

Design of Two-Tailed Swimmer to Swim at Low-Reynolds Number

by

Sarah Elizabeth Cole

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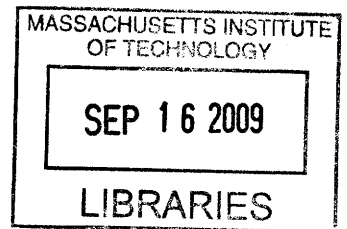
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ABSTRACT

In the realm of systems with Reynolds numbers less than 1, swimming is a difficult task. Viscous forces from the fluid dominate inertial forces. In order to propel itself, a mechanism must be designed to overcome the viscous forces from the fluid and satisfy the non-reciprocal, cyclic motion requirements of the Scallop Theorem. Furthermore, a swimmer must employ one of the three mechanisms stated by Purcell to be capable of swimming at low Reynolds number, a three link swimmer, a corkscrew, or a flexible tail. Three devices utilizing the flexible-tail paradigm of swimming were tested using silicon oil to simulate a Reynolds number of approximately 0.6. Design parameters were uncovered which determine the successfulness of the swimmer and can be used for creating future successful flexible-tail swimmers.

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I would also like to thank my parents for offering their continued support and encouragement throughout this project. Many times I called home frustrated with the outcome of an experiment and they would provide the motivation I needed to try another solution and often mailed various toys as inspiration for new mechanisms.

Lastly, I would like to thank Professor Anette Hosoi for giving me the opportunity to work in the Hatsopolous Microfluids laboratory, and exposing me to the challenges of those organisms that must move in the realm of low Reynolds number.

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1 Introduction

Any type of progressive movement through a liquid can be defined as swimming. Swimming is typically affected by both inertial and viscous forces. The way in which these forces relate to each other is known as the Reynolds number of the flow: $Re = \rho v L / \mu$, where ρ , is the density of the fluid, v is the characteristic velocity of the fluid, μ is the dynamic viscosity of the fluid, and L is a characteristic length. The Reynolds number can characterize flows as either laminar or turbulent. Low Reynolds number environments can exist when either the viscous term from the fluid, μ , is large, or the characteristic length of the swimmer, L , is small. For this research, a low Reynolds number is considered to be less than one.

Mechanisms with reciprocal movements are unable to propel themselves at low Reynolds numbers where the viscous forces in the fluid dominate the inertial forces and the flow is typically laminar.³ Because the inertial terms have essentially no effect on the movement of the mechanism, these types of objects are subject to the Scallop Theorem as presented by E. M. Purcell.³ If an organism moves by changing its position into a certain shape and then reversing the action to return to the initial position, at low Reynolds number, the organism would not be able to move.³ This would be similar to placing a scallop in a low Reynolds number environment. Because there is only one hinge present in the animal, only one movement can occur, opening and closing. These two movements are the reciprocal of each other and would cause the scallop to only move forwards and backwards indefinitely. Therefore, to create forward movement, a mechanism which will swim at low Reynolds number must be designed with both cyclic and non-reciprocal motion.³

2 Background

The simplest mechanism Purcell believed could move at low Reynolds number is that of a two hinged object.³ Purcell's three linked swimmer, as shown in Figure 1 has been researched by many others who have proved that the net direction of movement is horizontally with respect to

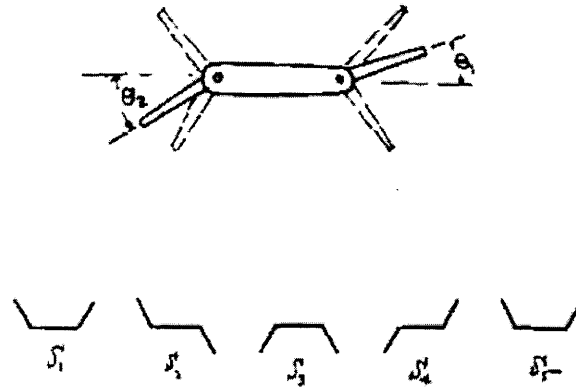


Figure 1. Purcell's Three Link Swimmer from Figure 5.³ If the links of the swimmer were actuated in the manner shown at the bottom of the figure, the swimmer would move horizontally across the page with an up and down deviance.⁵

the orientation of the swimmer in Figure 1. However, much energy is spent in the movement of the swimmer in the vertical deviations as can be seen in videos captured by Annette Hosoi and Brian Chan.⁵

Another example of a mechanism that can swim at low Reynolds number would be the bacteria, *Escherichia coli*. Because its typical length is about 2 microns, this microorganism is found to swim at a Reynolds number on the order of 10^{-5} .¹ While this is if the *E. coli* is swimming at full speed, Berg gives the comparison of a human swimming slowly in a swimming pool, creating a Reynolds number of 10^5 , much larger than that of the *E. coli*'s situation.¹ This is because the human's characteristic length is seven orders of magnitude larger than that of the bacteria. *E.*

coli employs a special method of movement, the corkscrew as seen in Figure 2, which is one of the three types Purcell³ defined to be productive at producing forward propulsion at low Reynolds numbers.

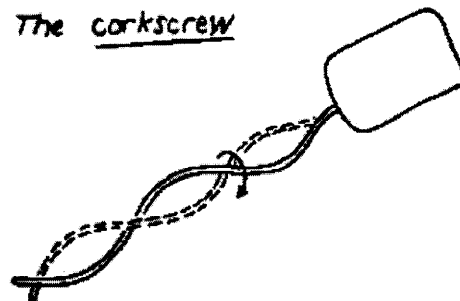


Figure 2. Purcell's Corkscrew Swimmer from Life at Low Reynolds Number³ (Figure 9). Notice that the tail is flexible and as it rotates, the body of the organism will rotate in the opposite direction while travelling forwards.

The third type of swimmer is the flexible oar as seen in Figure 3. This swimmer can progress because its tail is flexible. If the tail were stiff, the swimmer would be unable to move its position because it would have the same type of reciprocal movement as the scallop.

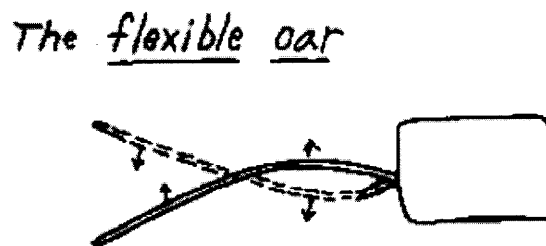


Figure 3. Purcell's Flexible Oar from Life at Low Reynolds Number³ (Figure 9). The flexible tail is required for movement.

For this project it is proposed that a flexible oar swimmer with multiple tails will create a near straight swimming path. Yu, Lauga, and Hosoi¹¹ presented experimental results of a single tailed swimmer's propulsive force at low Reynolds number. This swimmer, RoboChlam, as seen in Figure 4, has a spherical head and an elastic filament as tail. The tail is angularly actuated,

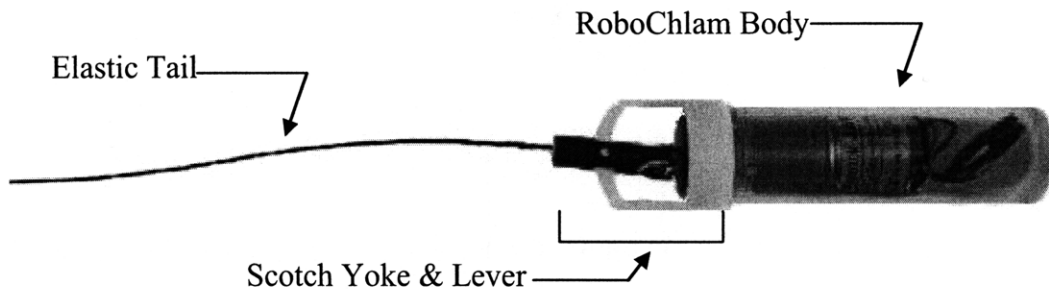


Figure 4. RoboChlam without spherical casing from Elastic Tail Propulsion at Low Reynolds Number¹⁰ (Figure 3-1).

by varying the base angle sinusoidally with a specified amplitude and frequency. RoboChlam, while moving in a generally straight direction, creates a sinusoidal trajectory in the forward direction. However, it is desired that the swimmer moves in a forward trajectory without expending “a lot of effort [...] into propelling the swimmer body in a direction which is different from the main swimming direction.”⁴ In order to develop a swimmer that spends more of its energy propelling itself forward than the RoboChlam, much theoretical work has already been done by Lauga and Yu.^{4,10} In Lauga’s article, *Floppy Swimming*, he presents theoretical data on the kinematics of swimming and the periodic shape of the elastic filament. Lauga discovers the characteristics of “optimal swimmers” in categories such as swimming speed, swimmer length per beat, body length per unity beat, and swimming efficiency.⁴ He has also looked into the theory of a swimmer with two symmetrically located tails operating 180 degrees out of phase

which resulted in a straighter path, but not the desired trajectory. Lauga believes that the optimal swimmer for straight line swimming would have three pairs of filaments located symmetrically on the swimmers body and operating out of phase.

2.1 Theory of low Reynolds number swimming

Lauga, in his *Floppy Swimming: Viscous locomotion of actuated elastica*⁴, characterizes the optimal swimmer by defining the efficiency of the swimmer motion as the “ratio of useful work (defined as work necessary to move the entire swimmer forward at the steady speed) by the total work done by the swimmer.”⁴ By working from the results presented by Lauga, the most efficient swimming mechanism would employ an optimal body radius, r , to tail length, L , ratio of 0.37.⁴

The dimensions of r and L are shown in Figure 5. Through the tests performed by Yu, the angle

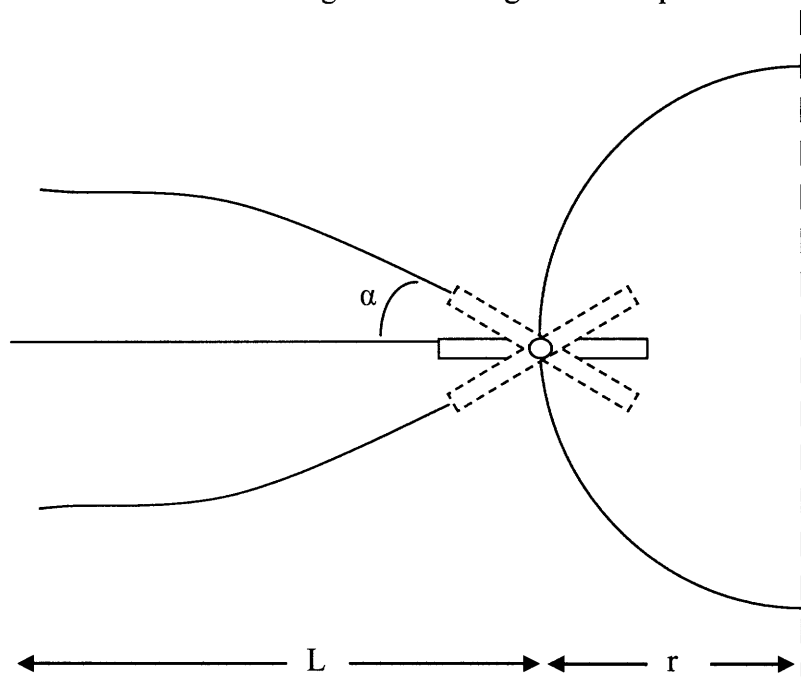


Figure 5. Detail of one side of swimmer actuation parameters. Optimal ratio, $r/L=0.37$. Optimal actuation angle, α , found to be 24.9 degrees. Mirror along dotted line.

of actuation desired, α , was found to be optimal at approximately 25 degrees which is also shown in Figure 5.¹⁰

2.2 Silicon oil

In Yu's¹⁰ previous work with RoboChlam, silicon oil was used to simulate a low Reynolds number environment. The problem with silicon oil is that its viscosity changes with temperature. With the characteristic length of the RoboChlam being on the order of 30 centimeters, travelling at approximately 1 centimeter/second¹⁰, the simulated Reynolds number was around 0.7 using the value for fluid viscosity, 3.18 Pa s as stated in Table 3.2 from Yu¹⁰ and the density value for silicon oil being 760 kg/m³ as found in The Engineering Tool Box⁶.

For the subject of this paper, silicon oil was found in the lab that was labeled to have kinematic viscosity of 3000 cSt, or $3000 \times 10^{-6} \text{ m}^2/\text{s}$ at what was assumed to be room temperature, 20 °C. Using this information along with the fact that the various swimmers tested were between 15 and 20 cm long, if it was assumed that the swimmer would also swim at the 1 cm/s rate as stated by Yu, the working Reynolds number range was to be between 0.6 and 0.5, which is comparable to Yu's environment.

Unfortunately, there was not enough silicon oil from the one labeled container in lab to fill the testing apparatus such that the swimmer would be completely covered. Two more containers were found labeled silicon oil, but with no other identifying information. Given the nature of silicon oil and that it can be reused, it was apparent that the fluids in the two other containers had been used before as there were small floating marker devices and random dust filaments present

in the liquid. Therefore it is only assumed that these liquids had a similar kinematic viscosity as the labeled container. It is also worth noting that the liquid from the three containers was well mixed before any test took place. Special care should be taken in the clean up of silicon oil. It should not be poured down the sink in large quantities. It also takes a large amount of soap and water and vigorous scrubbing to remove it from surfaces. As it is typically used as a lubricant, make sure to clean any spills on the laboratory floor right away as it could cause someone to slip and fall.

3 Design Iterations and Experiments

3.1 Windup Device

A challenge presented by the suggestion of having two tails operating out of phase is to create the small actuation method for propelling the swimmer. A Scotch yoke and lever device was used by Yu¹⁰ to change the rotational movement of a motor into a horizontal movement for the actuation of the tail as used in the RoboChlam pictured in Figure 4. Further research has shown the device used by Sir Geoffrey Taylor. The crank mechanism as seen in Figure 6 works in a

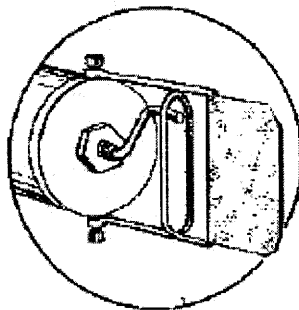


Figure 6. Taylor's Crank Mechanism from Figure 16.⁸

similar manner to the one developed by Yu, while taking up less space. With this mechanism, the pivot point is moved backwards behind the rotating object. Because the characteristic length of the mechanism is to be minimized, any space saving tactics are worth investigating. This mechanism was implemented in the first and third designs described below.

3.1.1 Design & Construction

In order to create the actuation for two tails inside a sphere near the size of a ping pong ball, the actuation method described above needs to be employed in two directions. A diagram of this can be seen in Figure 7. Because of the small characteristic length needed, this device was created

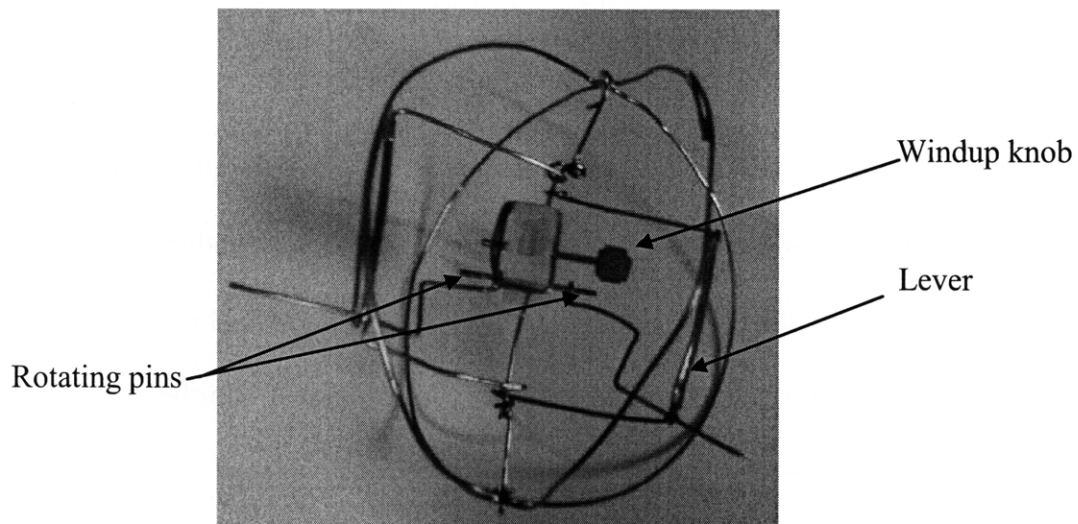


Figure 7. Initial proof of concept for multiple Scotch yoke and lever mechanism.

Characteristic length of this device is approximately 11 cm. Four pieces joined at the top of image form a temporary casing for the device.

first using 18-gauge copper wire. The power device was from a windup toy made by Tomy USA, headquarters located in Santa Ana, California.⁹ Multiple Tomy Pocket Pets were purchased from Borders, with the most useful being the Tomy Not So Grand Band, as it had two

protruding posts from either side of the wind up box that rotated in the same direction, with the wind up handle located decently far away from the posts.

Two main problems existed with the wind up device. First of all, the posts that extended from the wind up box were either plastic or stainless steel. In order to solder anything to the stainless steel posts, silver core solder was needed. Unfortunately, the silver core solder would not adhere well to the copper wire being used. With the plastic posts, it was easier to compress the copper wire around the posts, but it was also relatively easy to snap the posts in half rendering the windup box unusable.

The device was constructed one side at a time. After the first side was built as seen in Figure 7, the device was wound up to see if it functioned correctly. The device correctly translated rotational motion to linear motion, however as it was constructed with bent wire, it was not perfect. Once the other side was constructed, the wind up device would run, but not completely smoothly. It was discovered that if the device was held with one side upwards and one side downwards it would run more smoothly than if the two halves were held facing the right and left. This can most likely be attributed to a jamming issue that resolved itself due to the effect of the force of gravity when held in the vertical orientation.

Two generic plastic Easter eggs as shown in Figure 8 were taken apart and the rounder side sanded down such that they could be joined together. Care was taken to feed the tails and the wind up mechanism through holes drilled in the Easter eggs.

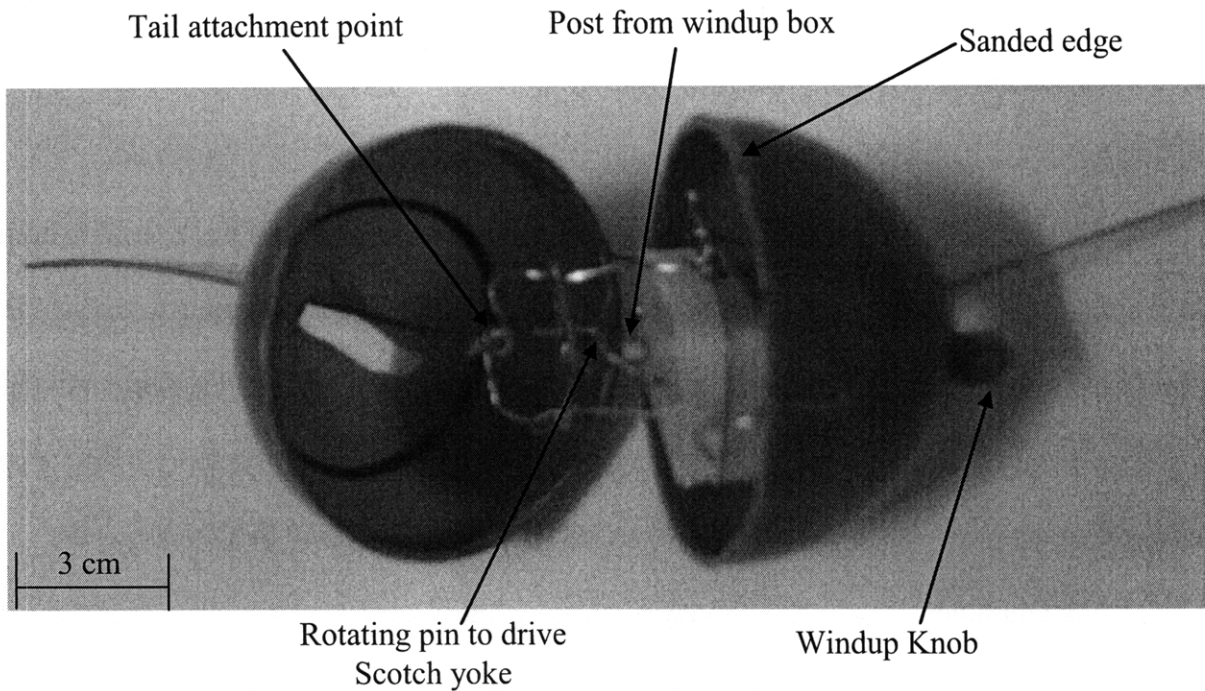


Figure 8. Partially Disassembled Swimmer. Notice the two plastic Easter egg halves which have been sanded to fit flush together.

Then the near-sphere was covered with two balloons in an attempt to make the device water tight as seen in Figure 9. It was found that a certain amount of oil had to be let inside the device in order to create the appropriate buoyancy.

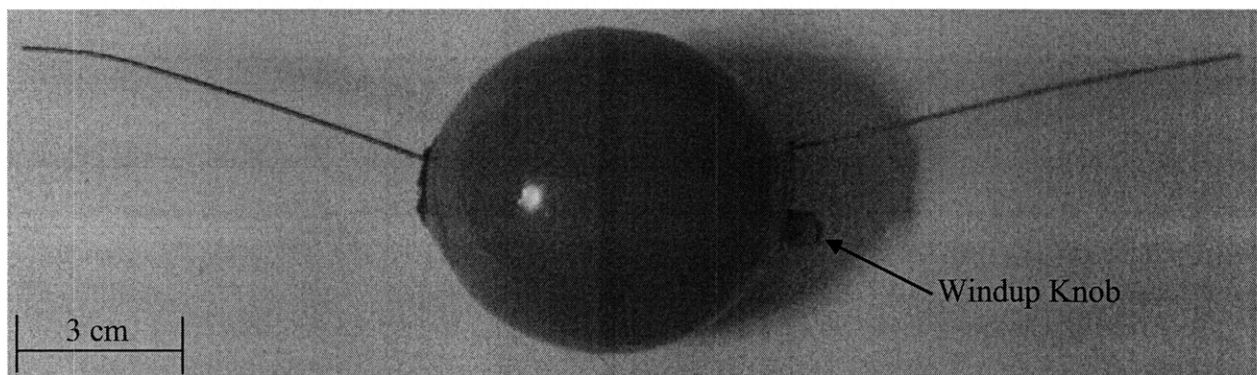


Figure 9. Assembled Windup Swimmer. Notice that two balloons were used in order to create the best seal. Also notice the protrusion of the windup knob. Characteristic length of device is slightly under 6 cm.

3.1.2 Windup Device Results

In order to test out the devices built, a tub was found that would allow the device enough depth to float in and would remove the walls of the tub from interfering with the motion of the tails. For the first device only, the tub was filled with water first. For the following trials, the tub was then filled with approximately six inches of silicon oil. A camera stand device was used to hold a camera parallel to the surface of the silicon oil such that any motion of the swimmer could be analyzed as being only in the parallel plane. An image of the set up can be seen in Figure 10.



Figure 10. Experimental set up for devices using the silicon oil. Camera is not shown as it was used to take the image.

This device had many jamming issues. Sometimes it would not even run in air. It was placed in water and the device ran very slowly. When placed in the silicon oil, the device could barely overcome the viscous forces from the fluid. This resulted in little to no movement of the tails of

the mechanism. Also, the device was too buoyant for the silicon oil. For a simple solution, the hole through which the wind up knob extruded was used to allow the silicon oil to fill in the inner compartment of the device. This increased the force required to move the tails in the silicon oil resulting in no movement when placed in the silicon oil.

The reasons this device failed were because of the inaccurate bending achieved by hand and that the maximum power output of the device was lower than the force required to move a tail through the silicon oil. As can be seen in Figure 8, without the use of machined pieces or an intricate jig, it becomes impossible to have completely similar mechanisms on both sides of the windup mechanism. Also, the copper wire when bent more than once begins to lose its stiffness and becomes unable to hold its shape. Therefore, when any minor adjustment needed to be made it became more fruitful to just remake the entire section.

3.2 Butterfly Device

After determining that building a device that creates symmetric linear movement from a rotating motion was not trivial, investigation into various types of mechanisms had to be pursued.

3.2.1 Design & Construction

After purchasing many toys that seemed as if they would create the motion desired, the Flytech™ Butterfly flyer by Wowwee,™ headquartered in the USA at Carlsbad, California, seen in Figure 11, was discovered.¹² It is actuated with a motion similar to the flexible oar device.

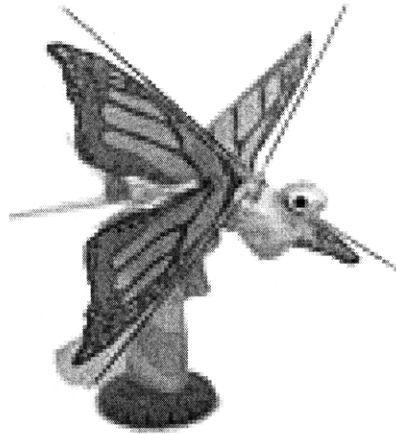


Figure 11. Flytech™ Butterfly flyer by Wowwee™ with charging stand. Notice that top left wing is coupled with the bottom right wing. Top wings are driven while bottom wings just follow along.

The major difference is that the desired motion moves between a positive and negative degree from the horizontal. Instead, the butterfly uses two sets of wings to create the full positive to negative degree range. The mechanism used inside the butterfly can be seen in Figure 12. As it is a little hard to distinguish in the image, the mechanism consists of a small gear attached to the output shaft of the pager motor.

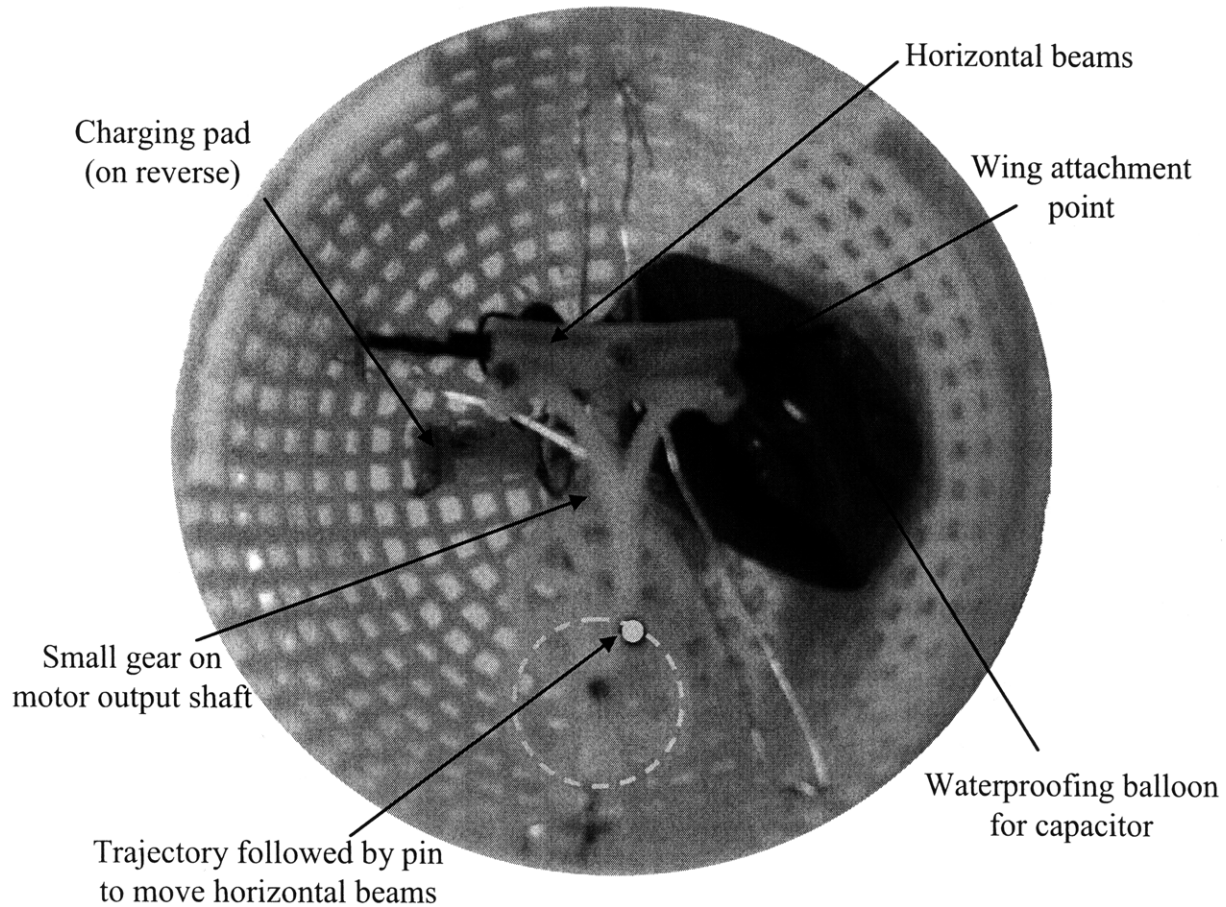


Figure 12. Mechanism powering the flying motion of the butterfly. Notice the motion created by one part of the wing originates from the horizontal line. Tails have been removed from device.

This gear also turns one of the two large gears visible from the front. Attached to the lower gear are two arms that move the horizontal beams in the figure. The beginning point of the devices wing structure can be seen in black. This is only one set of the wings that was present when the

device was purchased. The other set came out of the lower end of the horizontal pieces in the figure.

This device has the benefit of creating both sides of motion in a relatively compact front to back space. The system is powered by a handheld device that charges a small capacitor with two AA batteries. In order to make the device useable for a test in the silicon oil, the capacitor and its circuit board were encompassed in a balloon which was tightly tied off to be liquid tight. The motor remained attached to a casing of injection molded pieces as provided by the toy manufacturer. The wings were modified and 0.5 mm guitar string was attached to the beginning part of the wire used for the wings.

In order to test this device in the silicon oil, it needed to be encompassed by a spherical shell. The egg enclosure used in the previous model was too small to be used for this device. Instead, a plastic sphere used for making yarn ball decorations was found at Michael's Arts and Crafts store, headquarters located in Irving, Texas. This sphere came apart in two pieces but could snap together. It also consisted of number small square openings or roughly the same size which could be used to poke things through the surface. It was relatively easy to either melt or clip through some of the divider pieces in order to make room for larger pieces that needed to extend outside the spherical shell, such as the charging pads, a close up of which can be seen in Figure 13 below.

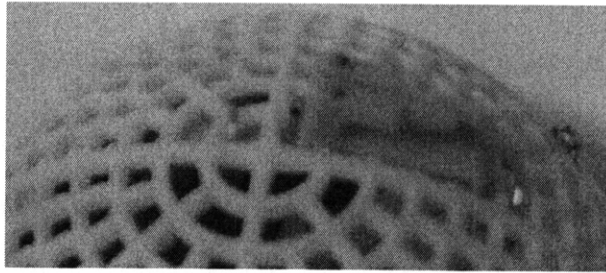


Figure 13. Close up of one piece placed outside the spherical shell. These are the charging pads used to connect to the handheld charger included with the purchase of the butterfly toy.

Once the pieces were all installed and secured inside the sphere, a space had to be cut so that the tails could move freely. Because of the actuation method of this device, the tails needed an entire slit opening in order to move. This is not ideal because then it is impossible to make the device watertight. Therefore the entire motor and its mechanism must be able to run in the environment of the silicon oil.

3.2.2 Butterfly Device Results

Because the mechanism had to run through the silicon oil, it was hard for the small pager motor to create the forces needed to propel the device for a decent amount of time. Typically, on a full capacitor charge, the device would run at a steady state for about thirty seconds. However, with capacitor was fully charged and the device in the silicon oil, the motor was typically only able to produce five or six cycles of tail flapping. This proved inadequate to measure any trajectory of the device.

Also, because of the nature of the mechanism, when the movement was slowed down in the silicon oil, it became apparent that one tail had a slight phase shift from the other. One tail would reach its maximum amplitude slightly before the other.

3.3 Powered Motor Device

After noticing that small pager motor was able to produce enough force to move the tails through the silicon oil, even though the duration was not as long as desired, it seemed that a constantly powered motor would have to be used for the device.

3.3.1 Design & Construction

A motor was harvested from a build your own hovercraft model kit similar to the one pictured below in Figure 14 which could be purchased from Interplay, headquartered in Buckinghamshire, United Kingdom.⁶



Figure 14. Hovercraft kit from which motor was used. As the kit was not purchased by the author, specifications for the motor were not available.

Attached to the shaft of the motor was a small round plastic piece. This piece was salvaged from the top of a pill bottle, and later sanded down to avoid jamming and rubbing issues. By using the same Scotch yoke mechanism on one side of the motor, a similar two point actuation method was created for moving the two tails at the same time as seen in Figure 15.

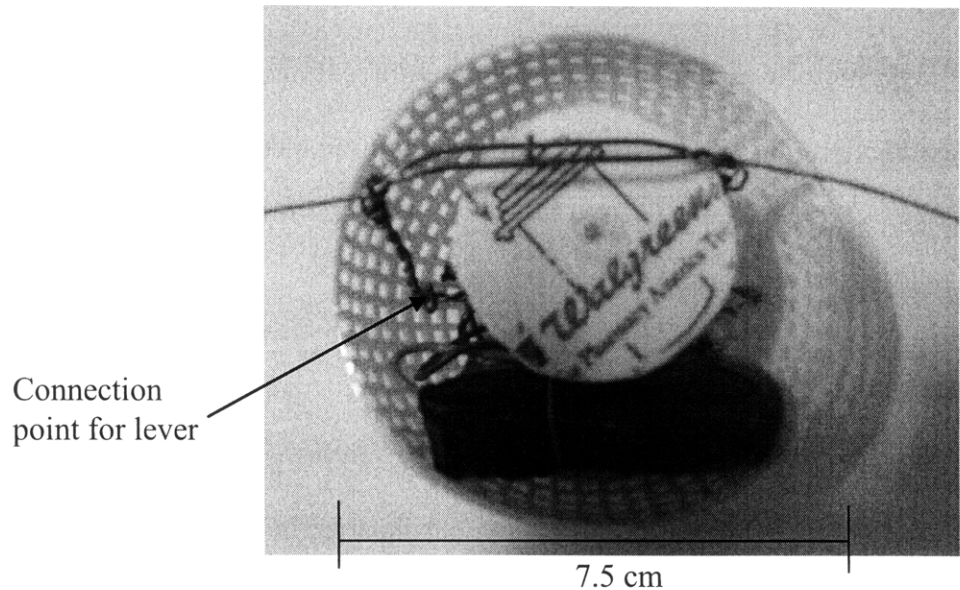


Figure 15. Mechanism for powered motor device. The black piece at the bottom is the battery pack for the motor, two AAA's are enclosed.

This design is not ideal because when the tail moves from its highest point to its lowest point, the distance between the attachment point and the lever point is changed by a small amount which changes the length of the tail which, in turn, changes ratio between the radius of the device and the tail length, which has a defined optimal value.

In order to encase this device in a spherical shell, the same plastic sphere casing was used as was utilized in the Butterfly design, giving the last two devices the same body length of 7.5

centimeters. This device was also made to be watertight by adding two balloons with a hole for the tails to protrude from as seen in Figure 16. Also, the on off switch remained inside the waterproofing, because it was still able to be switched through the balloons.

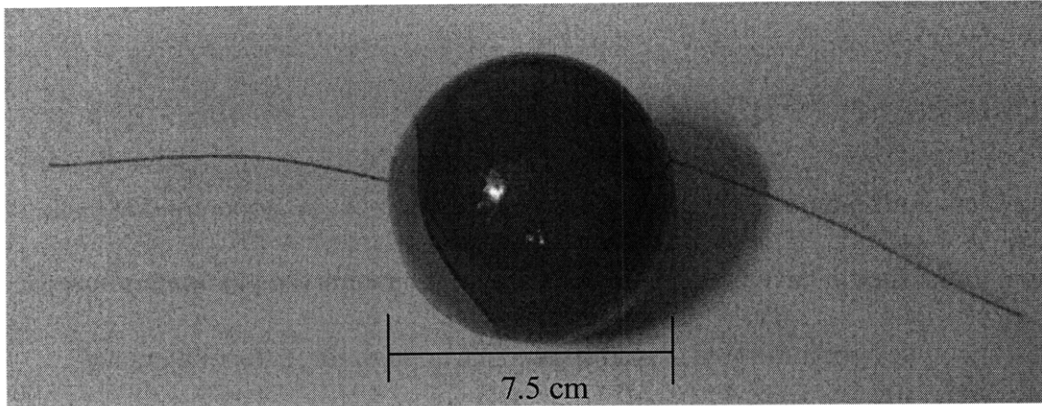


Figure 16. Powered motor device ready for immersion into silicon oil.

3.3.2 Powered Motor Device Result

In the air, this device would run at about two flaps per second. However, it would often get jammed because the 20 gauge copper wire used would not retain its exact shape when the force from the motor was applied to a piece that had become temporarily lodged. Once one piece was deformed, it would cause other pieces to become stuck and therefore increase the deformation on the inside of the device. Because the device was covered by two balloons, whenever the mechanism stopped functioning correctly, the entire device had to be dismantled and the inside evaluated and corrected. Because of this, finding the optimum configuration of the inside was quite frustrating and every time the device was moved, it had to be taken apart and reassembled.

With the watertight shell, the large amount of air trapped inside caused the device to float. In order to counteract this, a small balloon was added inside that was filled with spherical lead weights

until the correct buoyancy was achieved. After numerous adjustments inside the device, it was found that the larger motor was still unable to produce the force needed to propel the device in the silicon oil.

4 Conclusions and Future Work

After the swimmers built did not work as they were expected to, it was apparent that back of the envelope calculations should have been made before trying to match the parameters specified in earlier articles. Each design, in the end, failed to produce enough force to counteract the viscosity of the silicon oil.

In order to make sure that the next motor used is sufficient to power the swimmer through the silicon oil, it is helpful to know the drag force, F_d , on the swimmer in the liquid,

$$F_d = \frac{1}{2} \rho v^2 C_d A \quad (1)$$

where ρ is the density of the fluid, v is the velocity of the object relative to the fluid, C_d is the coefficient of drag based on the objects shape, and A is the surface area in contact with the fluid.

However, because this system operates in low Reynolds number and the device can be approximated as a sphere, it is more useful to use Stoke's law for the drag force,

$$F_d = 6\pi\mu RV \quad (2)$$

where μ is the dynamic viscosity of the fluid, R is the radius of the sphere and V is the velocity of the sphere relative to the fluid. Using 2.28 kg/m s as the dynamic viscosity, 3.75cm as R and 1cm/s as the velocity, the force from drag, F_d , is found to be approximately 16 mN. Using the design in the Powered Motor Device, the motor would need to be able to apply this force at the two sides where the tails attach. Assuming the attachment point is 2cm away from the motor

shaft, and adding a safety factor of 2, the motor needs to be able to provide at least 0.7 mNm of torque before stalling. This also assumes that the device previously fabricated by bending copper wire, could be machined out of a material that would hold its shape to prevent jamming. A quick internet search finds a suitable motor can be found at MicroMo Electronics, headquarters located in Clearwater, Florida.² This motor would provide 1.04 mNm of stall torque in a package that is 15mm in diameter and 16 mm long, plenty small enough to be incorporated into the third design.

It is also suggested that a better connection mechanism be made for the lever noted in Figure 15. Because the lever was a square U shape, the distance between the two arms was made for the size of the rotating disk. Previous attempts tried adding rings of copper wire that would sandwich either side of the attachment point to keep it from sliding. However it was found that the edges of the ring more often hindered the movement of the lever all together rather than preventing translation. Attempts in making the U shaped piece stay centered on the rotating disks proved inadequate, a better solution should be found before attempting the design again.

In conclusion, it was learned that it is always better to do the back of the envelope calculations before testing something. Sometimes it seems that it will be quicker to build a prototype and test it, however if the prototype fails, little information is gained to make a better next model. With the information gained during this research, future models can be made based on the calculations in this section that will be able to test if the theory set out by Lauga and Yu correctly defines the most efficient way to swim to at low Reynolds number.^{4,10}

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