Multi-period Optimal Network Flow and Pricing Strategy for Commodity Online Retailer

by

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Submitted to the School of Engineering
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Abstract

This thesis aims to study the network of a nationwide distributor of a commodity product. As we cannot disclose the actual product for competitive reasons, we will present the research in terms of a similar, representative product, namely salt for ice prevention across United States. The distribution network includes four kinds of nodes, sources, buffer locations at sources, storage points and demand regions. It also includes four types of arcs, from sources to buffer locations and to storage points, from buffer locations to storage points, and from storage points to demand regions. The goal is to maximize the total gross margin subject to a set of supply, demand and inventory constraints.

In this thesis, we establish two mathematical models to achieve the goal. The first one is a basic model to identify the optimal flows along the arcs across time by treating product prices and market demand as fixed parameters. The model is built in OPL and solved by CPLEX. We then carry out some numerical analyses and tests to validate the correctness of the model and demonstrate its utility.

The second one is an advanced model treating product prices and market demand as additional decision variables. The product price and market demand are related by an exponential function, which makes the model difficult to solve with the available commercial solver codes. We then propose several algorithms to reduce the computational complexity of the model so that we can solve with CPLEX. At last, we compare the algorithms to identify the best one. We provide additional numerical tests to show the benefit from including the pricing decisions along with the optimization of the network flows.

Thesis Supervisor: Stephen C. Graves

Title: Abraham J. Siegel Professor of Management Science

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1 Introduction

The project is done for a United States based third party logistic company (3PL). Due to confidentiality, we will refer to the company by the disguised name of ABC and the product by the disguised product road salt for ice prevention as it shares the following characteristics with the real product. First of all, road salt is consumed over the year with seasonal demand. Secondly, it costs a lot when it is transported from one region to another region and thus effective transportation is a major concern in the industry. Lastly, the road salt industry has many different sources and demand regions across the country, although most of the demand occurs in regions with harsh winter weather.

1.1 Company Background

Company ABC is a national wide distributor of road salt in United States and its main goal is to streamline both the flow of materials and the flow of information in the industry. Due to the special property of the product, transportation can be up to 50 percent of the cost of road salt and thus a good transportation network is essential to the success of the business in this industry. Company ABC does have an unrivaled carrier and distribution network and thus plays an important role in this industry. Through its network of affiliates, it can deliver road salt directly to the customers. It also provides consumers the opportunity to buy truckloads of bagged product anywhere in the country through its extensive network. The network flow problem is thus a top priority.

Company ABC has several sources for road salt as well as multiple demand regions. It also rents dozens of storage points to facilitate the flow requirements at different time periods. In addition, the manufacturers (or sources) also allow Company ABC to store some inventory at their plants with a very low storage cost as a marginal benefit. This gives Company ABC options to time its purchase of the salt and its transportation of the salt, so as to get the lowest purchasing and transportation costs. However, all these factors make the distribution network large and complicated.

1.2 Project Description

This thesis is to study company ABC's existing network and suggest the optimal commodity flow policy as well as pricing strategy for the company based on the current network structure. We will build a network model which would include four types of nodes, namely sources, buffer locations at sources, storage points and demand regions. It has four types of arcs, from sources to buffer locations and to storage points, from buffer locations to storage points, and from storage points to demand regions.

Since the market prices and transportation costs always vary across the year, it is sometimes more profitable to purchase the product in the off season and then store the products temporarily for future sales. For instance, since the transportation cost is very high in January due to the heavy snows, it is rationale to move the products into warehouses in summer for January sales. Due to this aspect, we will formulate the model as a multi-period model. According to the information provided, a reasonable time horizon would be 18 months with monthly time buckets.

The goal is to maximize the total gross margin subject to a set of supply, demand as well as inventory constraints. The decision variables would be the flows along the arcs over time. A basic model will be built in OPL and solved by CPLEX [1][2] and after that we will conduct the result analysis and sensitivity analysis to better understand the solution. Last but not least, we will incorporate the demand-price function into the basic model to find out the optimal pricing policy as well.

1.3 Literature Review

1.3.1 Linear Programming

One of the most important tools of optimization is "linear programming" (L.P.). A linear programming problem is specified by a linear, multi-variable function which is to be optimized (maximized or minimized) subject to a number of linear constraints. The mathematician, G. B. Dantzig [3] developed an algorithm called the "simplex method" to solve problems of this type. The original simplex method has been modified into an efficient algorithm to solve large L.P. problems by computer.

Problems from a wide variety of fields can be formulated and solved by means of L.P. The application includes resource allocation problems in government planning, network analysis for urban and regional planning, production planning problems in industry and the management of transportation distributive systems [4]. Hence L.P. is one of the successes of modern optimization theory.

The application of linear programming also arises in the financial industry. Sheldon D. Balbirer and David Shaw [5] published a paper describing a successful application of linear programming for assisting the management of Central Carolina Bank and Trust Company (CCB) in their financial planning process. The fundamental issues facing senior bank management revolve around the structuring of a bank's balance sheet. Since yields can be assigned to each asset category and costs to each liability category, the profits of the bank can be represented in terms of its balance sheet position. Thus, a bank's financial goal of maximizing returns to shareholders through maximizing profits can be translated into the operational goal of achieving some target end-of-period balance sheet position producing the greatest profits.

The mathematical structure of L.P. also allows important questions to be answered concerning the sensitivity of the optimum solution to data changes. Two fundamental ingredients in the economic analysis of an LP would be the shadow price with each constraint and reduced cost coefficient with each decision variable. Shadow price is defined as the change in the optimal value of the objective function if the right-hand side of the constraints is relaxed by one unit. Reduced cost, or opportunity cost, is the amount by which the objective value would have to improve (so increase for maximization problem, decrease for minimization problem) before it would be possible for a corresponding variable to assume a positive value in the optimal solution [6].

1.3.2 Multi-commodity Flow Problem

Multi-commodity flow problems appear frequently when dealing with the operation of communication or transportation networks. In telecommunication, the demands, or calls, on the networks are the commodities and the objective is to route the calls from their origin to their destination. In transportation like express package delivery, we require that shipments, each with a specific origin and destination, be routed over a transportation network [7]. These real life problems are usually solved by linear multi-commodity flow problems.

The difficulty with the applications in real life is that the problem size is so large that it is challenging to solve them by standard linear programming solution techniques like simplex. F. Babonneau, O. du Merle, and J.-P. Vial [8] proposed a partial Lagrangian relaxation to solve the large scale linear multi-commodity flow problem. The relaxation is restricted to the set of arcs that are likely to be saturated at the optimum and the partial relaxation is then solved by Proximal-ACCPM, a variant of the analytic center cutting-plane method.

1.4 Software Introduction

The commercial solver we use in the thesis is CPLEX embedded in OPL. Today, over 1,000 corporations and government agencies use OPL CPLEX, along with researchers at over 1,000 universities. OPL CPLEX helps solve planning and scheduling problems in virtually every industry. More than 100 of the world's leading software companies are also OPL CPLEX customers, including market leaders like SAP, Oracle and Sabre [9].

The algorithm we use to solve the model would be OPL CPLEX Simplex Optimizers, which provide the power to solve quadratic programs and linear programs with millions of constraints and continuous variables, at record-breaking speed. The optimizers include implementations of dual simplex and primal simplex, as well as a network simplex that can solve problems with side constraints. The OPL CPLEX algorithms for problem size reduction are integrated into the OPL CPLEX Simplex Optimizers [9].

1.5 Thesis Overview

In chapter 2 we present a detailed description of the assumptions used in the thesis. In Chapter 3 we establish the optimal network flow model with fixed product price and market demand while in Chapter 4 we take product price as an additional decision variable and develop an advanced model to find out the optimal pricing strategy. We also examine the exponential relationship between market demand and product price and propose several algorithms to solve the advanced model approximately. Chapter 5 then concludes the thesis.

2 Model Assumptions

Since company ABC's network is very large and complex, we make a set of assumptions to simplify the situation for the modeling purpose.

Firstly, we assume there are only two types of road salts in the market, namely H and S. In fact, there are many different types of products in terms of different efficiency of preventing ice. However, they can be generally classified into these two major categories. Since some customers prefer type H while others prefer type S, we have to distinguish between the two products to capture this aspect of the reality. Under this assumption, we are able to simplify the problem a lot without compromising too much on the reality. Each source could supply either type H or S, not both. Meanwhile, we assume there is no difference between the two products in terms of their occupation of the storage space

Secondly, we also assume that the sources and demand regions can be clustered into high level nodes. Since there are large number of manufacturers and retailers all over the States, the network would be too complex to model if we treat each plant or retailer as a separate node. So we will do some aggregation for the modeling purpose. For each aggregated demand region, we choose the center of that demand region as the geographical location of this demand region. Furthermore we assume that ABC has a storage point within each aggregated demand region; the storage point will hold inventory and will then serve the customers from this inventory. ABC typically will rent a public warehouse for these purposes. In the thesis, we will use 8 sources and 14 demand nodes to cover the whole region of United States. Note that the demand for road salt is highly concentrated in the northern parts of the United States, which is reflected in the choice of the demand regions.

Thirdly, we assume that all customers within a demand region are served from the storage point in the demand region. Since each demand region has a single, fixed storage point, we can ignore the transportation cost from the storage point to the customers, as these costs are invariant to the decisions in our model. However, we do allow product to be transported from a storage point in one demand region to another storage point in another demand region; in this way we can move product between demand regions and salt that is stored at one storage location might eventually be used to serve customers in other demand regions. The transportation cost from one storage point to another demand region is linear in the amount being shipped, as these shipments are primarily truckload moves. Similarly, we assume the transportation cost from a source to a storage point is linear because ABC ships everything in truckloads or rail cars.

Fourthly, we assume there is supply-volume agreement between company ABC and each of its affiliated suppliers. To ensure both parties' profitability, ABC will purchase a monthly volume from each supplier that is within a certain range around this agreed-to target volume. For example, shipments from vendor A must be between 70% and 100% of its monthly target, while those from vendor B must be between 85% and 110% of the volume target, and company ABC has a lot of leeway with vendor C, with a range of 30%-120%.

Fifthly, we assume there would be a penalty cost imposed if the total inventory exceeds a certain level. This is to penalize on over storage of the product. This sets a maximum level for aggregate inventory held at all buffer locations and storage points. For instance, the policy might state that if aggregate inventory exceed 15,000 tons in June then company ABC needs to pay $2 for each ton over the peak. So holding 17,000 tons of inventory would result in a charge of $2/ton x (17,000 – 15,000) = $4000 for that month. The maximum allowable total inventory is defined by month, because it is likely to vary over the year. For instance, it is acceptable to have high inventory level in late summer, but not in late fall when demand starts to wane.

Last but not least, we assume that company ABC has a quite accurate forecast about the market demand in each demand region. Moreover, we also assume that all demand regions have demand for both products and there is a particular split between the two products for each demand region. For example, demand region A may prefer type H and we may target a mix of 60% H and 40% S for that particular region.

3 Basic Model

In this chapter, we build a network flow model to maximize the gross margin over a time period of 18 months subject to a set of supply, demand and storage constraints. In this model, we assume company ABC has no control over the product price and thus market demand. At the same time, to make sure the market equilibrium would not be affected by oversupply, we assume that the total shipments into demand regions cannot be more than 1.05 times of the forecasted demand for that month. Based on the previous set of assumptions, we come up with the following mathematical formulation for the network flow model.

3.1 Problem Formulation

3.1.1 Notations

Indices

i: Index of source nodes, $i \in \{1,2,..,8\}$

j: Index of storage points , $j \in \{1,2,..,14\}$

k: Index of demand nodes, $k \in \{1,2,..,14\}$

t: Index of the time periods, $t \in \{1,2,..,18\}$

Input Parameters

C_{it}: Agreed-to volume (tons) from source i for month t (S or H)

L_i: Lower bound on actual shipments from source i, as a percent of the agreed-to volume

U_i: Upper bound on actual shipments from source i, as a percent of the agreed-to volume

u_{it}: Material cost ($/ton) at source i for month t

BC_i: Storage capacity (tons) at buffer location i (fixed over time)

w_i: Monthly storage cost ($/ton/month) at buffer location i (fixed over time)

δ_i: Material handling cost ($/ton) from source i to its buffer location

SC_j: Storage capacity (tons) at storage point j (fixed over time)

v_j: Storage cost ($/ton/month) at storage point j (fixed over time)

MI_t: Maximum allowable total inventory (tons) for month t

γ_t: Penalty cost ($/ton) if total inventory exceeds the maximum amount for month t

D_{kt}: Demand (tons) for road salt at demand region k for month t

α_k: Fraction of the forecasted demand that is for H at demand node k

P_{kt}: Market price ($/ton) at demand region k for month t (same for S and H)

c_{ijt}: Transportation cost ($/ton) from source i to storage point j for month t

c_{jkt}: Transportation cost ($/ton) from storage point j to demand region k for month t

Calculated parameters

BI_{it}: Inventory (tons) at buffer location i for month t

HI_{jt}: Inventory of product H (tons) at storage point j for month t

SI_{jt}: Inventory of product S (tons) at storage point j for month t

ω_t: Quantity (tons) by which total inventory exceeds the maximum amount for month t ie. $\max(\sum_j (SI_{jt} + HI_{jt}) + \sum_i BI_{it} - MI_t, 0)$

R_t: Total revenue ($) for month t

M_t: Total material cost ($) for month t

S_t: Total storage cost ($) for month t

H_t: Total transportation cost ($) for month t

π_t: Total margin ($) for month t

Decision variables

F_{it}: Quantity (tons) purchased from source i and stored at buffer location i for month t

F_{ijt}: Quantity (tons) purchased and shipped from source i to storage point j for month t

G_{ijt}: Quantity (tons) shipped from buffer location i to storage point j for month t

HF_{jkt}: Quantity (tons) of H shipped from storage point j to demand node k for month t

SF_{jkt}: Quantity (tons) of S shipped from storage point j to demand node k for month t

To illustrate the network model more clearly, we draw a representative graph of the model in Figure 3.1.

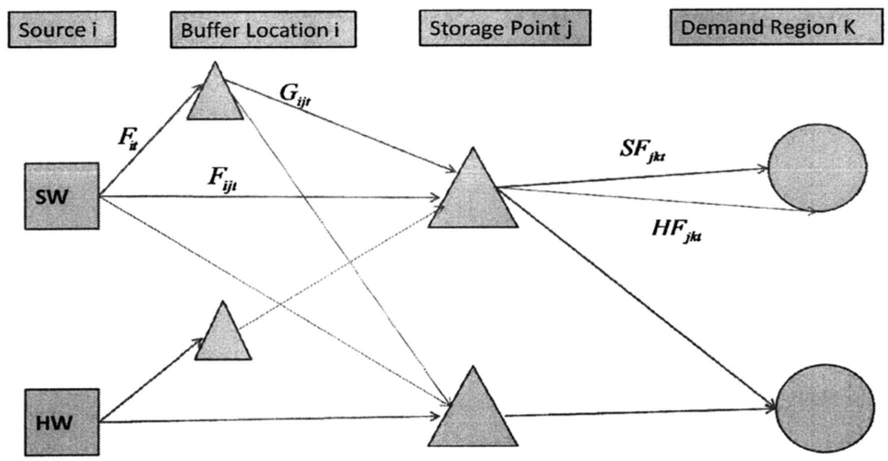

Figure 3.1 Network Model

3.1.2 Model Formulation

The basic model is easy to formulate as it is a multi-period two commodity flow problem and it is a linear program as the objective and constraints are all linear.

Objective

$$\max \sum_t \pi_t$$

Total margin for month t: $\pi_t = R_t - M_t - S_t - H_t - \gamma_t \omega_t$

$$R_t = \sum_k (P_{kt} \sum_j (SF_{jkt} + HF_{jkt}))$$

$$M_t = \sum_i u_{it} (\sum_j F_{ijt} + F_{it})$$

$$S_t = \sum_j v_j (SI_{jt} + HI_{jt}) + \sum_i w_i BI_{it}$$

$$H_t = \sum_i \delta_i F_{it} + \sum_i \sum_j c_{ijt} (F_{ijt} + G_{ijt}) + \sum_j \sum_k c_{jkt} (SF_{jkt} + HF_{jkt})$$

Constraints

Supply constraints

$$L_i C_{it} \le \sum_j F_{ijt} + F_{it} \le U_i C_{it} \qquad \forall i, t$$

Inventory constraints

$$HI_{j(t+1)} = HI_{jt} + \sum_{i \in HW} (F_{ij(t+1)} + G_{ij(t+1)}) - \sum_{k} HF_{jk(t+1)} \qquad \forall j,t$$

$$SI_{j(t+1)} = SI_{jt} + \sum_{i \in SW} (F_{ij(t+1)} + G_{ij(t+1)}) - \sum_{k} SF_{jk(t+1)} \qquad \forall j,t$$

$$BI_{i(t+1)} = BI_{it} + F_{i(t+1)} - \sum_{j} G_{ij(t+1)} \qquad \forall i,t$$

$$HI_{jt} + SI_{jt} \leq SC_j \qquad \forall j,t$$

$$BI_{it} \leq BC_i \qquad \forall i,t$$

$$\sum_{j} (HI_{jt} + SI_{jt}) + \sum_{i} BI_{it} - MI_t \leq \omega_t \qquad \forall t$$

$$HI_{jt}, SI_{jt}, BI_{it}, \omega_t \geq 0 \qquad \forall j,t$$

Demand constraints

$$\sum_{j} HF_{jkt} \leq 1.05 \alpha_k D_{kt} \qquad \forall k,t$$

$$\sum_{j} SF_{jkt} \leq 1.05(1-\alpha_k) D_{kt} \qquad \forall k,t$$

Nonnegative constraints

$$G_{ijt}, F_{ijt}, SF_{jkt}, HF_{jkt} \geq 0 \qquad \forall i,j,k,t$$

3.2 Result Analysis

To protect company ABC's confidential information, we use a representative input data set which is indicative of the real life scenarios. The input data set used in this thesis is attached in appendix A.

Meanwhile, we know the problem size will be very large due to large number of nodes and arcs as well as time periods. Actually, the model with the sample data in appendix A has 3,619 variables and 2,538 constraints in total. OPL CPLEX is a good tool to solve large size linear programs so we build the model in OPL and use CPLEX to solve it.

We build the OPL model according to the mathematical formulation above. Before running the model, we need to do some preprocessing of the input data to prepare it for the model and then convert it into a format compatible with the model. After the model is executed, we need to export the results to a format easy to understand. A detailed instruction for how to prepare the input file, how to run the program and how to extract the outputs into spreadsheets is attached in appendix B.

3.2.1 Objective

We run the model with the input data and Table 3.1 below is a summary of the objective function in the basic model.

Table 3.1 Results from the Basic Model

Components of Revenue	Value ($)	Percentage of Revenue
Total Transportation Cost	129,481	20.93%
Total Material Cost	373,957	60.45%
Total Inventory Cost	1,478	0.24%
Total Penalty	666	0.11%
Total Margin	113,068	18.27%
Total Revenue	618,646	100%

As the table suggests, the two major components of costs are material cost and transportation cost. The transportation cost (20.93%) is only approximate 1/3 of the material cost (60.45%). However, in the current operation, transportation may be as high as 50% of the total cost. The network flow model also optimizes the amount and timings of purchase over the time horizon so that the material cost is only 60.45% of the total revenue. Thus we have some confidence to say that this optimal network flow model would find some improvements to the current operations.

3.2.2 Optimal Flow

The optimal solution consists of four types of flows, namely, from sources to buffer locations (F_{it}), from sources to storage points (F_{ijt}), from buffer locations to storage points (G_{ijt}) and from storage points to demand regions (HF_{jkt} & SF_{jkt}). Each flow is displayed in a two dimensional array with rows representing feasible transportation routes and columns representing different time periods. However, it is not easy to see how the product actually flows along the arcs from the two-dimensional arrays. For example, it would be difficult for company ABC to see how the supply from a specific source is distributed over different storage points from the two dimensional array F_{ijt}. To make the solution easier to understand and interpret, we construct Pivot Tables which conceptually convert the solution from two dimensions (route and time) to three dimensions (source, destination and time). As a result, the flow for each month can be displayed in a two dimensional array with each row representing the source and each column representing the destination of the flow. The procedure to construct Pivot Table is as following.

1. Open a new excel sheet and then click insert→Pivot Table→select the data range→ select the location of the Pivot Table
2. Choose the Column Labels and Row Labels as well as the value to be displayed by dragging fields into the corresponding areas
3. Change the field displayed by simply unclick the current displayed field and click a new field
4. Right click the Pivot Table, choose value Field setting to change the setting of displayed data like the number format

The Pivot Table is able to display the optimal flows for different months by simply choosing the corresponding month. Meanwhile, we can choose the solution to be displayed in terms of actual flow or percentage of row sum or column sum by changing the field setting. To illustrate the idea, we display the flows for January 2010 in Pivot Table below. Table 3.2 is the flow from sources to buffer locations (F_{it}) while Table 3.3 is the flow from sources to storage points (F_{ijt}) for this month. Table 3.4 is the flow from buffer locations to storage points for the month (G_{ijt}). Table 3.5 and 3.6 are the shipments from storage points to demand regions for H and S respectively.

Table 3.2 Shipments from Sources to Buffer Locations for Jan 2010 (tons)

Row Labels	Sum of Jan-10
ARGT	0
CRTH	0
HASS	0
JAFF	0
MICH	14
PRIN	24
SCHY	14
STFC	69
Grand Total	121

Table 3.3 Shipments from Sources to Storage Points for Jan 2010 (tons)

Sum of Jan-10	Column Labels														
Row Labels	BERK	CJER	CTRI	DEME	HUDV	NCMA	NJER	OHIO	SCST	SEMA	SONH	UVLY	WTCT	WTMA	Grand Total
ARGT		0			17		12						21		50
CRTH				0		0			29	0	3				32
HASS		0			0	9	1	0		5			5	3	23
JAFF	0										0	0		0	0
MICH								0						22	22
PRIN	16	0	0	5	0	0				0	0	0	0	0	21
SCHY	5	0			6	0	0						0	0	11
STFC			14	0	0	34		0			0	41	4	37	131
Grand Total	21	0	14	5	22	43	13	0	29	5	3	41	30	63	289

Table 3.4 Shipments from Buffer Locations to Storage Points for Jan 2010 (tons)

Sum of Jan-10	Column Labels														
Row Labels	BERK	CJER	CTRI	DEME	HUDV	NCMA	NJER	OHIO	SCST	SEMA	SONH	UVLY	WTCT	WTMA	Grand Total
ARGT		0			0		0						0		0
CRTH				0		0			0	0	0				0
HASS		0			0	0	0	0		0			0	0	0
JAFF	0										0	3		0	3
MICH								0						0	0
PRIN	0	0	0	0	0	0				0	0	0	0	0	0
SCHY	0	0			0	0	0						0	0	0
STFC			0	0	0	0		0			0	0	0	0	0
Grand Total	0	0	0	0	0	0	0	0	0	0	0	3	0	0	3

Table 3.5 Shipments of H from Storage Points to Demand Regions for Jan 2010 (tons)

Sum of Jan-10	Column Labels														
Row Labels	BERK	CJER	CTRI	DEME	HUDV	NCMA	NJER	OHIO	SCST	SEMA	SONH	UVLY	WTCT	WTMA	Grand Total
BERK	5														5
CJER		0													0
CTRI			0			0				0			0		0
DEME				0											0
HUDV					6										6
NCMA						9									9
NJER							1								1
OHIO								1							1
SCST									0						0
SEMA										5					5
SONH											0				0
UVLY						0			0		0	3			3
WTCT													5		5
WTMA	0				0	0			7	0	16		0	3	26
Grand Total	**5**	**0**	**0**	**0**	**6**	**9**	**1**	**1**	**7**	**5**	**16**	**3**	**5**	**3**	**60**

Table 3.6 Shipments of S from Storage Points to Demand Regions for Jan 2010 (tons)

Sum of Jan-10	Column Labels														
Row Labels	BERK	CJER	CTRI	DEME	HUDV	NCMA	NJER	OHIO	SCST	SEMA	SONH	UVLY	WTCT	WTMA	Grand Total
BERK	16														16
CJER		0													0
CTRI			14			0				0			0		14
DEME				5											5
HUDV					32										32
NCMA						34									34
NJER							12								12
OHIO								0							0
SCST									29						29
SEMA										0					0
SONH											3				3
UVLY						0			0		34	7			41
WTCT													26		26
WTMA	0				0	0			0	26	0		0	11	37
Grand Total	**16**	**0**	**14**	**5**	**32**	**34**	**12**	**0**	**29**	**26**	**37**	**7**	**26**	**11**	**249**

We are able to construct the above five Pivot Tables for the flows in each month. In the Pivot Table, a blank cell means it is an infeasible route while zero means there is no physical flow even though the flow is feasible. From Table 3.5 and 3.6, we can see that most of the shipments to demand regions are along the diagonal of the Pivot Tables. This suggests that most of the demand regions are supplied by their own storage points since we assume zero transportation cost if it is within the same region. The off-diagonal transportation only occurs when one storage point does not have sufficient supply while another storage point has extra storage. For example, we can see storage point WTMA supplies product H to regions SCST and SONH. As we look back into the raw data in appendix A, we find out that there are four sources supplying H to storage point WTMA, namely HASS, JAFF, MICH and SCHY. On the other hand, only one source JAFF supplies H to SONH while no source supplies H to SCST. This could possibly be the reason why there are shipments from storage point WTMA to demand regions SCST and SONH.

3.2.3 Inventory Analysis

Besides the optimal network flows, the OPL model also keeps track of the inventory at buffer locations and storage points. It might be company ABC's concern to know how the inventory changes over time so that they know whether they should expand or reduce the current storage capacity. This is also important as inventory means cash is tied up until the inventory can be sold.

Inventory at Buffer Locations

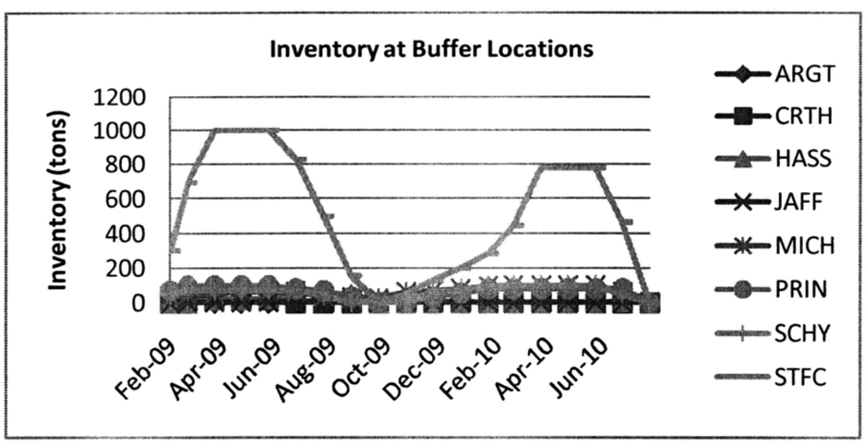

Figure 3.2 Inventory at Buffer Locations (tons)

Figure 3.2 shows that there is a general pattern for the change of inventory at all buffer locations. That is the inventory increases from March 2009 to June or July 2009 and then it starts to decrease from July to November. It repeats the same pattern for year 2010 too. It seems that company ABC should store the product at sources at the beginning of the year to satisfy the increase in the demand in the later stage of the year. This agrees with the fact that company ABC usually stores inventory in spring and summer for the huge demand in fall and winter.

We also observe that source 'STFC' has much higher inventory than other buffer locations. As we look back into the raw data, we find that the cumulative demand of S is very high for fall and winter and "STFC" is a major supplier of S. To satisfy the market demand for S, company ABC has to purchase S from "STFC" and store it at its buffer locations. Meanwhile, "STFC" has a large storage capacity of 1,000 tons and zero storage cost. Other buffer locations however have either small storage capacity (<=200 tons) or low supply volume. This explains why the inventory at 'STFC' increases so much.

<u>Inventory at Storage Points</u>

Figure 3.3 is the inventory of H at storage points while Figure 3.4 is that of S at storage points.

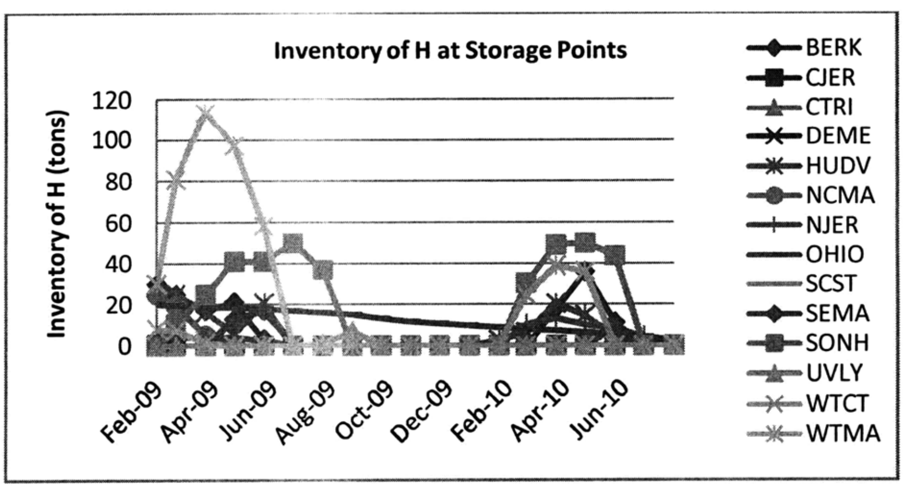

Figure 3.3 Inventory of H at Storage Points (tons)

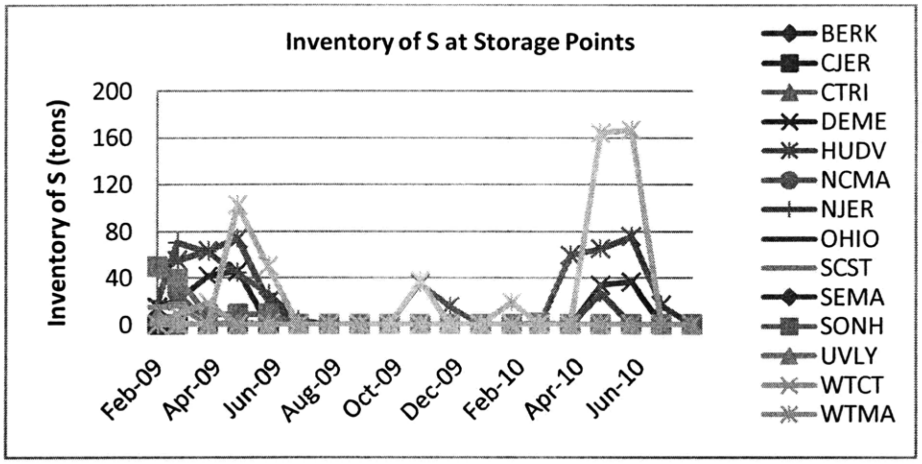

Figure 3.4 Inventory of S at Storage Points (tons)

Comparing the above two figures to Figure 3.2, we see that the inventory at storage points is much lower than that at buffer locations. This is reasonable since the storage cost at buffer locations is generally lower than that at storage points. Company ABC should only store product at the storage points when there is no space left at the buffer locations or there is a savings in the transportation cost from shipping early to a region.

The manager would also be interested to know the space utilization rate of the storage points, so we also calculate the total inventory at storage points by adding the S inventory and H inventory together. Figure 3.5 below shows the space utilization rate (total inventory/storage capacity) at each storage point over time.

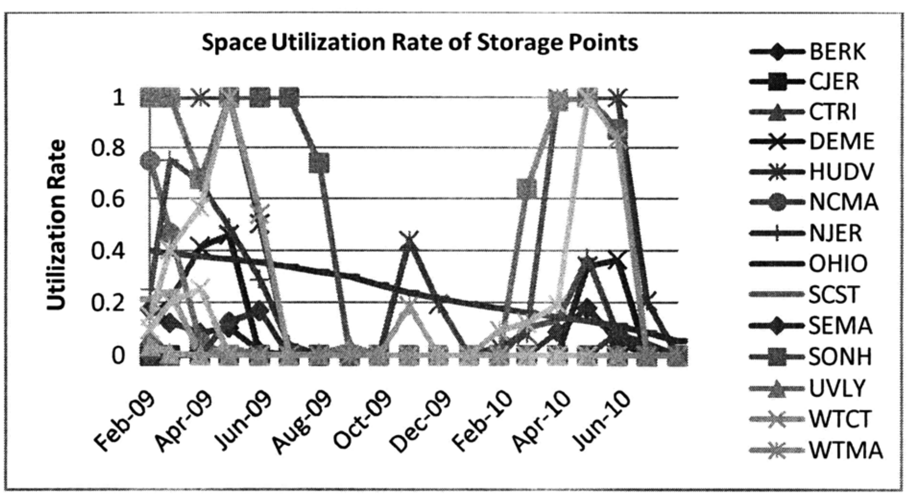

Figure 3.5 Space Utilization Rate of Storage Points

Generally speaking, the storage space is not fully utilized at some of regions and thus the storage capacity might be reduced accordingly. For example, the highest utilization rate at region "BERK" for the current solution is 18% in May 2010 and thus it is not necessary for company ABC to maintain a storage capacity of 200 tons at this region.

Total Inventory

Company ABC may want to know how the total inventory of H and S over all warehouses changes over time. We plot the graph as follows.

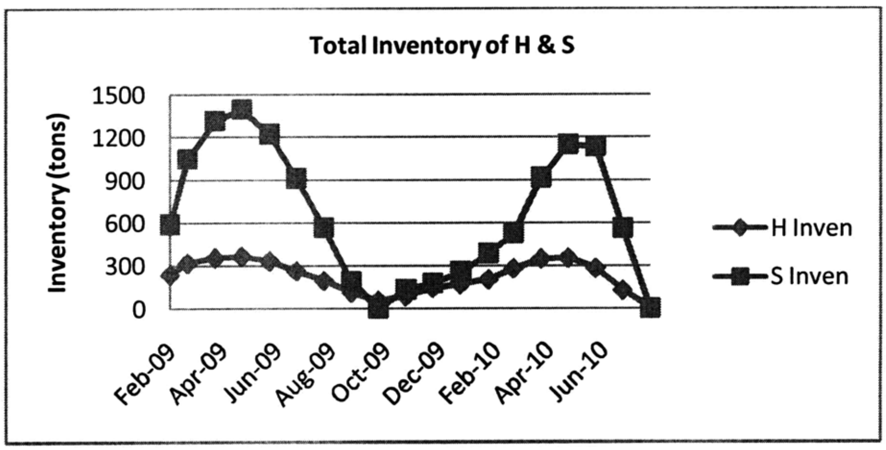

Figure 3.6 Total Inventory of H and S (tons)

Figure 3.6 show that both products have similar pattern of inventory change although product S has a larger scale than product H.

From Chapter 2, we know that there would be a penalty cost if the total inventory exceeds a certain level. To see whether and when the penalty is imposed during the process, we plot the actual inventory level and maximum allowable amount over the time.

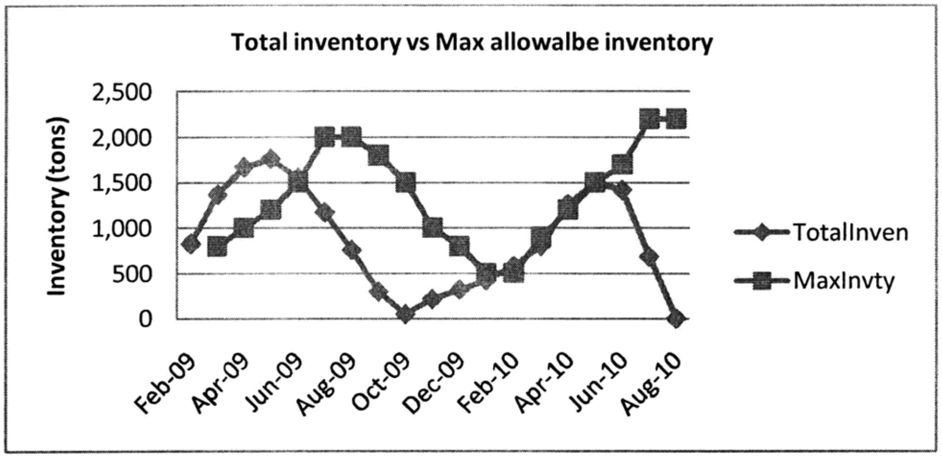

Figure 3.7 Total Inventory versus Maximum Allowable Inventory (tons)

From Figure 3.7, we see that the actual total inventory generally follows the pattern of the maximum inventory with some lagging. The actual inventory is below the maximum allowable inventory for most of the time periods except for the first few months. The reason is that the starting inventory is high and company ABC had built up the inventory a lot during spring. As a result, it needs to pay penalty cost for the high inventory at the beginning of 2009. But after that, the optimal plan has company ABC maintaining its inventory at or below the maximum allowable inventory to avoid the penalty cost.

3.3 Sensitivity Analysis

After the basic model is solved, company ABC also wants to see how the objective and optimal solution would change if there is a slight change in the supply volume or market demand. This leads to the following discussion on the sensitivity of the basic model.

3.3.1 Change in Supply Volume

From Chapter 2, we know that each source has an agreed-to volume with company ABC for each month and the actual shipments have to fall within a range around this agreed-to volume. If the supply constraint is tight at either the lower bound or upper bound, the manager might want to know by how much the profit would increase if they are able to decrease (tight at lower bound) or increase (tight at upper bound) the agreed-to volume by one ton. This information would help the company to decide how they should change the agreed-to volume with each source if they want to expand their businesses at some point of time. OPL CPLEX allows us to obtain the shadow price (dual variable) of the constraints as a marginal feature.

Figure 3.8 is the shadow price of the supply lower bound constraints and all the values are non-positive. The reason is that if we increase the lower bound, the feasible region would be smaller and thus the objective would decrease. If the value is zero, it suggests that the lower bound constraint is not tight. Otherwise, the lower bound is binding and a decrease in the lower bound would increase the objective.

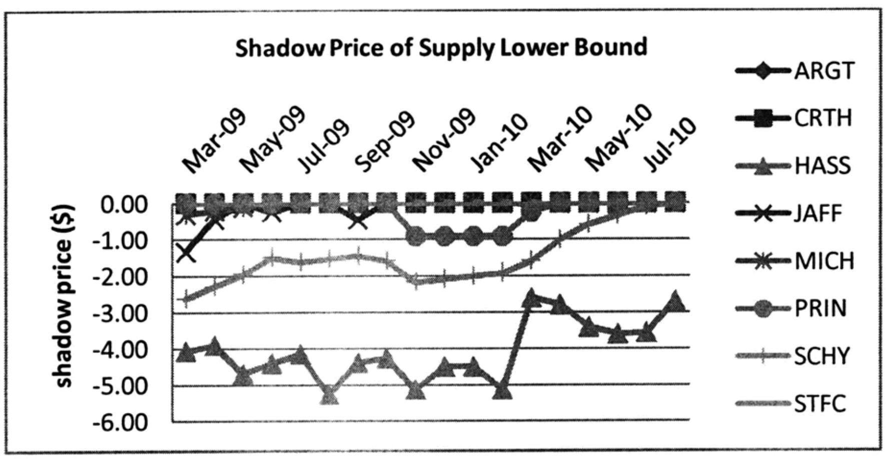

Figure 3.8 Shadow Prices of Supply Lower Bound Constraints ($/ton)

From the figure, we see that sources HASS & SCHY have relatively high negative shadow prices for their supply lower bound constraints for the entire time horizon. This suggests that company ABC should reduce the agreed-to volumes with these two sources to increase its gross margin. In other words ABC would prefer to buy less salt from these two sources.

Figure 3.9 is the shadow price of supply upper bound and all the values are non-negative. This is because if the upper bound is increased, the feasible region would increase and thus the objective would increase. If the value is zero, the upper bound is likely to be non-binding. Otherwise, the upper bound is binding and an increase in the upper bound would improve the objective.

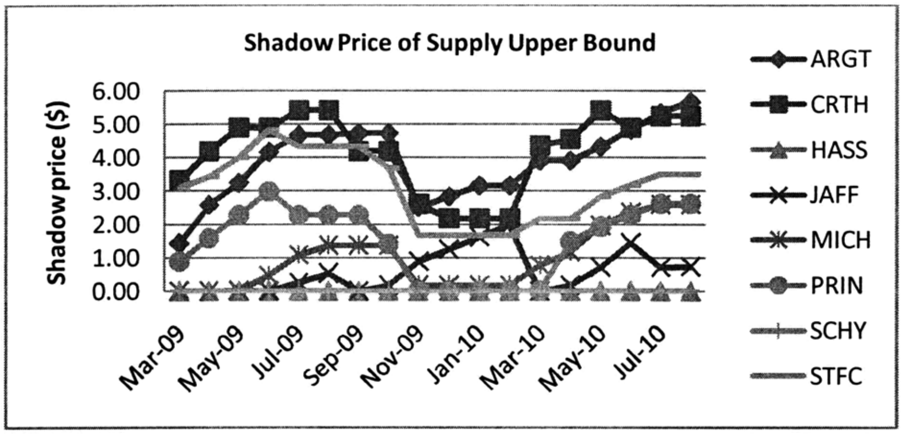

Figure 3.9 Shadow Price of Supply Upper Bound Constraints ($/ton)

Figure 3.9 suggests that most of the supply upper bound constraints are tight and thus increasing the agreed-to volume with most of the sources would help company ABC to improve its performance. Among all sources, ARGT, CRTH and STFC have higher values over the entire time horizon, so company ABC should try to increase its agreed-to supply amount with these three sources.

3.3.2 Change in Market Demand

Each demand region has a predicted demand for S and H across the time horizon. If company ABC wants to promote its product to increase the sales at some demand regions (e.g., by increasing advertizing or distributing coupons), they would probably want to know which region and which month would give them the best profit if they have certain budget constraints. That is why we also investigate the shadow price of demand constraints. Figure 3.10 and 3.11 are the shadow prices of demand constraints for product H and S.

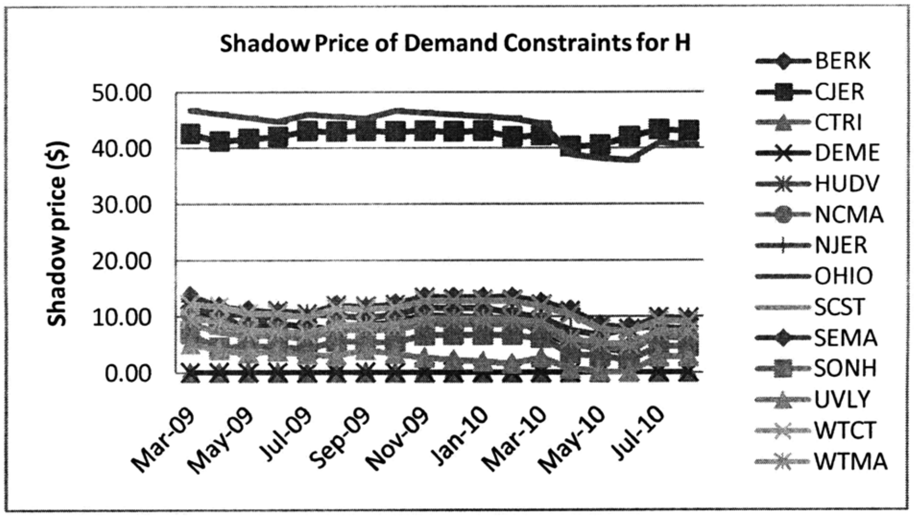

Figure 3.10 Shadow Price of Demand Constraints for H ($/ton)

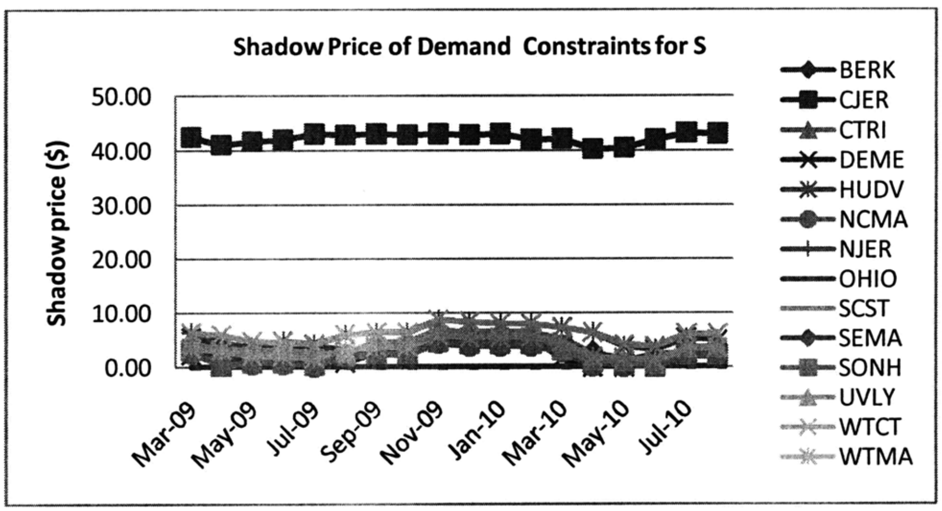

Figure 3.11 Shadow Price of Demand Constraints for S ($/ton)

From the plots, we can see that regions CJER and OHIO have very high shadow prices for product H while only region CJER has very high shadow prices for product S. This indicates that company ABC should consider how it might increase demand in the two regions CJER and OHIO.

3.3.3 Change in the Storage Capacity

If company ABC wants to expand its business, it may need to increase its inventory capacity. Thus it has to decide which storage point they should expand. This leads to the discussion of shadow prices of buffer inventory and storage inventory capacity constraints. Since each location has a fixed storage capacity over time, a change in the capacity would affect all of the 18(months) storage constraints. Thus it would be reasonable for us to add the shadow prices for the 18 months together for each buffer location and storage point to get a measure of the possible effect from increasing the storage capacity.

Table 3.7 Sum of Shadow Price of Buffer Location Storage Capacity over Time

SourceID	ARGT	CRTH	HASS	JAFF	MICH	PRIN	SCHY	STFC
Value ($/ton)	0	0	0	0	0.18	0	0.15	0.07

The number in Table 3.7 is the increase of total margin if the storage capacity at the corresponding buffer location increases by one ton. If the value is zero, the storage limit is never binding and thus the expansion is not necessary. Otherwise, the inventory reaches the capacity limit at some time and an increase in the capacity would improve the objective. The table shows only three sources have positive values. However, the values are all very small compared to that of supply and demand constraints. This implies that the inventory capacity should not be a major concern of company ABC.

Table 3.8 Sum of Shadow Price of Storage Point Storage Capacity over Time

RegionID	BERK	CJER	CTRI	DEME	HUDV	NCMA	NJER	OHIO	SCST	SEMA	SONH	UVLY	WTCT	WTMA
Value ($/ton)	0	0	0	0	1.69	0	0	0	0	0	1.28	0	0	0.17

Similarly, the number in Table 3.8 represents the increase in the total margin if the inventory capacity at the storage points is increased by one ton. Only HUDV, SONH and WTMA have positive shadow price and this implies that most of the storage points still have extra space. The amount is small, thus expansion of the storage capacity would not help a lot.

3.3.4 Reduced Cost of Non-basic Variables

Non-basic variables in this context are the feasible routes which do not have non-zero physical flows under the current optimal solution; that is the flow on the route is zero in the optimal solution. Since this is a maximization problem, all the reduced cost for non-basic variables would then be negative. So if company ABC wants to enforce flow along non-basic routes, it would reduce their gross margin. On the other hand, reduced cost gives company ABC insight on how much the market price should increase before a particular route can be profitable.

OPL CPLEX also provides the information about reduced cost. The reduced cost can also be analyzed by Pivot Table. The following two tables are two specific examples.

Table 3.9 Reduced Cost of Flow from Sources to Storage Points for Sep 2009 ($/ton)

Sum of Sep-09	Column Labels														
Row Labels	BERK	CJER	CTRI	DEME	HUDV	NCMA	NJER	OHIO	SCST	SEMA	SONH	UVLY	WTCT	WTMA	Grand Total
ARGT		-41			0		0						0		-40.73
CRTH				-0.65		-1.16			0	-1.05	-0.5				-3.36
HASS		-33			-0.69	-0.42	0	-29.8		0			-0.41	-0.84	-64.65
JAFF	-5.44										-0.5	0		-6.09	-12.03
MICH								-39.3						0	-39.28
PRIN	0	-43	-0.9	0	-5.77	-1.85				-0.9	-2.52	-1.18	-0.78	-1.07	-58.17
SCHY	0	-35			0	0	-0.5						0	-0.28	-35.58
STFC			0	-0.99	-5.27	0		-0.33			-1.43	0	0	0	-8.02
Grand Total	**-5.44**	**-151**	**-0.9**	**-1.64**	**-11.7**	**-3.43**	**-0.5**	**-69.4**	**0**	**-1.95**	**-4.95**	**-1.18**	**-1.19**	**-8.28**	**-261.82**

Table 3.10 Reduced Cost of Flow from Storage Points to Demand Regions for Sep 2009 ($/tons)

Sum of Sep-09	Column Labels														
Row Labels	BERK	CJER	CTRI	DEME	HUDV	NCMA	NJER	OHIO	SCST	SEMA	SONH	UVLY	WTCT	WTMA	Grand Total
BERK	0														0
CJER		0													0
CTRI			0			-10.4				-11.91			-10.74		-33.05
DEME				0											0
HUDV					0										0
NCMA						0									0
NJER							0								0
OHIO								0							0
SCST									0						0
SEMA										0					0
SONH											0				0
UVLY						-7.05			-4.34		-3.5	0			-14.89
WTCT													0		0
WTMA	-1.05				-4.55	-2.22			0	-3.73	-0.01		-2.56	0	-14.12
Grand Total	**-1.05**	**0**	**0**	**0**	**-4.55**	**-19.67**	**0**	**0**	**-4.34**	**-15.64**	**-3.51**	**0**	**-13.3**	**0**	**-62.06**

The above two tables can help company ABC to understand how much the gross margin would decrease if they were to force one ton to flow on a non-basic arc under current market situation. In other words, it indicates how the market condition needs to change in order for a non-basic route to become economic. For example, suppose that company ABC wants to move product H along route PRIN→CTRI→NCMA for September 2009. The sum of the reduced costs along this path for that month is -$11.3 (-$0.9-$10.4) per ton. So the market price of H at demand region NCMA has to increase by at least $11.3 per ton to make the physical flow along this path profitable. Alternatively, the purchase cost at source PRIN plus the transportation costs from PRIN→CTRI→NCMA would need to decrease by at least $11.3 per ton in order for it to be profitable to use this route.

4 Price Models

In Chapter 3, we build the network flow model to find the optimal flows along routes across the time horizon of 18 months. In that model, we assume that product price is fixed and company ABC has no control over it. We also assume that the market demand can be forecasted perfectly. However, the reality is that company ABC has certain control over the product price. They are able to adjust the product price within a reasonable amount. It is obvious that the product price they set will affect the market demand. To model this aspect of the issue, we set market demand as a function of the product price and rebuild the model to determine both the optimal prices and optimal flows at the same time.

4.1 Demand Function

According to the information provided by company ABC, there is a nonlinear negative relationship between the product price and the market demand. ABC has found that the following function captures the relationship between demand and price:

$$D(P) = \overline{D}(1-0.15)^{0.6(P-\overline{P})} \quad (4.1)$$

In the formula, D and P represent the market demand and the product price. $\overline{P}$ is the baseline price for the demand region, and $\overline{D}$ is the corresponding baseline market demand. Formula (4.1) means every increase of \$ 1.67 (1/0.6) in the price leads to a 15%

reduction of the market demand. Thus the actual demand D is a non-linear function of the actual price P, which dramatically increases the computation complexity.

4.2 New Notations

To embed the demand function into the model, we first have to introduce some new notation.

Parameters

$\overline{P}_{kt}$: Baseline market price ($) for both H and S at demand region k for month t

$\overline{D}_{kt}$: Baseline market demand (tons) for road salt at demand region k for month t

$\overline{HD}_{kt}$: Baseline market demand (tons) for H at demand region k for month t, where $\overline{HD}_{kt} = \alpha_k D_{kt}$

$\overline{SD}_{kt}$: Baseline market demand (tons) for S at demand region k for month t, where $\overline{SD}_{kt} = (1-\alpha_k)D_{kt}$

The two parameters $\overline{P}_{kt}$ and $\overline{D}_{kt}$ in the price model would have the same value as P_{kt} and D_{kt} in the basic model. What is taken as the fixed product price and market demand previously would now be the baseline price and baseline demand as company ABC can now change the price and thus the market demand. As a result, we also have to introduce a set of new decision variables.

Decision Variables

HP_{kt} : Actual price for product H at demand region k for month t

HD_{kt} : Resulting market demand for product H at demand region k for month t

SP_{kt} : Actual price for product S at demand region k for month t

SD_{kt} : Resulting market demand for product S at demand region k for month t

The baseline prices for H and S in each demand region are the same. However, the demand functions are different as the baseline market demand is different. So we have to distinguish the prices for the two products in the price model. That is the reason why we declare the prices for the two products separately. Except for these changes, the rest of the notation in the basic model would still be in use for the price model (*refer to section 4.1.1*).

4.3 Price Model 1---Gradient Line Linear Approximation

The exponential demand function (Equation 4.1) makes the model very difficult to solve with the available commercial solver codes. To reduce the computational complexity of the model and make it solvable by CPLEX, we introduce a linear approximation to the demand function; we will use the gradient line at the baseline price to replace the actual exponential demand function in this section. The resulting model would then be a quadratic program and thus can be solved by CPLEX directly.

4.3.1 Model Formulation

From equation (4.1), we can obtain the gradient at $\overline{P}$

$$\left.\frac{\partial D(P)}{\partial P}\right|_{P=\overline{P}} = 0.6\ln(0.85)\overline{D}(0.85)^{0.6(P-\overline{P})}\Big|_{P=\overline{P}} = 0.6\ln(0.85)\overline{D}$$

The gradient line at $\overline{P}$ would thus be

$$D(P) = \overline{D} + 0.6\ln(0.85)\overline{D}(P-\overline{P}) \qquad (4.2)$$

We apply equation (4.2) for each demand region k, each time period t and each product (H/S) to obtain the following set of equations.

$$HD_{kt}(HP_{kt}) = \overline{HD}_{kt} + 0.6\ln(0.85)\overline{HD}_{kt}(HP_{kt} - \overline{P}_{kt}) \text{ for H, where } \overline{HD}_{kt} = \alpha_k D_{kt}$$

$$SD_{kt}(SP_{kt}) = \overline{SD}_{kt} + 0.6\ln(0.85)\overline{SD}_{kt}(SP_{kt} - \overline{P}_{kt}) \text{ for S, where } \overline{SD}_{kt} = (1-\alpha_k)D_{kt}$$

To better understand the gradient line approximation, we provide an instance of the demand function with baseline price \$40 and baseline demand 5 tons and its gradient line at baseline price in Figure 4.1.

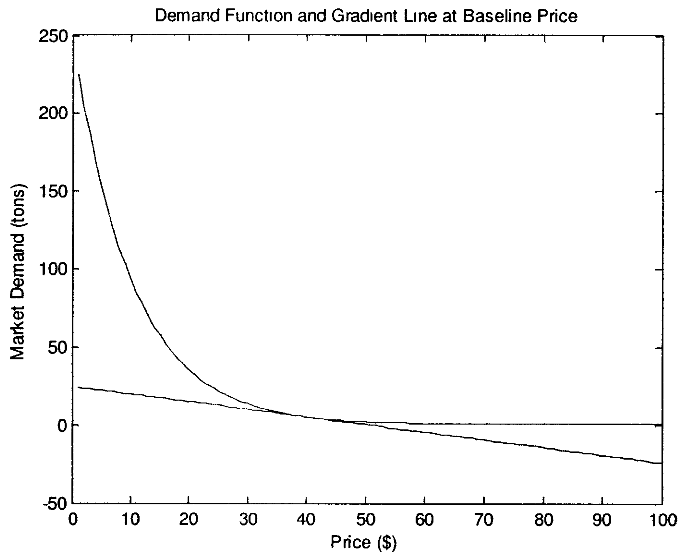

Figure 4.1 Example of Demand Function and Gradient Line at Baseline Price

From Figure 4.1, we can see that the gradient line is a good approximation to the demand function if the actual price is near the baseline price (\$40). Price Model 1 is based on this idea and uses the gradient line to replace the actual demand curve.

The mathematical model for the basic model in Chapter 3 still applies in the price model with some slight changes in the revenue function and demand constraints.

The revenue function would now be

$$R_t = \sum_k (HP_{kt}HD_{kt} + SP_{kt}SD_{kt})$$

$$= \sum_k (HP_{kt}(\overline{HD}_{kt} + 0.6\ln(0.85)\overline{HD}_{kt}(HP_{kt} - \overline{HP}_{kt})) + SP_{kt}(\overline{SD}_{kt} + 0.6\ln(0.85)\overline{SD}_{kt}(SP_{kt} - \overline{SP}_{kt})))$$

Demand constraints for the two products would now be

$$\sum_j HF_{jkt} = HD_{kt} = \overline{HD}_{kt} + 0.6\ln(0.85)\overline{HD}_{kt}(HP_{kt} - \overline{P}_{kt}) \text{ where } \overline{HD}_{kt} = \alpha_k D_{kt},\ \forall k,t$$

$$\sum_j SF_{jkt} = SD_{kt} = \overline{SD}_{kt} + 0.6\ln(0.85)\overline{SD}_{kt}(SP_{kt} - \overline{P}_{kt}) \text{ where } \overline{SD}_{kt} = (1-\alpha_k)D_{kt},\ \forall k,t$$

In this way, the sales in each demand region have to be exactly equal to the predicted demand that corresponds to the price set by company ABC. Except for the above changes, the rest of the basic model like the inventory constraints would still be the same in this price model (*refer to section 4.1.2*).

4.3.2 Result Analysis

We built Price Model 1 in OPL and run it with the same data set used for the basic model. This section is to analyze the new results and compare them to those obtained from the basic model.

4.3.2.1 Objectives

Since gradient line is only a good approximation to the demand function when the actual price is near the baseline price, we need to impose some range constraints on the actual prices HP_{kt} and SP_{kt} so that the linear approximation is indeed a valid approximation.

By setting different range constraints on HP_{kt} and SP_{kt} , we obtain different results. Table 4.1 below is a summary of the results.

Table 4.1 Objective Value for Different Range Constraints on Price from Price Model 1

Range of HP_{kt}, HP_{kt}	Result Message
$HP_{kt}, SP_{kt} \in \left[\overline{P}_{kt} \pm 10\right]$	Solution is unbounded or infeasible
$HP_{kt}, SP_{kt} \in \left[\overline{P}_{kt} \pm 15\right]$	Solution (optimal) with objective \$116,275
$HP_{kt}, SP_{kt} \in \left[\overline{P}_{kt} \pm 20\right]$	Solution (optimal) with objective \$116,275

From Table 4.1, we realize that if we enforce the price to be within $10 away from the baseline price, the model is unbounded or infeasible. The reason is that Formulation I enforces equality between flows and actual demand. By putting a range constraint on the actual price, we simultaneously put a range constraint on the actual market demand. By some point of time, if the actual flows fall out of the feasible range of the actual market demand due to the supply constraints, the model would become infeasible. However, by increasing the flexibility of the price, we increase the range of the actual market demand and thus it is unlikely for the flows to fall out of the feasible range of the market demand. Table 4.1 shows that if the price is allowed to drift as far as $15 away from the baseline price, the model would become feasible and be able to deliver an optimal solution with objective value $116,275. Actually, if we do not put any range constraints on the price, the objective value would still be $116,275.

Table 4.2 below shows results from Price Model 1.

Table 4.2 Results from Price Model 1

Components of Revenue	Value ($)	Percentage of Revenue
Total Transportation Cost	110,311	20.15%
Total Material Cost	319,047	58.28%
Total Inventory Cost	1,325	0.24%
Total Penalty	488	0.089%
Total Margin	116,275	21.24%
Total Revenue	547,447	100%

If we compare Table 4.2 to Table 3.1, we see an increase of around 3K in the total margin for the Price Model 1 than the basic model. The solution now has a decrease of around 55K in the material cost and a decrease of around 19K in the transportation cost and a decrease of 71K in the revenue. All the other components stay almost the same. This is how an increase of 3K (55K+19K-71K) appears in the profit. The idea behind is that company ABC now forsakes certain market demand by increasing the market price so that the scale of the business is smaller but the total profit is larger. This demonstrates the potential advantages for reducing the market demand by increasing the market price.

4.3.2.2 *Optimal Price*

Figure 4.2 and Figure 4.3 are the optimal prices for product H and S over time from Price Model 1. Figure 4.4 and Figure 4.5 show the discrepancy of the optimal price from their original baseline price.

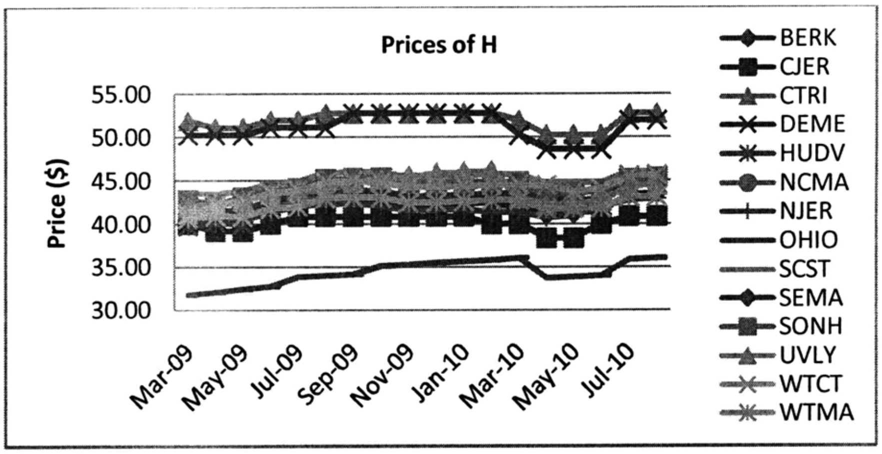

Figure 4.2 Optimal Prices of H from Price Model 1($)

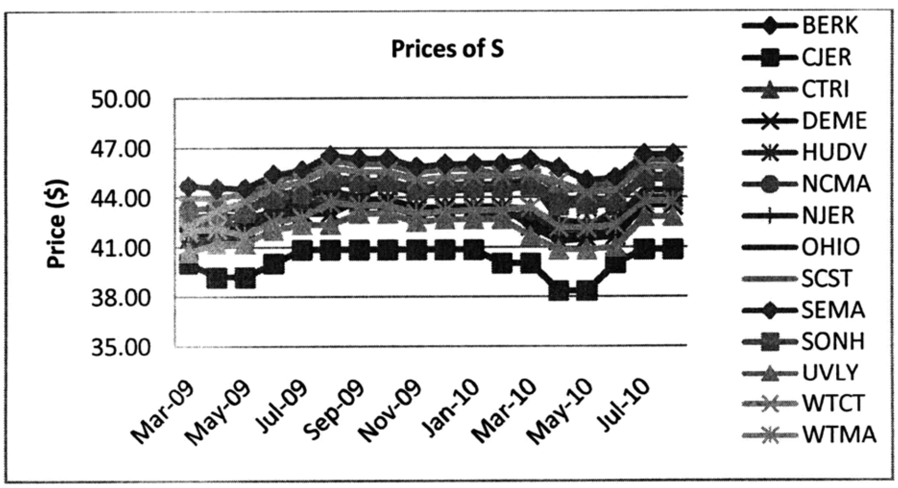

Figure 4.3 Optimal Prices of S from Price Model 1($)

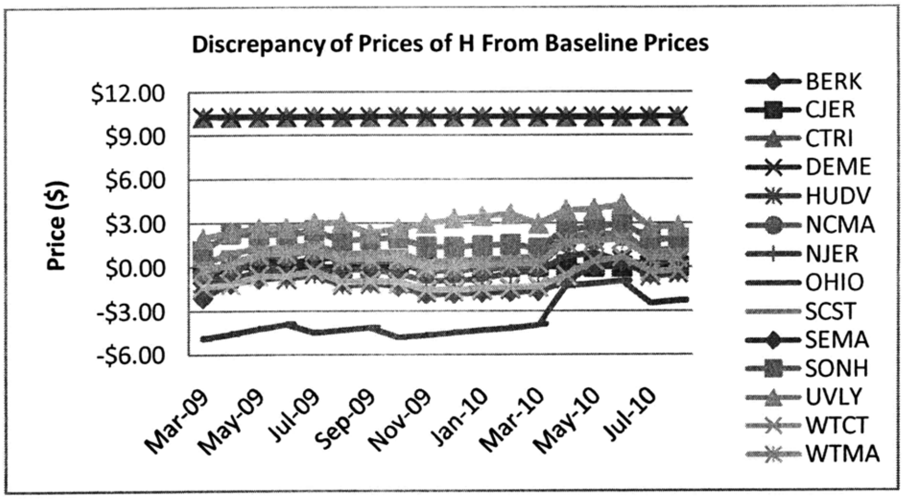

Figure 4.4 Change of Prices for H from Price Model 1 ($)

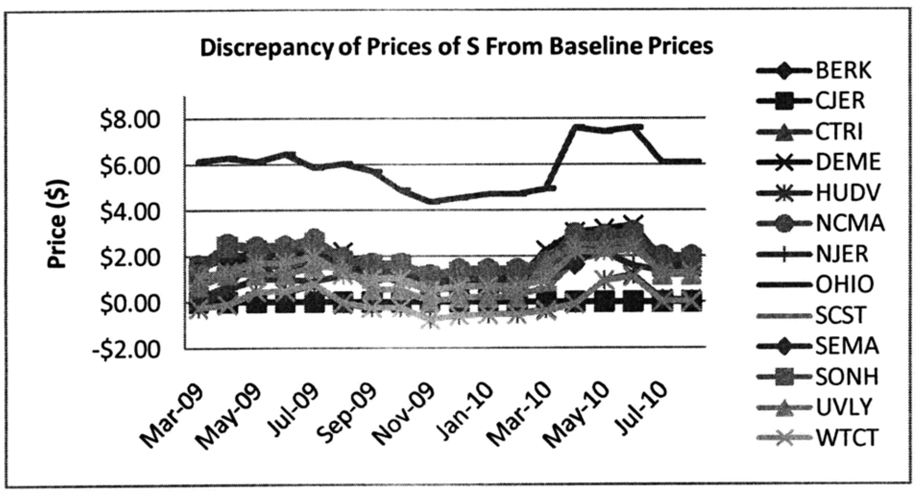

Figure 4.5 Change of Price for S from Price Model 1($)

Figure 4.2 shows that the optimal prices for product H in most of the demand regions stay within $40 and $45 except for two outliners CTRI and DEME, which always have higher price than other regions. By examining the discrepancy of optimal prices from the baseline prices in Figure 4.4, we find out that the prices in regions CTRI and DEME for product H increase by a significant amount. Meanwhile Figure 4.5 suggests that the price of product S in region OHIO increases more significantly than all the other regions.

The trends described above can be explained from the structure of the network. As we look into the network, we find out that the supply chain that ends at demand region CTRI and DEME all starts with sources which only supply product S. For instance, demand region CTRI can only be supplied by storage point CTRI which is supplied by sources PRIN and STFC. And both of the sources only provide product S. This implies that there can be no flow of H to these two regions while there is indeed baseline demand for H in the two regions. On the other hand, the equality demand constraint requires that the flow has to be equal to the resulting market demand in each demand region, which means that the market demand of H at these two regions has to be forced to be zero to maintain the model feasibility. As a result, the price of H at these two regions has to be increased by a large amount so that the corresponding market demand is zero. As we use the gradient line at the baseline price as the approximate demand-price function, the optimal price should be the horizontal intercept of the gradient line. This also explains why the model is infeasible when prices have to be within $\left[\overline{P}_{kt} \pm 10\right]$ but feasible when prices are allowed to be within range $\left[\overline{P}_{kt} \pm 15\right]$.

As for region OHIO, it has a relatively large increase in the price for S. One possible reason is that the baseline price of S in OHIO is much lower than those in other regions.

Another important trait of the solution is that the optimal prices for the products H and S in the same demand region are no longer the same although they share the same baseline price. The reason is that they have different baseline demand and thus different demand function, and they have different supply chain costs since they come from different sources. As a result, the optimal price will not be the same for the two products any more.

4.3.2.3 Other Results

Besides optimal price, Price Model 1 also determines the optimal flows and inventory as in Chapter 3. However, we skip the discussion about optimal flows, inventory and sensitivity for this model due to space constraints.

4.4 Price Model 2--- Piecewise Linear Approximation

Besides using the gradient line at baseline price as a linear approximation to demand function, we can also use a piecewise linear function to approximate the demand function. In this way, it would involve binary variables and thus the model becomes a MIQP (mixed integer quadratic programming). In this section, we will use the piecewise linear function around the baseline price to replace the actual demand function and solve the resulting MIQP. Also to simplify the formulation slightly, we temporarily forsake the difference between products in the following formulation. In other words, the formulation below applies to both products H and S.

4.4.1 Model Formulation

Figure 4.6 below gives an example of a piecewise linear approximation to the demand function.

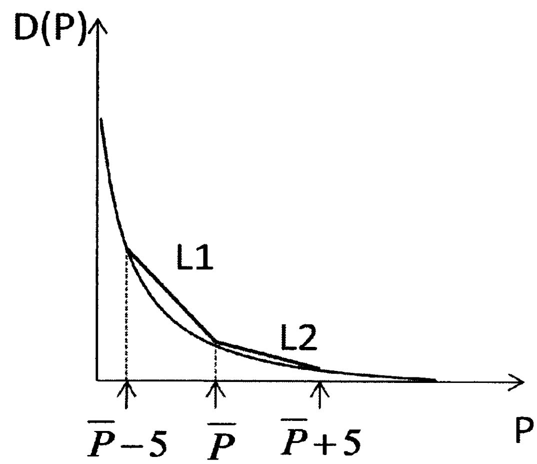

Figure 4.6 Piecewise Linear Approximation of Demand Function

We start with two line segments around the baseline price. We initially set the length of both intervals to be \$5. For each demand region k and time period t, the piecewise linear approximation function is different as they have different baseline price and baseline demand.

Gradient of the first line segment L1:

$$\frac{D_{kt}(\overline{P}_{kt})-D_{kt}(\overline{P}_{kt}-5)}{5}=\frac{\overline{D}_{kt}-\overline{D}_{kt}(1-0.15)^{0.6(-5)}}{5}=-0.1275\overline{D}_{kt}$$

Gradient of the second segment L2:

$$\frac{D_{kt}(\overline{P}_{kt}+5)-D_{kt}(\overline{P}_{kt})}{5}=\frac{\overline{D}_{kt}(1-0.15)^{0.6(5)}-\overline{D}_{kt}}{5}=-0.0772\overline{D}_{kt}$$

Function of the first segment L1

$$D_{kt}=\overline{D}_{kt}-0.1257\overline{D}_{kt}(P_{kt}-\overline{P}_{kt}) \qquad \overline{P}_{kt}-5\leq P_{kt}\leq \overline{P}_{kt}$$

Function of the second segment L2

$$D_{kt}=\overline{D}_{kt}-0.0772\overline{D}_{kt}(P_{kt}-\overline{P}_{kt}) \qquad \overline{P}_{kt}\leq P_{kt}\leq \overline{P}_{kt}+5$$

To decide whether the price falls within the first line segment or the second line segment, we introduce binary indicator variables Z_{kt1} and Z_{kt2} for the two line segments.

The revenue function would now be

$$R_t=\sum_k P_{kt}(\overline{D}_{kt}-0.1257\overline{D}_{kt}(P_{kt}-\overline{P}_{kt}))Z_{kt1}+P_{kt}(\overline{D}_{kt}-0.0772\overline{D}_{kt}(P_{kt}-\overline{P}_{kt}))Z_{kt2}$$

The demand constraint would now be

$$\sum_j F_{jkt}=(\overline{D}_{kt}-0.1257\overline{D}_{kt}(P_{kt}-\overline{P}_{kt}))Z_{kt1}+(\overline{D}_{kt}-0.0772\overline{D}_{kt}(P_{kt}-\overline{P}_{kt}))Z_{kt2}$$

The revenue function and demand constraint make the model very difficult to solve as each involves the product of two different decision variables P and Z. To avoid using the product of decision variables, we introduce a large positive integer M to the model.

The revenue function is replaced with the two inequality constraints

$$R_t \le \sum_k P_{kt}(\overline{D}_{kt} - 0.1257\overline{D}_{kt}(P_{kt} - \overline{P}_{kt})) + M*(1 - Z_{kt1})$$
$$R_t \le \sum_k P_{kt}(\overline{D}_{kt} - 0.0772\overline{D}_{kt}(P_{kt} - \overline{P}_{kt})) + M*(1 - Z_{kt2})$$

The two constraints work together to give us the correct upper bound on the revenue. For example, if the price is in the region $[\overline{P}_{kt} - 5, \overline{P}_{kt}]$ then Z_{kt1} =1 and Z_{kt2} = 0. The two constraints would become

$$R_t \le \sum_k P_{kt}(\overline{D}_{kt} - 0.1257\overline{D}_{kt}(P_{kt} - \overline{P}_{kt}))$$
$$R_t \le \sum_k P_{kt}(\overline{D}_{kt} - 0.0772\overline{D}_{kt}(P_{kt} - \overline{P}_{kt})) + M$$

Since the model is a maximization problem, revenue will take the value at its upper bound automatically. In this way, we avoid using product of two decision variables but achieve the same effect as the revenue function.

Similarly, we use a set of inequality constraints to replace the demand equality constraint

Demand Constraints

$$\sum_j F_{jkt} \le \overline{D}_{kt} - 0.1257\overline{D}_{kt}(P_{kt} - \overline{P}_{kt}) + M(1 - Z_{kt1})$$

$$\sum_j F_{jkt} \le \overline{D}_{kt} - 0.0772\overline{D}_{kt}(P_{kt} - \overline{P}_{kt}) + M(1 - Z_{kt1})$$

$$\sum_j F_{jkt} \ge \overline{D}_{kt} - 0.1257\overline{D}_{kt}(P_{kt} - \overline{P}_{kt}) - M(1 - Z_{kt1})$$

$$\sum_j F_{jkt} \ge \overline{D}_{kt} - 0.0772\overline{D}_{kt}(P_{kt} - \overline{P}_{kt}) - M(1 - Z_{kt1})$$

Meanwhile, we have to ensure P_{kt} falls into one of the line segment and set the corresponding Z equal to 1

$$Z_{kt1} + Z_{kt2} = 1$$

$$P_{kt} >= (\overline{P}_{kt} - 5) Z_{kt1}$$

$$P_{kt} >= \overline{P}_{kt} Z_{kt2}$$

$$P_{kt} \le \overline{P}_{kt} + M * (1 - Z_{kt1})$$

$$P_{kt} \le (\overline{P}_{kt} + 5) + M * (1 - Z_{kt1})$$

.

4.4.2 Result Analysis

We implement Price Model 2 in OPL and we were not successful at solving the problem for the base case. The error message is that the model is either unbounded or infeasible. The reason is the same as what we have discussed in the section 4.3.2.1 for the infeasibility when the range constraints are $HP_{kt}, SP_{kt} \in \left[\overline{P}_{kt} \pm 10\right]$. Due to supply and inventory constraints, shipments into demand regions may fall out of the expected range of actual market demand. Since we only consider the price interval $\left[\overline{P}_{kt} - 5, \overline{P}_{kt} + 5\right]$, it is very likely that the flow is not within the feasible region of the actual demand. To test whether this suspicion is valid, we enlarge the interval to be $\left[\overline{P}_{kt} - 40, \overline{P}_{kt} + 40\right]$ and rerun the model. The model tends to be feasible but it cannot be solved within reasonable time. The version of CPLEX that was available to us has limited capability for solving large scale integer programs and thus we could not obtain the optimal solution for this formulation. However, it might be solvable with more powerful software; in this case, , the objective from the piecewise linear approximation should serve as an upper bound of the actual optimal profit as the line segments always line above the demand curve.

4.5 Price Model 3 --- Iterative Algorithm Based on Gradient Line

In Price Model 1, we use the gradient line at the baseline price as the linear approximation to the demand curve. We also know that the linear approximation is only accurate if the actual price is near to the baseline price. However, as we examine the optimal solution delivered by Price Model 1, we find that the distance between optimal prices and baseline prices at some regions is large (>$10) and thus the gradient line at the baseline price is a very inaccurate approximation for the demand around those points. This is the reason why we propose the following iterative algorithm to ensure a good approximation to the demand function.

4.5.1 Algorithm

The algorithm for Price Model 3 is as below.

Step 1 Solve Price Model 1 with the original input data set and obtain current optimal solution P_{kt}^{c}

Step 2 Determine the demand regions and months (k,t) such that $\left|P_{kt}^{c} - \overline{P}_{kt}\right| \geq tol$ where tol is a user defined tolerance level. For each pair, reset the baseline price $\overline{P}_{kt}$ and market demand $\overline{D}_{kt}$ to P_{kt}^{c} and D_{kt}^{c} respectively and use the gradient line at the updated $\overline{P}_{kt}$ as the new linear approximation to the demand function. After that, go to step 3. If there are no such demand regions and months, go to step 4.

Step 3 Resolve Price Model 1 with the updated data set and linear approximations to obtain the new solution P_{kt}^{c} and go back to step 2

Step 4 The current solution is the solution for Price Model 3, ie $P^{*} = P^{c}$ and $D^{*} = D^{c}$.

From the algorithm, we know that Price Model 1 is a special case of Price Model 3 where only one iteration is applied.

4.5.2 Result Analysis

4.5.2.1 Objectives

We wrote a script file which calls Price Model 1 and the corresponding data file iteratively. To avoid infinite loop, we also impose a maximum number of iterations allowed in the file. We execute the script file with different parameter values and list the results in Table 4.3.

Table 4.3 Objective value with Different Parameters from Price Model 3

Tolerance	Max # of Iterations	Objective ($)	CPU time (sec)
0.001	10	118827.4976	7.30
0.001	30	118827.5012	20.66
0.01	10	118827.4424	7.22
0.01	30	118827.4425	20.18
0.1	10	118,822.7441	7.34
0.1	30	118,822.7441	20.82
1	10	118,532.5375	7.18
1	30	118,532.5375	21.15

Although all the cases above go until the maximum number of iteration is reached, we found that after a few iterations, all the pairs (k,t) such that $\left|P_{kt}^{c} - \overline{P}_{kt}\right| \geq tol$ are from regions CRTI and DEME for product H. This means the prices in all the regions except CRTI and DEME converge to their optimal solution very fast. The two regions are special regions where the demand of H can never be met under the current network structure as we have discussed in section 4.3.2.2. However, this will not affect the convergence of the profit because there are no sales of H in the two regions and thus they do not contribute to the total profit. This is the reason why the objective converges very fast.

Table 4.3 also shows that the objective value mainly depends on the value of the tolerance. The smaller the tolerance is, the higher the objective would be. Moreover, as the tolerance value decreases from 0.01 to 0.001, the objective remains the same. This implies the objective converges for the tolerance level 0.01 already. On the other hand, if price is ensured to be within $0.01 away from baseline price, the gradient line at baseline price can be considered as a fairly good approximation to the demand function. Meanwhile, we can see that for a fixed level of tolerance, the solution converges very fast and it usually takes less than 10 iterations to converge to a fairly good solution.

The rest of the result analysis is based on the tolerance of 0.01 and maximum number of iterations of 10.

The decomposition of the total revenue is illustrated in Table 4.4.

Table 4.4 Results from Price Model 3

Components of Revenue	Value ($)	Percentage of Revenue
Total Transportation Cost	107,243	20%
Total Material Cost	308,107	57.48%
Total Inventory Cost	1,317	0.24%
Total Penalty	508	0.095%
Total Margin	118,827	22.17%
Total Revenue	536,006	100%

If we compare the results to that from Price Model 1, we find out that the total revenue decreases by approximate 11.5K while total material cost decreases by 11K and total transportation cost decreases by 3K. All the other components stay almost the same. Consequently, the total margin still increases by around 2.5K (11K+3K-11.5K). This indicates that company ABC now forsakes more market demand by maintaining a relatively high price for the products. In this way, they can actually increase the total gross margin by balancing off the product price and market demand.

4.5.2.2 Optimal Prices

Figure 4.7 and Figure 4.8 are the optimal prices for product H and S from Price Model 3.

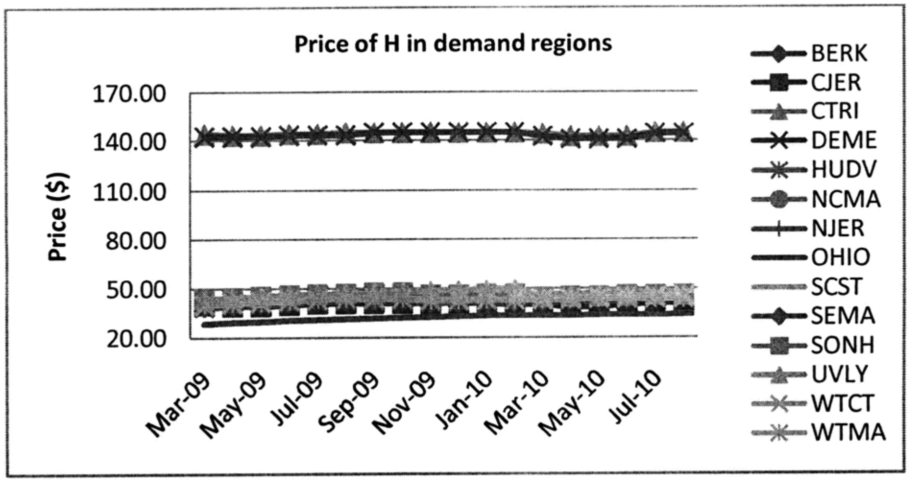

Figure 4.7 Optimal Prices of H from Price Model 3 ($)

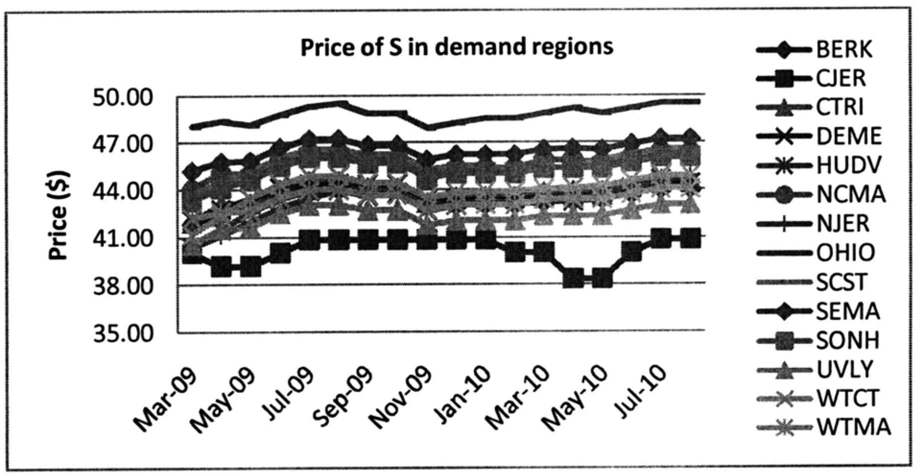

Figure 4.8 Optimal Prices of S from Price Model 3($)

Comparing the above two figures to those from Price Model 1, we can see one major difference in the price for product H in regions CTRI and DEME. The optimal prices in the two regions increase to a very high level compared to that from Price Model 1. This can be explained by the same reason described in section 4.3.2.2. The difference is that in Price Model 1, the gradient line only applies once and the horizontal intercept of the gradient line is the optimal solution. However, in Price Model 3, we iteratively update the gradient line and horizontal intercept and the intercept in the last iteration is taken as the optimal price. The idea can be illustrated in the following graph.

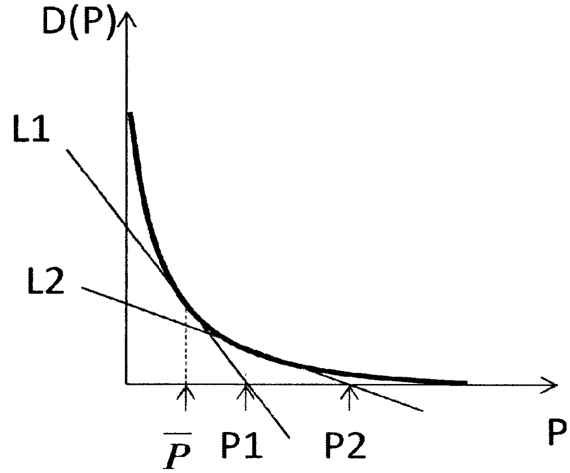

Figure 4.9 Illustration of Iterative Gradient Line Approximation

In the first iteration, the linear approximation of demand function is the gradient line around baseline price L1. Since the demand has to be zero in the two regions, the optimal price is updated to P1. In the second iteration, the gradient line is updated to L2, which is the gradient line around P1. By the same reason, the optimal price would then increase from P1 to P2. So we can see that the optimal price for H in region CRTI and DEME would increases as the algorithm proceeds. This can also be explained mathematically as following.

From equation (4.1) we know that the slope of the demand function is always negative and thus the horizontal intercept in the next iteration would always be larger than that in the previous iteration for these two regions. After several iterations in Price Model 3, the optimal price would definitely be larger than that obtained from applying only one iteration in Price Model 1.

Except for the two regions, we also want to see the change of prices in other region. Figure 4.10 is the difference of optimal price of H from the baseline price for all regions except CTRI and DEME. Figure 4.11 displays the discrepancy of the optimal price from its original baseline price for product S at all demand regions over time.

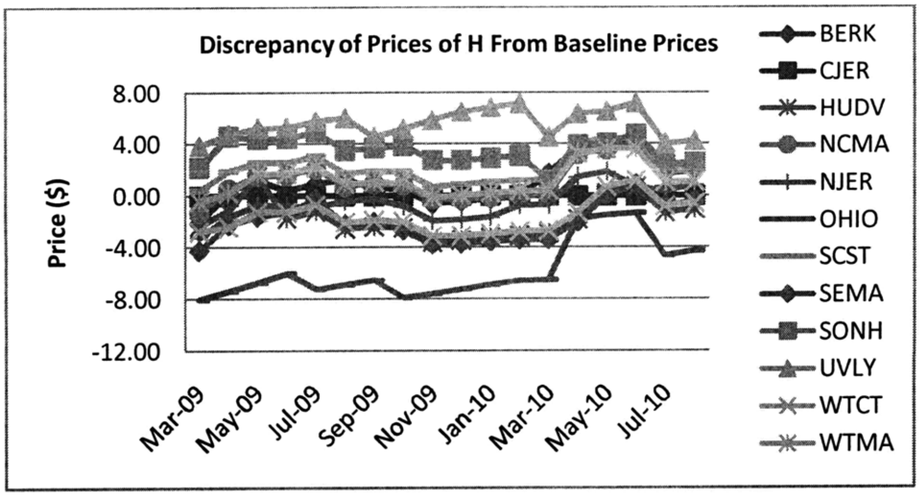

Figure 4.10 Change of Price for H except CTRI & DEME from Price Model 3($)

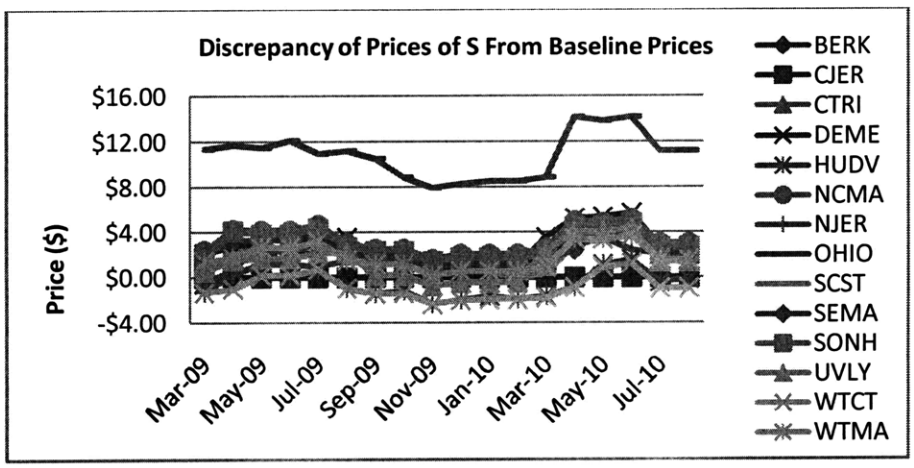

Figure 4.11 Change of Price for S from Price Model 3 ($)

From comparing these two graphs to Figure 4.4 and 4.5, we see that the change in the product price for Price Model 3 is higher than that for Price Model 1. As we mentioned earlier, Price Model 1 is actually the first step in Price Model 3. From the intermediate solutions from Price Model 3, we see that if the price increases in the first iteration, it would also increase in consequent iterations until the optimal price from the next iteration is within the neighborhood of the baseline price in current iteration. However, we could not prove this so far although Figure 4.9 could be a specific example of this idea since horizontal intercept keeps increasing. As a result, the change in price in Price Model 3 is larger than that in Price Model 1.

4.6 Price Model Performance Comparison

As Figure 4.1 shows, the gradient line would always lie under the convex demand curve, which implies that the actual market demand for a given price would always be higher than that obtained from the gradient line (except at the baseline price). As a result, the objective from Price Model 1 would serve as a lower bound of the actual total margin.

From Figure 4.6 we see that the piecewise linear approximation always lies above the demand curve. Thus the actual demand for a given price would be lower than that obtained from the piecewise linear approximation (except at the intersection of the line segments). Consequently, the actual total margin would be lower than the objective from Price Model 2. Unfortunately, the current solver is not able to solve Price Model 2 effectively. However, if a more powerful solver were available to solve MIQP, the objective from Price Model 2 would then serve as an upper bound to the actual total margin.

As for Price Model 3, the objective is expected to be a very accurate approximation to the real gross margin. From the intermediate solutions obtained, we can see the objective converges very fast to the final solution. We suspect the final solution is very close to the real optimal solution although we do not prove it. The proof would be left to further studies.

Table 4.5 is a summary of the objectives to all the Price Models.

Table 4.5 Performance Comparison of Basic Model and Price Models

Model	Objective ($)	Improvement	CPU time (s)
Basic Model	113,068	0%	1.29
Price Model 1	116,275	2.8%	1.09
Price Model 2	N.A	N.A	N.A
Price Model 3	118,827	5.1%	7.08

As we compare the result from the price models to that from the basic model, it is obvious that the solution is better if the demand function is taken into consideration. Using the gradient line linear approximation once would increase the objective by 2.8% while iteratively using the gradient line approximation would result in an increase of 5.1% of the objective. Due to the limitations of CPLEX, we are not able to solve price model 2; however, the objective from price model 2 is expected to be higher than actual objective. On the other hand, all the solvable models can be solved pretty fast (usually within a few seconds) and thus we skip the discussion of computational complexity in this section.

5 Conclusion

This project determines the optimal commodity flow policy as well as pricing strategy for a nationwide distributor of road salt.

We have established in Chapter 3 the basic network flow model and then have solved it by CPLEX. We saw that the transportation cost can be reduced to only 1/3 of the material cost if the optimal flow policy is adopted. We also found that the current network structure should be modified to better serve the market if possible. For example, there are two demand regions (CTRI and DEME) where the market demand for product H is always lost as there are no feasible routes for product H to be transported to these two demand regions. Thus company ABC should add new routes from sources providing H to demand regions CTRI and DEME if possible. Meanwhile, the sensitivity analysis gives company ABC's manager an insight on from which source they should increase the supply volume and in which market region they should increase the sales if possible.

In Chapter 4, we took product price as a new decision variable and examined the non-linear relationship between market demand and product price. We proposed several algorithms to solve the newly developed price model and the result from the best performing algorithm shows that the total gross margin can be improved by 5% if company ABC has the control over product price. We also found that the prices of the two products H and S should be set differently as they have different supply volume and market demand.

However, the models we developed in the thesis do have several limitations. For instance, both the basic model and the price model treat decision variables (flows along the arc) as float instead of integer while the quantity of shipments may have to be integer values due to some physical constraints in real life scenarios. In addition, our price model assumes that all the demand regions share the same demand function which does not vary over time. However, the market situation may change over demand regions and time. In the future, a more complex price model which has a specific demand function for each demand region and each time period can be built based on the Price Model 3 in Chapter 4.

6 Reference:

[1] ILOG. Retrieved July 2, 2009, from Wikipedia: http://en.wikipedia.org/wiki/ILOG

[2] CPLEX. Retrieved July 2, 2009, from Wikipedia: http://en.wikipedia.org/wiki/CPLEX

[3] Dantzig, G.B., *Linear Programming and Extensions*, Princeton University Press,Princeton, N.J., 1963.

[4] Jim Caldwell and Douglas K. S. Ng, *Mathematical Modeling Case Studies and Projects, page 173,* Springer Netherlands, 2004

[5] Sheldon D. Balbirer and David Shaw, An Application of Linear Programming to Bank Financial Planning**,** *Interfaces*, Vol. 11, No. 5 (Oct., 1981), pp. 77-83

[6] Jeremy F. Shapiro. *Modeling the Supply Chain,* chapter 3 page 103.

[7] A. Ouorou, P. Mahey, and J.-Ph. Vial.2000. "A Survey of Algorithms for Convex Multicommodity Flow Problems". *Management Science,* January 2000; 46: 126 - 147.

[8] F. Babonneau, O. du Merle, and J.-P. Vial .2006. "Solving Large-Scale Linear Multi-commodity Flow Problem with an Active Set Strategy and Proximal-ACCPM," *Operations Research,* January-February 2006; 54: 184 - 197.

[9] IBM ILOG Products and Solutions. (2003). Retrieved July 2, 2009, from official ILOG website: http://www.ilog.com/products/cplex/

Appendix A Input Data Set

Table A.1: Source Characteristics

SourceID	PostalCode	H-S	Storage Capacity (tons)	Monthly Storage Cost/Ton	Min Avail	Max Avail
ARGT	07107	SW	100	0.33	90%	100%
CRTH	04427	SW	200	0.17	75%	105%
HASS	38450	HW	500	0.17	75%	120%
JAFF	03452	HW	100	0.33	90%	110%
MICH	49424	HW	100	0.00	30%	120%
PRIN	V0X 1W0	SW	1,000	0.00	90%	105%
SCHY	13502	HW	100	0.17	50%	100%
STFC	G8K 0A1	SW	1,000	0.00	80%	100%

Table A.2: Source Costs

SourceID	Mar-09	Apr-09	May-09	Jun-09	Jul-09	Aug-09	Sep-09	Oct-09	Nov-09	Dec-09	Jan-10	Feb-10	Mar-10	Apr-10	May-10	Jun-10	Jul-10	Aug-10
ARGT	$30	$30	$30	$30	$30	$30	$30	$30	$30	$30	$30	$30	$30	$30	$30	$30	$30	$30
CRTH	$31	$31	$31	$32	$32	$32	$33	$33	$33	$33	$33	$33	$31	$31	$31	$32	$32	$32
HASS	$20	$20	$22	$22	$22	$23	$23	$23	$23	$23	$23	$23	$21	$21	$23	$23	$23	$23
JAFF	$33	$33	$33	$34	$34	$34	$36	$36	$36	$36	$36	$36	$34	$34	$34	$34	$35	$35
MICH	$27	$27	$27	$27	$27	$27	$27	$27	$27	$27	$27	$27	$27	$27	$27	$27	$27	$27
PRIN	$18	$18	$18	$18	$19	$19	$19	$20	$20	$20	$20	$20	$20	$18	$18	$18	$18	$18
SCHY	$31	$31	$31	$31	$32	$32	$32	$32	$32	$32	$32	$32	$32	$31	$31	$31	$31	$31
STFC	$26	$26	$26	$26	$27	$27	$27	$28	$28	$28	$28	$28	$28	$28	$28	$28	$28	$28

Table A.3 Source Volume

SourceID	Mar-09	Apr-09	May-09	Jun-09	Jul-09	Aug-09	Sep-09	Oct-09	Nov-09	Dec-09	Jan-10	Feb-10	Mar-10	Apr-10	May-10	Jun-10	Jul-10	Aug-10
ARGT	100	100	100	100	100	150	150	150	150	50	50	50	50	150	150	150	150	200
CRTH	30	30	30	30	30	30	30	30	30	30	30	30	30	100	100	100	100	100
HASS	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30
JAFF	20	20	20	20	20	20	20	0	0	0	0	0	40	40	40	40	40	40
MICH	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30
PRIN	50	50	50	50	50	50	50	50	50	50	50	50	50	100	100	100	100	100
SCHY	120	120	140	140	140	140	140	140	140	140	50	50	50	150	150	150	150	150
STFC	400	400	400	400	400	400	400	400	300	200	200	200	200	500	500	500	500	500

Table A.4: Storage Characteristics

StorID	Region Name	Cost/Mo	StorCapacity (tons)
BERK	Berkshire	0.50	200
CJER	Central NJ	0.17	70
CTRI	East CT-RI	0.83	100
DEME	Downeast ME	0.33	100
HUDV	Hudson Valley	0.33	80
NCMA	N-Central Mass	0.67	100
NJER	North NJ	0.50	100
OHIO	Ohio	0.33	50
SCST	MA-ME Seacoast	0.50	70
SEMA	SE Mass-RI	0.83	100
SONH	NH South	0.17	50
UVLY	NH Upper Valley	0.67	200
WTCT	West CT	0.83	80
WTMA	West Mass	0.33	200

Table A.5: Region Demand

Region ID	HW	Mar-09	Apr-09	May-09	Jun-09	Jul-09	Aug-09	Sep-09	Oct-09	Nov-09	Dec-09	Jan-10	Feb-10	Mar-10	Apr-10	May-10	Jun-10	Jul-10	Aug-10
BERK	25%	20	29	43	68	60	58	90	90	38	20	20	16	30	44	64	103	90	87
CJER	10%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
CTRI	15%	9	22	31	45	68	92	76	57	27	16	16	13	13	32	47	67	102	138
DEME	30%	8	12	33	63	33	25	23	19	9	6	6	5	12	18	50	94	50	37
HUDV	15%	32	55	67	104	131	132	108	105	64	36	36	29	48	82	101	156	197	198
NCMA	20%	27	68	94	134	192	211	229	174	80	41	41	33	40	102	141	201	288	316

NJER	10%	4	9	19	19	24	28	34	44	19	13	13	10	5	13	29	29	36	43
OHIO	15%	3	4	3	4	5	7	6	10	10	7	7	5	5	6	4	6	8	10
SCST	20%	9	58	110	169	199	164	156	130	59	34	34	27	13	87	165	253	298	247
SEMA	15%	9	29	44	56	107	119	142	85	42	30	30	24	13	43	66	83	161	179
SONH	30%	17	42	73	132	154	129	117	110	55	90	50	40	26	62	109	199	231	194
UVLY	30%	5	12	19	37	32	33	37	23	12	10	10	8	8	17	28	55	48	49
WTCT	15%	4	37	54	69	109	127	143	106	45	29	29	23	6	55	81	103	164	191
WTMA	20%	6	11	13	21	34	41	43	32	20	13	13	11	9	17	20	32	51	61

Table A.6: Region Price

RegionID	Mar-09	Apr-09	May-09	Jun-09	Jul-09	Aug-09	Sep-09	Oct-09	Nov-09	Dec-09	Jan-10	Feb-10	Mar-10	Apr-10	May-10	Jun-10	Jul-10	Aug-10
BERK	$42	$40	$40	$42	$42	$42	$43	$43	$43	$43	$43	$43	$41	$39	$39	$39	$43	$43
CJER	$40	$39	$39	$40	$41	$41	$41	$41	$41	$41	$41	$40	$40	$38	$38	$40	$41	$41
CTRI	$42	$41	$41	$42	$42	$43	$43	$43	$43	$43	$43	$43	$42	$40	$40	$40	$43	$43
DEME	$40	$40	$40	$41	$41	$41	$43	$43	$43	$43	$43	$43	$40	$38	$38	$38	$42	$42
HUDV	$41	$41	$39	$43	$43	$45	$45	$45	$45	$45	$45	$45	$45	$44	$43	$43	$45	$45
NCMA	$42	$41	$41	$42	$42	$43	$43	$43	$43	$43	$43	$43	$43	$41	$41	$41	$43	$43
NJER	$42	$41	$41	$42	$43	$43	$43	$43	$43	$43	$43	$42	$42	$40	$40	$42	$43	$43
OHIO	$37	$37	$37	$37	$38	$38	$38	$40	$40	$40	$40	$40	$40	$35	$35	$35	$38	$38
SCST	$43	$43	$43	$43	$43	$45	$45	$45	$45	$45	$45	$45	$45	$43	$43	$43	$45	$45
SEMA	$43	$43	$43	$43	$43	$45	$45	$45	$45	$45	$45	$45	$45	$44	$43	$43	$45	$45
SONH	$42	$40	$41	$42	$42	$43	$43	$43	$43	$43	$43	$43	$43	$41	$41	$41	$43	$43
UVLY	$40	$40	$40	$41	$41	$41	$43	$43	$43	$43	$43	$43	$40	$38	$38	$38	$42	$42
WTCT	$43	$43	$43	$44	$44	$46	$46	$46	$46	$46	$46	$46	$46	$45	$43	$43	$46	$46
WTMA	$41	$41	$40	$41	$41	$43	$43	$43	$43	$43	$43	$43	$43	$40	$40	$40	$43	$43

Table A.7 Direct Transportation Cost

SourceID	StorID	Base Cost/ton	Mar-09	Apr-09	May-09	Jun-09	Jul-09	Aug-09	Sep-09	Oct-09	Nov-09	Dec-09	Jan-10	Feb-10	Mar-10	Apr-10	May-10	Jun-10	Jul-10	Aug-10
ARGT	CJER	$6	1	1	1.1	1.1	1.1	1.1	1	1	1	1	1	1	1	1	1.1	1.1	1.1	1.1
ARGT	HUDV	$4	1	1	1.1	1.1	1.1	1.1	1	1	1	1	1	1	1	1	1.1	1.1	1.1	1.1
ARGT	NJER	$4	1	1	1.1	1.1	1.1	1.1	1	1	1	1	1	1	1	1	1.1	1.1	1.1	1.1
ARGT	WTCT	$5	1	1	1.1	1.1	1.1	1.1	1	1	1	1	1	1	1	1	1.1	1.1	1.1	1.1
CRTH	DEME	$4	1	1	1	1	1.05	1.05	1	1	0.9	0.9	0.9	0.9	1	1	1	1	1.05	1.05
CRTH	NCMA	$6	1	1	1	1	1	1	1	1	0.95	0.95	0.95	0.95	1	1	1	1	1	1

CRTH	SCST	$5	1	1	1	1	1	1	1	1	0.9	0.9	0.9	0.9	1	1	1	1	1	1
CRTH	SEMA	$7	1	1	1	1	1	1	1	1	0.9	0.9	0.9	0.9	1	1	1	1	1	1
CRTH	SONH	$5	1	1	1	1	1	1	1	1	0.95	0.95	0.95	0.95	1	1	1	1	1	1
HASS	CJER	$14	1	1.05	1.05	1.1	1.15	1.1	1.05	1	1	0.95	0.95	1	1	1.05	1.05	1.1	1.15	1.1
HASS	HUDV	$16	1	1.05	1.05	1.1	1.15	1.1	1.05	1	1	0.95	0.95	1	1	1.05	1.05	1.1	1.15	1.1
HASS	NCMA	$18	1	1.05	1.05	1.1	1.15	1.1	1.05	1	1	0.95	0.95	1	1	1.05	1.05	1.1	1.15	1.1
HASS	NJER	$15	1	1.05	1.05	1.1	1.15	1.1	1.05	1	1	0.95	0.95	1	1	1.05	1.05	1.1	1.15	1.1
HASS	OHIO	$8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
HASS	SEMA	$16	1	1.05	1.05	1.1	1.15	1.1	1.05	1	1	0.95	0.95	1	1	1.05	1.05	1.1	1.15	1.1
HASS	WTCT	$17	1	1.05	1.05	1.1	1.15	1.1	1.05	1	1	0.95	0.95	1	1	1.05	1.05	1.1	1.15	1.1
HASS	WTMA	$17	1	1.05	1.05	1.1	1.15	1.1	1.05	1	1	0.95	0.95	1	1	1.05	1.05	1.1	1.15	1.1
JAFF	BERK	$4	1	1	1	1	1	1	1.05	1.05	1	1	1	1	1	1	1	1	1	1
JAFF	SONH	$3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
JAFF	UVLY	$3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
JAFF	WTMA	$5	1	1	1	1	1	1	1.05	1.05	1	1	1	1	1	1	1	1	1	1
MICH	OHIO	$7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
MICH	WTMA	$7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
PRIN	BERK	$18	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
PRIN	CJER	$22	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
PRIN	CTRI	$19	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
PRIN	DEME	$19	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
PRIN	HUDV	$23	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
PRIN	NCMA	$22	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
PRIN	SEMA	$22	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
PRIN	SONH	$22	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
PRIN	UVLY	$18	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
PRIN	WTCT	$19	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
PRIN	WTMA	$19	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SCHY	BERK	$6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SCHY	CJER	$6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SCHY	HUDV	$5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SCHY	NCMA	$7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SCHY	NJER	$5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

OriginID	DestID	Base Cost/ton	Mar-09	Apr-09	May-09	Jun-09	Jul-09	Aug-09	Sep-09	Oct-09	Nov-09	Dec-09	Jan-10	Feb-10	Mar-10	Apr-10	May-10	Jun-10	Jul-10	Aug-10
SCHY	WTCT	$6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SCHY	WTMA	$6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
STFC	CTRI	$8	1.05	1.05	1	1	1	1	0.95	0.95	0.95	1	1	1	1.05	1.05	1	1	1	1
STFC	DEME	$10	1.05	1.05	1	1	1	1	0.95	0.95	0.95	1	1	1	1.05	1.05	1	1	1	1
STFC	HUDV	$13	1.05	1.05	1	1	1	1	0.95	0.95	0.95	1	1	1	1.05	1.05	1	1	1	1
STFC	NCMA	$10	1.05	1.05	1	1	1	1	0.95	0.95	0.95	1	1	1	1.05	1.05	1	1	1	1
STFC	OHIO	$13	1.05	1.05	1	1	1	1	0.95	0.95	0.95	1	1	1	1.05	1.05	1	1	1	1
STFC	SONH	$12	1.05	1.05	1	1	1	1	0.95	0.95	0.95	1	1	1	1.05	1.05	1	1	1	1
STFC	UVLY	$7	1.05	1.05	1	1	1	1	0.95	0.95	0.95	1	1	1	1.05	1.05	1	1	1	1
STFC	WTCT	$9	1.05	1.05	1	1	1	1	0.95	0.95	0.95	1	1	1	1.05	1.05	1	1	1	1
STFC	WTMA	$8	1.05	1.05	1	1	1	1	0.95	0.95	0.95	1	1	1	1.05	1.05	1	1	1	1

Table A.8: DC to DC Transportation Cost

OriginID	DestID	Base Cost/ton	Mar-09	Apr-09	May-09	Jun-09	Jul-09	Aug-09	Sep-09	Oct-09	Nov-09	Dec-09	Jan-10	Feb-10	Mar-10	Apr-10	May-10	Jun-10	Jul-10	Aug-10
CTRI	NCMA	$3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
CTRI	SEMA	$3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
CTRI	WTCT	$3	1	0.95	0.95	0.95	1	1	1	1	1	1	1	1	1	0.95	0.95	0.95	1	1
UVLY	NCMA	$4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
UVLY	SCST	$4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
UVLY	SONH	$3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
WTMA	BERK	$1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
WTMA	HUDV	$4	1	1	1.05	1.05	1.05	1	1	1	1	1	1	1	1	1	1.05	1.05	1.05	1
WTMA	NCMA	$3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
WTMA	SCST	$3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
WTMA	SEMA	$3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
WTMA	SONH	$4	1	1	1	1	1	1	1.05	1.05	1.05	1	1	1	1	1	1	1	1	1
WTMA	WTCT	$3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Table A.9: Inventory On Hand

SourceID	StorID	Landed Cost	Inventory
JAFF	BERK	$225	30
SCHY	HUDV	$215	30
MICH	MICH	$157	40
SCHY	NCMA	$225	25
SCHY	NJER	$215	5
HASS	OHIO	$165	20
SCHY	SCHY	$187	50
SCHY	WTCT	$223	8
MICH	WTMA	$200	30
PRIN	BERK	$220	5
PRIN	CTRI	$224	7.5
PRIN	DEME	$222	15
ARGT	HUDV	$205	50
STFC	NCMA	$218	50
ARGT	NJER	$205	20
PRIN	PRIN	$105	70
CRTH	SCST	$215	15
CRTH	SONH	$215	10
STFC	SONH	$226	40
STFC	STFC	$160	300
STFC	UVLY	$198	7.5

Table A.10: Total Inventory

	Mar-09	Apr-09	May-09	Jun-09	Jul-09	Aug-09	Sep-09	Oct-09	Nov-09	Dec-09	Jan-10	Feb-10	Mar-10	Apr-10	May-10	Jun-10	Jul-10	Aug-10
MaxInvty	800	1,000	1,200	1,500	2,000	2,000	1,800	1,500	1,000	800	500	500	900	1,200	1,500	1,700	2,200	2,200
Penalty	0.33	0.33	0.33	0.17	0.17	0.17	0.33	0.33	0.33	0.50	0.50	0.50	0.33	0.33	0.33	0.17	0.17	0.17

Appendix B Data Processing

B.1 Preprocessing of Input Data

We do some preprocessing of the raw data in EXCEL to prepare it for the OPL model. Below is the steps needed.

1. Reconstruct the data about transportation cost in the sheet "TransportCosts-DCtoDC". Firstly, add in rows for transportation from storage point to its own demand region and set the cost to be zero. Next, insert one column "Product" on the left and key in "H" in this column for all the existing rows. Duplicate all the current data and append them below the existing data. Key in "S" in product column for all the new added rows. As a result, we would have two rows for each route, one for H and the other for S.

2. Add in one column "Product" on the left in the sheet "TransportCosts-Direct". The product depends on the source as each source provides either H or S. This information can be found from the sheet "SourceChar".

3. Add in one column "product" on the left in the sheet "RegionalPricing" and key in "H" in this column for all the current rows. Duplicate all the current rows and append them below the existing rows. Key in "S" in the product column for the duplicated

rows. In this way, we have prices for product H and S in each region separately though the prices would be same for both products in this case.

4. Rearrange the order of rows in all data sheets so that the information is arranged in compatible order. In sheets “SourceChar”,”SourceCosts” and “SourceVolume”, sort the data by SourceID. In sheets “StorChar” and “RegionalDemand”, sort the data by StorID or RegionID. In sheet “RegionalPricing” we sort the data by Product first and then by RegionID. In sheet “TransportCosts-DCtoDC”, we sort the data by Product first and then by DestID. In sheet “TransportCosts-Direct”, we sort the data by SourceID.

B.2 Importing Input Data

Since OPL is not able to read in large size of data from EXCEL directly, we write a MATLAB function to convert the data from EXCEL to TXT which is compatible with the OPL model. The steps are as following.

1. Run the MATLAB M file ‘DataReading’ to read the data from EXCEL and generate a TXT file “input”. The MATLAB file and EXCEL file should be under the same directory.
2. Copy the contents of ‘input’ into the data file “RoadSalt.dat” in the OPL project
3. Run the configuration including mod file “RoadSalt.mod” and data file “RoadSalt.dat’

B.3 Exporting Output Data

After the model is solved, the solutions would be stored in separate TXT files under the project. To make it easier to analyze, we export the results from TXT to EXCEL by following the steps below.

1. After the execution, the model would generate four separate output files under the current directory of the OPL project.

2. To open the output files in excel, we open a new empty excel sheet. Click Data→Get External Data → From Text. Change the file types to 'All files' and then browse for one output file from the directory of the project Road Salt. Then Click open, under 'original data type', choose 'Delimited', then click next. Under Delimiters, unclick 'Tab', click 'Space' then click next. Now choose the location you want to position the data, and then click ok. The data in the output file would then be displayed in the excel worksheet.