XIV. GASEOUS ELECTRONICS*

Academic and Research Staff

Prof. G. Bekefi
Prof. W. P. Allis
Prof. S. C. Brown
Prof. J. C. Ingraham
J. J. McCarthy
W. J. Mulligan

Graduate Students

W. B. Davis
G. A. Garosi
L. D. Pleasance
T. T. Wilheit, Jr.
B. L. Wright

A. EFFECTS OF HYDRODYNAMIC TURBULENCE ON A WEAKLY IONIZED ARGON PLASMA

An apparatus has been built to study the effects of hydrodynamic turbulence on a weakly ionized plasma. The plasma that is being studied is the positive column of an argon discharge run at pressures of 15-40 Torr and with a discharge current of 0.1-1.0 amps. The experimental arrangement is shown in Fig. XIV-1; it is based on that used by Gentle, Ingard, and Bekefi,[1] but is capable of much higher flows.

One of the most important parameters in this experiment is the Reynolds number defined as

$$\mathrm{Re} = \frac{\rho U L}{\nu}, \tag{1}$$

where ρ is the mass density, μ is the viscosity, and L and U are characteristic length and velocity parameters, respectively. For this apparatus L is the tube diameter, and U is the mean flow velocity in the discharge tube. Because of the temperature dependence of μ, it was necessary to measure the gas temperature. The measurement was made by using a method described by Gentle and Ingard.[2] By measuring the wall temperature with a thermocouple, we found that at very low mass flows the temperature drop from tube center to wall can be as large as 200°C ($T_g \approx 250$°C and $T_\omega \approx 75$°C for $I_D = 0.5$ amp and p = 30 Torr). At the higher flows, however, the gas temperature was very close to room temperature.

Ion density fluctuations can be studied by means of Langmuir probes biased to draw ion saturation current. Results obtained in this experiment by means of negatively biased probes are in good agreement with those obtained by V. L. Granatstein and his co-workers.[3-5]

A theory for an electrostatic probe in a medium-pressure discharge, with the mean-free path much smaller than the probe size ($\lambda \ll r_p$), has been given by Zakharova and his co-workers,[6] and by Boyd.[7] The ion saturation current, I_{s+}, to a cylindrical

*This work was supported by the Joint Services Electronics Programs (U.S. Army, U.S. Navy, and U.S. Air Force) under Contract DA 28-043-AMC-02536(E).

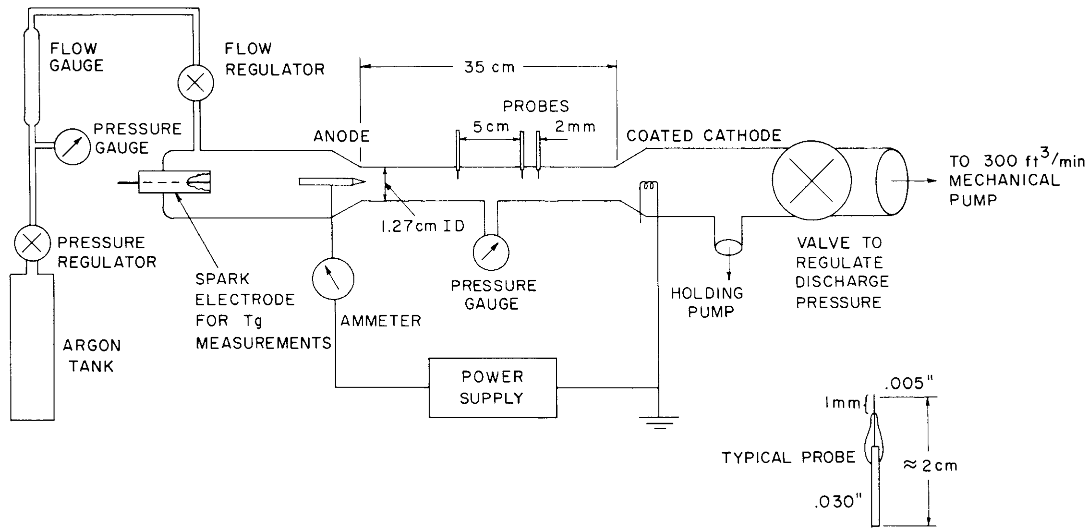

Fig. XIV-1. Experimental arrangement.

probe of radius r_p and length ℓ is given by

$$I_{s+} = \mu_+ \frac{2\pi\ell}{\ell n(\ell/r_p)} kT_- n_+ \qquad \text{for } T_- \gg T, \tag{2}$$

where T_- is the electron temperature, k is Boltzmann's constant, n_+ is the ion density, and μ_+ is the ionic mobility for pressure p and ion temperature T, under conditions in which the probe sheath is small and when the abnormal diffusion region[7] can be neglected. These two conditions[7,6] are given by

$$n \gg 10^8 \frac{V_e}{\lambda_+} \tag{3}$$

and

$$\frac{\lambda_+}{r_p} \frac{T_-}{T_+} < 5, \tag{4}$$

where V_e is the electron temperature in volts, and λ_+ is the ion mean-free path in cm. These conditions were satisfied in this experiment.

From Eq. 2, we see that when fluctuations in μ_+T_- can be neglected, fluctuations in I_{s+} will be directly proportional to fluctuations in n. The conditions for which the fluctuations in μ_+T_- can be neglected are given by Granatstein.[4] These conditions are satisfied in this experiment. [A plot of $\Delta I_{s+}/\sqrt{I}_{s+}$ against flow is given in Fig. XIV-5.]

The electron temperature was measured by using the V-I characteristics of a

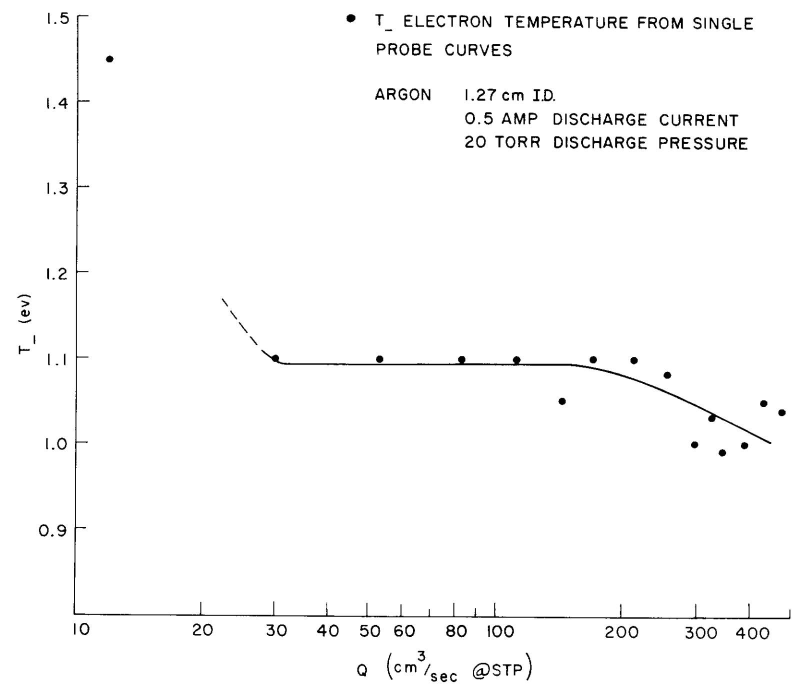

Fig. XIV-2. Results of electron temperature measurement.

cylindrical probe. Although this method has been questioned by Zakharova and his co-workers,[6] the results yielded an electron temperature in agreement with the double probe results of Granatstein.[4] The results in Fig. XIV-2 show a slight effect attributable to flow. The main result is that the change in T_- ($<10\%$) can be neglected, as compared with the change in Tg ($\approx 40\%$) in the expression for I_{s+}. In order to eliminate the effect of Tg on the behavior of I_{s+}, we define with Granatstein[4]

$$I_{so} = \left(\frac{273}{Tg}\right)^{1/2} I_{s+} \tag{5}$$

and thus have

$$I_{so} = \mu_o\left(\frac{760}{P}\right) \frac{2\omega\ell}{\ln \ell/rp} kT_-n, \tag{6}$$

where μ_o is the ion mobility at STP.

A plot of I_{so} against flow will thus show the change in density in the vicinity of the probe. This is shown in Fig. XIV-3. This behavior has not yet been explained.

The axial electric field was obtained by measuring the potential difference of two floating probes separated by 5 cm, and is shown in Fig. XIV-3. The increase in E_z with flow begins at a lower Reynolds number than that of Gentle, Ingard, and Bekefi[1] and Granatstein and Buchsbaum.[3] This discrepancy is being investigated. It

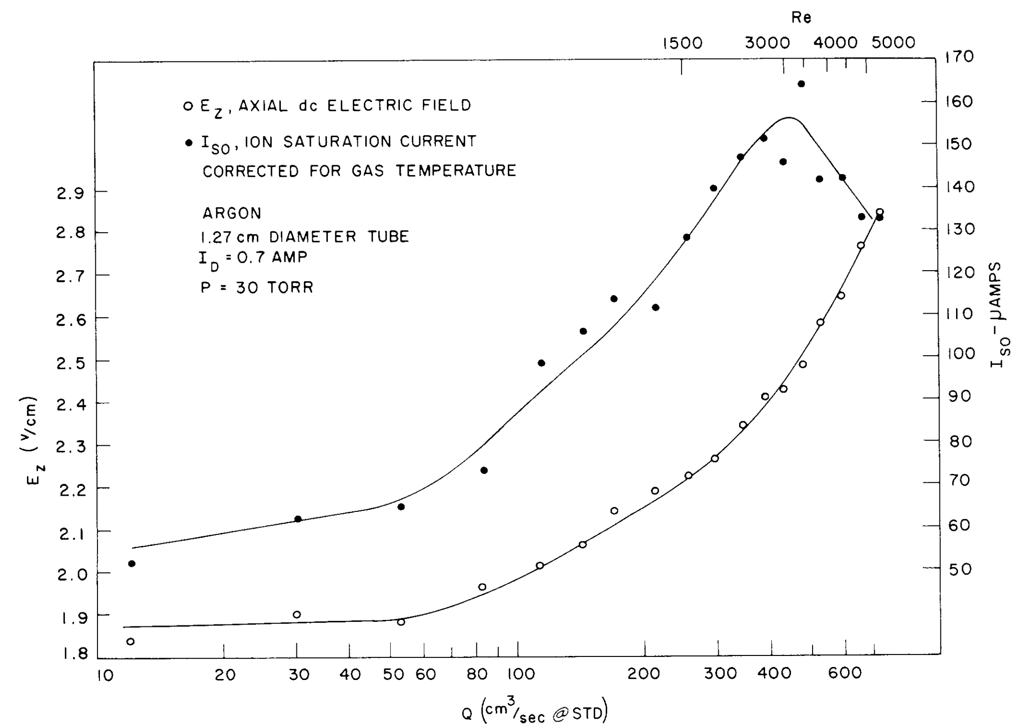

Fig. XIV-3. I_{so} vs flow.

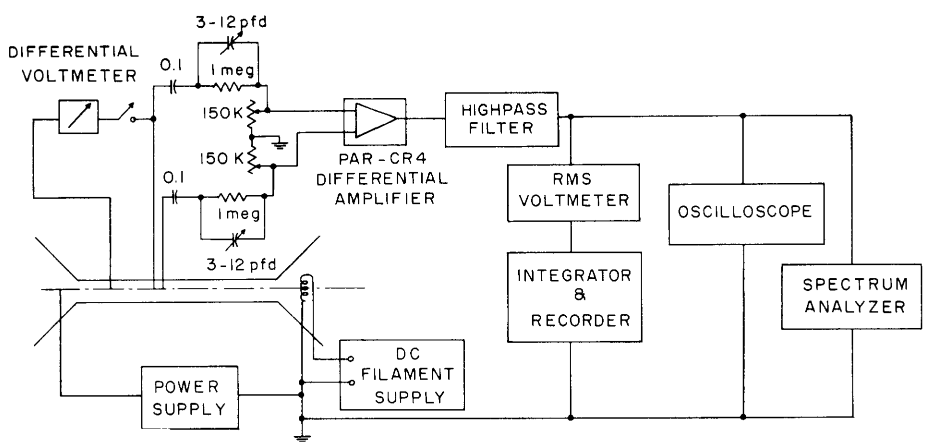

Fig. XIV-4. Circuit for measuring the axial electric field.

may possibly be due to the perturbing effects of the anode.

The fluctuations in the axial electric field were studied by means of two floating probes separated by 2 mm. The circuit used is shown in Fig. XIV-4 and is based on the circuit of F. F. Chen.[8] The attenuation circuit is needed to keep the signal from saturating the amplifier input stage. The variable capacitors and potentiometers are needed to compensate for output capacitance and to balance the attenuators for maximum common mode rejection.

In Fig. XIV-5 there is also shown a plot of $\Delta E_z/\bar{E}_z$ against flow. The flows for which

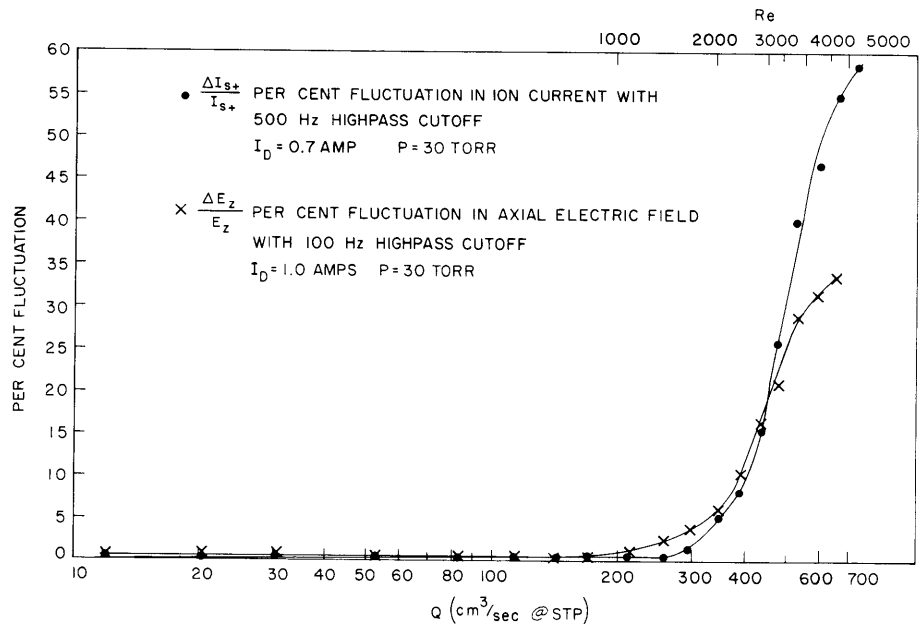

Fig. XIV-5. $\Delta I_{s+}/\bar{I}_{s+}$ vs flow.
$\Delta E_z/\bar{E}_z$ vs flow.

the fluctuations in both density and axial electric field begin to increase coincide closely. The agreement is possibly better than the data indicate, since the density fluctuations were taken with a 500-Hz highpass filter, and thus fail to reflect low-frequency fluctuations.

Both curves show a tendency to saturate. The fact that the electric field fluctuations begin to saturate at a lower Reynolds number than the density fluctuations is probably due to the larger discharge current that was used for the electric field measurements. The higher current results in a less constricted discharge and thus a smaller density gradient to drive the plasma fluctuations. The crossing of the two curves may also be the result of the difference in the measuring parameters.

Future work includes study of the frequency spectra of both density and electric field, and also a study of the effect of flow on the loss of electrons in the afterglow of an argon discharge. Preliminary measurements pointed up many problems which need to be solved before meaningful data can be taken.

G. A. Garosi

References

1. K. W. Gentle, Uno Ingard, and G. Bekefi, Nature 203, 1369 (1964).
2. K. W. Gentle and Uno Ingard, Appl. Phys. Letters 5, 105 (1964).
3. V. L. Granatstein and S. J. Buchsbaum, Appl. Phys. Letters 7, 285 (1965).
4. V. L. Granatstein (to appear in Physics of Fluids).
5. V. L. Granatstein, S. J. Buchsbaum, and D. S. Bugnolo, Phys. Rev. Letters 16, 504 (1966).
6. V. M. Zakharova, Yu. M. Kagan, K. S. Mustafin, and V. I. Perel, Soviet Physics — Tech. Phys. 5, 411 (1960).
7. R. L. F. Boyd, Proc. Phys. Soc. (London) B64, 795 (1951).
8. F. F. Chen, "Electric Probes" in Plasma Diagnostic Techniques, edited by Richard H. Huddlestone and Stanley L. Leonard (Academic Press, New York, 1965), p. 197.

B. CALCULATIONS OF THE TIME-DEPENDENT ELECTRON VELOCITY DISTRIBUTION FUNCTION IN AN ARGON AFTERGLOW PLASMA

In a series of previous reports,[1-3] we have presented details of measurements of the time-dependent electron velocity distribution function in an argon afterglow plasma. Although complete plots of f(v) at various afterglow times were provided by the experimental technique, our interest was primarily in the electron mean energy (U) and a collection of parameters (H, Q_n, and A_o) which measured the departure of f(v) from a Maxwellian. By studying the time dependence of these quantities, characteristic rates for electron-atom collisional cooling and electron-electron relaxation could be determined.

In support of the experimental results, a simple model of the discharge plasma and its afterglow was examined.[2] On the basis of this model, theoretical curves were obtained for f(v) and its time derivative at the instant of removal of the applied DC electric field ($t = 0$). As has already been indicated, good agreement was obtained between the experimental and theoretical electron velocity distribution functions in the steady-state ($t \leq 0$) case. In examining the initial time dependence of f(v, t), however, a fundamental difference was observed. According to the model, in the range of argon pressures and electron densities considered, electron-electron relaxation is overwhelmed by the effects of electron-atom collisions, and initially f(v) is driven farther away from a Maxwellian with time. This conclusion seems at variance with the experimental results which indicate a fairly uniform decay of f(v) toward a Maxwellian. In this report, we show that the problem can be resolved by computing from the model the full afterglow time dependence of f(v, t).

Because calculation of f(v, t) is necessarily more involved than determining f(v) in the steady state, a slight idealization of the model equation previously given was required. In particular, the term describing inelastic collisions of electrons with atoms was deleted, because of its strong dependence on velocity and relatively short time scale. The resulting model equation for the isotropic component of $f(\vec{v}, t)$ is given below in MKS units.

$$\frac{\partial}{\partial t} f(v,t) = \frac{1}{3}\left(\frac{eE}{m}\right)^2 \frac{1}{v^2}\frac{\partial}{\partial v}\left[\frac{v^2}{\nu}\frac{\partial f}{\partial v}\right] + \frac{m}{M}\frac{1}{v^2}\frac{\partial}{\partial v}\left[v^3 \nu f\right]$$

$$+ n\left(\frac{e^2}{\epsilon_0 m}\right)^2 \ln\left(12\pi n \ell_D^3\right)\left\{f^2 + \left[\frac{1}{v}\int_0^v V^2 f\, dV - \frac{1}{3v^4}\int_0^v V^4 f\, dV\right.\right.$$

$$\left.\left. + \frac{2}{3v}\int_v^\infty Vf\, dV\right]\frac{\partial f}{\partial v} + \left[\frac{1}{3v^2}\int_0^v V^4 f\, dV + \frac{1}{3}\int_v^\infty Vf\, dV\right]\frac{\partial^2 f}{\partial v^2}\right\}.$$

Here, e is the charge of the electron, m is its mass, M is the mass of a neutral atom, E is the applied (DC) electric field, ℓ_D is the Debye length, n is the electron density, $\nu(v)$ is the electron-atom collision frequency for momentum transfer, and $4\pi\int_0^\infty v^2 f\, dv = 1$. The three components on the right represent the driving force of the applied electric field, elastic recoil in electron-atom collisions, and electron-electron interaction. In keeping with the idealized character of the problem, we have used a power-law approximation for $\nu(v)$ in argon:

$$\nu(v) = 2.00 \times 10^{-11}\, p\, v^{3.3}\ \text{sec}^{-1},$$

where p is in Torr, and v is in m/sec.

The afterglow behavior of f(v, t) was obtained by numerically integrating $\partial f/\partial t$ with E = 0 from an initial value, f(v, 0), provided by solution of the steady-state problem ($\partial f/\partial t = 0$, $E \neq 0$). The numerical integrating technique that was employed is known as the Kutta-Simpson one-third rule. Two such calculations were made: the first with $E(t \leq 0) = 100$ volts/m, p = 1.0 Torr, and $n = 10^{10}\ \text{cm}^{-3}$; the second with the same field and pressure but with no electron-electron interaction (n = 0). In both cases, the solution was carried out to an afterglow time of 4.3 μsec. Although complete tables of f(v, t) and $\partial f/\partial t$ were obtained, our concern here will be with the values and rates of change of the various parameters determined from f(v).

To begin with, the time dependence of the mean electron energy, $U(t) = 2\pi m\int_0^\infty v^4 f(v,t)\, dv$, was analyzed. We found that neither the initial value nor the subsequent temporal behavior of U(t) was significantly affected by the change in electron density from zero to 10^{10} cm^{-3}. Starting at 4.08 eV, U decreased with time at a rate very closely given by

$$\dot{U}/U = -0.102\ (U/4.08)^{1.65}\ \mu sec^{-1},$$

a result which is not surprising in view of the assumed power-law dependence of the collision frequency ($\nu \sim v^{3.3}$).

As far as the relaxation of f(v) to a Maxwellian is concerned, consider the parameter $Q_1(t)$ given by

$$Q_1(t) = 4\pi \int_0^{\infty} z^3 g(z,t)\, dz,$$

where $z = v/w$ and $g(z,t) = w^3 f(wz,t)$ constitute a dimensionless velocity and distribution function written in terms of the characteristic velocity

$$w(t) = \left[\frac{4}{3m} U(t)\right]^{1/2}.$$

As f(v) approaches a Maxwellian, Q_1 approaches $\hat{Q}_1 = 2/\sqrt{\pi}$. Thus the difference

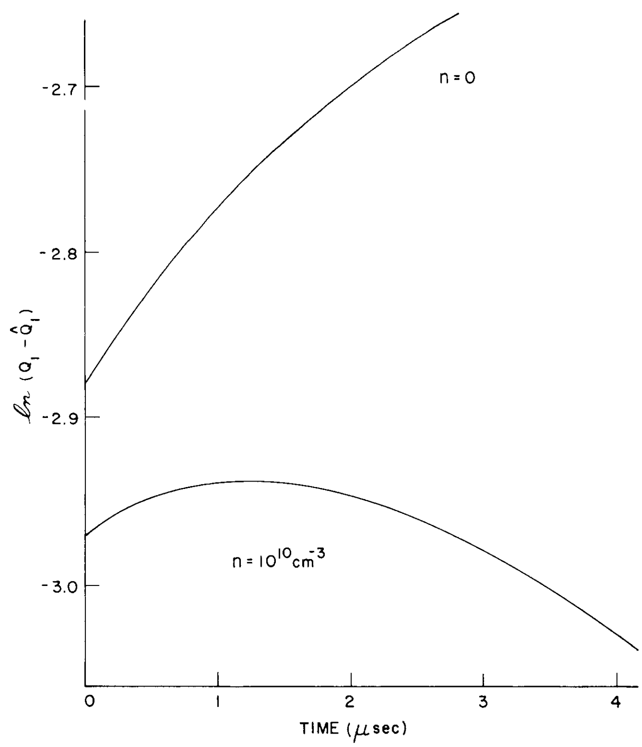

Fig. XIV-6. Computer time dependence of $Q_1 - \hat{Q}_1$. (E = 100 v/m, p = 1.0 Torr.)

$Q_1 - \hat{Q}_1$, can be used as a measure of the departure of f(v) from a Maxwellian.

The theoretical time dependence of $Q_1 - \hat{Q}_1$ in the two cases is illustrated in Fig. XIV-6. As we have noted, the positive slopes at t = 0 indicate that initially f(v) is becoming less Maxwellian with time. In the case for no electron-electron interaction, this behavior continues, but with 10^{10} electrons per cm^3 the trend is quickly reversed after 1.2 μsec, and thermalization of f(v) occurs. The basis for this reversal is illustrated by Fig. XIV-7. The quantities plotted are the electron-atom contribution to the relaxation rate of Q_1 and the negative of the electron-electron contribution. Thus the

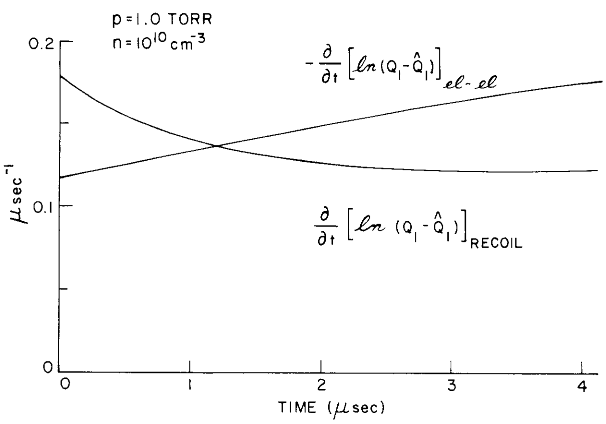

Fig. XIV-7. Computed time dependence of the component rates of relaxation.

total time derivative of ln $(Q_1-\hat{Q}_1)$ is given by the difference between the two curves. It is seen that the reversal in sign of $\dot{Q}_1$ is the result of both a decrease in the first component and an increase in the magnitude of the second. Indeed, the uniform rise in the electron-electron portion is found to correspond quite well to $U(t)^{-3/2}$, as expected from the velocity dependence of the electron-electron interaction rate. Figure XIV-7 illustrates the difficulty of determining the long-term rate of Maxwellianization from a solution of only the steady-state (t = 0) problem. The ultimate value ($-0.051\ \mu sec^{-1}$) at t = 4 μsec bears little relation to the computed rates at t = 0. Although we have used the parameter Q_1 for these illustrations, all of the other measures of the departure of f(v) from a Maxwellian exhibited similar behavior.

Because the experimental electron velocity distribution functions previously reported represent averages over a 1-μsec sample interval, and because cutoff of the applied DC electric field was, in fact, not instantaneous, it is not surprising that no initial increase in $Q_1 - \hat{Q}_1$ was observed. Thus calculation of the full afterglow time dependence of f(v,t) has helped resolve the apparent discrepancy between the measured time dependence of the departure of f(v) from a Maxwellian and the theoretical instantaneous rate at t = 0. The calculation for n = 0 suggests the possibility of lowering the degree of ionization of the discharge to a level at which the experimentally determined f(v,t) can become less Maxwellian with time over a long interval. Investigation of that possibility is a potential topic for further research in this area.

B. L. Wright

References

1. B. L. Wright, "Microwave Measurements of a Time-Dependent Electron Velocity Distribution Function," Quarterly Progress Report No. 80, Research Laboratory of Electronics, M. I. T., January 15, 1966, pp. 99-103.
2. B. L. Wright, "Comparison of Measured Time-Dependent Electron Velocity Distributions with a Theoretical Model," Quarterly Progress Report No. 83, Research Laboratory of Electronics, M. I. T., October 15, 1966, pp. 59-64.
3. B. L. Wright, "Electron-Electron Relaxation Rates as Determined from the Observed Time-Dependent Electron Velocity Distribution," Quarterly Progress Report No. 84, Research Laboratory of Electronics, M. I. T., January 15, 1967, pp. 137-140.

C. DOPPLER BROADENING OF ELECTRON-CYCLOTRON RESONANCE ABSORPTION OF MICROWAVES

In two previous reports, Ingraham[1,2] has reported and discussed a microwave measurement of the electron-neutral atom collision frequency in cesium. These data showed little agreement with other published data. This was not considered serious because the other data did not agree among themselves. More recently, Nighan[3] has reported results of a DC measurement. By taking into account the electron-ion collisions, he was able to bring many of the cesium data into substantial agreement with his; however, Ingraham's data still do not agree with Nighan's. Ingraham's collision frequencies are generally larger than Nighan's. One conceivable source of this discrepancy is Doppler

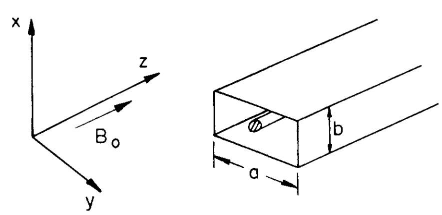

Fig. XIV-8. Experimental arrangement.

broadening of the cyclotron absorption peak. The purpose of this report is to examine this possibility.

Consider the following model: A uniform plasma column of electron density n_o and radius ρ, inserted along the axis of a rectangular waveguide of dimensions a, b (a > b) as shown in Fig. XIV-8. There is also a magnetic field, B_o, along the axis of the waveguide. Assume that the plasma is tenuous enough so that the electric field is not significantly altered by its presence.

The electric field is assumed to be that of the lowest waveguide mode:

$$\vec{E} = \hat{x} E_o \sin \pi y/a \cos (kz-\omega t),$$

where

$$k^2 = \omega^2/c^2 - \pi^2/a^2.$$

The ions are assumed to be immobile. We are only interested in the region of frequency near the electron-cyclotron frequency. That is,

$$\omega \approx \Omega = |eB_o/mc|,$$

where e is the magnitude of the electronic charge, and m is the electron mass.

We shall assume that the electron-cyclotron radius $r_c \ll a$, so that we may ignore the cyclotron harmonic terms. We shall also assume that the unperturbed distribution function is a Maxwellian.

$$f_o = \left(\frac{m}{2\pi KT}\right)^{3/2} \exp - \left\{\frac{mv^2}{2KT}\right\} \qquad \text{for } x^2 + y^2 \leq \rho^2$$

$$f_o = 0 \qquad \text{for } x^2 + y^2 > \rho^2$$

where K is Boltzmann's constant, and T is electron temperature.

The linearized Vlasov equation for the system is

$$\frac{\partial f_1}{\partial t} + \vec{v} \cdot \frac{\partial f_1}{\partial \vec{r}} - \frac{e}{mc} (\vec{v} \times \vec{B}) \cdot \frac{\partial f_1}{\partial \vec{v}} = \frac{e}{m} E_o \cdot \frac{\partial f_o}{\partial \vec{v}} - \nu_c f_1,$$

where ν_c is the collision frequency for momentum transfer. This equation can be integrated in a tedious but straightforward way. The time asymptotic result, with terms with factors of v_y or sin (kz-ωt) ignored, which do not contribute to the final answer, is

$$f_1 = \frac{eE_o}{2KT} \sin \frac{\pi y}{a} V_x \nu_c f_o \cos (kz-\omega t) \left\{\frac{1}{\nu_c^2 + (\omega - kv_z - \Omega)^2} + \frac{1}{\nu_c^2 + (\omega - kv_z + \Omega)^2}\right\}.$$

We shall drop the second term, since it is small for $\omega \approx \Omega$.

The energy dissipation per unit volume is $\langle \vec{E} \cdot \vec{J} \rangle$, where $\vec{J}$ is the current density, and the brackets indicate a time average.

$$\vec{J} = -en_o \int \vec{v} f_1 \, d^3\vec{v}$$

$$<E \cdot J> = \frac{e^2 n_o E_o^2 \sin^2 (\pi y/a)}{4KT} \int \frac{d^3\vec{v} \, (v_x)^2 \, \nu_c f_1}{\nu_c^2 + (\omega - kv_z - \Omega)^2}.$$

We can compare this with the electromagnetic energy flux down the waveguide,

$$\frac{k\epsilon_o c^2 abE_o^2}{4\omega},$$

where E_o is the permittivity of free space, and c is the velocity of light. So if we define a power absorption, a, such that

$$P(z) = P(0) \, e^{-az}$$

$$a = \frac{\omega_p^2 \omega}{kc^2 v_T ab} \int_0^{x^2+y^2=\rho^2} dxdy \sin^2 \left(\frac{\pi y}{a}\right) \int \frac{d^3\vec{v} \, (v_x)^2 \, \nu_c f_o}{\nu_o^2 + (\omega - kv_z - \Omega)^2},$$

where ω_p is the electron plasma frequency $\left(n_o e^2/E_o m\right)^{1/2}$, and $v_T = (KT/m)^{1/2}$.

If we ignore the Doppler shift term in the denominator (kv_z), a reduces to

$$\frac{4\pi\omega_p^2 \omega}{3kc^2 v_T^2 ab} \int_0^{x^2+y^2=\rho^2} dxdy \sin^2 \left(\frac{\pi y}{a}\right) \int_0^\infty \frac{dvv^4 \nu_c f_o}{\nu_c^2 + (\omega-\Omega)^2}.$$

This is essentially the form used by Ingraham.[1,2] An improvement can be made, however, to approximate the Doppler broadening. The integrations over velocity directions can be done exactly without ignoring the Doppler term,

$$\int \frac{d^3\vec{v} \, (v_x)^2 \, \nu_c f_o}{\nu_c^2 + (\omega - kv_\epsilon \; \Omega)^2} = \pi \int_0^\infty dv \, v^4 f_o \; R.F.(\omega - \Omega).$$

R. F. ($\omega-\Omega$) is a resonance function similar to a Lorentzian,

$$R.F.(\omega-\Omega) = \nu_c \left[\frac{\omega - \Omega}{k^3 v^3} \log \left\{ \frac{\nu_c^2 + (\omega - \Omega + kv)^2}{\nu_c^2 + (\omega - \Omega - kv)^2} \right\} - \frac{2}{k^2 v^2} \right.$$

$$\left. + \frac{k^2 v^2 - (\omega-\Omega)^2 + \nu_c^2}{k^3 v^3} \left\{ \tan^{-1} \left(\frac{kv + \omega - \Omega}{\nu_c} \right) + \tan^{-1} \left(\frac{kv - \omega + \Omega}{\nu_c} \right) \right\} \right]$$

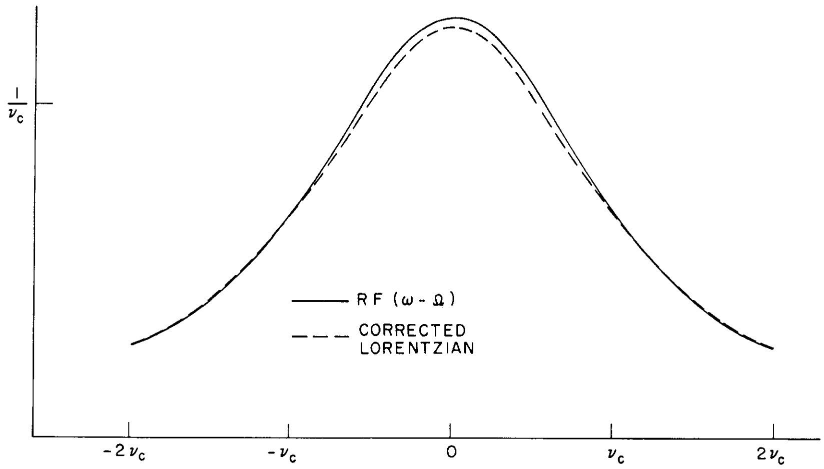

Fig. XIV-9. Comparison of the R. F.(ω–Ω) solution and Lorentzian approximation for $kV = \frac{1}{2}\nu_c$.

The area under this resonance function when integrated over Ω (or ω) from −∞ to +∞ is $4/3\pi$. Its half-width at half-maximum is

$$\nu_c + \left(\frac{3}{10}\right)\frac{k^2v^2}{\nu_c} + O\left(\frac{k^4v^2}{\nu_c^3}\right).$$

So we can approximate R. F. (ω–Ω) by a Lorentzian resonance function

$$\frac{\frac{4}{3}\nu_c'}{(\nu_c')^2 + (\omega-\Omega)^2},$$

where $v_c' = v_c + \frac{3}{10}\left(\frac{k^2v^2}{\nu_c}\right)$. In Fig. XIV-9 R. F.(ω–Ω) and the Lorentzian approximation for $kv = \frac{1}{2}\nu_c$ are compared. This gives

$$\alpha \cong \frac{4\pi}{3}\frac{\omega_p^2\omega}{kc^2v_T^2ab}\int_0^{x^2+y^2=\rho^2} dxdy\,\sin^2\frac{\pi y}{a}\int_0^\infty \frac{d\nu\,\nu_c' f_o}{(\nu_c')^2 + (\omega-\Omega)^2}.$$

To estimate the significance of the Doppler broadening, note that

$$(P_c')^2 = \left(P_c^2 + \frac{3k^2}{5p_o^2}\right) + O\,\frac{k^4}{P_o^4P_c^2},$$

where P_c is the collision probability per unit length normalized to a pressure of 1 Torr, and p_o is the pressure in Torr. The lowest P_c measured by Ingraham for cesium is

~1000 cm^{-1} $Torr^{-1}$. The pressures were all greater than 0.01 Torr. For ω = 5.5×10^9 Hz in C-band waveguide, $k^2 \approx 0.9\ cm^{-2}$.

$$k^2/p_o^2 = 9000\ cm^{-2}\ Torr^{-2}$$

and

$$P_c^2 \geq 10^6,$$

so the error involved in ignoring the Doppler shift is less than 0.3 per cent. A similar experiment is being performed with argon. Frost and Phelps[4] give a lowest P_c for argon of ~0.5 cm^{-1} $Torr^{-1}$. The lowest pressure in the argon experiments is 10 Torr, so the error is approximately 1 per cent.

The 0.3 per cent error in cesium is not sufficient to bring Nighan's and Ingraham's data into agreement.

T. T. Wilheit, Jr.

References

1. J. C. Ingraham, Quarterly Progress Report No. 77, Research Laboratory of Electronics, M.I.T., April 15, 1965, p. 112.
2. J. C. Ingraham, Quarterly Progress Report No. 81, Research Laboratory of Electronics, M.I.T., April 15, 1966, p. 63.
3. W. L. Nighan, "Low Energy Electron-Cesium Atom Collision Probability Investigations," United Aircraft Research Laboratories Report, 1965.
4. L. S. Frost and A. V. Phelps, Phys. Rev. 136, A1538 (1964).