XIX. NEUROPHYSIOLOGY*

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A. NEW METHODS FOR TESTING CAMERA LENSES (Part II)

In Quarterly Progress Report No. 73 (pages 209-216) we described a new method for testing camera lenses and cameras. This method, which makes use of an especially designed analyzer lens placed in front of the camera lens, together with a polar coordinate test chart, permits simple diagnosis of the focal properties of the lens-camera combination. In particular, our method was shown to be sensitive to range-finder error, misalignment of lens-to-film plane, and to astigmatism and curvature of field.

Further tests with this method have indicated the need for the improved construction of the analyzer lens element which is reported here. Also, we evidently did not exhaust the potentialities of the method for the diagnosis of lens aberrations, since, with slight modification, our earlier work provides a sensitive test for spherical aberration and for coma. An additional, somewhat similar, test is also given which permits the determination of lateral chromatic aberration. Using both of these methods, we can determine five of the seven primary lens aberrations; the exceptions are distortion and axial chromatic aberration. Since these tests do not require the use of a precision optical bench, they should be particularly useful for amateur and professional photographers.

1. Improved Construction of the Analyzer Lens

In Fig. XIX-1 we show the basic method of lens testing. The analyzer lens CC is a crossed cylinder, i.e., a spherocylinder having equal positive and negative powers about two orthogonal axes, Y and Z. A narrow horizontal slit S is oriented at 45° to the axes of the crossed cylinder. For our previous tests we used ±1/8, ±1/4, and ±1/2 diopter ophthalmic cylinder lenses. Recent tests, however, indicate that the best available quality of such lenses of powers ±1/8 diopter or smaller give uncertain results in this application.

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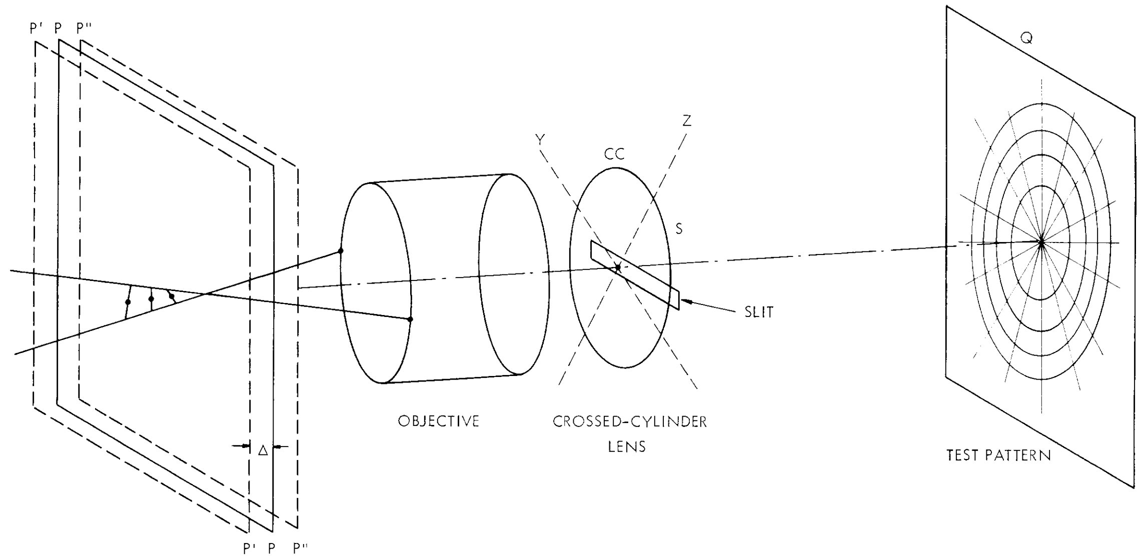

Fig. XIX-1. Arrangement of optical elements for testing the lens objective.

Accordingly, we have devised an alternative method of fabricating weak analyzer lenses, which makes use of the fact that the small section of the crossed cylinder that we actually use (that portion visible through the slit) can be approximated by a surface of a narrow rectangular glass prism stressed in torsion. A prism, $3 \times 7 \times \sim 100$ mm is cut from the best quality of optical flat glass and stressed in torsion. Flexible couplings are used to eliminate bending moments. While the torsional stress is held constant, the prism is securely glued to a thicker optically flat glass window with a clear, well-mixed, epoxy cement (see Fig. XIX-2). The slit (parallel to the prism) is glued to the other side of this window.

Satisfactory long-term bonding can only be achieved if the glass surfaces are pretreated with a 2% solution of Union Carbide A-1100 Silane bonding agent because the glue joint must withstand indefinitely the torsional prestressing of the prism. Satisfactory bonds of 6-months endurance have been obtained by using R-314 Epoxy bonding agent (Carl H. Biggs Company) and also Epon 825 cement with a 20% Epon 2807 hardening agent. The maximum equivalent cylindrical power of the lens that we have fabricated in this way is $\pm 1/8$ diopter. We have found that analyzer lenses with 2-inch apertures and of successively halved powers of $\pm 1/2$, $\pm 1/4$, $\pm 1/8$, and $\pm 1/16$ diopters will suffice to test most camera lenses. We recommend the use of ophthalmic cylindrical lenses at the two strongest powers and the stressed-prism construction for the weaker powers.

We have examined the diffraction images of these new analyzer lenses when used with our best 200-mm telephoto lenses. Our tests with a high-power microscope

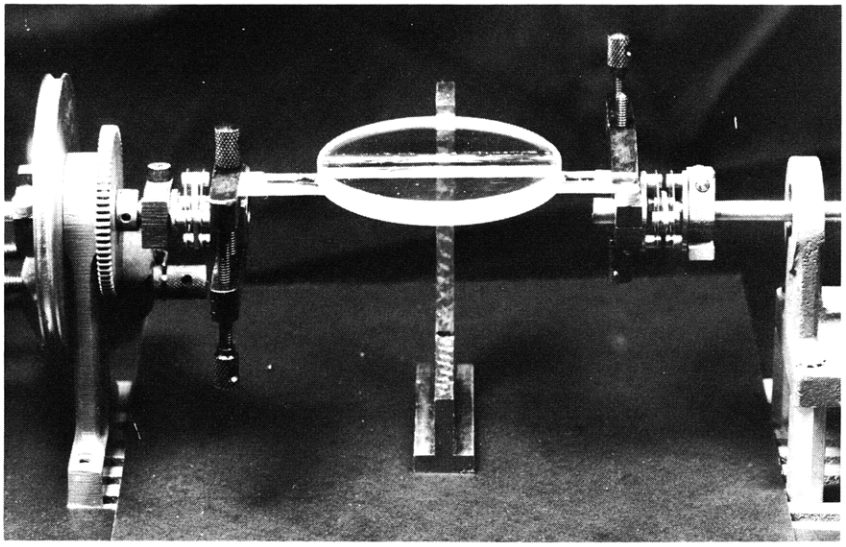

Fig. XIX-2. Epoxy gluing of flat glass disc to prism of optical glass deformed by torsion. Flexible couplings minimize bending moments.

revealed some irregularities of performance which could be traced to the telephoto lenses, but no defects that could be attributed to the new construction of the analyzer lens.

2. Tests for Spherical Aberration and Coma

We are concerned with a lens test in which the image of a point source is a line segment. The angular orientation of this line-segment image is a sensitive function of the focus of the lens. In testing for spherical aberration, we introduce a complication, in that the various zones of the lens may have different focal lengths. The image of a point source in this instance will not be a line segment, but with simple spherical aberration it will be a sigmoidal curve. In order to provide satisfactory imaging of the polar-coordinate chart in this new situation, it is preferable that the chart be ruled with white lines on a black background. With this modification, we tested a lens known to have large spherical aberration; the results are shown in Fig. XIX-3. (In Fig. XIX-4, we show a similar test with a well-corrected lens.) The effect of the spherical aberration is evident both in the appearance of the radial lines, and in the splitting of the images of the circles. The appearance and angular extent of the split images of the circles provides an indication of the spherical aberration of the lens (both in sign and magnitude); however, before precise measurements are attempted by this means, it will be necessary to standardize the film exposure and development conditions and to obtain correlation with other methods

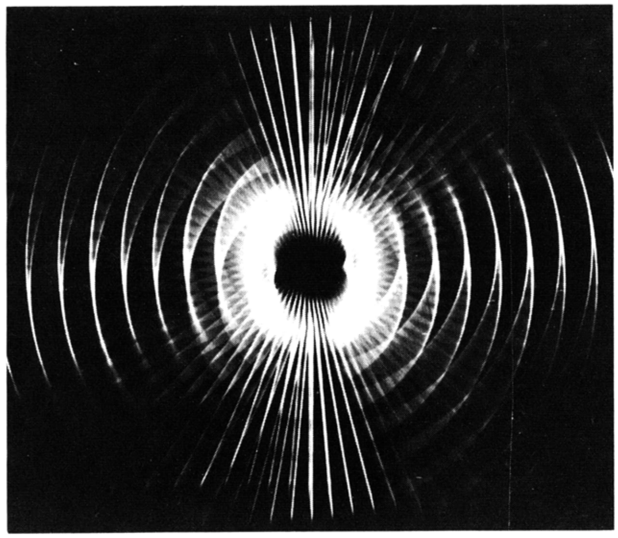

Fig. XIX-3. Lens with appreciable spherical aberration. (Compare with Fig. XIX-4.)

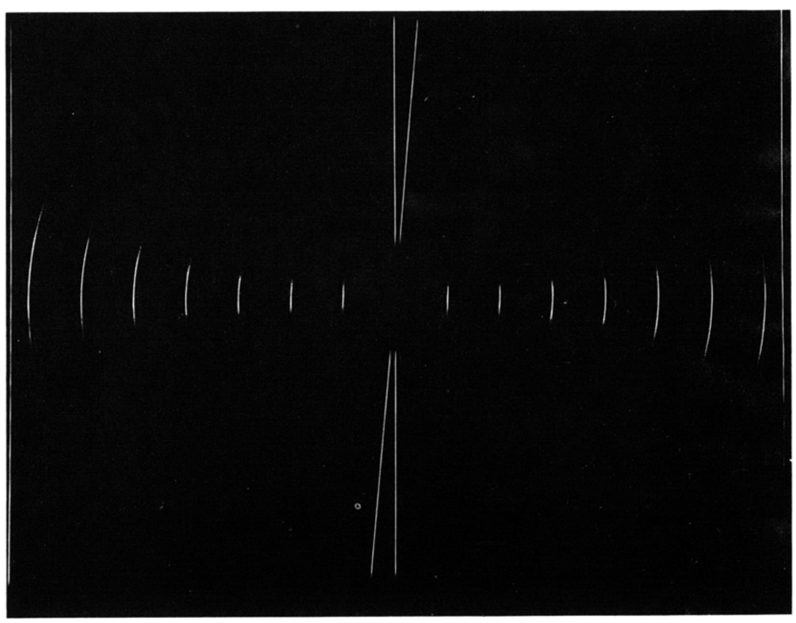

Fig. XIX-4. Well-corrected lens.

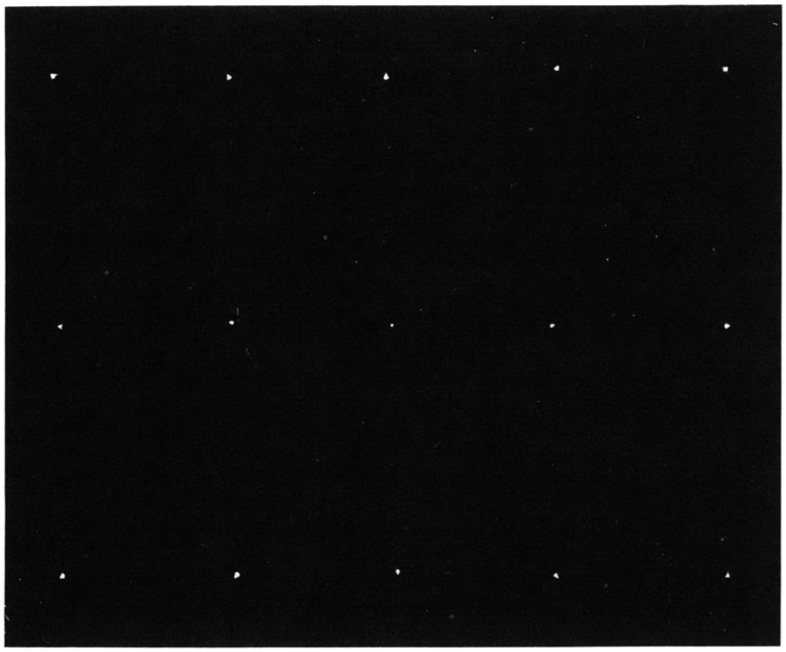

Fig. XIX-5. Array of point sources photographed with lens system having coma.

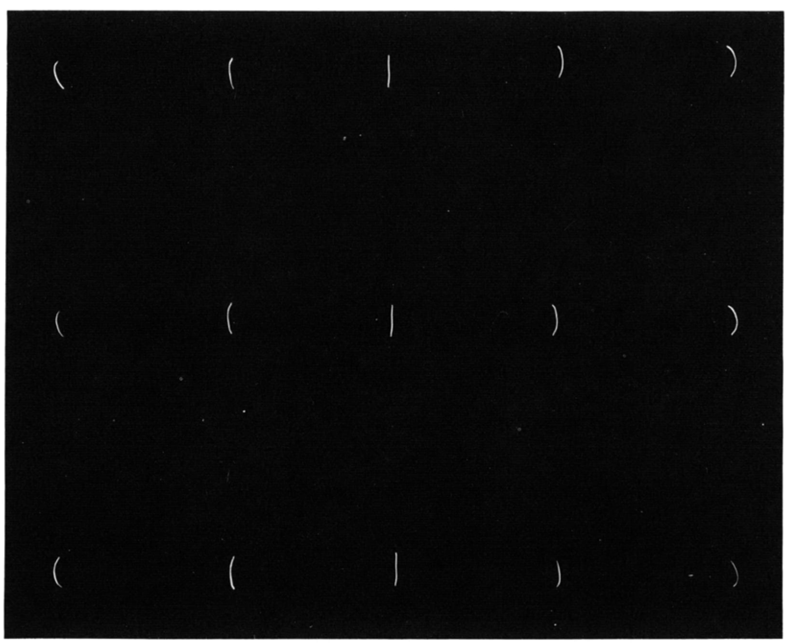

Fig. XIX-6. Use of lens analyzer to test for coma.

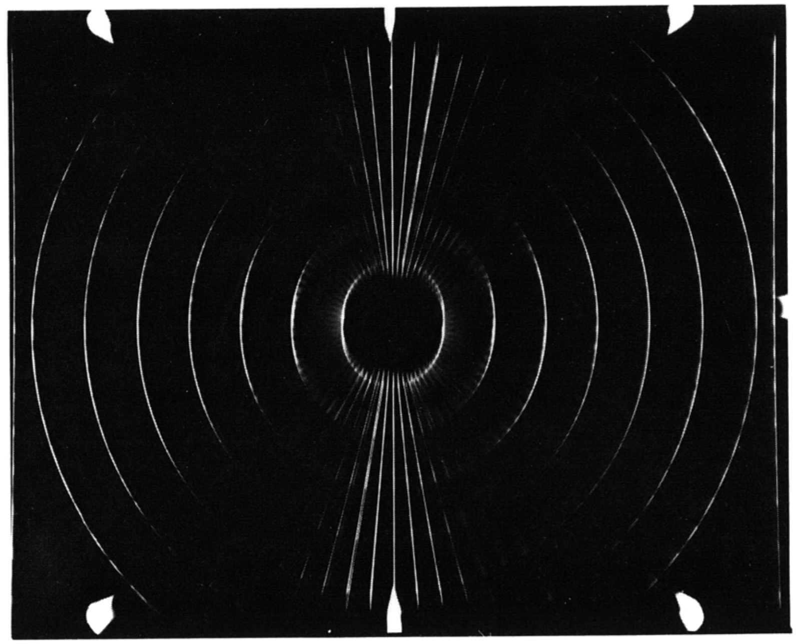

Fig. XIX-7. Test for coma with polar chart. (Compare with Fig. XIX-4).

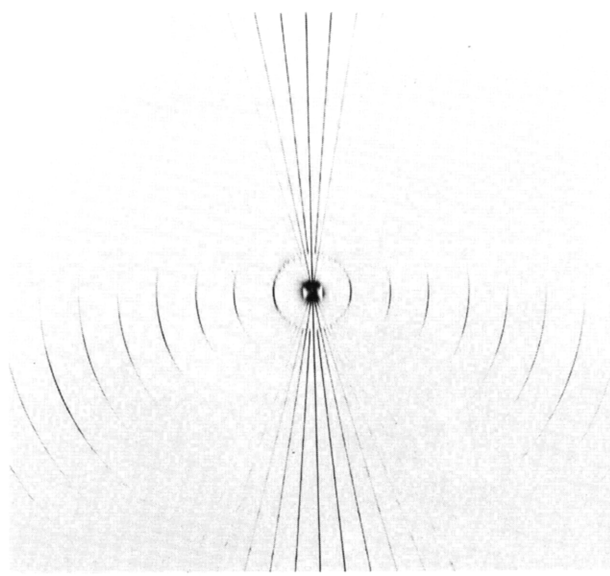

Fig. XIX-8. Lateral chromatic aberration.

of measurement of the aberration of the lens.

The use of this method to determine coma is similar. We have simulated this aberration by the use of a TelXtender-type image amplifier, a Barlow-type negative amplifying lens interposed between the telephoto lens and the film plane. Such a combination has been found to generate almost pure coma, if the "TelXtender" is at full aperture. This is perhaps evident from the image of an array of 15 dots shown in Fig. XIX-5 (15 ball bearings illuminated with a single source). In Fig. XIX-6 we show the corresponding images photographed with the analyzer lens in place. This method is evidently a sensitive test for coma. In Fig. XIX-7 we show the appearance of the polar chart when photographed with the same apparatus. It is evident that the angular extent of the segments of the images of the circles becomes larger toward the edge of the field. Measurements of this angular broadening should give the magnitude — but not the sign — of the comatic distortion.

In summary, we note that this test determines four lens aberrations and also checks the focus and alignment of the camera. Critical focus is not necessary for this lens test to be effective.

3. Test for Lateral Chromatic Aberration

The method described here can be varied to permit determination of lateral chromatic aberration. The polar chart (black lines on white background) is photographed in white light with panchromatic high-contrast film through a weak dispersive analyzer prism oriented to provide spectral dispersion along the vertical axis of the chart. The exposure, development, and printing variables are so adjusted that only those line segments that point in the direction of the spectral dispersion will register on the prints. The spectral blurring in the vertical direction, caused by the analyzer prism. acts in combination with the radial spectral blurring of the image, which is the effect known as lateral chromatic aberration. The resultant of these two factors is a blur that obscures all but a selected set of the circumferential line segments. The figure formed by this set of line segments indicates the extent of the lateral chromatic aberration by the sign and magnitude of its curvature.

Since lateral chromatic aberration is not a common defect of modern lenses, we have simulated it by means of a large, single-element field lens placed one meter in front of our polar chart. This introduces a noticeable change in magnification with color, without significant focal error. For our analyzer prism, we used an ophthalmic prism that provides 10-cm deviation at one meter, and is made of glass with $n = 1.523$ and $\nu = 58$. In Fig. XIX-8 we show that the results of this test. The pattern has a similarity to our earlier test in which horizontal lens misalignment was indicated. It is important to note that in this test care must be taken with the focal adjustment.

B. Howland

B. QUANTUM THEORY OF MEASUREMENT IN RELATION TO A GENERAL THEORY OF OBSERVATION AND CONTROL*

1. Problem Statement

The quantum theory of measurement, due in its formal aspects to Dirac and von Neumann, is well known to have logical flaws, as well as to involve technical difficulties. Both originate from the nonrelativistic theory of stationary states; for it is immediately obvious that actual physical observables represent transitions between states with finite lifetimes which therefore cannot be stationary.

Logically, this shows up in von Neumann's account of measurement as the reduction of the triadic relation observer-instrument-object to a dyadic relation instrument-object. A standard example illustrating this point is the hydrogen atom; the electron, as object, is projected into a set of discrete states by the field of the proton as instrument. Nothing could actually be observed, however, without the perturbing effect of the radiation field (RF) which must be reckoned as the observer and form an integral part of the theory of measurement. The present perturbation theoretic treatment fails to be satisfactory because quantized fields such as RF cannot be defined unambiguously.

Apart from the theory of measurement, this last difficulty may be regarded as a technical one. Existing theory leans heavily on the use of self-adjoint operators which, with real eigenvalues, must represent stationary states. As a result, one practically has to adopt not only microcausality

$$[\phi(x), \phi(x')] = 0 \qquad (x-x')^2 < 0 \tag{1}$$

but also equal time commutation relations

$$[\phi(x), \pi(x')]_{t=t'} = \delta(x-x') \tag{2}$$

for field operators $\phi(x)$ and their conjugates $\pi(x)$. It is a well-known result[1] that there are uncountably many inequivalent representations satisfying (1) and (2). There then follows from the requirements of scattering theory the result known as Haag's theorem, namely the only unitarily equivalent representations are the free fields.

Our problem can now be stated as follows:

(A) Reformulate the theory of measurement in terms of triadic relations. In practice this means, find three term analogs of (1), (2).

(B) Show that (A) leads to a unique, or at least manageable equivalence classes of representation.

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There is a distinct advantage in looking at this problem in a wider context than the physical, one can then ask what is unique about physics as a theory of observation; this will be implicit in the sequel. Before stating the approach specifically, some requisite background will be given.

2. Logic

There is a well-known isomorphism[2] between combinatory logic and computability, considered in terms of Turing machines. There is also the further reduction now known to exist (Arbib, Papert, unpublished lectures) to the theory of infinite, but finitely presented, semigroups. A basic building block of this theory is the Kleene three-term predicate T(R, A, S) which states the existence of a logic (alternatively, a machine) with rules of inference R, axiom A, derivations S (in the machine's case corresponding indices, x, y, z, say). It is a well-known result that $\{x\} = (x \mid \exists_y\ T(x, x, y))$ is a recursively enumerable but nonrecursive set; that is, no finite amount of equipment will certainly decide whether $x_o \in \{x\}$; nevertheless, $\{x\}$ is finitely generated and therefore has a well-defined structure. This is taken to indicate that in a theory of observation we may reasonably speak of nonobservable entities having a structural significance.

The discussion above may be exemplified in the logic of quantum mechanics represented in the conventional way[3] as the lattice of all closed subspaces of Hilbert space. This is known to be a relatively orthocomplemented lattice which is not modular,[4] that is, it has sublattices of the form shown in Fig. XIX-9.

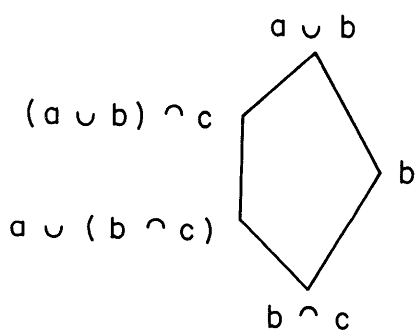

Fig. XIX-9.

Here

$$(a \cup b) \cap c \neq a \cup (b \cap c); \tag{3}$$

however, the essential features of existing quantum mechanics depend only on modularity (i.e., equality in (3)) without distributivity

$$a \cup (b \cap c) \neq (a \cup b) \cap (a \cup c) \tag{4}$$

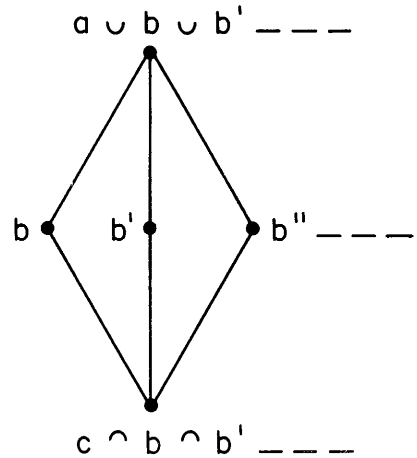

Fig. XIX-10.

as in Fig. XIX-10. It seems very likely then that the ambiguity of field theory arises from insufficient restriction on the appearance of nonmodular sublattices, thereby indicating ordered indiscernibles rather than equivalent indiscernibles (as in the modular case, Fig. XIX-10), and this restriction represents some symmetry condition on the predicate T(R, A, S).

3. Quantum Field Theory

The typical problem[5] of field theory is the solution of the nonlinear equation

$$(\Box + k^2)\Phi = \epsilon^2 \Phi^3 \tag{5}$$

where Φ is an operator-valued function subject to conditions (1) and (2). The existing approach is to treat the right-hand side as a perturbation and expand the integral form of solution

$$\Phi(x) = \Phi_o(x) + \epsilon^2 \int \Delta_o(x-x')\, \Phi(x')\, dx', \tag{6}$$

where Φ_o, Δ_o are solutions of the 'free' equation ($\epsilon=0$). To remove divergences from the expansion, it is necessary to interpret it in terms of Feynman diagrams; while this can be done consistently, it is ad hoc and leaves us with no theroy about what determines the experimental values of (m, g). There are other equal methods such as the program known as 'Reggeization' in scattering theory of removing ambiguities from field theory.

It seems that there are at least two other approaches that could lead to a deeper theory. The first is to consider (5) as an equation in the non self-adjoint operators

$$A^{\pm} = (J \pm ik \pm \epsilon\, \Phi). \tag{7}$$

To justify this one would have to show that the A's give a representation of a group that expresses, in some basic way, a theory of measurement. This will be indicated in section 4.

Alternatively, one can notice that if ϕ is a c-number, (5) has a solution in terms of elliptic functions with modulus τ (period ratio) which determines $(m(\tau), g(\tau))$. The last need not be constant off the light cone (or mass shell) and the modular transformation group Γ gives a basis for introducing what is currently known as the renormalization group.[6] To go over to a g-number theory one can consider the Hilbert space of differentials on a Riemann surface[7] and its various orthogonal decompositions. The importance of this approach is that it establishes a connection with the powerful methods of algebraic field theory when $K(\tau)$ is algebraic. In particular, we shall have in some limit

$$\omega g|\tau|_v = (x-x')^2, \tag{8}$$

where v is a valuation that may not be Archimedean.[8] This crucial point explains why nonlocal field theories have failed up till now. A typical[9] form of it requires relations of form

$$\begin{aligned} &[P^\mu[P_\mu, \psi]] = k^2\psi \\ &[X^\mu[X_\mu, \psi]] = \lambda^2\psi; \end{aligned} \tag{9}$$

however, if X is chosen to be a self-adjoint (or even normal) operator (therefore diagonalizable, with eigenvalues x^μ, say) we are back essentially to a localizable theory, since there is no way of distinguishing the domains $x^\mu x_\mu \gtrless \lambda^2$.

4. Non Self-Adjoint Operators and Group Representations

If A is not a normal operator (i.e., does not commute with its adjoint) then it cannot be unitarily equivalent to a diagonal, but only a triangular form. The eigenfunctions are not orthogonal and this has the apparently unpleasant consequence that probability is not conserved, but there is no evidence that this should be so in every bounded domain. Non self-adjoint operators have forced themselves on the attention of physicists,[10] in particular, in the theory of scattering by metastable systems. Here we should ask only that the A's give a representation, necessarily nonunitary, of some group that is uniquely associated with a theory of measurement. Mackey[11] has pointed out that the most natural infinite representations of groups are not necessarily unitary and that nonunitary representations can be induced for subgroups by unitary representations of larger ones. Two questions now arise which can be seen as reformulations of (B) and (A), respectively.

(C) What is the source of nonunitary representations? The answer is in algebraic groups,[12] that is, groups whose field of definition is restricted to algebraic numbers. This is discussed further in section 5.

(D) What group structure expresses the uniqueness of physical measurement? The answer here, which I have discussed in a preliminary fashion elsewhere,[13] lies in a phenomemon occurring uniquely in the exceptional simple Lie groups which is known as the 'Principle of Triality'.[14,15]

This principle can be formulated as a physical one by regarding it as a far-reaching extension of the 'Dirac trick,' by which is meant the following – in the relativistic theory of the electron, the angular momentum operators M (as generators of the Lorentz group D_2) are extended to $M' = (M+\sigma)$, so that M' becomes a constant of the motion. In this case M' acts on a direct product of two representations (Infinite and Spin) of the same group D_2. We have a degeneracy which would be lifted if the direct product representation were replaced by a single representation of a larger group; it seems that there is a unique candidate for the latter, namely the exceptional Lie group F_4. Operators of this group can be put in the form $A = (Z+\sigma_1+\sigma_2)$, where the terms operate respectively on a vector space V and two spin spaces S_1, S_2. There exists, then, as an automorphism of the group, a triality operator J such that $V = JS_1 = J^2S_2$. The existence of J is due to the fact that V is an 8-dimensional vector space (relativistic phase space) that can be coordinatized by the nonassociative division algebra $Q_8(z)$ of octonions which, in addition to the involution $Tz = \bar{z}$, has another $S(z_1z_2) = z_1(z_2\cdot)$ because of the nonassociativity. Then J = ST.

Triality is the mathematical expression of what has been called[16] 'the elementarity of measurement,' since it says – no matter how V is distinguished, the information obtained is the same. The group F_4 thus contains within it not only coordinate but gauge transformations that at present are considered separately, for example,[17] as the group A_2 of strong interaction symmetries.

It follows from this that, unlike the Dirac case, the representations of σ_1, σ_2 must be infinite dimensional, at least if that of Z is. But, in any case, there are no finite dimensional representations of $Q_8(z)$ on account of the nonassociativity. The nature of these representations is at present a matter of speculation, but in view of what was said in the previous section it seems likely that they may be totally discontinuous p-adic representations. If this is the case, possible substitutes for (1) would be

$$[\sigma_1, \sigma_2] = Jv(z-z'), \tag{10}$$

together with two further relations obtained by application of J, since this can be assumed to act on the valuation of v(z).

5. Algebraic Field Theory

The key to representation theory is algebraic groups.[20] There is a classical approach to these through Abelian functions that represent an algebraic field on a Riemann surface. A surface of genus g (i.e., g cuts) is completely characterized by (3g-3) moduli. The transformation group Γ on these moduli establishes a correspondence[19] between surfaces which, when the moduli are from an algebraic number field $K(\tau)$, may be an automorphism (for elliptic functions this is known as complex multiplication). To represent a Lie group G on the variables of these functions one has to embed the Weyl group W(G) in Γ. Elsewhere[18] I have given some conjectures about this for F_4. An essential role here is played by the class field theory of $K(\tau)$, that is, the equivalence classes of ideals in $K(\tau)$, hence the connection with p-adic valuations. Since Γ is an infinite group, it may contain finitely presented groups with an unsolvable word problem, hence a connection also with the remarks on logic of section 2.

It seems quite essential to preserve the preferred role of time as implied by (2), and therefore another condition imposed on $K(\tau)$ will be that it causes just such a splitting of the group structure.

6. Global Viewpoint

The group F_4 represents only half, which we may consider the local part, of the picture. If we write the relativistic wave equation in the form

$$\left\{\Gamma^{\mu}\left(\nabla_{\mu} + i\frac{\delta}{\delta J^{\mu}}\right) + i\kappa\right\}\psi = 0, \tag{11}$$

where the covariant derivative

$$\nabla_{\mu} = \partial_{\mu} + i\langle A_{\mu}\rangle \tag{12}$$

is determined by the expectation value $\langle A_{\mu}\rangle$ of the electromagnetic potential, then the triality $\left(\frac{\delta}{\delta J^{\mu}}, \nabla_{\mu}, \kappa\right)$ is local in the sense that it describes events in the hypersurface determined, apart from $\langle A_{\mu}\rangle$, by the element of support Γ^{μ} considered as a constant. If the last is no longer assumed, we have the point of view of general relativity and all the arguments about triality now apply to Γ^{μ}.

F_4 is not the largest group that acts on an octonion structure. This structure is E_8 which acts on a tensor product of octonion algebras in the same way that D_6 acts on the Dirac algebra (of Γ's) as a tensor product of quaternions. It is evident that the splitting relations here are important just as they are in the Dirac case, where

$$(\Gamma^{\mu}, \kappa) = (\sigma_1, \rho_i, 1, \rho_3) \tag{13}$$

(σ_i, ρ_i, elements of complex quaternion algebras). In particular, it is clear, in view of (12), that the 'trialities' will not be independent.

The relation between the groups is[15]

	m = 1	2	3	8
n = 1	B_1	A_2	C_3	F_4
2	A_2	A_2	A_5	E_6
4			D_6	E_7
8				E_8

Here, (m, n) are the dimensions of the algebras; B_1 is the rotation group of real 3-space; A_2, A_5 (=SU(3), SU(6)) have been proposed as gauge groups for strong interactions. If A_5 turns out to be the correct identification, then we have the possibility of the approximate splitting, $E_8 \sim A_5 + B_3$, where B_3 is the smallest group admitting a nontrivial triality, for which I have given an interpretation elsewhere in terms of a cosmological model.[22]

M. C. Goodall

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