XII. SPONTANEOUS RADIOFREQUENCY EMISSION FROM HOT-ELECTRON PLASMAS*

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A. INSTABILITIES IN THE EXTRAORDINARY WAVES ACROSS THE MAGNETIC FIELD

We have completed the stability analysis for propagation across the magnetic field in a plasma with energetic electrons having an unperturbed distribution function

$$f_0(\bar{v}) = \frac{1}{2\pi v_{0\perp}} \delta(v_\perp - v_{0\perp})\, \delta(v_{\parallel}), \tag{1}$$

where the subscripts $\perp$ and $\parallel$ refer to directions across and along the applied magnetic field B_0.

The dispersion relation for small-amplitude perturbations with dependence

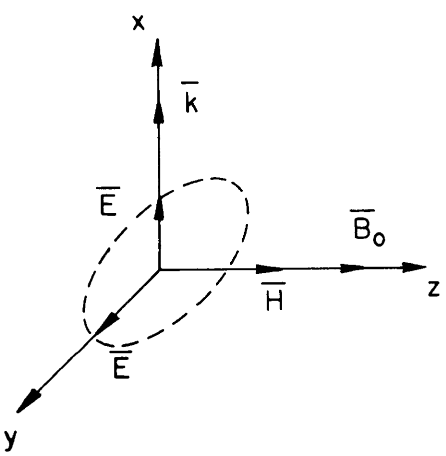

Fig. XII-1. Extraordinary wave propagation across the applied magnetic field $\bar{B}_0$.

exp(jωt−jkx) (see Fig. XII-1) was obtained from the relativistic, collisionless, Vlasov equation and Maxwell's equations.[1,2] The dispersion relation for the extraordinary wave (Fig. XII-1) and the electron distribution function of Eq. 1 is

$$\frac{c^2k^2}{\omega^2} = \frac{K_{xy}^2}{K_{xx}} + K_{yy}, \tag{2}$$

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where

$$K_{xx} = 1 - \frac{a^2}{\gamma^2} \sum_{n=-\infty}^{\infty} \left[\frac{\gamma n^2 \left(J_n^2\right)'}{\nu(\nu-n)} - \frac{\beta^2 n^2 J_n^2}{(\nu-n)^2} \right] \tag{3}$$

$$K_{yy} = 1 - a^2 \sum_{n=-\infty}^{\infty} \left\{ \frac{[\gamma^2 (J_n')^2]'}{\gamma\nu(\nu-n)} - \frac{\beta^2 (J_n')^2}{(\nu-n)^2} \right\} \tag{4}$$

$$K_{xy} = j \frac{a^2}{\gamma} \sum_{n=-\infty}^{\infty} \left[\frac{n(\gamma J_n J_n')'}{\nu(\nu-n)} - \frac{\beta^2 n J_n J_n'}{(\nu-n)^2} \right], \tag{5}$$

and we have used the following abbreviations: $\nu = \omega/\omega_b$, with ω_b the relativistic cyclotron frequency; $a = \omega_p/\omega_b$, with ω_p the relativistic plasma frequency; $\gamma = kv_{0\perp}/\omega_b$, $J_n = J_n(\gamma)$ is the ordinary Bessel function of order n and argument γ; the prime indicates a derivative with respect to γ; and $\beta = v_{0\perp}/c$, with c the velocity of light in free space.

With the aid of the CTSS of Project MAC, and of the Newton-Raphson technique for finding roots of a transcendental equation, Eq. 2 was programmed to give the complex ω solutions for real wave numbers k. The results are summarized in Figs. XII-2 and XII-3. Three distinct types of instabilities can be identified: (i) fast-wave, relativistic, (ii) slow-wave, relativistic; and (iii) electrostatic. The terms "fast-wave" and "slow-wave" refer to the phase velocity regimes of the instability, being faster or slower than the velocity of light in free space. The term "relativistic" indicates that in the interaction the change in electron mass is crucially important. The term "electrostatic" refers to the approximation in which $\bar{k} \parallel \bar{E}$ is valid.

The fast-wave, relativistic instability is illustrated in Fig. XII-2. It can be seen to arise from the interaction between the fast-wave extraordinary mode that would exist in a cold plasma and the cyclotron harmonic wave branches that exist for finite $v_{0\perp}/c$. The instabilities occur near the velocity-of-light line where the extraordinary wave is essentially linearly polarized and the wavelength is large compared with the electron's Larmor radius. The physical description of these instabilities and their relativistic nature can be understood from the simple model shown in Fig. XII-4 for $\omega \gtrsim \omega_b$. Electrons that are in phase with respect to the electric field so as to give up energy (Fig. XII-4a) have their mass reduced and therefore their cyclotron frequency increased. But since the frequency of the field is slightly greater than the cyclotron frequency, these electrons remain in the same phase (Fig XII-4b) with respect to the electric field and continue to give up energy. Electrons of opposite phase (Fig. XII-4c), which initially

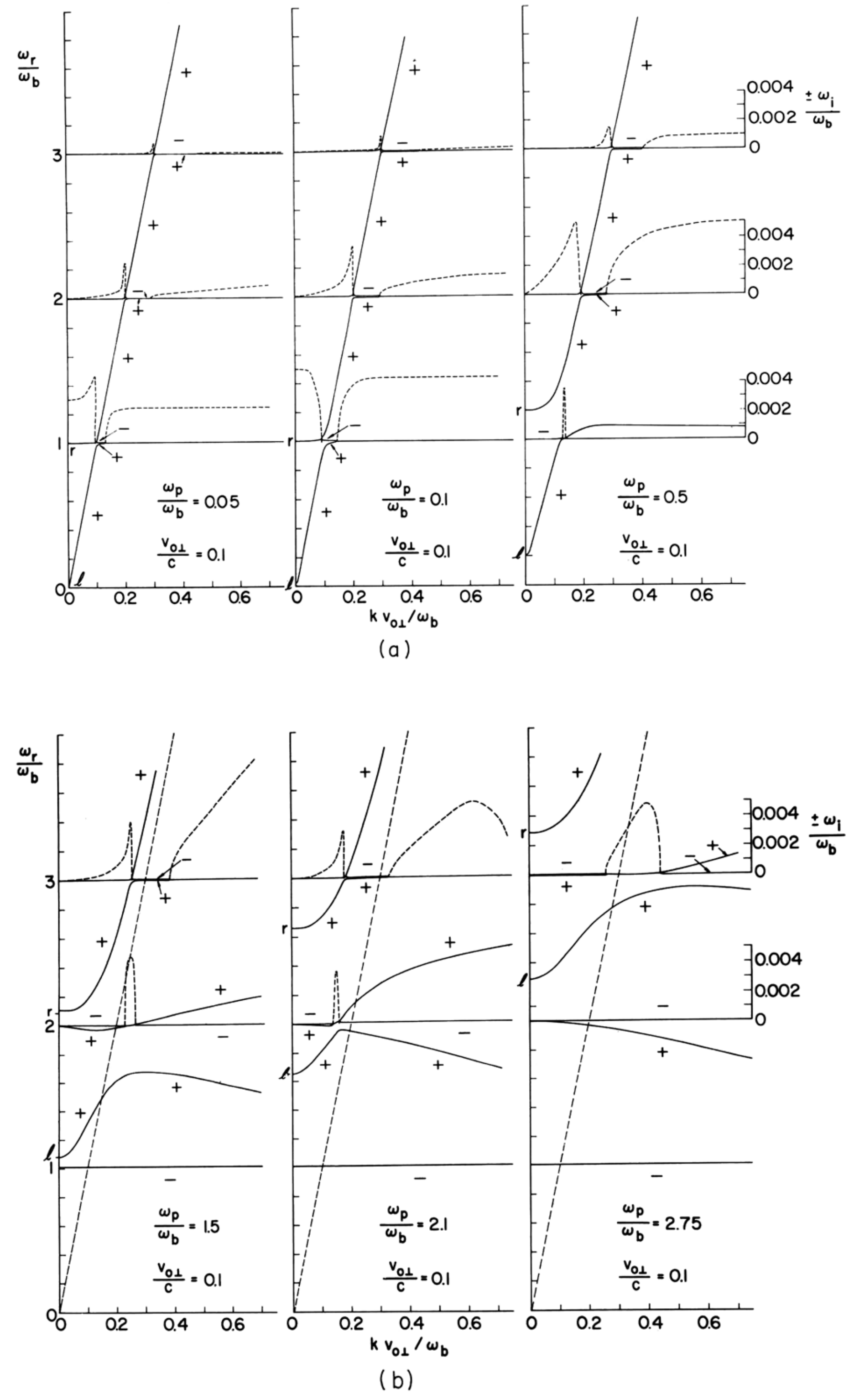

Fig. XII-2. Dispersion diagrams showing the fast-wave relativistic instabilities as a function of (ω_p/ω_b) for fixed $(v_{0\perp}/c) = 0.1$. The frequencies marked "ℓ" and "r" are the cold-plasma cutoff frequencies of the extraordinary wave which are left- and right-circularly polarized.

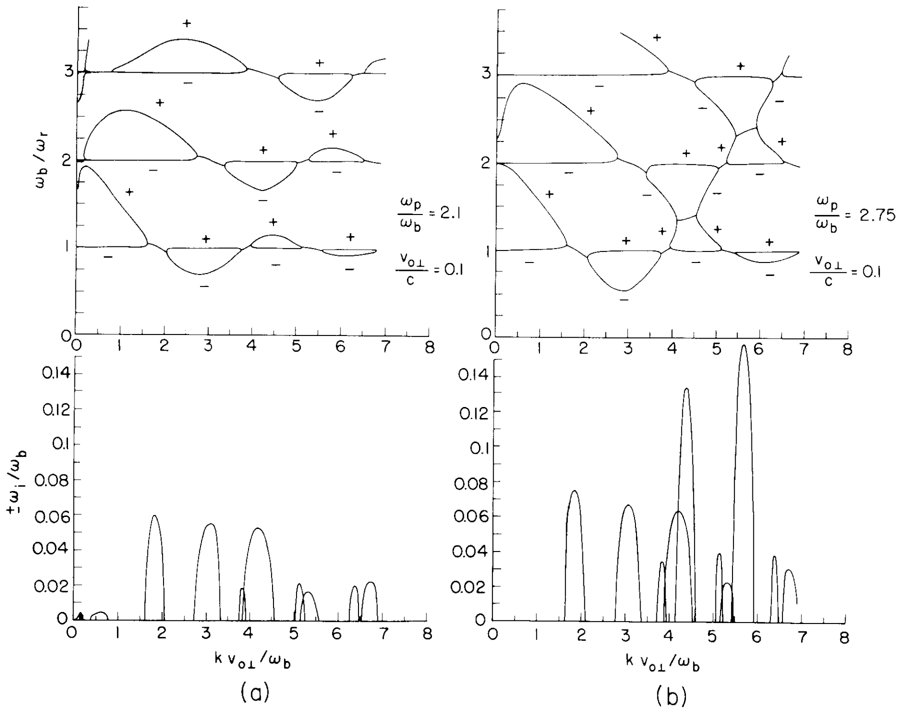

Fig. XII-3. Dispersion diagrams showing the slow-wave, relativistic instabilities (at cyclotron harmonics) and electrostatic instabilities (in between cyclotron harmonics). (a) (ω_p/ω_b) is below the threshold for the electrostatic instabilities. (b) Both types of instability are present. (For detail of the fast-wave region see Fig. XII-2b.)

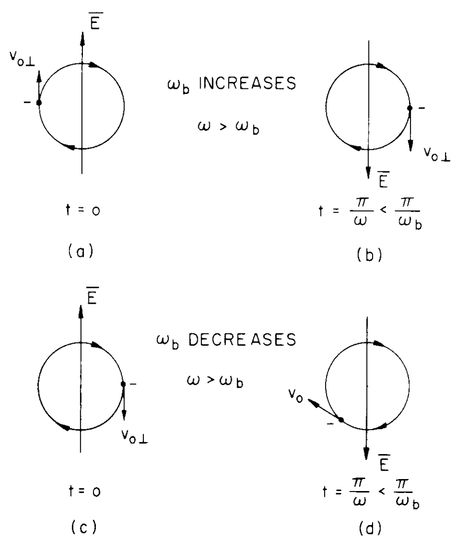

Fig. XII-4.

Model for the fast-wave, relativistic instability. The magnetic field B_0 is into the paper. The wavelength is assumed large compared with the electron-cyclotron orbit, and ω is slightly greater than ω_b.

take energy from the field, become heavier and come into phase with respect to the electric field so as to take less energy from the field (Fig. XII-4d). Thus phase conditions for a net loss of transverse energy from the electrons, and a consequent build-up of the fields, is established. As we have remarked previously,[3] these instabilities are seen to vanish as (ω_p/ω_b) is increased to a value that makes the (cold-plasma) cutoff frequency exceed the (hot-plasma) cyclotron mode frequency. Figure XII-2a illustrates this for the unstable mode at $\omega \gtrsim \omega_b$, and Fig. XII-2b for the unstable modes at $\omega \gtrsim 2\omega_b$ and $\omega \gtrsim 3\omega_b$.

The slow-wave, relativistic, and the electrostatic instabilities are illustrated in Fig. XII-3. We have discussed certain aspects of these instabilities in previous reports.[4-6] These can be understood in terms of wave-wave coupling in the presence of negative-energy modes that are due to finite $v_{0\perp}$. The slow-wave, relativistic instability (Fig. XII-3) occurs at the cyclotron harmonic frequencies; it depends upon $(v_{0\perp}/c)^2$ and occurs for arbitrarily low (ω_p/ω_b). As (ω_p/ω_b) increases, the range of wave numbers over which this instability exists shrinks. The electrostatic instabilities, on the other hand, occur at frequencies in between cyclotron harmonics, and only if $(\omega_p/\omega_b) \gtrsim 2.5$. These instabilities occur even in the absence of relativistic mass effects.[5,7] Relativistic effects produce a reduction of this instability.[4].

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