Research Staff

Martha M. Pennell
T. H. Brooks
Gail M. Fratar
Veronica E. McLoud
R. M. Nacamuli

A. FURTHER COMPUTATIONS USING NEWTON'S METHOD FOR FINDING COMPLEX ROOTS OF A TRANSCENDENTAL EQUATION*

Because of our prior success[1] with Newton's method, we used the same technique on the following, more complicated equation. Given real values for α and β, solve for ν as a function of γ $(0 \leq \gamma \leq 10)$:

$$\frac{\gamma^2}{\beta^2\nu^2} = \frac{K_{xy}^2}{K_{xx}} + K_{yy},$$

where

$$K_{xy} = j\frac{\alpha^2}{\gamma}\sum_{N=-\infty}^{\infty}\left[\frac{N\{\gamma J_N J_N'\}'}{\nu(\nu-N)} - \frac{\beta^2 N J_N J_N'}{(\nu-N)^2}\right]$$

$$K_{xx} = 1 - \frac{\alpha^2}{\gamma^2}\sum_{N=-\infty}^{\infty}\left[\frac{\gamma N^2\left(J_N^2\right)'}{\nu(\nu-N)} - \frac{\beta^2 N^2 J_N^2}{(\nu-N)^2}\right]$$

$$K_{yy} = 1 - \alpha^2\sum_{N=-\infty}^{\infty}\left[\frac{\left\{\gamma^2\left(J_N'\right)^2\right\}'}{\gamma\nu(\nu-N)} - \frac{\beta^2\left(J_N'\right)^2}{(\nu-N)^2}\right].$$

$J_N = J_N(\gamma)$, is the n^{th}-order Bessel function of the first kind. All derivatives are taken with respect to γ.

Because we expected roots near integer values of ν, we chose the same method[1] as before to estimate their value. The resulting sixth-degree polynomial yielded roots which, when used as initial guesses in Newton's method, approximated the roots in most cases to two significant figures. Once a root had been determined to the desired accuracy, γ was varied by a small amount (usually 0.1 or 0.05) and the root used as the new initial guess. This eliminated solving the sixth-degree polynomial at each step.

This procedure broke down in the region where γ changed from a pure real to a complex root (but not in the reverse situation). If the initial guess were pure real, the

*This work was supported by the National Science Foundation (Grant GK-614).

computer program[2] could only generate real computations because none of the arithmetic operations led to complex numbers. In the reverse situation, however, starting with a complex initial guess and performing several iterations of Newton's method, the program would make the imaginary part of the root disappear.

A second difficulty with this procedure was keeping on the same branch of the root. In this particular problem we were more interested in seeing the change in the root as we varied γ than in finding all roots for a given γ. In the regions where Newton's method could not converge or in the transition regions described above, the program jumped branches to find roots outside our range of interest. We had to keep a constant watch on the calculations. These difficulties might possibly have been solved by judicious programming, but the computer time and human effort involved would outweigh the advantages. Here the availability of a time-sharing computer solved a problem which otherwise might not have been attempted.

Martha M. Pennell

References

1. Quarterly Progress Report No. 80, Research Laboratory of Electronics, M. I. T., pp. 263-266.
2. All computations were performed on the Project MAC time-sharing system.