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SAT Mathematics Standardized Test Manual for High-Performing High School Students

By

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ABSTRACT

Most high school standardized testing preparation materials are geared towards the average student scoring in the 50th percentile. There are few resources available to lower and higher scoring students who have different needs. The SAT is a standardized test that American high school students must take as part of the college application process. It includes subsections on Mathematics, Writing, and Critical Reading. The goal of this project is to provide a comprehensive SAT Mathematics study guide for high-achieving high school students initially scoring in the 70th percentile. Unlike other study materials, this guide focuses on the content and strategies needed for this particular group of students to perform better on the SAT. In the future, this study guide will include material for the Writing and Critical Reading subsections, and provide students with more sample SAT problems.

Thesis Supervisor: Barbara Hughey
Title: Instructor
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Introduction

My first job was in sixth grade tutoring my friend LaToya Porter. LaToya was struggling in Spanish, and her mom was willing to pay me big bucks to help her do better: $6 an hour. For someone whose greatest expenses were candy and video games it was more money than I could imagine. Mrs. Porter would drive to my house, pick me up, and then drive both LaToya and me to the library so we had a quiet place to study. Every session followed the same routine: thirty minutes of vocabulary study, thirty minutes of grammar, thirty minutes of workbook assignments, and the last thirty minutes were spent reading Spanish tabloid magazines that I would borrow from my mother. LaToya loved Hollywood gossip, and reading about her favorite celebrities Will Smith and Michael Jordan in Spanish kept her focused and learning. One of the keys to teaching, I discovered, is to keep your students engaged and to make the material relevant to their lives.

Eight years later, I was in search of a part-time job and saw an ad for teaching at a local standardized test preparation company. I was quickly hired and became certified to teach both ACT and SAT courses, the two standardized tests high school students must take in order for admission to college. A full course entailed 10 sessions that were 3 hours long and followed a set syllabus. This setup was problematic from the start. Each class had upwards of 20 students all at different ability levels, it was scheduled late at night at the end of long school days, and it was too brief to ensure that students were fully prepared. The testing materials were geared towards the average test-taker, leaving struggling students at a loss and high-performing students uninterested.

A year later I left the company and started teaching on my own. I no longer taught in large classrooms but in private sessions tailored to every student. I could teach each student at a
comfortable pace and at the appropriate level. I used readily available test preparation materials but would always supplement with material of my own. With every student I gained insight into how to better prepare each student for these standardized tests. I came up with better techniques and provided extra content that was not readily available. I spent much time searching for more comprehensive preparation materials to no avail.

The majority of students I teach are high-performing: honors students with high grades and a thirst for knowledge. These students initially score higher than average students on the ACT and SAT, scoring in the 70th percentile. They have high dreams and aspirations of attending an Ivy League university or winning a college scholarship. They understand the competitiveness of college admissions and want to ensure they have the best preparation possible. Ensuring a good chance at admission to these highly competitive schools requires a standardized test score in the 90th percentile. How could I do my best to guarantee each student performed well? By writing a standardized testing manual of my own.

This SAT Mathematics study guide is geared towards the high-performing student. It includes techniques and content that cannot be found in other materials. The study guide is written in plain, applicable language that students can easily grasp. Standardized testing is one of the most daunting, nerve-wracking experiences every high school student must face. It is imperative to present the necessary content and techniques in a comfortable, relatable manner.

Over the course of the semester, each chapter in the study guide has gone through at least three iterations and been independently tested with current and former students. The goal of the manual is to provide students with a thorough discussion of the content and strategies necessary to perform well on the SAT, utilize clear and concise language that is easily understood, and to ensure a high-quality learning experience. Strategies and content that are not familiar to high-
achieving students are presented in detail with step-by-step examples. Basic mathematics content that is well known by these students is given limited discussion with solutions to practice problems placed at the end of every chapter. In early versions of the study guide every problem had detailed solutions after every example, but many students found this cumbersome and unnecessary. They already knew how to solve easy problems but needed more guidance on the more difficult material.

This study guide presents the Math content and strategies necessary for students to perform well on the SAT Mathematics subtest. In the future, I hope to supplement this material with more sample problems and content and strategies for the SAT Writing and Critical Reading sections. This thesis is the start of a work in progress; I envision a final study guide containing around 400 pages that has gone through many drafts and more user testing.

I could have never imagined that what was my first job as a child would be the same job I would have many years later. I found a passion and love for teaching that I have not been able to find in any other career. My early experience with LaToya laid the foundation for my philosophy in teaching: focus on the individual needs of every student, give each student the necessary tools for success, and to create an enjoyable learning experience that inspires a love for learning.
SAT Introduction

You're an A student. You take the hardest classes at your high school and are a superstar in all of them. (Well, except for maybe English...) You're fluent in Swahili and are a master at the didgeridoo that you play in your weekend garage fusion band. You've reached Level 50 in World of Warcraft and are morally offended when someone pronounces Linux "Line-ucks." You're on your way to finding the cure for cancer and have dreams of attending MIT. But there is one thing in the way, only one thing stopping you: that dreadful SAT!
You are probably reading this book because you know you have to take the SAT, and want to improve your Math score and learn more about the test. So before we delve into the finer nuances of synthetic division and try to crack Fermat's Last Theorem, let's take a few moments to discuss some facts about the SAT.

**What does the SAT test?**

Even though the SAT has math and reading problems it does not reflect what you have learned in school. While you have memorized every proof in multivariable calculus and are the next Yo-Yo Ma in training, you still struggle with the SAT.

"How come I'm not doing well on the SAT when I do so well in school?"

"Why am I only getting a 650 on Math, when I should easily be getting a perfect score?"

There is only one simple explanation to all the above: the SAT is not school. Many students and parents believe there is a direct correlation between how well a student performs in their classes to how well they perform on standardized tests. This is simply not true. The types of questions asked on the SAT are not the types of questions you are asked in school. School is meant to help and guide you. The SAT is not. The test writers claim that the SAT measures a student's "reasoning ability" but all it tests is how well you take the SAT. This is not a test of how intelligent you are or how well you will do in college.
Who writes the SAT?

Educational Testing Service (ETS for short) is a nonprofit company based in Lawrenceville, New Jersey that administers and writes the SAT. Many believe that college professors or other high school teachers write each exam question, but in fact ETS has a full-time staff of professional test writers whose only job is to write more SAT questions! ETS writes not only the SAT, but also many other standardized tests as well including Advanced Placement (AP) tests and the Graduate Record Examination (GRE) for graduate school. And you thought you were done with standardized tests after the SAT!

What is on the SAT, and how is it scored?

The SAT is divided into three subject areas: Math, Writing, and Critical Reading. The SAT is 3 hours and 45 minutes long, and the subject areas are divided as follows:

Math
* two 25-minute sections, and one 20-minute section containing multiple-choice questions and student-produced response questions which ETS affectionately calls "grid-ins"

Writing
* one 25-minute Essay section in which you present your point of a view on a question ETS asks.
* one 25-minute section, and one 10-minute section containing multiple-choice questions testing you on grammar and parts of speech
Critical Reading
* two 25-minute sections, and one 20-minute section containing multiple-choice questions testing reading comprehension and vocabulary

Included on the SAT is an additional 25-minute Experimental section, which may be Math, Writing or Critical Reading. This Experimental section is not counted toward your overall score but there is no way of knowing what section it is, and when it will appear during the test! For this reason, it is important to treat every section as if it will count toward your score!

The Essay section always comes first on the SAT, while the 10-minute Writing section always comes last. The other sections can come in any order, and you will not know until your test day.

The Math, Writing, and Critical Reading areas are scored on a scale of 200 to 800, and the three scores are then totaled for a composite score between 600 and 2,400. The average score is about 500 for each area, or about 1,500 total. Each student receives a score report a few weeks after he takes the test, and the report includes your individual scores along with a percentile rank. If you score in the 85th percentile, it means that you scored better than 85% of test takers.
Why should I use this book to help me prep for the SAT?

This textbook is designed to help students who are already high performing and initially scoring around the 70th percentile or higher. You have done decently well on the Math test, but are shooting for a perfect 800. Other textbooks are geared towards the average test taker and simply do not go into the necessary depth needed to ace this test. In this book, not only will you learn all of the basic math skills that you will find in other books, but also you will be exposed to the most difficult math problems and concepts found on the SAT that you will master to achieve that perfect score.

So sit back, relax, and we'll be on our way!
Math Introduction

Most of the math concepts tested on the SAT are from math classes taken during your freshman and sophomore year of high school. That includes Pre-Algebra, Elementary Algebra, and Geometry. There are also a few easy problems testing you on Advanced Algebra concepts as well. There are no logarithms, no trigonometry, no hyperbolas or ellipses, so you can rejoice!

What makes SAT Math problems so difficult is not that they test difficult concepts; it is that the problems are designed to trick you. Math problems in school are designed to reinforce and help you remember concepts. Math problems on the SAT are designed to hinder and confuse you.
The Question Types

There are two types of Math questions on the SAT: multiple-choice and grid-ins. Multiple-choice questions are the most common on the SAT and each question has five possible answer choices. Grid-Ins are ETS’s cute name for free response questions, where you are not given answer choices and must come up with a response on your own. We’ll explain the grid-ins more in depth later.

There are 3 individual Math sections on the SAT, and they are as follows:

- one 25-minute, 18 question section containing multiple-choice and grid-ins
- one 25-minute, 20 question section containing only multiple-choice problems
- one 20-minute, 16 question section containing only multiple-choice problems

Each individual chapter will focus on how to solve the various types of problems you will find on these sections.
Basic Strategy

Before we delve into the important math concepts you need to know for the SAT, it is important to lay down the basic strategy for approaching SAT Math problems.

“Strategy? What strategies could possibly be important for the SAT?”

There are many strategies that are imperative to understand in order to perform well on this test. What many students do not realize is that the key to performing well on the SAT is 50% knowledge and 50% strategy. You may have memorized the distance formula and derived the Pythagorean theorem all on your own, but if you do not know how to apply the concepts in the manner SAT expects then it is impossible to do well on this test. But that’s why you’re reading this book! Let’s lay out these strategies now, shall we?

SAT Math problems are arranged in increasing order of difficulty

and

Do the easiest problems first

Unlike the SAT Writing and Critical Reading sections, SAT Math problems within each section are arranged from easiest to hardest. On the section containing both multiple-choice and grid-in problems, difficulty is arranged within each section part.
Many students want to know if they should do the harder problems first because they will take more time, or save them for later. Always get all your easy points first, and that way you’ll have more time for the harder problems later. If you try and do the more difficult problems initially, you may run out of time and not get all the easy points you should.

Here is a nice graph explaining how SAT Math problems are arranged:
Reading on SAT Math is just as important as reading on any other section

To better emphasize this, let's start with a typical SAT problem:

1. Professor Chun buys a delicious vegetable quiche from his favorite bakery. On Saturday he eats \( \frac{1}{3} \) of the quiche. On Sunday, he eats \( \frac{1}{3} \) of what is remaining. How much of the quiche did Professor Chun eat?

   (A) \( \frac{1}{3} \)
   (B) \( \frac{2}{3} \)
   (C) \( \frac{4}{9} \)
   (D) \( \frac{5}{9} \)
   (E) \( \frac{7}{9} \)

Did you pick B? Makes sense right? The good professor ate \( \frac{1}{3} \) on Saturday and \( \frac{1}{3} \) on Sunday, giving us a grand total of \( \frac{2}{3} \) of the quiche eaten. But this choice is incorrect! If you picked answer choice B, you didn’t read the problem carefully. He does eat \( \frac{1}{3} \) of the quiche on Saturday, but on Sunday he eats \( \frac{1}{3} \) of what is remaining. Let’s go through this problem step-by-step:
<table>
<thead>
<tr>
<th>Professor Chun</th>
<th>What does he eat?</th>
<th>Eaten</th>
<th>Remains</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saturday</td>
<td>$\frac{1}{3}$ of the quiche</td>
<td>$\frac{1}{3}$ of the quiche</td>
<td>$1 - \frac{1}{3} = \frac{2}{3}$ remains</td>
</tr>
<tr>
<td>Sunday</td>
<td>$\frac{1}{3}$ of what remains</td>
<td>$\frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$</td>
<td>$\frac{2}{3} - \frac{2}{9} = \frac{6}{9} - \frac{2}{9} = \frac{4}{9}$</td>
</tr>
<tr>
<td>Total Eaten</td>
<td>$\frac{1}{3} + \frac{2}{9} = \frac{3}{9} + \frac{2}{9} = \frac{5}{9}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Doing this problem carefully, we arrive at the correct answer of D. But alas, some of you have done these very steps, but marked your answer as C. What happened? Remember, the question asks how much of the quiche Professor Chun has *eaten*, not how much *remains*.

The above example illustrates the most common mistake high-scoring students make on the SAT. More than half the mistakes you will make are because you misread the problem. Just because this is a Math test does not mean you can ignore the words and go straight to the numbers. Let Professor Chun and his deliciously flaky quiche remind you of the importance of reading on SAT Math!
Your calculator is your friend

While ETS says calculators are “optional” on the SAT exam, in reality they are required. Calculators help prevent careless math errors, are faster than we in performing calculations and are just all around awesome. On the other hand, do not use your calculator as a constant crutch. Calculators are meant to help your brain solve math problems, and not serve as a replacement for thinking!

Use a graphing calculator on the SAT that you already feel comfortable using. (If your calculator has a QWERTY style keyboard, find a replacement because these types are banned on the SAT.) Why a graphing calculator? Some problems are easier to solve visually, and having the ability to graph parabolas and lines is quite useful. More importantly, with whatever calculator you choose, you must be familiar with its functions and how to use it properly! Some students go out right before the test and buy a fancy new calculator but get stuck on the test because they don’t know how to use it. Don’t be one of these people!

Oh, and don’t forget a fresh set of batteries before the test!
Sometimes guesstimating is better than solving

Let's look at another example:

2. Square $ABCD$ is circumscribed by circle $O$. The shaded region inside the square is composed of the intersection of two circles centered on $B$ and $D$. If circle $O$ has a radius of 5, then what is the area of the entire shaded region?

(A) $25\pi - 50$
(B) $50\pi - 100$
(C) $25\pi$
(D) $50\pi$
(E) $50\pi + 5\sqrt{2}$
If this problem makes you break out into a cold sweat, trust us, you are not alone. There are a number of things that make this problem universally scary to SAT students everywhere: weird shapes, odd shading, funky use of $\pi$ and square roots. The list goes on and on. Solving this would take pages of work and cost you too much valuable time when you could be solving other problems.

Guesstimating is where you make an educated estimate on a math problem when you are not sure how to go about solving. This is a prime example where guesstimating and eliminating answer choices will help us more than actually solving. Let’s go step by step on how to approach this problem.

**Step 1: Figure out what you know**

We know that circle $O$ has a radius of 5, giving us an Area = $\pi r^2 = 25\pi$. Since the shaded regions are inside the circle, there is no possible way that the shaded area could be larger than the actual circle!

Looking at circle $O$, we can reasonably guesstimate that about 75% of the circle itself is shaded. Knowing the area of the circle and estimating what percent of the circle is shaded, we can now solve the problem.

Shaded Area of Circle $O = 75\% \times 25\pi = \frac{75\pi}{4} = 58.9$
**Step 2: Eliminate bad answer choices**

By making a reasonable estimate that the shaded area will be close to 58.9. Approximating all the values of our answer choices:

(A) $25\pi - 50 \approx 28.5$
(B) $50\pi - 100 \approx 57.1$
(C) $25\pi \approx 78.5$
(D) $50\pi \approx 157$
(E) $50\pi + 5\sqrt{2} \approx 164.1$

The only answer choice that is even close to our estimate is B. Answer choice A is much too small for our shaded region, answer choice C is the area of Circle $O$, and choices D and E are much too large.

Voila! Guesstimating helped us arrive at the correct answer without even solving.
If at first you don’t succeed, try, try again.

If you cannot solve a problem using your preferred way, always look for alternative methods to help you find the solution. Did you try graphing the equation on your calculator? Did you draw a figure? In this book we will learn alternative methods of solving such as Plug & Chug or Plugging in the Answers to help us solve SAT problems.

One goal of this book is to reinforce math concepts you learned in school and to teach you multiple approaches in attacking SAT Math problems.

Remembering these basic strategies will guide us through each Math section on the SAT, and now we’re ready to learn some concepts!
Math Vocabulary

Vocabulary is not important just for Critical Reading, but for Math as well. ETS knows students can calculate arithmetic problems using a calculator, so they design Math questions testing your knowledge of basic Math terms. Here is a list of common Math Vocabulary you must know for the SAT.
Definitions

**Integer** - Integers are the counting numbers; no fractions nor decimals. 3, -1, 0, 1, 8, and 100 are all integers. $\frac{1}{2}$, 0.60, and $\sqrt{5}$ are not integers.

**Positive** - Numbers more than zero are positive.

**Negative** - Numbers less than zero are negative. Zero is neither positive nor negative and is considered "neutral."

**Non-negative** - Numbers that are positive and include zero. Note the difference positive and nonnegative; ETS likes to trick students on this definition!

**Greater** - Greater means further to the right on the number line. For example, 4 is greater than -1, and -3 is greater than -7.

**Less** - Less means farther to the left on the number line. For example, 6 is less than 9, and -12 is less than -8.

**Distinct** - Distinct means different; do not count more than once! For example, if set A = \{-1,0,3,4,5\}, then set A has 5 distinct members.

**Even** - Even integers are divisible by 2. For example, -4, -2, 0, and 6 are even. Note that zero is even.

**Odd** - Odd integers are not divisible by 2. For example, -3, -1, 3, and 7 are odd. Only integers can be even or odd.
Consecutive - Consecutive means in order, one right after the other. For example, -2,-1,0,1 and 2 are consecutive integers, and 3, 6, 9, and 12 are consecutive multiples of three.

Rational - Rational numbers are numbers that can be expressed as a fraction or decimal. Decimals that repeat without end are rational. For example, 0.35, $\frac{1}{3}$, and $\frac{5}{7}$ are rational numbers.

Irrational - Irrational numbers are decimals that never end or repeat. The most commonly used irrational numbers on the SAT are $\pi$ and square roots, such as $\sqrt{5}$.

Sum - Sum means the result of addition. For example, the sum of 7 and 4 is $7 + 4 = 11$.

Difference - Difference means the result of subtraction. For example, the difference between 27 and 15 is $27 - 15 = 12$.

Product - Product means the result of multiplication. For example, the product of 5 and 4 is $5 \times 4 = 20$.

Quotient - Quotient means the result of division. For example, the quotient of 20 and 5 is $20 \div 5 = 4$ and not $5 \div 20$. Here order matters!

Remainder - The remainder is the part left over when two numbers won't divide evenly. For example, $17 \div 3 = 5$ with a remainder of 2.

Factor (Divisor) - The factors of a number are the things you can multiply to get to that number. For example, the factors of 8 are 1, 2, 4, and 8. Note that the factors of a number always include 1 and itself.
**Multiple** - The multiples of a number are the things that are divisible by that number. For examples, the multiples of 4 are 4, 8, 12, 16, etc. Note that the multiple of a number always includes the number itself.

**Prime** - A prime number is a positive integer with two distinct factors, 1 and itself. The first 10 prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29. Note that 0 and 1 are not prime!

**Prime factor** - Prime factors are the factors of a number that are also prime. For example, the prime factors of 10 are 2 and 5.

**Reciprocal** - The reciprocal of a number is 1 divided by that number. Another way to think of a reciprocal is to turn a fraction upside-down. For example, 2 and 5 are reciprocals, and 6 and 1 are reciprocals.

**Rules of Zero** – Zero is neither positive nor negative. Zero is even. Any number multiplied by zero is zero. Any number divided by zero is undefined. ($\frac{0}{0}$ is undefined!)

Think you know your vocabulary well? Test your knowledge with a Mini-Quiz!
Mini-Quiz!

1. How many nonnegative integers are even and less than or equal to 12?
   (A) 5
   (B) 6
   (C) 7
   (D) 8
   (E) 9

2. Set A is comprised of the factors of 36. Set B is comprised of the multiples of 12. How many numbers do Set A and Set B have in common?
   (A) 1
   (B) 2
   (C) 3
   (D) 4
   (E) 5

3. Which of the following numbers is irrational?
   (A) \( \frac{1}{3} \)
   (B) 0.67
   (C) 5.72839
   (D) \( \sqrt{8} \)
   (E) \( \sqrt{9} \)
4. What are the prime factors of 24?

(A) \{1, 2, 3, 4, 6, 8, 12, 24\}
(B) \{1, 2, 3\}
(C) \{2, 3\}
(D) \{2, 4, 6, 8, 12, 24\}
(E) \{1, 3\}

5. \(|-12 - -4| - 10\)

(A) -2
(B) 0
(C) 2
(D) 4
(E) 6

6. The product of odd integers from 1 to 9 exclusive is how much greater than the sum of the even integers from 4 to 10 inclusive?

(A) 1
(B) 9
(C) 56
(D) 70
(E) 896

7. A number q has a remainder 3 when divided 5. What is the remainder when 4q is divided by 5?

(A) 4
(B) 5
(C) 6
(D) 7
(E) 8
8. What is the greatest common prime factor of 26 and 91?

(A) 1
(B) 2
(C) 7
(D) 13
(E) 26

9. The sum of a set of consecutive even integers is 14. The least number in the set is -12. How many numbers are in this set?

(A) 10
(B) 11
(C) 12
(D) 13
(E) 14

10. If \( x = | -4 - 5 | \) then what is the sum of \( x \) and the next even integer less than \( x \)?

(A) 17
(B) 18
(C) 19
(D) 20
(E) 21

You can check your answers on the next page.
Solutions to Mini-Quiz

1. C

Nonnegative integers include positive integers and zero, meaning the nonnegative integers less than or equal to 12 are \{0, 2, 4, 6, 8, 19, 12\} giving us a total of 7 integers.

2. B

The factors of 36 are \{1, 2, 3, 4, 6, 9, 12, 18, 36\}. The multiples of 12 are \{12, 24, 36, 48, 60, 72, \ldots\}. The numbers 12 and 36 are common to both sets, showing 2 numbers in common.

3. D

Irrational numbers are decimals that never repeat or end. Answer choice A is rational since \(\frac{1}{3}\) is a repeating decimal. Choices B and C are decimals that end, and answer choice E reduces to 3.

4. C

To begin, we find all the factors of 24 to be \{1, 2, 3, 4, 6, 8, 12, 24\}. The only factors from this list that are prime are 2 and 3. Remember, 1 is never a prime number!

5. A

For absolute values, solve inside the absolute value first.

\[ | -12 - 4 | - 10 = | - 8 | - 10 = 8 - 10 = - 2 \]
6. C

Inclusive means “including.” Exclusive means “not including.” The product of odd integers from 1 to 9 exclusive is $3 \times 5 \times 7 = 105$. The sum of even integers from 4 to 10 inclusive is $4 + 6 + 8 + 10 = 28$. The difference between 105 and 28 is $105 - 28 = 67$

7. A

First, we have to pick a number for $q$ that fits the criteria. If let $q = 8$, we see that 8 has a remainder of 3 when divided by 5. $4q = 4 \times 8 = 24$. When 24 is divided by 5, we get a remainder of 4.

8. D

List the factors of 26 and 91. For 26 the factors are {1, 2, 13, 26}. The factors of 91 are {1, 7, 13, 91}. The greatest common prime factor between 26 and 91 is 13.

9. E

This problem is a bit tricky but can be solved with a bit of tinkering. You know the smallest number in your set is -12 and the sum of all numbers is 14. To negate the -12, you know you need a positive 12 to balance out. The next consecutive even integer higher than -12 is -10. To balance out the -10 you need a positive 10. The next consecutive integer higher than -10 is -8, and you need a positive 8 to balance out the ---- 8. Continuing with this logic you know the sum of all consecutive numbers from -12 to 12 is zero! Add 14 to your set of numbers and you have found your set! The numbers in the set are {-12, -10, -8, -6, -4, -2, 0, 2, 4, 6, 8, 10, 12, 14} giving a grand total of 14 numbers.
10. A

Solving for $x$, we determine $x = |-4 - 5| = |-9| = 9$. The next even integer less than 9 is 8. The sum of 9 and 8 is 17.

**How many did you get right?**

0-5  D’oh! Try making flashcards of your terms and try again.
6-7  Study those terms a bit more.
8-9  Not bad! But we know you can do better!
10  You’re a Math Vocabulary master!
Grid-Ins

As you may recall, not all questions on the SAT Math are multiple choice. ETS affectionately calls these “Grid-Ins.” Why so? You must come up with the answers on your own and enter them on a special grid. A typical grid-in looks like this:
There are some key things to keep in mind as you do your grid-in problems. ETS has a very confusing, long explanation on how to answer these problems, but we'll try to do it as briefly as possible. Here are the Nine Rules of Grid-Ins:

1. **Approach every Grid-In like you would a multiple-choice problem.**

   All the math techniques you will learn are valid for Grid-In problems. You can still use your calculator and your knowledge of order of difficulty to answer these questions.

2. **No points are deducted for wrong answers.**

   As consolation for taking away all the answer choices, ETS does not deduct for incorrect responses. Because of this you must, without fail, **answer every grid-in question**!

3. **Always write your answer at the top of the grid, and bubble in accordingly.**

   Even though the scoring machine will only grade your bubbled responses, you should always write your answer in the grid itself. Why is that? It helps prevent making careless error when you bubble in. Plus, if you have to go back and change your answer, you can better identify your initial response.

4. **You can grid-in fractions or decimals, but not mixed numbers!**

   One nice thing about grid-ins is that ETS gives you flexibility in marking your answers by allowing you to use fractions or decimals. For example, if the correct answer is $\frac{3}{4}$, you may enter it as:
On the other hand, the scoring machine will not recognize mixed numbers. If your answer is $2\frac{1}{4}$, you must grid in either 2.25 or $\frac{9}{4}$ as an improper fraction.

If you try to jam in $2\frac{1}{4}$, it will be read as $\frac{21}{4}$, and marked wrong!

5. **Start gridding-in at the far left, and use each of the four spaces as necessary.**

Even though not every answer will use every space in the grid-in, it is important to start at the far left every time. Why is this? Say your answer is $\frac{2}{3}$. Here are the possible choices that ETS would mark as correct:

But say you’re feeling extra lazy, and mark your answer as such:

Both of these answers will be marked wrong! ETS wants the most accurate value possible, and does not reward lazy test takers! (Never be lazy on any test!)
6. Chop, don’t round!

Suppose your answer is some long decimal like .18375. ETS will award credit for the following responses:

\[ \boxed{.1 8 3} \] or \[ \boxed{.1 8 4} \]

ETS does not care if you round up to the last digit of a decimal that fits in the grid-in or not. So why make a big to-do about rounding? It’s one more place you can make a mistake, and save yourself the extra time for solving more problems.

7. The only possible answer choices for Grid-Ins are positive integers, fractions, or decimals.

Say your solution is \( \sqrt{2} \), but nowhere in the grid-in box do you see a square root sign! The grid-in box only can accept positive integers, fractions, or decimals. It cannot handle negative numbers, square roots, or variables. The solution box can also not handle mathematical constants like \( \pi \). If you get an answer with \( \pi \) in it, go back and check your work and do not round your answer. ETS would consider it unfair to students who do not have calculators to know what number \( \pi \) rounds to.

So what do you do if you get an answer like \( \sqrt{7} \) or \( 8\pi \)? It means you made a mistake in your calculation and should rework the problem.
8. **Do not reduce fractions that fit in the box.**

If your final answer is \(\frac{22}{4}\), your instinctual response is to reduce it. What have we learned thus far? Never waste time on a standardized test! ETS will give you credit if you grid-in that ugly looking \(\frac{22}{4}\), so why waste time figuring out that you can put in \(\frac{11}{2}\) or 5.5?

On the other hand, if your solution is \(\frac{12}{60}\), the box cannot fit the response so you must reduce to a fraction or a decimal.

9. **Always chop the percent sign.**

This seemingly trivial piece of advice is where high scoring students make the biggest mistakes on grid-ins. If you get 35% as your answer, always chop off the percent sign and grid in 35 like so:

\[
\begin{array}{c|c}
3 & 5 \\
\end{array}
\]

Do not convert to a fraction; do not convert to a decimal. If you put in .75, the scoring machine will interpret this as .75 percent and mark your answer incorrect.
Arithmetic is the branch of mathematics that deals with numerical calculations. You may think arithmetic is simply adding, subtracting, multiplying, and dividing. In fact, there are many, many arithmetic concepts that are important for the SAT! We’ll discuss all the ones you need to know to get a perfect score. The answers to each question in this chapter are at the end of the section.
Order of Operations

The order of operations tells the order in which we need to perform mathematical operations in arithmetic problems. You may remember the little mnemonic:

- **P**lease Parenthesis
- **E**xcuse Exponents
- **M**y Multiplication
- **D**ear Division
- **A**unt Addition
- **S**ally Subtraction

You perform these operations in the following order working from left to right:

1. Parenthesis
2. Exponents
3. Multiplication and Division
4. Addition and Subtraction

Remember, for multiplication and division you perform the operations in order from left to right even if division comes before multiplication! This same line of thinking applies to addition and subtraction; work from left to right even if subtraction comes before addition.

1. \[
[(7 + 3)^2 - (12 \times 5)] + (10 - 8)^3
\]

(A) 5
(B) 10
(C) 15
(D) 20
(E) 25
Exponents

Exponents mean repeated multiplication. “Five to the fourth power” means you multiply the number 5 four times.

\[ 5^4 = 5 \times 5 \times 5 \times 5 = 625 \]

The number “5” is known as the base. The number “4” is known as the exponent. Whenever you multiply, divide, or raise to a power expressions with exponents, use the following exponent rules when the bases are the same.

**Multiplying** exponents means

**Dividing** exponents means

**Power** to an exponent means

**Addition**

\[ n^4 \times n^3 = n^7 \]

**Subtraction**

\[ \frac{n^9}{n^3} = n^6 \]

**Multiply**

\[ (n^5)^4 = n^{20} \]

2. Which of the following is equivalent to the expression

\[ 3x^2 y^3 (2xy^2) \]?

(A) \( 5x^3 y^5 \)  
(B) \( 5x^2 y^6 \)  
(C) \( 6x^2 y^6 \)  
(D) \( 6x^3 y^5 \)  
(E) \( 6x^2 y^3 \)

3. If \( x = 12 \) and \( x^3 \cdot x^2 \cdot x^7 \cdot x^a = 1 \), then what is the value of \( a? \)

(A) \(-12\)  
(B) \(-8\)  
(C) \(4\)  
(D) \(8\)  
(E) \(12\)
Roots

If \( x^2 = 16 \), then \( x = +4 \) or \( x = -4 \).

However, on the SAT the square root of a number is defined as its positive root only.

\[ \sqrt{16} = +4 \]

Roots may be added or subtracted when numbers under the square root sign are the same.

When roots are multiplied or divided, put everything under the square root sign.

4. If \( x^2 = 9 \), then \( x^3 \) could be which of the following numbers?

(A) 3 only 
(B) 3 or -3 only 
(C) 27 only 
(D) -27 only 
(E) 27 or -27 only 

5. \( [3(\sqrt{2})^4 - 4(\sqrt{2})^3]^2 = ? \)

(A) \(-2\sqrt{2}\) 
(B) \(\sqrt{2}\) 
(C) 4 
(D) 16\sqrt{2} 
(E) 32
Sets

A set is a group of numbers where each number belonging to the set are called members or elements. We usually use brace notation.

Set A = {-2, -1, -1, 0, 3, 4} Set A has 6 members, 5 of which are distinct.
Set B = {-2, -2, 1, 3, 4, 5, 5} Set B has 7 members, 5 of which are distinct.

When referring to sets, often ETS asks for the union of a set or the intersection of a set.

To find the union of a set, combine all members of a set into a new set of distinct members. Union is denoted by the symbol ∪.
To find the intersection of a set, combine the members that belong to both sets into a new set of distinct members. Intersection is denoted by the symbol ∩.

A ∪ B = {-2, -1, 0, 1, 3, 4, 5} A ∩ B = {-2, 3, 4}

6. If Set X = {2, 3, 5, 7} and set Y = {3, 6, 7, 8}, which of the following numbers is in the union of set X and set Y, but not in the intersection of set X and set Y?

(A) {3, 7}
(B) {2, 3, 6, 8}
(C) {3, 6, 7, 8}
(D) {2, 5, 6, 8}
(E) {2, 3, 5, 6, 7, 8}

7. Set Z consists of all the prime numbers less than 11, then how many members are in set Z?

(A) 2
(B) 3
(C) 4
(D) 5
(E) 6
Mean, Median, and Mode

ETS will ask problems dealing with the mean, median, and the mode of a group of numbers. To help us better understand these definitions, we will use the following set of numbers.

Set A = {-6, -3, 1, 1, 3, 5, 8, 10}

The mean is the sum of all the numbers in the set divided by the number of numbers in the set.

Set A has a sum of 16 with 8 numbers in the set. The mean of Set A is 2.

The median is the middle number in the set. Arrange all numbers in the set from least to greatest; the middle number is the median. If there is an even number of numbers in the set, the median is the mean of the two middle numbers.

Since there is an even number of numbers in Set A, we must take the mean of the middle numbers, which are 1 and 3. The mean of these two numbers, and therefore the median, is 2.

The mode is the number that appears the greatest number of times in the set.

The mode of Set A is 1.

8. If the mean of six numbers is 28, and the average of four of these numbers is 33, what is the average of the other two numbers?

(A) 17
(B) 18
(C) 19
(D) 20
(E) 21
For which of the following values of $x$ is the median of the list of numbers above equal to the mean?

(A) 7  
(B) 11  
(C) 13  
(D) 15  
(E) 19

If $x + y + z = 15$, what is the mean of $x$, $y$, $z$, 7, and 8?
Percent Change

Percent change can be determined by the following equation

\[
Percent \ Change = \frac{Difference}{Original \ Number} \times 100
\]

This equation is valid for calculating percent increase and percent decrease.

11. What is the percent increase from 4 to 5?

The difference of 4 and 5 is 1. 4 is the original number.

\[
Percent \ Change = \frac{(5 - 4)}{4} \times 100 = 25\% \ increase
\]

12. What is the percent decrease from 5 to 4?

The difference of 5 and 4 is 1. 5 is the original number.

\[
Percent \ Change = \frac{(5 - 4)}{5} \times 100 = 20\% \ increase.
\]

Note that increasing from 4 to 5 is a 25\% increase, but going from 5 to 4 is a 20\% decrease. The percent change differs even though we are going back to the same number! ETS will commonly trick students in this way.
13. A dress costs $x$ dollars. If the price of the dress increases 30%, then decreases 30% from the higher cost, how much does the dress cost now in terms of the original price?

(A) 0.3x  
(B) 0.7x  
(C) 0.91x  
(D) $x$  
(E) 1.3x

14. For what value of $x$ does the percent increase from $x$ to 60 equal the percent decrease from 50 to 25?

(A) 15  
(B) 25  
(C) 35  
(D) 40  
(E) 60

15. A coffee table that normally sells for $400 is on sale for $320. Ming purchased the stereo at the normal price and Jared purchased the coffee table during the sale. By what percent was the price that Ming paid more than the price that Jared paid?
Probability

Probability is the chance that something will happen. The probability of an event can never be less than 0 or greater than 1. If an event has a probability of 0, then it will never happen. If an event has a probability of 1, it means that it has a 100% chance of happening.

\[ \text{Probability} = \frac{\text{# of outcomes fulfilling requirements}}{\text{total # of outcomes}} \]

16. Kathryn owns 40 Pink Floyd shirts: 6 of them are sleeveless, 22 of them are short-sleeved, and 12 of them are long-sleeved. If she chooses a shirt at random, what is the probability that the shirt will not be long-sleeved?

(A) \( \frac{3}{20} \)

(B) \( \frac{3}{10} \)

(C) \( \frac{9}{20} \)

(D) \( \frac{11}{20} \)

(E) \( \frac{7}{10} \)
Cindy has a bag of lollipops. 35 percent of the lollipops are cherry, 15 percent are orange, 20 percent are grape, 5 percent are apple, and the other 15 lollipops are lemon. How many lollipops are in the bag?

(A) 24  
(B) 30  
(C) 36  
(D) 45  
(E) 60

Rates and the “Dirt” Formula

Rate word problems involve either an object moving at a constant speed, or work performed at a constant rate. Whenever a problem asks for “how fast”, “how far”, or “for how long” it usually indicates a rate problem. The rate formula is:

\[ d = r \times t \]

where \( d \) is the distance traveled or work performed

\( r \) is the rate

\( t \) is the time

This is also known as the “dirt” formula since the formula itself looks like it’s spelling the word dirt!

Whenever you deal with rate problems, it is important that units for rate and time agree with the units for distance. Did the problem give you a rate in miles per hour and time in hours, but asks for the total number of minutes? ETS will trick you by using the wrong units so make sure you convert to the right quantities!
18. Cyril can make 20 chocolate chip cookies in 40 minutes. Phil can make 30 chocolate chip cookies in 15 minutes. If Cyril and Phil work together, how many hours does it take for them to make 600 chocolate chip cookies?

19. A plane leaves Miami at 11:00 AM and arrives in New York at 2:30 PM. Another plane leaves Miami at 1:30 PM headed towards Boston. If both planes travel at the same speed and New York is 1050 miles from Miami while Boston is 1200 miles from Miami, at what time does the plane arrive in Boston? (Miami, New York, and Boston are all in the same time zone.)

(A) 2:30 PM
(B) 3:30 PM
(C) 4:30 PM
(D) 5:30 PM
(E) 6:30 PM
20. A train from Chicago to St. Louis is traveling at 80 miles per hour. An airplane traveling from Chicago to St. Louis is traveling at 400 miles per hour. If the distance between Chicago and St. Louis is 250 miles. How much longer does it take to get travel to St. Louis by train then by airplane, in minutes?

21. It normally takes Jason 30 minutes to run 4 miles. How long, in hours, does it take Jason to run 2 miles if he is traveling at half his normal speed?

(A) 0.5 hours
(B) 1 hour
(C) 1.5 hours
(D) 2 hours
(E) 2.5 hours
Solutions to Arithmetic

1. A

Taking care of the exponents and parenthesis first, the equation simplifies to
\[ 40 + 5 = 8 \]

2. D

When multiplying complex exponents remember to multiply like terms. The
coefficients multiply to 6, but the exponents are added!

3. B

The fact that \( x = 12 \) is not necessary. Using the rules of exponent multiplication,
\[ x^3 \cdot x^2 \cdot x^7 \cdot x^a = 1 \] simplifies to \( x^{12+a} = 1 \). What value of \( a \) will give us a value of 1?
Whenever you raise a number to the 0\(^{th} \) power, the solution is 1. We must find
the value of \( a \) that will make the entire exponent 0. \(-12\) does the trick.

4. E

\( x^2 = 9 \) indicates that \( x \) could be \(+3\) or \(-3\). Plugging both these values into the
equation we find solutions of 27 or \(-27\).

5. E

Simplifying the bracket leaves us with \( (4\sqrt{2})^2 \), which reduces to 32.
6. D

First, find the union of set X and set Y. \( X \cup Y = \{2, 3, 5, 6, 7, 8\} \). Then find the intersection of set X and set Y. \( X \cap Y = \{3, 7\} \). Now, take out the members that are common to both sets and you’re left with \{2, 5, 6, 8\}.

7. C

Do you know your prime numbers? The prime numbers less than 11 are 2, 3, 5, and 7. Remember, 1 is not prime!

8. B

We solve by finding the total of the other two numbers first. If six numbers have an average of 28, their total is 168. If four of those numbers have an average of 33, their total is 132. This means the total of the other two number is 168 – 132 = 36. The average of these two numbers is 36 / 2 = 18.

9. E

Read careful and test your numbers! When \( x = 19 \), the median and the mean are 11.

10. 6

The mean is defined as the number total divided by the number of numbers. We do not need to find the individual values of \( a, b \), and \( c \). The total is 30 and there are 5 numbers in the list giving a mean of 6.

11. & 12.

Solutions in workbook.
13.  C

Plug in a value for $x$. Pick an easy number like $100!$ If the dress price increases 30%, the new cost of the dress is $130. The price then decreases 30% from $130. This gives us a price of $91. What answer choice gives us $91 when we plug in $100? Letter C!

14.  D

First, we determine there is a 50% decrease from 50 to 25. Next, plug in the answer choices. From what number to 60 gives a 50% increase? Answer choice D, 40.

15.  25%

Be careful! We are comparing the price Ming paid to the price Jared paid. When plugging into our percent change formula, the difference is $80 but the original number is $320. We want to see Ming’s price in relation to Jared’s. This gives us a 25% increase

16.  E

The number of shirts that are not long-sleeved is 28. Probability is $\frac{28}{40}$ which reduces to $\frac{7}{10}$

17.  E

Find the percentage of other lollipops in Cindy’s bag. 75% of the lollipops are cherry, orange, grape and apple, while 15 are lemon. This tells us that 25% of the lollipops are lemon. Plug in 25% and 15 into the probability equation, and solve for the total. There are 60 lollipops in the bag.
18. 4

Convert Cyril and Phil's rates to cookies per hour. Cyril makes 30 cookies per hour while Phil makes 120 cookies per hour. When Cyril and Phil work together, their combined rate is 150 cookies per hour. Plug in 150 cookies per hour and 600 cookies into the rate equation and solve. It takes 4 hours to make 600 cookies.

19. D

First, find the rate the airplanes travel. It takes 3.5 hours to travel from Miami to New York a distance of 1050 miles, which means the airplanes travel at a rate of 300 miles per hour. At this rate, to travel a distance of 1200 miles to Boston it would take 4 hours. Four hours of travel time means the airplane arrives at 5:30 PM

20. 2.5

Find the total length of time it takes the train to travel to St. Louis. Using the dirt equation, a train traveling a distance of 250 miles at 80 miles an hour takes 3.125 hours. An airplane traveling this same distance at 400 miles an hour takes 0.625 hours. What’s the difference in time? 2.5 hours!

21. B

Again, careful with your units! If Jason runs 4 miles in 30 minutes he runs 8 miles per hour. Half his normal speed is 4 miles per hour. How long does it take then to travel 2 miles? 0.5 hours!
Before we discuss the algebra strategies of Plug & Chug and Plugging in the Answers, we’ll review some of the basics. Solutions are at the end of the chapter.
Solving for a Variable

Whenever you have a variable in an equation, you have to get the variable by itself. Isolate the variable to one side of the equation and then you can solve.

1. If $8 - 2y = 16$, then $y =$

   (A) -8
   (B) -4
   (C) 0
   (D) 4
   (E) 8

2. In the equation $2a - 3b - 5a + 6b = 0$. What is the value of $b$ if $a = -7$?

   (A) -7
   (B) -1
   (C) 0
   (D) 1
   (E) 7
3. If \( \frac{3n}{4} = \frac{n+2}{8} \), what is the value of \( n \)?

(A) \( \frac{3}{32} \)

(B) \( \frac{1}{6} \)

(C) \( \frac{2}{5} \)

(D) \( \frac{1}{2} \)

(E) \( \frac{7}{9} \)

Sometimes you don’t have to solve for the variable itself. Make sure you answer what ETS asks you to find!

4. If \( 100x - 20y = 30 \), what is \( 10x - 2y \)?

(A) 0

(B) 3

(C) 9

(D) 30

(E) 100

5. If \( x^2 - y^2 = 120 \) and \( x + y = 12 \), what is the value of \( (x - y)^2 \)?

(A) 10

(B) 12

(C) 60

(D) 100

(E) 120
Decoding English into Math

Many times you are not given an equation, and must make one up yourself. Know how the following words translate into math!

*is* means equals =
*of* means multiply
*what* is the variable

6. If \( \frac{1}{2} \) of a number is 30 less than twice that number, what is the original number?

   (A) -5  
   (B) -1  
   (C) 10  
   (D) 20  
   (E) 30

7. Which of the following represents the statement: “The square of a number \( a \) multiplied by one-half the square root of a number \( b \) is equal to the square of the difference of \( a \) and \( b \).”

   (A) \( a^2 + \frac{\sqrt{b}}{2} = (a - b)^2 \)
   (B) \( a^2 + \frac{\sqrt{b}}{2} = (a + b)^2 \)
   (C) \( \frac{a\sqrt{b}}{2} = (a - b)^2 \)
   (D) \( \frac{a^2 \sqrt{b}}{2} = (a + b)^2 \)
   (E) \( \frac{a^2 \sqrt{b}}{2} = (a - b)^2 \)
Inequalities

Inequalities can be solved just like regular equations with one important difference:

Whenever you multiply or divide by a negative number, you must change the direction of the inequality sign.

8. Which of the following expressions defines the values of $x$ if $-2x + 3 \geq 2x + 15$?

(A) $x \geq -3$
(B) $x \geq -4$
(C) $x \leq -4$
(D) $x \leq -3$
(E) $x \leq 3$

9. Which of the following ranges of values of $x$ satisfies $2(x + 3) \leq 6(x + 1) - 8$?

(A) $x \leq \frac{2}{5}$
(B) $x \leq \frac{5}{2}$
(C) $x \leq \frac{1}{2}$
(D) $x \leq 2$
(E) $x \geq 2$
Distributing and FOIL

The FOIL rule is useful in remembering how to multiply binomials

First terms are multiplied
Outer terms are multiplied
Inner terms are multiplied
Last terms are multiplied

\[(a + b)(c + d) = ac + ad + bc + bd\]

10. For all values of \(x\), \((4x - 4)(4x - 4) = ?\)

(A) \(16x^2 - 32x + 16\)
(B) \(16x^2 - 16x - 32\)
(C) \(16x^2 + 16\)
(D) \(4x^2 - 16\)
(E) \(2x - 2\)

11. For all values of \(x\), \((3x + 4)(2x - 4) = ?\)

(A) \(6x^2 - 16\)
(B) \(6x^2 - 4x - 16\)
(C) \(6x^2 - 8x - 16\)
(D) \(12x^2 - 4x - 16\)
(E) \(12x^2 + 8x - 16\)
Solutions to Algebra

1. B

Move the 8 over to the right-hand side and you’re left with \(-2y = 8\). Dividing both sides by \(-2\) gives \(y = -4\).

2. A

Plug in \(-7\) for \(a\). Rewriting the problem gives you \(21 + 3b = 0\). Solving for \(b\), \(b = -7\).

3. C

To solve we must cross-multiply. This gives us the equation \(24n = 4n + 8\). Solving for \(n\), we find \(n = \frac{8}{20}\) which reduces to \(\frac{2}{5}\).

4. B

\(10x - 2y\) is one-tenth of \(100x - 20y\). We do not need to solve for \(x\) nor \(y\) at all; simply need to divide the right-hand side by 10, giving an answer of 3.

5. D

The key to this problem is recognizing that \(x^2 - y^2\) is the difference of two squares. Therefore, \(x^2 - y^2 = (x+y)(x-y)\) showing that the quantity \((x-y) = 10\). Since we want \((x-y)^2\), simply square 10 getting a final value of 100.
6.  D

Converting the problem into a math equation we get \( \frac{1}{2}x = 2x - 30 \). Solving for \( x \), \( x = 20 \).

7.  E

Carefully translating this problem gives us answer choice E.

8.  D

Collecting like terms simplifies the equation to \(-12 \geq 4x\). Simplifying we obtain \( x \leq -3 \). We do not flip the sign since we divided by 4 and not negative 4!

9.  E

Collecting like terms simplifies the equation to \(-4x \leq -8\). Since we must divide by a negative number to isolate the variable, we flip the sign leaving \( x \geq 2 \).

10. A

Remember the quantity \((4x - 4)^2\) is a binomial, and we must separate it out to \((4x - 4)(4x - 4)\). FOIL gives us the solution \(16x^2 - 32x + 16\).

11. B

Use FOIL and find that \((3x + 4)(2x - 4)\) simplifies to \(6x^2 - 4x - 16\).
Plug & Chug is one of the most important strategies you will learn on the SAT to solve algebra problems. But before we dive right in, let’s start with a little story…

You just aced your AP Chemistry test, and decide to reward yourself with some delicious ice cream. Your grandma was so proud, that she gave you $5 to splurge at the local ice cream shop. If you’re anything like the rest of us, standing in an ice cream store surrounded by gallons of ice cream is right up there with getting a brand new laptop, or having dinner with Angelina Jolie. So much ice cream; so little time!
1. A scoop of Mint Chocolate Chip ice cream costs $1, and you buy 4 scoops. Ignoring tax, if you pay the cashier $5, how much change do you get back, in dollars?

That’s an easy problem. 4 scoops cost $4, you hand the cashier $5 and you get $5 - $4 = $1 in change.

2. A scoop of Rocky Road ice cream costs $1 while a scoop of Neapolitan ice cream costs 50 cents. You decide to buy 2 scoops of Rocky Road ice cream, and 3 scoops of Neapolitan ice cream. Ignoring tax, if you pay the cashier $5, how much change do you get back, in dollars?

A little bit harder, but not by much. Two scoops of Rocky Road cost $2. Three gallons of Neapolitan cost $1.50. All the ice cream together costs $2 + $1.50 = $3.50. The cashier hands you back $5 - $3.50 = $1.50 in change.

3. A scoop of Moose Tracks ice cream costs $c$ cents, and you decide to buy $s$ scoops. If you pay the cashier $d$ dollars, and assuming you pay the cashier more than the total cost of the ice cream, how much change do you get back, in dollars?

Umm…

Where do we even begin? And why is this question so much harder than the rest? (And who knew buying ice cream could be so difficult!) It is far easier for us to understand quantifiable values than unknown variables. We can easily imagine what we would buy with $1,000,000, but with $d$ dollars we’re quite perplexed. If we ate 50 chicken wings we would probably have a tummy ache, but if we ate $c$ wings we wouldn’t know if that was a lot or a little.

So how does this help solve our Moose Tracks dilemma? By using Plug & Chug!

Plug & Chug turns algebra and geometry problems with unknown values into simple arithmetic problems. You can use Plug & Chug whenever you have variables in the answer choices. There are 5 easy steps to solving Plug & Chug problems. Let’s show the full ice cream problem in its entirety and show how Plug & Chug can help us solve:
1. A scoop of Moose Tracks ice cream costs \( c \) cents, and you decide to buy \( s \) scoops. If you pay the cashier \( d \) dollars, and assuming you pay the cashier more than the total cost of the ice cream, how much change do you get back, in dollars?

\[
\begin{align*}
(A) & \quad c - sd \\
(B) & \quad s - cd \\
(C) & \quad 100d - cs \\
(D) & \quad c - \frac{sd}{100} \\
(E) & \quad d - \frac{cs}{100}
\end{align*}
\]

**Step 1: Identify unknown values.**

Our unknown values are
- \( c \) cents
- \( s \) scoops
- \( d \) dollars

**Step 2: Substitute numbers for each variable.**

How do we know what numbers to pick? Numbers that make the math easy! We’ll delve into the specifics a little later but for now we’ll start with the following values:

- \( c = 50 \) cents
- \( s = 4 \) scoops
- \( d = 5 \) dollars
Step 3: Solve the problem and find your target.

Now that we chose numbers for each of our variables, we can solve the resulting arithmetic problem! Reread the question, and substitute the numbers you picked for each of the variables.

1. A scoop of Moose Tracks ice cream costs 50 cents, and you buy 4 scoops. If you pay the cashier 5 dollars, and assuming you pay the cashier more than the total cost of the ice cream, how much change do you get back, in dollars?

This problem just got a lot easier to solve! If each scoop is 50 cents, then 4 scoops cost 200 cents, or $2. If we hand the cashier $5, we get $5 - $2 = $3 in change.

What’s so special about the $3? $3 is our target answer. The target answer is the result of the arithmetic problem you solved by picking your own numbers and will lead us to our final answer.

Step 4: Plug-in your numbers into every answer choice.

Substituting our numbers, we get the following:

(A) \( c - sd = 50 - (4 \times 5) \)
(B) \( s - cd = 4 - (50 \times 5) \)
(C) \( 100d - cs = 5 - (50 \times 4) \)
(D) \( \frac{sd}{100} = \frac{50 - 4 \times 5}{100} \)
(E) \( \frac{cs}{100} = \frac{5 - 50 \times 4}{100} \)
Step 5: Solve for your target, and find your answer!

If we solve for each of our answer choices we will find our target answer from Step 3, which will be the correct answer to the question!

(A) \( c - sd = 50 - (4 \times 5) = 30 \)

(B) \( s - cd = 4 - (50 \times 5) = -246 \)

(C) \( 100d - cs = 500 - (50 \times 4) = 300 \)

(D) \( c - \frac{sd}{100} = 50 - \frac{4 \times 5}{100} = 49.8 \)

(E) \( d - \frac{cs}{100} = 5 - \frac{50 \times 4}{100} = 3 \)

Using Plug & Chug, we find that the correct answer is E. You may have noticed that in the question the price of ice cream is given in cents but ETS wants the answer in dollars. When units change on Plug & Chug problems do not try and convert to the final units ahead of time! The correct answer will do the conversion for you. If we plugged in $0.5 instead of 50 cents for \( c \), we would obtain the wrong answer. Answer choice C gives you a value of 300, which would be the solution if ETS asked for the answer in cents, not dollars! The unit swap is a common way ETS will try and trick you on Plug & Chug problems.

What would have happened if you picked different numbers for the variables initially? Would you still arrive at answer choice E? Different numbers would give us different target answers, but ultimately the answer would still be the same. It does not matter if a scoop of ice cream costs 25 cents, 50 cents, or 100 cents; our answer would still be E.
What numbers should I pick?

Plug & Chug numbers that make the math easy. Beyond that, here are some good guidelines to follow when choosing numbers to plug in:

1. **Never pick 0 or 1**

   If you were to pick 0 on plug and chug problems, it wouldn’t make much sense logically speaking. Looking at the above problem, could you have 0 scoops of ice cream that costs 50 cents? If you were to pick one 1, you will obtain identical values if in the Plug & Chug problem there was any multiplication or division.

2. **Do not use the same number for different variables.**

   To avoid confusion over which number you assigned to each variable, choose different numbers to keep track of your work.

3. **If you pick a bad number, you can always change and resolve.**

   Sometimes when you pick numbers, the resulting arithmetic problem gets too messy to solve. Other times when you Plug & Chug you find your same target answer in multiple answer choices! When this happens choose different numbers and repeat the steps for Plug & Chug.
4. Pick numbers that satisfy initial conditions in the problem.

Did the problem say $x$ is an odd multiple of 3? Must $y$ be a factor of 12? The numbers you pick must agree with what is stated in the problem, or your solution will be meaningless.

5. Use integers 2 and up.

Avoid negative numbers, fractions, decimals, square roots, etc. Remember, the point is to make the math easy, not more difficult!

That’s the low-down on Plug & Chug; you are now off to Plugging and Chugging on your own!
PAGES (S) MISSING FROM ORIGINAL

Pages 71–74
Geometry is the branch of Mathematics that deals with questions of size, shape, and position of figures in space. Let’s start with the basics.
Line and Angles

A line has no width and extends infinitely in both directions. It has a degree measure of $180^\circ$. A line that contains the points A and B is called $\overline{AB}$.

A ray has one endpoint and extends infinitely in the other direction. A ray with endpoint A that passes through point B is called $\overrightarrow{AB}$.

A line segment is a part of the line that contains two endpoints. A line segment with endpoints A and B is called $\overline{AB}$.

Two rays or endpoints sharing a common endpoint called the vertex form an angle. The measure of the angle is the amount of rotation between the two measured in degrees. Line segments $\overline{AB}$ and $\overline{AC}$ form $\angle ABC$.

A right angle is an angle that measures $90^\circ$. Two lines are perpendicular when they meet at a $90^\circ$ angle. The symbol for perpendicular is $\perp$. 
Parallel Lines and Transversals

When two parallel lines are cut by a third, called the transversal, three important things happen

1) Two types of angles are created, small angles and big angles. 
   \( \angle s \) and \( \angle b \)

2) All the small angles are equal to each other. All the big angles are equal to each other.

3) Any small angle plus a big angle equals 180°. \( \angle s + \angle b = 180° \)

Lines \( m \) and \( n \) are parallel. Transversal \( t \) passes through \( m \) and \( n \)
1. Given that lines $M$ and $N$ are parallel, what is the value of $x$ in the figure?

(A) 30°
(B) 40°
(C) 90°
(D) 120°
(E) 140°

2. In the figure above, line $A$ is parallel to line $B$, and line $C$ intersects them both. Which of the following lists 3 equal angles?

(A) angle $q$, angle $r$, angle $s$
(B) angle $q$, angle $s$, angle $t$
(C) angle $r$, angle $u$, angle $s$
(D) angle $r$, angle $t$, angle $u$
(E) angle $s$, angle $t$, angle $u
Triangles

A triangle is a three-sided shape whose angle measures add up to 180°.

The area of a triangle is given by the formula

\[ A = \frac{1}{2}bh \]

where \( A \) is the area, \( b \) is the length of the base, and \( h \) is the height.

In an isosceles triangle two sides are equal, and angles opposite the equal sides are equal.

In an equilateral triangle all three sides are equal, and each angle is equal to 60°.

3. In triangle \( ABC \), if \( AB = AC \), what is \( m\angle C \) if \( m\angle A = 40° \)?
   (A) 40°
   (B) 70°
   (C) 90°
   (D) 120°
   (E) 150°

4. A triangle has a height that is one-third the length of its base. What is the length of the base if the area is 24?
   (A) 4
   (B) 8
   (C) 12
   (D) 16
   (E) 20
**Right Triangles and the Pythagorean Theorem**

A right triangle is any triangle that has a 90° angle. The two sides that form 90° angle are called the legs. The side opposite the 90° angle is called the hypotenuse. The hypotenuse is the longest side of the triangle.

Pythagorean Theorem states:

\[ a^2 + b^2 = c^2 \]

where \( a \) and \( b \) are the legs, and \( c \) is the hypotenuse.

5. If \( AD \) is perpendicular to \( CB \), and \( AD = 6 \), \( AB = AC = 10 \), what is the area of triangle \( ABC \)?

(A) 3
(B) 6
(C) 12
(D) 24
(E) 48
Similar Triangles

Two triangles are similar if the three angles in one triangle are identical to the three angles in the other triangle. Similar triangles are denoted by the notation \( \sim \). The corresponding sides of similar triangles are proportional in length. You can setup a proportion to find the unknown side.

\[ \triangle ABC \sim \triangle DEF \]
\[
\frac{AC}{DF} = \frac{AB}{DE}
\]
\[
\frac{4}{6} = \frac{8}{x}
\]
\[
x = 12
\]

6. In the figure above, \( BC \) and \( DE \) are parallel. If \( DE = 6 \), \( AB = 4 \) and \( AD = 8 \), then what is the length of \( BC \)?

(A) \( \frac{4}{3} \)

(B) 2

(C) 3

(D) 4

(E) \( 4\sqrt{2} \)
Special Triangles

45° – 45° – 90° and 30° – 60° – 90°

There are no trigonometry problems on the SAT. (Woo hoo!) But you still need to know the relationship between angle measure and length for 45° – 45° – 90° and 30° – 60° – 90° triangles.

For a 45° – 45° – 90° triangle, the sides opposite 45° have length \( x \), and the side opposite 90° has length \( x\sqrt{2} \). Another name for this triangle is an isosceles right triangle.

For a 30° – 60° – 90° triangle, the side opposite 30° has length \( x \), the side opposite 60° has length \( x\sqrt{3} \), and the side opposite 90° has length 2\( x \).
7. In the figure above, AC and BC are perpendicular. What is the length of AC if \( m\angle A = 60^\circ \) and CB = 6?

(A) 1  
(B) \( \sqrt{2} \)  
(C) 2  
(D) 2\( \sqrt{3} \)  
(E) 6

8. Triangle ABC is an isosceles right triangle. If the \( m\angle A = 90^\circ \) and BC is length 4, what is the length of AB?

(A) 2  
(B) 2\( \sqrt{2} \)  
(C) 4  
(D) 5  
(E) 5\( \sqrt{2} \)
Circles

A circle is a shape whose points are the same distance from a given point called the center.

The \textit{radius} is the distance from the center to any point on the edge of the circle. It is usually denoted by \( r \).

The \textit{diameter} is the distance from one point on the circle to another, passing through the center point. It is usually denoted by \( d \). The diameter is equal to twice the radius.

\[ d = 2r \]

The \textit{circumference} of a circle is the distance around the circle. It is denoted by \( C \). The circumference is equal to pi times twice the radius, or pi times the diameter.

\[ C = 2\pi r \]
\[ C = \pi d \]

The \textit{area} of a circle is the amount of space the circle covers. It is denoted by \( A \). The area is equal to pi times the radius squared.

\[ A = \pi r^2 \]

9. The area of circle \( M \) is \( 16\pi \), and the radius of circle \( R \) is twice the radius of circle \( M \). What is the area of circle \( R \)?

(A) 8
(B) \( 8\pi \)
(C) \( 32\pi \)
(D) 64
(E) \( 64\pi \)
You got me going in circles

Most circle questions ask about the radius, diameter, circumference and area. Sometimes you also need to know about arc length and tangent lines.

The arc length of a circle is any part of the circles circumference. The arc measure is proportional to the size of the interior angle of the arc length.

10. On a circle $O$, arc length $\overline{QR}$ has a degree measure of 40, and the circumference of circle $O$ is 90. What is the distance measure of $\overline{QR}$?

(A) 10  
(B) 40  
(C) $20\pi$  
(D) 90  
(E) $90\pi$

A line tangent to a circle touches the circle at one point and is perpendicular to the radius at that point.
11. Line $BC$ is tangent to circle $A$ at point $B$. If $AB = BC$, and $AC = 10$, what is the area of triangle $ABC$?

(A) $5\sqrt{2}$
(B) 10
(C) 25
(D) 50
(E) $50\sqrt{2}$

12. Side $AC$ of $\triangle AOC$ is tangent to the circle with center $O$ and radius 3 at point $B$. If $m\angle AOB$ is $45^\circ$ and $\angle AOB \cong \angle BOC$, what is the perimeter $\triangle AOC$?

(A) 3
(B) 6
(C) $3 + 3\sqrt{2}$
(D) 80
(E) $6 + 6\sqrt{2}$
Quadrilaterals

A quadrilateral is a four-sided figure whose degree measure adds up to 360°. There are 3 commonly tested quadrilaterals on the SAT: parallelograms, rectangles, and squares.

A parallelogram has opposite sides that are equal and parallel, and opposite angles are equal.

A rectangle has opposite sides that are equal and parallel, and every angle is 90°.

A square has four equal, parallel sides and every angle is 90°.

13. In parallelogram ABCD, $m \angle ABC = 112°$ and $m \angle ABD = 47°$.
What is the $m \angle ADB$?
(A) 37°
(B) 45°
(C) 47°
(D) 65°
(E) 68°

14. Susie is building a garden and bought a wrought iron fence to surround the garden. If the length of the garden is 10, and the width is three times the length, how long is the fence surrounding the garden?
(A) 10
(B) 30
(C) 40
(D) 80
(E) 300
Degrees in a Polygon

On geometry problems, we mostly deal with triangles, circles and quadrilaterals. Sometimes we deal with other shapes such as pentagons and octagons.

The total number of degrees in a polygon is defined as:

\[(n-2)\cdot 180'\]

where \(n\) is the number of sides of the polygon

15. Hexagon ABCDEF has one angle that measures 145' and the other five angles are identical. What is the degree measure of one of the smaller angles?

(A) 115'
(B) 120'
(C) 145'
(D) 160'
(E) 180'

16. A regular polygon has interior angles whose sum is 1980'. How many sides does the polygon have?

(A) 11
(B) 12
(C) 13
(D) 14
(E) 15
Distance and Midpoint Formulas

Coordinate geometry questions often ask about the distance between two points that lie on the same plane.

The distance $d$ between two points $(x_1, y_1)$ and $(x_2, y_2)$ is defined as:

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

17. If the distance between an unknown point and $(4, 0)$ is $5$, which of the following could be the coordinate of the unknown point?

(A) $(0, 3)$
(B) $(5, 0)$
(C) $(1, 4)$
(D) $(4, 5)$
(F) $(3, 4)$

You also need to know the midpoint formula

The midpoint between two points $(x_1, y_1)$ and $(x_2, y_2)$ is the new point:

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

18. On the coordinate plane, the diameter of circle $O$ is marked by the points $(-4, 7)$ and $(12, -11)$. What is the coordinate point of the center of circle $O$?

(A) $(2, -4)$
(B) $(4, -2)$
(C) $(-4, 12)$
(D) $(7, -11)$
(E) $(7, 12)$
Geometry Strategies

Now that you know the geometry basics, we can focus some more on strategy.

Every geometry figure on the SAT is drawn to scale unless otherwise noted.

Does that angle look acute? It is! Do two lines look parallel? They are! We can use this information to guesstimate and eliminate answer choices that are extreme.

1) Write down any information from the problem on the figure given.
2) Write down any formulas you need and Plug & Chug any information you have.
3) If a figure is not provided, draw one! If a figure is not drawn to scale, draw it correctly!

19. If the radius of a circle that has an area of $64\pi$ is increased by 50%, what is the area of the resulting circle?

(A) 4  
(B) 12  
(C) 100\pi  
(D) 121\pi  
(E) 144\pi

20. If $\triangle DEF$ is isosceles, and the measure of $\angle DEF = 40^\circ$, the measure of $\angle EDF$ could be which of the following?

(A) 25°  
(B) 50°  
(C) 60°  
(D) 65°  
(E) 70°
21. Rectangle ABCD has length 6 and width 8. What is the area of the circle?

(A) 10π
(B) 20π
(C) 25π
(D) 64π
(E) 100π

It can be determined!

One answer choice that comes up fairly often is “(E) It cannot be determined from the information given.” This answer choice is often wrong. Why would ETS give you a problem you could not solve?

22. What is the area of a square with sides of lengths 3x − 1 and 2x + 3?

(A) 4
(B) 8
(C) 9
(D) 16
(E) It cannot be determined from the information given.
Geometry guesstimating

As was shown in the Math Introduction, you can guesstimate on geometry problems to eliminate incorrect answers.

23. In the figure, $A$, $B$, and $C$ are the centers of three circles, each with radius 2. What is the sum of the measure of arc lengths $\widehat{AB}$, $\widehat{BC}$, and $\widehat{CA}$?

(A) $\frac{2\pi}{3}$

(B) $\pi$

(C) $\frac{4\pi}{3}$

(D) $2\pi$

(E) $\frac{8\pi}{3}$
Weird Shapes

Many times ETS will find strange ways to ask questions about basic geometry rules. You can use the same rules to solve these difficult problems like you would an easier problem.

Always look for basic shapes hidden in the figure. Then use those rules to help you solve!

24. In the figure above, $AB$ is parallel to $CD$, and $BC$ is parallel to $ED$.

If the measure of angle $ECD$ is $50^\circ$, and the measure of angle $EDC$ is $60^\circ$, what is the measure of angle $BCE$?

(A) $50^\circ$
(B) $60^\circ$
(C) $70^\circ$
(D) $80^\circ$
(E) $90^\circ$
Divide odd shapes into shapes that you know.

25. In the figure above, AB has length 6, AD has length 6 and BC has length 10. What is the area of the figure?

(A) 24
(B) 30
(C) 36
(D) 48
(E) 60

It's not what it seems

The SAT writers love special triangles and they are often hidden in problems. Remember these two rules:

A square divided along its diagonal forms a $45^\circ - 45^\circ - 90^\circ$ triangle.

An equilateral triangle divided along its height forms a $30^\circ - 60^\circ - 90^\circ$ triangle.
26. The diagonal of a square has length 12. What is the area of the square?

Note: Figure not drawn to scale

27. Triangle $ABC$ is an equilateral triangle with length 6. What is the area of triangle $ADC$ if $AD$ is perpendicular to $CB$?

(A) 3
(B) $3\sqrt{3}$
(C) $\frac{3\sqrt{3}}{2}$
(D) $9\sqrt{3}$
(E) $\frac{9\sqrt{3}}{2}$
Solutions to Geometry

1. E

A big angle plus a small angle is 180°. Solving for the big angle, we find x = 140°.

2. B

Answer choice B has the three small angles in the figure.

3. B

Since AB = AC, the triangle is isosceles. This means that the measure of angle B and angle C are equal, and all three angles add up to 180°. Therefore, the measure of angle B must be 70°.

4. C

Start with each answer choice in this problem and work backwards. Starting with C, if the base is 12, then the height is 4. The area then must be 24, which matches the question!

5. E

Using the Pythagorean Theorem, we find the lengths CD and DB to be 8. Since the triangle has a base of 16 and a height of 6, the area of the triangle is 48.
6. **C**

Triangles ABC and ADE are similar. Setup a proportion to solve for BC.

\[
\frac{AB}{AD} = \frac{BC}{DE} \quad 4x = 6 \quad BC = x = 3
\]

7. **D**

Using the special triangle rules, the side opposite 60° is \(x\sqrt{3}\). Solving for \(x\), we find \(x = 2\sqrt{3}\). Since side AC is opposite the 30° angle in the triangle, AC has length \(2\sqrt{3}\).

8. **B**

Start by drawing the figure. Since ABC is an isosceles right triangle, it is a 45°-45°-90° triangle. BC is opposite the 90° angle, so \(4 = x\sqrt{2}\). Solving for \(x\), and therefore AB, we find \(x = 2\sqrt{2}\).

9. **E**

Using the circle formulas, the radius of circle \(M\) is 4 which means the radius of circle \(R\) is 8. Therefore, the area of circle \(R\) is \(64\pi\).

10. **A**

Arc length is a proportion of the circumference. Since there are 360 degrees in a circle, and arc length \(\widehat{QR}\) is 40 degrees, the arc measure of \(\widehat{QR}\):

\[
\frac{40°}{360°} \cdot 90 = 10
\]
11. D

The information in the problem tells us that triangle ABC is an isosceles right triangle. Using our 45° - 45° - 90° triangle rules, we see that lengths of AB and BC are $5\sqrt{2}$. Plugging in for the area of the triangle we get 50.

12. E

To manage this problem, start by drawing a figure:

From the information given, we know that triangles AOB and AOC are 45° - 45° - 90°. Therefore, AO and OC are length $3\sqrt{2}$. Since AB and BC are length 3, the perimeter of triangle AOC is $6 + 6\sqrt{2}$. 
13. D

Opposite angles in a parallelogram are equal. Because of this, $m\angle ADB + m\angle ABD = m\angle ABC$. Plugging in we find $m\angle ADB = 65^\circ$.

14. D

The question is asking about the perimeter of fence. If the length is 10, the width is 30. The perimeter is 80.

15. A

Use the formula to find the total number of degrees in the hexagon. A hexagon has 720°. Since one angle is 145° and the other 5 angles are equal, each of the smaller angles is 115°.

16. C

Setup the polygon formula and solve for $n$. $n$ must be 13.

17. A

The best way to approach this problem is to test each answer choice. Points (0, 3) and (4, 0) have a distance of 5.

18. B

Plug in the points (-4, 7) and (12, -11) into the formula for midpoint. The center is (4, -2)
19.  E

Draw it out! The original circle has a radius of 8. If it is then increased 50%, the new circle has a radius 12. Solving for area we arrive at answer choice E

20.  E

Start by drawing triangle DEF. We do not know which two sides of the triangle are congruent, so there are two possibilities: Angle EDF could be equal to angle DEF, or it could be one of the two larger, identical angles. Therefore, angle DEF can be either 40° or 70°, but only 70° is given.

21.  C

Draw the figure to scale. The diagonal of the rectangle is equal to the diameter of the circle. Using the Pythagorean theorem, we find the diameter of the circle to be 10, making the radius 5. This gives an area of $25\pi$.

22.  D

Remember, choice E is usually wrong! What do you know about the sides of a square? They’re equal! Set up the equation $3x - 1 = 2x + 3$, and solve for $x$. The side of the square is 4, making the area 16.

23.  D

Solving directly for this figure would be a nightmare, so guesstimate! You know the lengths AB, BC, and AC are the radii of the circles which have length 2. Connecting the 3 points we then have an equilateral triangle with side length 2, and a total perimeter of 6. Since arc lengths $AB$, $BC$, and $CA$ form a region slightly larger than the triangle, the correct answer should be close to 6. Choice D works well!
24. C

What’s the hidden shape in the figure? Parallel lines cut by transversals! You can extend the lines to convince yourself. Therefore, \( \angle EDC + \angle ABC = 180^\circ \) by knowledge of big and small angles. This means \( \angle ABC = 120^\circ \). Continuing, 
\[ \angle ABC = \angle ECD + \angle BCE, \]
and solving for \( \angle BCE \) we find that it equals 70°.

25. E

Draw a vertical line to divide the figure into a square and a triangle. The square has side lengths of 6 and an area of 36. Using the Pythagorean theorem, the triangle has a base of 8 and a height of 6. Its area is 24. Adding each area, the figure has a total area of 60.

26. 72

The diagonal of the square forms the hypotenuse of a 45°–45°–90° triangle. Solving for side length, we find the square has sides of \( 6\sqrt{2} \), giving a total area of 72.

27. 72

First, draw the figure to scale. Next, convince yourself that AD cuts the equilateral triangle into a 30°–60°–90° triangle. If the hypotenuse, length AC, is equal to 6, side AD is equal to \( 3\sqrt{3} \) and side CD is equal to 3. Solving for the area we arrive at choice E.
A relationship between two variables, typically $x$ and $y$, is called a function, if there is a rule that assigns each input value, $x$, to an output value, $y$. Here is an example of a typical function seen on the SAT:

$$y = 3x + 4$$

In this case, $x$ is the value we put into the function (our input) and $y$ is the value we get out of the function (our output). When our input value is $x = 1$, then our output value is $y = 7$. When our input value is $x = 3$, then our output value is $y = 13$. We can write this same function another way:

$$f(x) = 3x + 4$$

When students see the notation $f(x)$ in an equation it usually sends chills down their spines! Don’t worry; $f(x)$ is another way to express the output value $y$. It’s no different than the previous equation; just written in another way. When our input value is $x = 1$, then our output value is $f(1) = 7$. When our input value is $x = 3$, then our output value is $f(3) = 13$. 


Now let’s try using this notation:

1. What is \( f(4) \) if \( f(x) = x^2 - 4x + 8 \)?

   (A) 0
   (B) 2
   (C) 4
   (D) 6
   (E) 8

\( f(4) \) simply means find the output value of the function when the input is 4. Just substitute \( x = 4 \) into the problem and solve.

\[
f(4) = (4)^2 - 4(4) + 8
\]
\[
f(4) = 8
\]

which gives us answer choice E. Now try another:

2. When \( f(x) = 4 \), what is the value of \( x \) if \( f(x) = x^2 - 4x + 8 \)?

   (A) 0
   (B) 2
   (C) 4
   (D) 6
   (E) 8

This problem may look similar to the previous one, but there is a big difference. When \( f(x) = 4 \), this tells you that the output value of the function is 4 and you need to solve for the input. Substitute 4 into the left-hand side of the equation and solve for \( x \).

\[
4 = x^2 - 4x + 8
\]
\[
0 = x^2 - 4x + 4
\]
\[
0 = (x-2)(x-2)
\]
\[
x = 2
\]

Our final answer is B.
Graphing with functions

Many times you are given the graph of the function rather than the equation itself.

3. The figure above shows the graph of the function $f$. Which of the following is closest to $f(-1)$?

(A) -2  
(B) -1  
(C) 0  
(D) 1  
(E) 2

You can solve this problem just like you did previously! $f(-1)$ tells us that the input value is $x = -1$ and we’re looking for the output. Graphically, we see the output is $f(-1) = 2$ giving us answer choice A. Here is another problem:
4. The graph of $y = g(x)$ is shown above. If $g(x) = 1$, which of the following is a possible value of $x$?

(A) -3  
(B) -1  
(C) 0  
(D) 1  
(E) 2

Here we are given the output value of $g(x) = 1$ and we are looking for possible input values for $x$. Graphically, we determine $x = -2$ or $x = 2$. This indicates that the answer is choice E.
Domain and Range

There are two important definitions when it comes to functions.

The domain of a function is all the possible values you can put into the function. (The $x$ values.)

The range of a function is all the possible values you can get out of the function. (The $y$ or $f(x)$ values.)

To find the domain of a function, Plug & Chug values from your answer choices and eliminate answers if the denominator of a fraction becomes zero.

5. If $f(x) = \frac{1}{2x - 2}$, then which of the following describes the domain of $f(x)$?
   (A) all real numbers $x$
   (B) $x \geq 0$
   (C) $x \leq 0$
   (D) all real numbers $x$, $x \neq 1$
   (E) all real numbers $x$, $x \neq 2$

When we Plug & Chug each answer choice, we find when $x = 1$ our denominator becomes zero making D the correct answer.

To find the range of a function, graph the equation on your calculator to find possible output values.
6. What is the range of \( f(x) \) if \( f(x) = |x| - 3 \)?

(A) all real numbers \( x \)
(B) \( x \geq -3 \)
(C) \( x \leq -3 \)
(D) \( x \leq 3 \)
(E) \( x \geq 3 \)

Using our calculator, we graph \( f(x) \):

The range is the list of possible output values. From our graph we see our lowest output value is \(-3\), and all output values must be greater than or equal to \(-3\) making choice B correct.
Funky Functions

Instead of using $y$ or $f(x)$, ETS often will use weird symbols to try and confuse you. Be especially careful of trick answers on these types of problems!

7. For integers $x$, define $\langle x \rangle$ by the equation $\langle x \rangle = x^2 + 2x - 3$.

What is $\langle 10 \rangle - \langle 9 \rangle$?

(A) $\langle 1 \rangle$
(B) $\langle 4 \rangle$
(C) $\langle 6 \rangle$
(D) $\langle 8 \rangle$
(E) $\langle 21 \rangle$

What is the misleading answer choice? Answer choice A! We know $10 - 9$ is equal to 1, but this question is asking for $\langle 10 \rangle - \langle 9 \rangle$. We have to substitute each number into the crazy function and solve.

$\langle 10 \rangle = (10)^2 + 2(10) - 3 = 117$
$\langle 9 \rangle = (9)^2 + 2(9) - 3 = 96$
$\langle 10 \rangle - \langle 9 \rangle = 117 - 96 = 21$

Great! We see our answer is 21 which makes answer choice E look pretty good…not so fast! The answer is 21, not $\langle 21 \rangle$. We have to now have to Plug in the Answers to see which gives us the correct solution.
Remembering our rules for Plugging in the Answers, we start with choice C.

\[(C) \quad \langle 6 \rangle = (6)^2 + 2(6) - 3 = 45\]

Choice C gives us a solution of 45, which is higher than our answer. We move up the list to letter B and try again.

\[(B) \quad \langle 4 \rangle = (4)^2 + 2(4) - 3 = 21\]

Answer choice B is correct!

These seemingly simple problems trap many students because they get lazy when solving. Make sure you are thorough in solving every problem!