Ray Scan

Before evaluating how the servomotor needs to rotate the LRF in order to meet the functional requirements concerning obstacles, an analysis of the geometric motion of the LRF’s scan of the environment which is due to the motion of the P3-AT and the rotation by the servomotor is necessary. Actually, there is already research on this available. In particular, Alonzo Kelly, a graduate from Carnegie Mellon university, has the exact same research except that the effect due to the movement of the mobile robot is not taken into account; hence, for his research, \( V = 0 \).

Because, as shown in figure 2-1, the plane scan emitted from the LRF is really composed of several individual ray scans which are all subsequently 0.25° apart from each other for the UTM-30LX (there are a total of 1,440 scan rays in the 270° azimuth scan range; \(-135° < \beta < 135°\)), the complete 3-D analysis of the geometric motion of the LRF’s scan as the robot moves in the \( \hat{y} \) direction (refer to figure 2-2 for illustration of directions) and the LRF rotates in the \( \hat{\theta} \) direction is complicated (HOKUYO Automatic CO.).

![Azimuth Scan Diagram](image)

*Figure 2-1. The plane scan that is emitted through the LRF’s screen is composed of 1,440 individual ray scans and each is assigned a \( \beta \) angle. The above diagram shows how 9 of these ray scans intersect with the ground as \( \theta \) decreases with time, the center one following a linear path of intersection and the others following hyperbolic paths. Notice how the resolution gets worse for the \( \beta \neq 0° \) region as the scan becomes skewed (Alonzo).*
Nonetheless, the analysis of the scan of the ground surface at azimuth angles $\beta \neq 0^\circ$ (refer to figure 1-3) is more important than the analysis of the specific scan of the surface at $\beta = 0^\circ$ since the crucial obstacles attack in front of the wheels which does not correspond to the $\beta = 0^\circ$ 2-D planar region illustrated in figure 2-2. In addition, noting the fact that the $\beta = 0^\circ$ ray scan travels in a straight line along the ground surface while all the other $\beta \neq 0^\circ$ travel in hyperbolic paths, the worst case scenario corresponds to the $\beta \neq 0^\circ$ region since the resolution is worse there as the scan becomes skewed as shown by figure 2-1. However, in spite of these facts, the analysis is done only for the $\beta = 0^\circ$ plane symmetrically cutting through the width and along the length of the robot. This is because the complicated analysis in all other regions can be built off of this specific 2-D analysis and, also, the complete 3-D analysis can only be properly dealt with by the creator of the software.

The geometric motion of the LRF’s scan is driven by the movement of the P3-AT on which the LRF is installed as shown in figure 2-2 and by the LRF’s rotation by the servomotor.

![Figure 2-2. The schematic of the ray scan at $\beta = 0^\circ$. Analysis is done for the motion of this ray scan as the 3-AT moves forth in the $\hat{y}$ direction and LRF rotates in the $\hat{\theta}$ direction. Notice that angular velocity is positive in the clockwise direction and also that the reference coordinate frame is attached to the ground.](image)

The LRF is shown installed at a height $h$ on the top of the P3-AT which moves forward at a constant velocity $V$. Also, as far as the length of the P3-AT is concerned, the LRF will be installed at the very forward edge of the robot such that its view of the ground right in front of the robot will not be blocked by the robot. And, as far as the width of the P3-AT is concerned, the LRF will be installed symmetrically in the center of the robot. Moreover, the angle of the scan ray
is measured with respect to the horizontal where angular velocity $\dot{\theta}$, the tiltable speed at which the LRF is rotated by the servomotor, is considered positive in the clockwise direction. Also, notice that the coordinate frame has been established to a starting point on the ground and does not move with the LRF. After a time $t$ and from a starting position $y(t = 0) = 0$, the position of the LRF’s scan ray becomes

$$y = \frac{h}{\tan \theta} + V \cdot t \quad (2-1)$$

where $\theta$ is

$$\theta = \theta(t) = \theta_0 + \dot{\theta} \cdot t \quad (2-2).$$

The equations can be verified geometrically. Indeed, these equations are the same as the ones given by Kelly except that a factor for velocity $V$ is included (Kelly). For this project, the height $h$ has been considered at 30 cm in order to account for the extra height added by the tilt-stage. And the velocity $V$ has been considered constant at 1.2 m/s, the maximum velocity of the P3-AT, since the design of the tilt-stage system needs to be tailored to mitigate the flaws that show up in worst case scenario (MobileRobots Inc.).

2.3 Resolution Analysis

Of particular interest for the tilt-stage design are the differential resolutions in the $y$ and $z$ directions. The equation for the $dy$ differential resolution is just the time derivative of equation 1:

$$dy = \left[ V - h \cdot \dot{\theta} \cdot \csc^2 \theta \right] dt \quad (2-3).$$

It is important to note that every $y$ position established from the stationary reference frame is assigned a corresponding $dy$ differential resolution value when the ray scan intersects with it. More specifically, in the case where the LRF is not rotating, the $y$ position that the ray scan is intersecting with at the ground surface at some angle $\theta_0$ at time $t_1$ is not the same $y$ position being intersected with at the same angle $\theta_0$ at some later time $t_2$. This is due to the fact that the robot is still moving at a velocity $V$. The differential resolutions depend on both the motion of the robot and on the rotation of the LRF. $dz$ can be geometrically derived from $dy$:

$$dz = dy \cdot \tan \theta = \left[ V - h \cdot \dot{\theta} \cdot \csc^2 \theta \right] dt \cdot \tan \theta \quad (2-4).$$

$dy$ and $dz$ and their geometric relation are illustrated in figure 2-3. These equations also agree with those given by Kelly for differential resolutions except that these account for velocity $V$ (Kelly).
Figure 2-3. A schematic of the movement of the ray scan as time $dt$ passes. The ray scan's movement is influenced by the motion of the robot and by the rotation of the LRF. Notice that the ray scan is rotating counterclockwise.

Figure 2-3 shows how the scan ray moves as time $dt$, which is really small, passes. It turns out that $dt$ corresponds to a specification of the UTM-30LX: its scan time, 25 ms/scan, the time taken for each planar sweep. It is important to take this value into consideration because it affects how good the resolution is. Also, even though 1 ms occurs between scans, it can approximately be said that they follow directly one after another (HOKUYO Automatic CO.).

### 2.4 Functional Requirements for 3-D Scan Function of Laser Rangefinder

There are functional requirements that the proposed tilt-stage system needs to meet in order to detect crucial obstacles that can hamper the P3-AT's motion. The functional requirements are defined by the interactions of the P3-AT's wheel of radius $r$ with pits in which the wheels can get stuck and with steps over which the wheels cannot step over. These two interactions have been selected for analysis because, for the most part, all other types of obstacles are composites of these two basic obstacle models.

Figure 2-4 shows the model for the gap obstacle and the conditions for which the wheel can get stuck in it.

Figure 2-4. The distance between the points of intersection of two consecutive ray scans with the ground is called the differential resolution $dy$. The differential resolution $dy$ is larger than the...
diameter \( d \) of the wheel. In this case, the LRF fails to detect the gap obstacle and the wheel is about to fall into the gap.

If there is a gap of a size greater than the diameter of the wheel \( d \) which is 21.5 cm for the P3-AT, then it is possible for it to get stuck in it whereby the robot will be rendered functionless (MobileRobots Inc.). Therefore, it is necessary for the LRF to detect any such gaps. In particular, the distance along the y-axis between two consecutive ray scans \( dy \), otherwise known as the LRF’s differential resolution in the \( y \) direction, needs to be smaller than the size of the smallest possible gap that can get the wheel stuck, a gap with a size of \( d \). This is important because, as shown in figure 2-4, if the resolution were larger than the size of the gap, it is possible for the LRF to totally leave the gap undetected. If it remains undetected, the robot will not be able to take the appropriate measures to overcome this obstacle. Figure 2-5 illustrates the two possible readings that can occur when \( dy > d \): case 1 shows how the scan rays can fail to detect the gap; and case 2 shows how the scan rays can still at times read some form of the gap as a ray scan can still fall into it. Note that the software must connect all points of scan with straight lines in order to optimally reconstruct what is viewed by the LRF.

**Figure 2-5.** Cases 1 and 2 are both possible when \( dy > d \). Only case 2 is possible when \( dy < d \). For the \( dy > d \) case, it is actually still possible for a ray scan to fall into the gap. The concern is case 1, the possibility of no ray scan falling into the gap thereby leaving it undetected to the robot.

When \( dy < d \), only case 2 is possible since the resolution is small enough such that at least one scan ray will always fall into the gap. Moreover, though case 2 illustrates the LRF’s raw vision of some form of the gap, what the robot decides to do with this information will depend on the in-built algorithm. Actions can range from stopping and running several more scans over this area to get more detailed information of an obstacle that is beginning to look like a gap to completely avoiding it altogether. Actually, according to the specification sheet for the P3-AT, the longest traversable gap for its wheels is of length 15.2 cm. For the analysis, instead of using the diameter value \( d \), the value given by the specification sheet is used.
Figure 2-6 shows the model for the step obstacle and the conditions for which the wheel fails to overcome it.

Figure 2-6. Theoretically, if the height of the step \( s \) is larger than the wheel’s radius \( r \), the wheel cannot go over it because the wheel cannot securely reach the corner point of the step about which it can rotate onto the upper platform.

If there is a step with a height \( s \) greater than the radius \( r \) of the wheel, then the wheel will not be capable of overcoming the step. It is therefore important for the LRF to detect step obstacles. The distance along the z-axis between two consecutive ray scans \( dz \), otherwise known as the LRF’s differential resolution in the z direction, needs to be smaller than the size of the smallest possible step that can stop the wheel, a step with a height \( s \) equivalent to radius \( r \). Actually, no matter what, the LRF should detect a change in elevation once it scans over a step obstacle. The problem is whether it detects the step obstacle for what it is or not. Even for the \( dz < r \) case, as shown in case 1 in figure 2-7, the ray scans may detect some slope form of the step obstacle instead of the exact form of the step obstacle thus misleading the robot to see a slope. Note that case 2 is also possible when \( dz < r \).
Figure 2-7. Case 1 is possible when \( dz > r \). Both cases 1 and 2 are possible when \( dz < r \). In reality, the protocols on what the robot should do depend on the algorithms that the software engineer designs.

However, the algorithm can be designed to raise a red flag if it detects a vertical portion of the step as shown in case 2. The robot can then run more scans to get more information about the obstacle that is starting to look like a step obstacle. The functional requirement for the \( dz \) resolution for a vertical portion to always be detected is that \( dz \) needs to be less than half the radius \( r \), \( dz < \frac{r}{2} \). However, whether or not to have this protocol in the algorithm depends on the robot’s programmer.

\( dy < d \) and \( dz < r \) are the two minimum functional requirements that are to be met in the design of the tilt-stage in order to avoid failure at detection of the two most crucial obstacles. However, according to the P3-AT’s specification sheet, the maximum height \( s \) of the step obstacle that the P3-AT can viably transverse is 10 cm while the maximum size of the gap obstacle that the robot can viably transverse is 15.2 cm (MobileRobots Inc.). For the analysis, the values given by the specification sheet are used and are listed in table 1.

<table>
<thead>
<tr>
<th></th>
<th>Analysis values</th>
<th>Specification sheet values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum length of traversable gap obstacle</td>
<td>21.5 cm</td>
<td>15.2 cm</td>
</tr>
<tr>
<td>Maximum height of traversable step obstacle</td>
<td>10.75 cm</td>
<td>10 cm</td>
</tr>
</tbody>
</table>

2.5 Observations from Analysis

A simulation has been made on MATLAB in order to analyze the \( dy \) and \( dz \) differential resolutions. Figures 2-8, 2-9, and 2-10 illustrate the \( dy \) and \( dz \) resolutions of the LRF mounted on the P3-AT which moves at maximum velocity \( V \) and being rotated at different angular velocities \( \dot{\theta} \) by the servomotor. Moreover, the conditions for figure 2-10 are the same as the ones for figure 2-8 except that the LRF rotates in the clockwise direction instead of counterclockwise direction. The main points to draw from the graphs are that wherever the curve of the differential resolution falls below the line of the obstacle size limit, then the 3-D scan function is performing safely and viably since, meeting the functional requirements, crucial obstacles will most likely be detected. And, conversely, wherever the curve of the differential resolution lies above the
obstacle size limit, then the 3-D scan function is not performing acceptably in that if the ray scan hits a crucial obstacle at any of these critical angles, then there is a high possibility that the obstacle can go completely unnoticed by the robot. Also, it is important to note that, according to equation 2-1, $y$ does not only depend on $\theta$, also a function of time, but also on the velocity $V$; hence, each angle $\theta$ corresponds to a $y$ position not by trigonometry alone (first term in equation 2-1) but also by the fact that the LRF is translating at velocity $V$ (second term).

Figure 2-8. The differential resolution $dy$ of the LRF's scan at different angles as the P3-AT moves forward at maximum velocity $V$ and the LRF is rotated at different positive angular velocities $\dot{\theta}$: 30°/sec, 60°/sec, 90°/sec, 120°/sec, 150°/sec, in increasing order according to arrow in the right of the graph. Moreover, the scan begins at $\theta_0 = 85^\circ$ and ends at $5^\circ$ where $\theta$ is a function of time. Note that the size for the maximum traversable gap is also graphed as the horizontal line for comparison.
Figure 2-9. The differential resolution $dz$ of the LRF's scan at different angles as the P3-AT moves forward at maximum velocity $V$ and the LRF is rotated at different positive angular velocities $\dot{\theta}$: 30°/sec, 60°/sec, 90°/sec, 120°/sec, 150°/sec, in increasing order according to arrow in the right of the graph. Moreover, the scan begins at $\theta_0 = 85^\circ$ and ends at $5^\circ$ where $\theta$ is a function of time. Note that the size for the maximum traversable step is also graphed as the horizontal line for comparison.

![Figure 2-9](image)

Figure 2-10. The differential resolution $dy$ of the LRF's scan at different angles as the P3-AT moves forward at maximum velocity $V$ and the LRF is rotated at different negative angular velocities $\dot{\theta}$: -30°/sec, -60°/sec, -90°/sec, -120°/sec, -150°/sec, in increasing order according to arrow in the left of the graph. Moreover, the scan begins at $\theta_0 = 5^\circ$ and ends at $85^\circ$ where $\theta$ is a function of time. Note that the size for the maximum traversable gap is also graphed as the horizontal line for comparison.

Moreover, the resolutions are given for different angular velocities as the LRF begins to rotate at $\theta_0 = 85^\circ$ below horizontal and rotates toward the horizontal for figures 2-8 and 2-9. For figure 2-10, the LRF instead begins to rotate at $\theta_0 = 5^\circ$ toward the vertical. Being that the LRF is moving with the P3-AT at velocity $V$, the direction of pitch rotation for figures 2-8 and 2-9 corresponds to the worst case scenario for the LRF because a bigger, worse resolution size is produced when the LRF rotates in the counterclockwise direction than in the clockwise direction. This is evident when comparing figures 2-8 and 2-10 in that the angle range where the resolution size is smaller than the obstacle size is shorter, worse, in the counterclockwise case than in the clockwise case (refer to table 2 below). This is akin to, while moving in a car, catching a larger but less detailed glimpse of your view when you move your vision further along the street ahead of you (bigger, worse resolution). This is opposed to catching a smaller but more detailed glimpse when you move your vision the other way like when your eyes follow cars traveling in the opposite direction (smaller, better resolution).
Also, the size of the smallest gap in which the P3-AT’s wheel can get stuck is illustrated as the line labeled max gap limit in the graph for \(dy\) resolution. And the size of the smallest step over which the wheel can no longer mount over is illustrated as the line labeled max step limit in the graph for \(dz\) resolution. Having the differential resolutions be smaller than the corresponding obstacle size limits over the whole angle \(\theta\) range, from 0° to 90°, is desirable because the likelihood of failure to detect the obstacles becomes impossible. However, this is not possible because both differential resolutions always continuously approach an infinite singularity whenever the LRF is pointing near the horizontal. In other words, there will always be an angle “below” (refer to figure 2-2) which the resolution size becomes too large to detect these crucial obstacles. This angle will henceforth be referred to as the minimum critical angle \(\theta_{cr-min}\). As can be expected, the angle \(\theta\) range over which the resolution is good becomes smaller as the LRF rotates at greater angular velocities. Also, for the \(dz\) resolution, there is always an angle “above” (again, refer to figure 2-2) which the resolution size begins to become larger than the obstacle size limit. This angle will henceforth be referred to as the maximum critical angle \(\theta_{cr-max}\). Actually, it turns out that there also exists for the \(dz\) resolution an infinite singularity at the 90°. Table 2 gives the valid angle ranges for which the resolutions are valid for different angular velocities.

\textit{Table 2. Provides the acceptable angle \(\theta\) ranges over which the differential resolutions satisfy the functional requirements concerning the obstacles. This is done not only for different angular velocities but also for different directions. Note that a larger range is better more desirable since it means that the resolution is acceptable over a greater range. In other words, the dy resolution for CCW case is worse (worst case scenario) than that for the CW case. Also, note that the resolutions get worse at greater angular velocities.}

<table>
<thead>
<tr>
<th>Angular Velocities</th>
<th>CCW valid (dy) resolution range</th>
<th>CCW valid (dz) resolution range</th>
<th>CW valid (dy) resolution range</th>
</tr>
</thead>
<tbody>
<tr>
<td>30°/sec</td>
<td>8.40°&lt;(\theta) &lt;90°</td>
<td>5°&lt;(\theta) &lt;56.44°</td>
<td>7.28°&lt;(\theta) &lt;90°</td>
</tr>
<tr>
<td>60°/sec</td>
<td>11.96°&lt;(\theta) &lt;90°</td>
<td>9.41°&lt;(\theta) &lt;51.44°</td>
<td>10.40°&lt;(\theta) &lt;90°</td>
</tr>
<tr>
<td>90°/sec</td>
<td>14.72°&lt;(\theta) &lt;90°</td>
<td>15.74°&lt;(\theta) &lt;45.16°</td>
<td>12.81°&lt;(\theta) &lt;90°</td>
</tr>
<tr>
<td>120°/sec</td>
<td>16.96°&lt;(\theta) &lt;90°</td>
<td>28.72°&lt;(\theta) &lt;32.21°</td>
<td>14.83°&lt;(\theta) &lt;90°</td>
</tr>
<tr>
<td>150°/sec</td>
<td>19.22°&lt;(\theta) &lt;90°</td>
<td>No where valid</td>
<td>16.51°&lt;(\theta) &lt;90°</td>
</tr>
</tbody>
</table>

A main benefit to note is that the maximum velocity of the P3-AT is slow enough such that for a majority of the angle range the differential resolutions are validly under the corresponding obstacle size limit even for large angular velocities. For example, as illustrated in
figure 2-11, if $V$ were above 7 m/s, then with the car running at this velocity, the 3-D scan function would never meet the defined functional requirements.

![Graph showing differential resolution $dz$ for different angular velocities and depression angles.](image)

**Figure 2-11.** Same conditions as figure 2-8's except that the velocity $V$ is set at 7 m/s instead of 1.2 m/s. Notice that nowhere is the differential resolution $dy$ below the value of the max gap limit; therefore, if the robot were to hit such a high velocity, the 3-D scan function would never meet the functional requirements.

### 2.6 Functional Requirements for Tilt-Stage Design

There are two main situations that can be recommended to the programmer of the servomotor/LRF system to watch out for: with regard to the differential resolution $dz$, he/she should be careful about the possibility of there being obstacles at $y$ positions that correspond to angles above the minimum critical angle $\theta_{cr-min}$, angles where failure to detect crucial obstacles is possible; also, with regard to both differential resolutions, the programmer should be careful about the possibility of there being obstacles at $y$ positions that correspond to angles below the minimum critical angle $\theta_{cr-max}$, angles where failure to detect crucial obstacles is possible.

For the first situation, there is a safety protocol that can be installed through an algorithm which has the robot run one scan of the environment at least for the range $0^\circ < \theta < 90^\circ$ right after it’s turned on but before it even begins moving. This way it can have a good preemptive view of the environment without the resolution noise that comes from the movement of the robot. Figure 2-12 illustrates the differential resolution $dz$ for $V = 0$ m/s and for different angular velocities.
Figure 2-12. The differential resolution $dz$ of the LRF’s scan at different angles as the P3-AT is still and the LRF is rotated at different positive angular velocities $\dot{\theta}$: 10°/sec, 15°/sec, 20°/sec, in increasing order according to arrow in the right of the graph. Moreover, the scan begins at $\theta_0 = 88^\circ$ and ends at 2° where $\theta$ is a function of time. Note that the size for the maximum traversable step is also graphed as the horizontal line for comparison.

For most of the angle range $\theta$ at these low angular velocities, the scan function meets the functional requirements thus allowing the robot to have preemptive knowledge of the existence/location of upcoming obstacles it should watch out for. In other words, with this safety protocol built into the algorithm, there is no need to be concerned about leaving undetected any crucial obstacles that lie above the maximum critical angle $\theta_{cr-max}$ right when the robot is booted up. Ultimately, however, there are many other ways to take care of this situation and it is up to the programmer whether or not to have such an algorithm installed.

For the second situation, obstacles that the P3-AT is approaching in $y$ positions corresponding to angles $\theta$ lying below the minimum critical angle $\theta_{cr-min}$ are of concern. But, as long as the ray scan approaches from angles $\theta$ above $\theta_{cr-min}$ to meet them inside the valid range then there should be no problem. However, if an obstacle were to enter the valid angle range towards the robot just as the ray scan is leaving the valid angle range, the only place where it can detect obstacles, then there is a possibility that the obstacle remain undetected until it reaches the robot as illustrated in Figure 3-13. This corresponds to a worst case scenario.
There is a possibility that, if the angular velocity of the ray scan is too slow, the ray scan will not make it back in time from its routine cycle scans to catch the obstacle in the valid angle range before it reaches the robot. The assumption that the LRF repeatedly executes a routine scan for the angle range $0^\circ < \theta < 90^\circ$ will be made. Furthermore, the case where the P3-AT is moving forward at maximum velocity $V$ and where the LRF is being rotated at angular velocity $150^\circ$/sec will be assessed for the differential resolution $dy$. Referring to table 2, the ray scan just entered the region below the minimum critical angle $\theta_{cr-min} = 19.22^\circ$ and will not start on its way back until it has reached $\theta = 0^\circ$. At the same time, the obstacle enters the valid angle $\theta$ range being about 86 cm away from the robot. Now, it takes about 0.71 seconds for the obstacle to reach the robot while it takes the ray scan 0.72 seconds to come back to $90^\circ$. Then, for this worst case scenario, it can be concluded that the angular velocity of the LRF must be at least $150^\circ$/sec in order for the ray scan to detect the approaching obstacle. However, because there is a tradeoff between a shortening valid angle range and an increasing angular velocity, there is a limit to how high the angular velocity can be set to prevent the worst case scenario. An easy remedy for this situation is to have the LRF routinely sweep scans over a shorter angle range when it detects that the P3-AT is moving at a high velocity. Also, there is the issue that if indeed the LRF is being rotated at angular velocity $150^\circ$/sec, then in the time of one 25 ms plane scan the LRF would have rotated about $3.75^\circ$. Unless the software corrects for this, the reconstructed view will come out really skewed.

In reality, there are many ways to deal with these two crucial situations and it will be up to the programmer to create sturdy safety protocols through the software. As far as the project is concerned, there are few functional requirements that need to be imposed on the design of the tilt-stage. Because the robot will not always be traveling at its maximum velocity $V$, the possibility of a dangerous occurrence is not as high as depicted by the analysis. That the LRF be rotated by the
servomotor at angular velocities $\dot{\theta}$ ranging from $0 \, ^{o}/sec$ to $150 \, ^{o}/sec$ provides a fair margin of safety for the performance of the LRF. The design of the active tilt-stage system will be developed with regard to this margin of safety. Ultimately, how the critical situations will be dealt with is up to the programmer. With observations through the analysis, there are many methods that can be used to protect the robot from being hampered by the obstacles.

3. Proposed Tilt-stage Design

3.1 Objective for Tilt-stage Design

The objective for the design of the tilt-stage is to have it hold the LRF such that rotation by the selected motor is possible at the required angular velocity ($0 \, ^{o}/sec \leq \left| \dot{\theta} \right| < 150 \, ^{o}/sec$).

Moreover, appropriate dimensions, material type, and build must be selected for the tilt-stage such that it is stable for static and non-static conditions. Also, a motor that is capable of rotating the LRF/tilt-stage system at the required angular velocities must be selected. That the tilt-stage does not block the view of the LRF is another important requirement.

3.2 Tilt-stage Design

Figures 3-1 and 3-2 illustrate the footprints for the tilt-stage design.

*Figure 3-1. Drawing of the tilt-stage system with motor installed. The left one is side view as will be seen from figure 2-2 and the right one is the front view as will be seen when facing the front of the robot. The dimensions are shown in centimeters.*
The tilt-stage system is made up of ¼ in. aluminum sheet for lightness in weight and will be assembled together with screws as shown in figure 3-2. The stage on which the LRF is installed on is rotatable and supported by the motor and a rod. Moreover, the points at which the stage is supported coincide with the axis of rotation. The weight of the movable stage along with the LRF totals to about $m_{\text{total}} = 461$ grams while the moment of inertia about the center of mass of the LRF/stage system is about $I = 480,231$ g-mm$^2$. Moreover, the center of mass of the LRF/stage system lies about $l_{\text{off}} = 7.6$ mm below the axis of rotation. And, between the active stage and the external platform that is to be attached to the robot there is enough clearance for the rotation of the active tilt-stage. Also, the external stage is made to support the system at the back of the motor and at the other side of the rod. While one end of the rod is press-fitted into the active stage, the other end can rotate relative to the external platform with the aid of bearings. And, finally, the last functional requirement for the tilt-stage design is met by the fact that no matter at what angle LRF is relative to the active stage, its view is not blocked. Also, the dimensions affecting the installation of the rod satisfy Saint Venant’s Principle.

3.3 Selection of Motor

The Dynamixel DX-117 motor has been selected mainly because its speed and position (resolution of 1,024 steps) can be controlled externally. It is illustrated in Figure 3-3.
Moreover, it is able to sustain huge torque loads about the axis of rotation and about the horizontal axis perpendicular to the axis of rotation. Referring to the motor’s specification sheet, it has a maximum no-load speed of 460 °/sec and a θ resolution of 0.35° ($R_{motor}$). Considering the LRF’s time scan, according to equation 3-1, this means that the minimum angular velocity at which the LRF can be rotated at while still functioning properly is about 14 °/sec. Rotating at angular velocities lower than this will cause for multiple scans to occur at the same angle since the angle resolution can’t get any finer.

$$t_{scan} \dot{\theta} = R_{motor}$$  \hspace{1cm} (3-1)\hspace{1cm}

Furthermore, the motor is able to sustain the torque imposed on it by the off-center center of mass of the LRF/stage system. The torque at its worst is given by equation 3-2:

$$\tau_{load} = I_{off} m_{load} g$$  \hspace{1cm} (3-2)

where $g$ is the acceleration due to gravity. $\tau_{load}$ for the LRF/stage system is about 0.0344 N·m.

According to the specification sheet for the DX-117, the max holding torque is 0.0393 N·m which is larger than $\tau_{load}$ (ROBOTIS). Therefore, it can be concluded that this motor is capable of sustaining the load along its axis of rotation. As for the load about the other horizontal axis perpendicular to the axis of rotation, it has been experimentally verified that the motor is also able to sustain this load.

4. Experimental Validation

At the moment, the fabrication of the tilt-stage is still in progress. Ultimately, whether or not the set tilt-stage will be able to allow the function of the LRF to meet the functional requirements will be determined and the results will be reported to the thesis advisor.
5. Conclusion

A good design for the active tilt-stage has been developed considering the functional requirements imposed by the obstacles. Physically, the requirements that the design of the LRF/tilt-stage system needs to meet is that the resolution must be good enough such that crucial obstacles will not go undetected. Though an analysis was done to see how the differential resolution of the LRF’s scan is sensitive to different parameters, it will ultimately depend on the programmer on what routines will be installed. At least, it is now observable that even at the maximum velocity $V$ of the P3-AT the resolution is still good. However, it is at high angular velocities that the resolution becomes worse. Also, the analysis is very much the same as that from Alonzo Kelly’s research. The only factor that makes the analysis for this project valuable is the fact that it accounts for the movement of the mobile robot on which the LRF is installed. However, the analysis is far from complete because it still needs to be developed such that it accounts for the differential resolutions along other azimuth $\beta$ angles where the resolution becomes even worse. Also, the analysis has not yet been designed to account for other types of movements for the mobile robot like rotation. Ultimately, there are a lot possible situations that the mobile robot can be in and these situations can only be properly prepared for through the software. Ultimately, the analysis can prepare the programmer so that he can make safety protocols in the algorithm of the P3-AT/LRF/DX-117 system. The main requirement that has been met by the tilt-stage design and that the programmer will most likely find very useful is the capability of the servomotor to rotate the LRF at angular velocities ranging through $0 \; ^{\circ}/\text{sec} < |\dot{\theta}| < 150 \; ^{\circ}/\text{sec}$ since this allows the availability of several methods that the programmer can use to handle worst case scenarios.

Also, the tilt-stage model has not been fabricated yet but will be soon and then shown to thesis supervisor.
6. Appendix

Specifications

<table>
<thead>
<tr>
<th>Product name</th>
<th>Scanning Laser Range Finder &quot;SOKUKYO Sensor&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>UTM-30LX</td>
</tr>
<tr>
<td>Power source</td>
<td>12VDC ± 10% (Current consumption: Max: 1A, Normal: 0.7A)</td>
</tr>
<tr>
<td>Light source</td>
<td>Semiconductor laser diode (λ = 785nm), Laser safety Class 1 (FDA)</td>
</tr>
<tr>
<td>Detection Range</td>
<td>0.1 ~ 30m (White Square Kent Sheet 130mm or more), Max. 60m 27º</td>
</tr>
<tr>
<td>Accuracy</td>
<td>0.1 ~ 15m : ± 0.3mm under 3,000x, White Square Kent Sheet</td>
</tr>
<tr>
<td></td>
<td>0.1 ~ 15m : ± 0.5mm under 100,000x, White Square Kent Sheet</td>
</tr>
<tr>
<td>Angular Resolution</td>
<td>0.25° (360° / 1440 steps)</td>
</tr>
<tr>
<td>Scan Time</td>
<td>25msec/scan</td>
</tr>
<tr>
<td>Sound level</td>
<td>Less than 25dB</td>
</tr>
<tr>
<td>Interface</td>
<td>USB2.0 (Full Speed)</td>
</tr>
<tr>
<td>Synchronous output</td>
<td>NPN open collector</td>
</tr>
<tr>
<td>Command system</td>
<td>Exclusively designed command SCIP Ver.2.0</td>
</tr>
<tr>
<td>Connection</td>
<td>Power and Synchronous output, 2m flying lead wire</td>
</tr>
<tr>
<td>Amb. (Temperature/Humidity)</td>
<td>10 ~ +55°C, Less than 85%RH (without dew and frost)</td>
</tr>
<tr>
<td>Vibration Resistance</td>
<td>Double amplitude 1.5mm 10 ~ 55Hz, 2 hours each in X, Y and Z direction</td>
</tr>
<tr>
<td>Impact Resistance</td>
<td>196 m/s², 10 times each in X, Y and Z direction</td>
</tr>
<tr>
<td>Weight</td>
<td>Approx. 370 g (with cable attachment)</td>
</tr>
</tbody>
</table>

Interface

- Cable 1, power and output
  - Color: Brown, Blue, Green, White
  - Signal: 12 VDC, 0V, Synchronous output, GND

- Cable 2 (4pin) Type-A USB for communication

External dimension

Figure A-1. Specification sheet for UTM-30LX laser rangefinder
DYNAMIXEL [ ] [ ] ROBOTIS

Metal Gear
All gears are made with metal to ensure durability.

Axis Bearing
A bearing is used at the final axis to ensure no efficiency degradation with high external loads on the output shaft.

Status LED
The LED can indicate the error status to the user.

1-2. Main Specifications

<table>
<thead>
<tr>
<th></th>
<th>DX-116</th>
<th>DX-117</th>
<th>DX-113</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight (g)</td>
<td>66</td>
<td>66</td>
<td>58</td>
</tr>
<tr>
<td>Gear Reduction Ratio</td>
<td>142.5</td>
<td>192.6</td>
<td>192.6</td>
</tr>
<tr>
<td>Input Voltage</td>
<td>12</td>
<td>16</td>
<td>12</td>
</tr>
<tr>
<td>Final Max Holding Torque (kgf.cm)</td>
<td>21.38</td>
<td>28.50</td>
<td>29.89</td>
</tr>
<tr>
<td>Sec/60degree</td>
<td>0.127</td>
<td>0.095</td>
<td>0.172</td>
</tr>
</tbody>
</table>

Resolution: 0.35°
Operating Angle: 300°
Voltage: DX116,117: 12V~16V (Recommended voltage: 14.4V)
DX113: 12V
Max. Current: 1200mA
Operating Temp.: -5°C ~ +85°C
Command Signal: Digital Packet
Protocol Type: Half duplex Asynchronous Serial Communication (8bit, 1stop, No Parity)
Link (Physical): RS 485 Mutli Drop (daisy chain type Connector)
ID: 254 ID (0~253)
Communication Speed: 7343bps ~ 1 Mbps
Feedback: Position, Temperature, Load, Input Voltage, etc.
Material: Full Metal Gear, Engineering Plastic Body
Motor: Swiss MAXON Motor (DX-116, DX-117). DX-113 uses a cored motor

Figure A-2. Specification sheet for DX-117.

MATLAB Code for analysis: plots dy or dz differential resolution, depending on input function dy2(n,w) % n = 1 means positive angular velocity; n = -1 is opposite

if n == 1
\[ p = 1; \]
\[
\theta_I = 5; \quad \text{degrees}
\]
\[
\theta_F = 85; \quad \text{degrees}
\]
\[
\text{elseif } n == -1
\]
\[
p = -1;
\]
\[
\theta_I = 85; \quad \text{degrees}
\]
\[
\theta_F = 5; \quad \text{degrees}
\]
end
\[
\theta_I = 5; \quad \text{degrees}
\]
\[
\theta_F = 85; \quad \text{degrees}
\]
\[
\text{AVI} = 15; \quad \text{degrees/sec, LRF velocity, this is for the main loop}
\]
\[
\text{AVF} = 45; \quad \text{degrees/sec, LRF velocity}
\]
\[
\text{dtAV} = 10; \quad \text{interval size}
\]
\[
\text{AV} = p*[\text{AVI}:\text{dtAV}:\text{AVF}];
\]
\[
\text{v} = 1.2; \quad \text{m/s, speed of vehicle}
\]
\[
h = 0.3; \quad \text{m, height of LRF on vehicle}
\]
\[
\text{tscan} = 0.025; \quad \text{ms/scan}
\]
\[
\text{rad} = 21.5/(2*100); \quad \text{m, radius of wheel}
\]
\[
\text{dy} = [];
\]
\[
\text{dz} = [];
\]
\[
\text{for } i = 1: \text{length(AV)}
\]
\[
dt = \text{tscan};
\]
\[
dtheta = \text{tscan*AV(i)}; \quad \text{still in degrees}
\]
\[
\theta = [\theta_I: \text{abs(dtheta)}: \theta_F];
\]
\[
\text{for } j = 1: \text{length(\theta)}
\]
\[
dy(j) = v*dt - h*(dtheta*pi/180)./(\sin((\theta(j))*pi/180))_.^2;
\]
\[
dz(j) = (v*dt - h*(dtheta*pi/180)./(\sin((\theta(j))*pi/180))_.^2)\times(\tan((\theta(j))*pi/180))
\]
end
\[
\text{if } (i == 1) \& (n == -1)
\]
\[
\text{set(axes,'XDir','reverse', 'YDir', 'default'); ........ & reverse x axis}
\]
end
\[
\text{if } i == 1
\]
\[
\text{hold}
\]
end
\[
\text{if } w == 5
\]
\[
\text{plot(\theta,dy)};
\]
\[
\text{elseif } w == -5
\]
\[
\text{plot(\theta,dz)};
\]
end
\[
\text{dy} = [];
\]
\[
\text{dz} = [];
\]
dtheta = []; theta = []; end
le = [thetaI:l:thetaF];

if w == 5
    plot(le,2*rad*ones(1,length(le)));
elseif w == -5
    plot(le,0.5*rad*ones(1,length(le)));
end
%the following code has to be edited for different initial conditions
if w == 5
    if n == 1
        legend('LRF falling at 15°/sec','LRF falling at 25°/sec','LRF falling at 35°/sec','LRF falling at 45°/sec','2 x radius');
        xlabel('depression angle with respect to horizontal (degrees)');
        ylabel('dy size (meters) at different angular speeds'); %change "dy" label to appropriate differential resolution label
        title('dy vs angle'); %change dy label to appropriate differential resolution label
    elseif n == -1
        legend('LRF rising at 15°/sec','LRF rising at 25°/sec','LRF rising at 35°/sec','LRF rising at 45°/sec','2 x radius');
        xlabel('depression angle with respect to horizontal (degrees)');
        ylabel('dy size (meters) at different angular speeds'); %change "dy" label to appropriate differential resolution label
        title('dy vs angle'); %change dy label to appropriate differential resolution label
    end
elseif w == -5
    if n == 1
        legend('LRF falling at 15°/sec','LRF falling at 25°/sec','LRF falling at 35°/sec','LRF falling at 45°/sec','1/2 x radius');
        xlabel('depression angle with respect to horizontal (degrees)');
        ylabel('dz size (meters) at different angular speeds'); %change "dy" label to appropriate differential resolution label
        title('dz vs angle'); %change dy label to appropriate differential resolution label
    elseif n == -1
        legend('LRF rising at 15°/sec','LRF rising at 25°/sec','LRF rising at 35°/sec','LRF rising at 45°/sec','1/2 x radius');
        xlabel('depression angle with respect to horizontal (degrees)');
        ylabel('dz size (meters) at different angular speeds'); %change "dy" label to appropriate differential resolution label
        title('dz vs angle'); %change dy label to appropriate differential resolution label
    end
end
end
7. References


