

COMMUNICATION SCIENCES
AND
ENGINEERING

XIV. STATISTICAL COMMUNICATION THEORY*

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A. WORK COMPLETED

1. A CLASS OF NONLINEAR FILTERS

This study has been completed by E. Jeenicke. It was submitted as a thesis in partial fulfillment of the requirements for the Degree of Master of Science, Department of Electrical Engineering, M. I. T., January, 1966.

A. V. Oppenheim

B. AN EFFICIENT TRANSVERSAL EQUALIZER FOR TWO-PATH CHANNELS

The intersymbol interference arising in digital communication over time-variant dispersive channels imposes practical limitations on the data rates that one can attain by using fixed modem structures. Using an adaptive modem – that is, one capable of measuring the channel dispersion and attempting to compensate for it by altering its own structure – it is possible, however, to achieve marked improvement in data rates, as demonstrated by Lucky in his recent work with dispersive telephone lines.¹

In this report we shall consider reduction of the multipath distortion for a discrete two-path channel, in which the signals received via the two paths may vary arbitrarily in amplitude with respect to each other. Such a situation might occur, for example, in communication over the HF ionospheric channel if one is operating sufficiently below the MUF. The discussion here will be limited to modem structures that one might employ for dispersion correction for this channel, under the assumption that its characteristics are known, although in practice an adaptive modem would have to obtain estimates of these characteristics through channel measurement.

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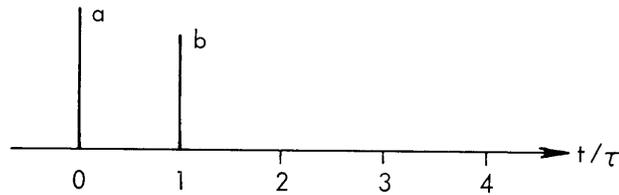


Fig. XIV-1. Impulse response of a two-path channel.

The channel of interest has the impulse response shown in Fig. XIV-1, where the first pulse position has been taken as time reference, and the time axis normalized to the separation, τ , between the pulses. One may represent this output, using z-transform notation, as

$$H(z) = az^0 + bz^{-1}.$$

As discussed by Hulst for the HF ionospheric channel,² one would like to construct the "equalization" or "inverse" filter defined by the z-transform

$$F(z) = \frac{1}{H(z)},$$

for then cascading this filter with the channel would give the desired over-all output z-transform

$$H(z) F(z) = 1$$

and the multipath would have been eliminated.

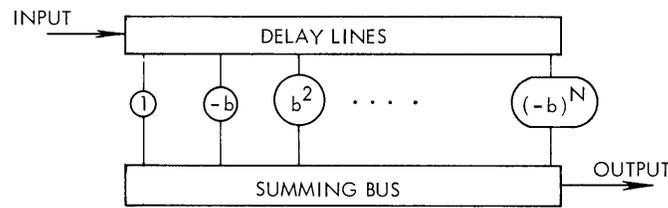
Before proceeding to the general case, we consider first the special case in which the longer path is always weaker than the shorter path — that is, $|b| < |a|$. Since there is no loss in generality in normalizing the input, we shall take $a = 1$, and require that $|b| < 1$. Defining the distortion to be the magnitude of the weaker path strength to that of the stronger path, we have at the input to the equalization filter a distortion

$$D_{\text{input}} = \frac{|b|}{|a|} = |b|.$$

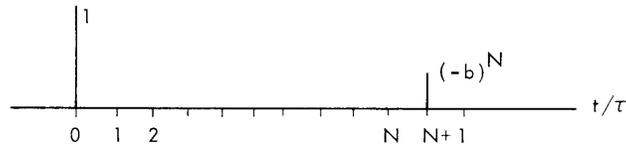
Now the desired equalization filter is described by its z-transform

$$F(z) = \frac{1}{H(z)} = \frac{1}{1 + bz^{-1}} = \sum_{n=0}^{\infty} (-bz^{-1})^n.$$

While the numbers of terms in this series expansion is infinite, in practice one



(a)



(b)

Fig. XIV-2. (a) Conventional transversal equalizer for the channel in Fig. XIV-1, where $a = 1$, $|b| < 1$.
 (b) Result of passing channel impulse response through this equalizer.

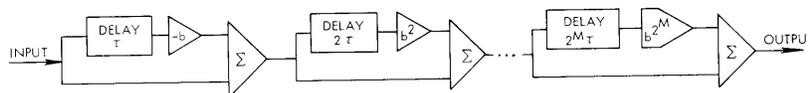
truncates the series after $n = N$, and the result is the well-known transversal equalizer structure shown in Fig. XIV-2a. This equalizer has a z-transform

$$F_N(z) = \frac{1 - (-bz^{-1})^{N+1}}{1 + bz^{-1}},$$

and so cascading it with the channel renders an over-all output having a z-transform

$$H(z) F_N(z) = 1 - (-bz^{-1})^{N+1}$$

which is shown in Fig. XIV-2b. This transform corresponds to an output distortion



(a)



(b)

Fig. XIV-3. (a) Cascade transversal equalizer for the channel of Fig. XIV-1, where $a = 1$, $|b| < 1$.
 (b) Result of passing channel impulse response through this equalizer.

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$$D_{\text{output}} = |b|^{N+1}$$

which has reduced the effective multipath under our assumption $|b| < 1$.

We next derive an alternative equalizer structure for the case considered above, using an alternative expansion of the filter z-transform:

$$F(z) = \frac{1}{H(z)} = \frac{1}{1 + bz^{-1}} = \prod_{m=0}^{\infty} \left[1 + (-bz^{-1})^2 \right].$$

By truncating this product after $m = M$, the resulting filter becomes the cascade of transversal filters shown in Fig. XIV-3a. This transversal equalizer has a z-transform

$$F_M(z) = \frac{1 - b^2 z^{-2M+1}}{1 + bz^{-1}},$$

and so cascading it with the channel renders an over-all output having a z-transform

$$H(z) F_M(z) = 1 - b^2 z^{-2M+1}$$

which is shown in Fig. XIV-3b. This transform corresponds to an output distortion

$$D_{\text{output}} = b^2 z^{-2M+1}$$

which again is less than the input distortion $|b|$, since $|b| < 1$ was assumed.

The performance of the equalizers of Figs. XIV-2a and XIV-3a will be identical if one takes

$$N + 1 = 2^{M+1},$$

for then both filters render an output distortion of $b^2 z^{-2M+1}$. Note, however, that the number of cascaded transversal filters required with the second approach, $M+1$, will in general be considerably smaller than the number of taps on a single transversal filter, $N+1$. The actual values required, of course, depend upon the maximum input distortion and the desired maximum output distortion. For example, with a maximum input distortion of .9 and a maximum output distortion of .01, $M+1 = 6$, while $N+1 = 44$. Thus while the delay requirements and distortion reductions of the equalizers of Figs. XIV-2a and XIV-3a are equivalent, it is seen that the second equalizer is more efficient in terms of the number of variable-gain amplifiers required. Moreover, it has the additional practical advantage of requiring (except for the first cascade stage) only amplifiers to provide positive gains, while in the conventional approach nearly half of the amplifiers must be capable of providing both positive and negative gains.

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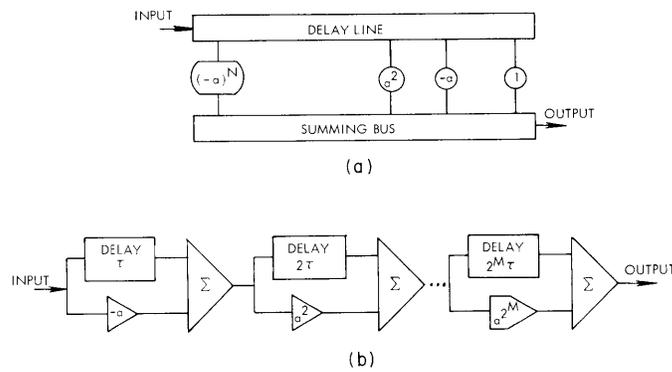


Fig. XIV-4. (a) Conventional transversal equalizer.
 (b) Cascade transversal equalizer. (Both for the channel of Fig. XIV-1, where $b = 1$, $|a| < 1$.)

Thus far, we have restricted the discussion to channels for which the longer path was always weaker than the shorter. Similarly, if the shorter path were always the weaker, one would normalize the input so that $b = 1$, and proceed as described above with bz^{-1} replaced by az , and so forth, to derive the structures shown in Fig. XIV-4, where the cascade approach is again seen to be more efficient than the conventional

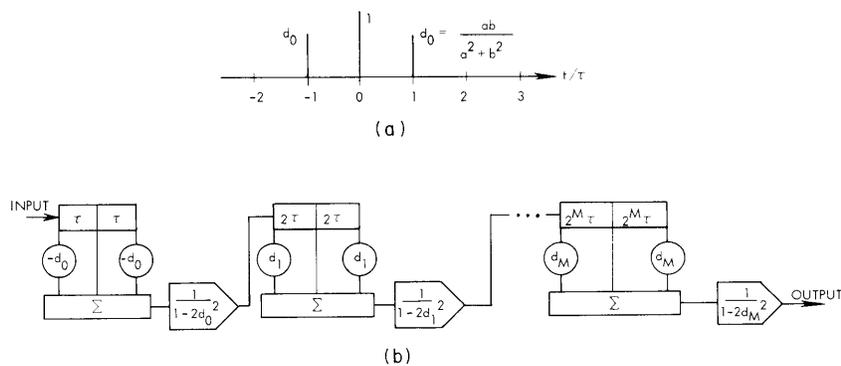


Fig. XIV-5. (a) Normalized matched-filter output for the channel of Fig. XIV-1.
 (b) Cascade transversal equalizer for this output.

approach. We next consider equalization for the general case, that is, where a and b may vary arbitrarily with respect to each other. By matched filtering to the channel output and again normalizing, one has the matched filter output shown in Fig. XIV-5a, in which now the input distortion is

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$$D_{\text{input}} = 2 |d_o| = \frac{2 |ab|}{a^2 + b^2},$$

and the input z -transform is the matched filter output

$$H_{\text{MF}}(z) = d_o z + 1 + d_o z^{-1},$$

and now the center pulse has been taken as time reference. The desired equalization filter is

$$F(z) = \frac{1}{H_{\text{MF}}(z)} = \frac{1}{d_o z + 1 + d_o z^{-1}}.$$

One could divide this out, expanding $F(z)$ as the sum of a sufficient number of terms, and realize the inverse filter approximately with a single transversal equalizer. Instead we shall follow the more efficient cascade approach introduced above, by first expanding $F(z)$ in the product

$$F(z) = \prod_{i=0}^{\infty} \left[\frac{d_i (-z)^{2^i} + 1 + d_i (-z)^{-2^i}}{1 - 2d_i^2} \right],$$

where

$$d_i = \frac{d_{i-1}^2}{1 - 2d_{i-1}^2},$$

and the distortion after the i^{th} filter is

$$D_{\text{output}}(i) = 2d_i.$$

The structure for this general case is shown in Fig. XIV-5b. The normalization gains after each stage could be combined into a single gain. It is straightforward to show that

$$|d_i| < |d_{i-1}|$$

whenever

$$|d_{i-1}| < 1/2$$

which is satisfied for every stage whenever $|d_o| < 1/2$. Since d_o can have magnitude $1/2$ only in the event that the path strengths $|a|$ and $|b|$ are identical, an event of probability zero, it is clear that this cascading of filters renders a transversal equalizer which reduces the multipath distortion with probability one. It is also clear that this

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cascaded transversal equalizer structure enjoys the same efficiency, in terms of implementation requirements, relative to the conventional transversal equalizer as that found for our earlier example.

M. E. Austin

References

1. R. W. Lucky, "Automatic Equalization for Digital Communication," Bell System Tech. J. 44, 547-588 (April 1965).
2. G. D. Hulst, "Inverse Ionosphere," IRE Trans., Vol. CS-8, No. 1, pp. 3-9, March 1960.

