XIV. SPONTANEOUS RADIOFREQUENCY EMISSION FROM HOT-ELECTRON PLASMAS^{*}

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A. EXPERIMENTAL STUDY OF ENHANCED CYCLOTRON RADIATION FROM AN ELECTRON-CYCLOTRON RESONANCE DISCHARGE

We are continuing the study of the intense instability occurring in the afterglow of a pulsed electron-cyclotron resonance discharge. In this report we enlarge upon the radiation measurements presented previously, interpret the density evolution of the plasma in terms of a simple model, and present a comparison between a number of the basic characteristics of the discharge.

1. Radiation Characteristics

a. Field Pattern of the Instability Radiation

As previously reported,¹ the radiation accompanying the instability is strongly peaked at a frequency of 2498 MHz. Mode perturbation techniques had shown that the frequency was that of the TE₂₃₁ resonance of our approximately cylindrical cavity in the absence of the plasma. In order to gain an understanding of the field structure of the instability radiation, it has been assumed that the cavity is a perfect cylinder. It is also assumed that the field pattern of the radiation in the presence of the plasma is unchanged from that of the empty cavity. The first assumption is based on the observed poor coupling of the mode to the open waveguide at the midplane of the cavity. Since all of the diagnostic ports are located at the midplane, it is expected that their effect on the fields, especially near the center of the cavity, will be small. The second assumption follows from the fact that the density of the plasma is small at the time of the instability. The main effect of the plasma is to produce a slight shift of the resonant frequency by changing the effective dielectric constant within the cavity. The change in the fields, being of second order, should be small. Subject to these approximations, the TE₂₃₁ fields, in complex notation may be written

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$$\begin{split} \overline{\underline{E}}_{\mathrm{T}} &= \mathrm{C} \cos\left(\frac{\pi_{Z}}{\mathrm{L}}\right) \left\{ j\frac{2}{\mathrm{r}} J_{2} \left(\frac{\mathcal{H}_{23}\mathbf{r}}{a}\right) \overline{i}_{\mathrm{r}} + \frac{\mathcal{H}_{23}}{a} J_{2}^{\prime} \left(\frac{\mathcal{H}_{23}\mathbf{r}}{a}\right) \overline{i}_{\phi} \right\} \mathrm{e}^{-j2\phi} \\ \overline{\underline{H}}_{\mathrm{T}} &= j\frac{\mathrm{C}\pi}{\mathrm{Lk}} \sqrt{\frac{\epsilon_{\mathrm{o}}}{\mu_{\mathrm{o}}}} \sin\left(\frac{\pi_{Z}}{\mathrm{L}}\right) \left\{ \frac{\mathcal{H}_{23}}{a} J_{2}^{\prime} \left(\frac{\mathcal{H}_{23}\mathbf{r}}{a}\right) \overline{i}_{\mathrm{r}} - \frac{j2}{\mathrm{r}} J_{2} \left(\frac{\mathcal{H}_{23}\mathbf{r}}{a}\right) \overline{i}_{\phi} \right\} \mathrm{e}^{-j2\phi} \\ \overline{\underline{H}}_{Z} &= \frac{\mathcal{H}_{23}^{2}}{a^{2}} \frac{\mathrm{C}}{\mathrm{jk}} \sqrt{\frac{\epsilon_{\mathrm{o}}}{\mu_{\mathrm{o}}}} \cos\left(\frac{\pi_{Z}}{\mathrm{L}}\right) \mathrm{e}^{-j2\phi} J_{2} \left(\frac{\mathcal{H}_{23}\mathbf{r}}{a}\right), \end{split}$$

where a is the radius of the cavity, L is its length, J_2 is the second-order Bessel function of the first kind, J'_2 is the derivative of J_2 with respect to its argument, and \mathcal{R}_{23} is the third zero of J'_2 . The wave number k is defined by

$$k = \sqrt{\frac{\Im C_{23}^2}{a^2} + \left(\frac{\pi}{L}\right)^2}.$$

In writing these fields we have considered only the component of the TE_{231} mode with an $e^{-j2\phi}$ azimuthal dependence. As shown below, it is this component that has an electric field rotating with the electrons (under the assumption that the static mirror field is directed along the +z axis). Thus, subject to the stated assumptions, Fig. XIV-1 illustrates the radial dependence of the instability electric field by showing the amplitude of the two terms in the bracket of the expression for \overline{E}_r . These terms have been evaluated for the dimensions of our cavity. Furthermore, note that the radial term



Fig. XIV-1. Radial variation of the electric field of the TE₂₃₁ mode in a cavity of radius equal to 20 cm.

within the bracket is pure imaginary, while the azimuthal term is pure real. Thus, in general, the electric field is elliptically polarized. An indication of the polarization is found by dividing the amplitude of the radial component by that of the azimuthal one. This is shown in Fig. XIV-2. From these two figures the following observations may be made.



Fig. XIV-2. Polarization, $\underline{E}_r/j \underline{E}\phi$, of the electric field illustrated in Fig. XIV-1.

The electric field is zero at the center of the cavity but increases as we move out in radius. Both E_r and E_{ϕ} increase with radius. E_{ϕ} reaches its maximum at approximately 3.1 cm, while E_r is a maximum at approximately 4.5 cm. The fields are circularly polarized at the center and remain essentially circular, even as far out as 4 cm where the ratio of the radial component to the azimuthal component is approximately 1.6. Since the two components have the same sign in this region, the direction of rotation of the fields is the same as that of an electron gyrating in a magnetic field directed along the +z axis. Further increases in radial position result in increasingly more eccentric elliptical polarization with the major axis along the radial direction. Beyond 6.1 cm where the field is linearly polarized along $\overline{i_r}$, the direction of rotation changes

sign. Further variations of the fields are evident from Figs. XIV-1 and XIV-2, but we are interested in the maximum field region between r = 3 cm and r = 4.5 cm.

As discussed in our previous report, the instability occurs most repeatedly at a magnet current of 74 A. Referring to that discussion, it will be found that this current results in a contour within the cavity upon which the local electron-cyclotron frequency equals that of the instability radiation, 2498 MHz. This contour passes through the midplane of the cavity (z = 0) at a radius r = 3.75 cm. In terms of the cavity mode at 2498 MHz discussed above, this contour passes through the region where the RF fields would be strongest and where they are almost circularly polarized with rotation in the same direction as the gyrating electrons. If the magnet current is altered from this value, the instability becomes less repeatable and the amplitude of the radiation decreases. Similarly, if the magnet system is shifted slightly with respect to the cavity, the instability becomes less repeatable. Thus we conclude that the instability results from a coupling between these resonant electrons and the cavity mode at the location where the RF fields are greatest. It should be noted that this interaction takes place at the midplane of the cavity. Here the RF magnetic field is purely z-directed. This field structure is very similar to that of the fast-wave branch of the extraordinary wave predicted by infinite plasma theory. Near its cutoff this wave also displays a zdirected magnetic field along with a circularly polarized transverse electric field. Although we have shown in a previous report that this wave is unstable in a hot infinite plasma,² in the experimental observation the presence of the eavity is important and further theoretical work is needed.

b. Amplitude of the Instability Radiation

In order to obtain an estimate of the power radiated by the plasma during the instability, a simple experiment was performed to determine the amplitude of the radiation coupled out of the cavity. As previously discussed,¹ the instability radiation is observed on the reverse power arm of a dual directional coupler in the waveguide that transmits the magnetron heating power to the cavity.³ The burst of radiation is detected by a microwave diode and the output displayed on an oscilloscope. In this experiment the amplitude of the burst was noted. The crystal was then excited by a well-padded microwave oscillator of the same frequency as the instability radiation. The amplitude of the oscillator was adjusted to yield the same peak output at the crystal as the instability. The value of the incident power at the crystal was then measured by replacing the crystal with a bolometer that determines the absolute power level. The peak power of the instability radiation in the waveguide was then determined by correcting the power measured at the detecting crystal for the known attenuation of the power sampling system (directional coupler, transmit/receive switch, and associated pads). This resulted in a value of 4.8 W peak of instability radiation. This value is well below estimates previously reported,⁴ but is consistent with other plasma energy measurements.

The total power radiated by the plasma during the instability must also include the losses in the cavity walls. The power measured above is only that fraction which is coupled out of the cavity. In order to determine this quantity, the cavity was excited by a small coupling loop on its side wall. An absolute measurement of the power coupled into the cavity was made with a dual directional coupler and a bolometer. Similarly, the absolute power coupled out of the cavity into the waveguide was measured. From these measurements it was found that for each Watt coupled into the waveguide, 8.63 W were dissipated in the cavity walls. Thus whereas 4.8 W peak was radiated into the waveguide by the instability, 46.2 W peak was radiated by the plasma. Again, as with the field-pattern prediction, these measurements are valid only if the plasma causes a slight change of the TE₂₃₁ cavity mode. If we assume a nominal width of 2 µsec for the instability burst, the total energy radiated from the plasma during the instability amounts to 9.24 × 10⁻⁵ J.

2. Interpretation of Density Curves

Previously, we reported measurement of the total electron density during the afterglow of the discharge.¹ Here we attempt to understand these measurements in terms of a simple model. This model assumes that the plasma electrons may be divided into two groups. The first, the hot-electron component, is taken to obey the rate equation

$$\frac{\mathrm{d}\mathbf{n}_{\mathbf{h}}}{\mathrm{d}\mathbf{t}} = -\boldsymbol{a}_{\mathbf{h}}\mathbf{n}_{\mathbf{h}},$$

where n_h is the hot-electron density, and a_h is its loss rate. These hot electrons are assumed to create the second group, the cold-electron component, by means of the ion-ization of the background gas. This component is also lost at its own characteristic rate. Thus the cold electrons obey the equation

$$\frac{\mathrm{dn}_{\mathrm{c}}}{\mathrm{dt}} = v_{\mathrm{i}}n_{\mathrm{h}} - a_{\mathrm{c}}n_{\mathrm{c}},$$

where n_c is the density, v_i the ionization frequency, and a_c the cold-electron loss rate. The analysis is divided into two time intervals.

The first time interval is between the end of the heating pulse at t = 0 and the occurrence of the instability at t = 175 μ sec. It is assumed that the heating pulse has completely heated all electrons present at t = 0. Thus the initial conditions are that $n_c(o) = 0$ and $n_h(o) = n_T(o) \equiv n_o$. Here, $n_T = n_h + n_c$ is the total electron density. Subject to these conditions, the solution to the equations above is

$$n_h = n_o e^{-a_h t}$$

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$$n_{c} = \frac{v_{i}n_{o}}{a_{c} - a_{h}} e^{-a_{h}t} - e^{-a_{c}t}.$$

The sum of these two terms was fitted to the experimental data by fixing $n_T(o)$, $\frac{dn_T}{dt}$, and n_T (175 µsec). The ionization frequency, v_i , was varied to obtain the best fit. The results of the analysis are shown in Fig. XIV-3. Although the model appears to yield a good fit to the experimental data, two questions arise. First, the predicted hot-electron loss rate is extremely rapid, corresponding to a time constant of 30.9 µsec. This



Fig. XIV-3. Comparison between density models and the measured total density variation before and after the instability. Experimental data indicated by x's and error bars.

is much faster than is to be expected from small-angle scattering of energetic electrons in a mirror magnetic field. But, since the energy density also shows a rapid decay with roughly the same time constant, it is possible that this is an indication of an unstable decay. The second question arises from the extremely long time constant of 662 μ sec predicted for the cold-electron loss. If the electrons are truly cold, their loss from the plasma should be determined by the slowly moving ions, or else extremely large space-charge potentials would result. Thus the loss rate of the cold electrons should occur on a time scale roughly equal to the time required for a room-temperature ion to move from the center of the plasma to the end wall. This requires approximately 46 µsec in our cavity. To overcome this difficulty, it must be proposed that the cold electrons be warm enough to be effective at ionizing the background gas, but not so energetic as to exceed the plasma potential. Thus we take

$$a_{\rm c} = a_{\rm co} - v_{\rm i}$$

where a_{co} is the "intrinsic" loss rate of cold electrons. Taking $v_i = 2 \times 10^4$ as representative of the ionization frequency at a pressure of 2×10^{-5} Torr of hydrogen, we find $\tau_{co} = a_{co}^{-1} = 36.4 \,\mu\text{sec}$. The conditions on the ionization frequency require that the cold-electron temperature be greater than approximately 30 V.

The second interval begins at the end of the instability transient at $t = 235 \mu sec$ and lasts until the plasma is regenerated. Here it is assumed that at $t = 235 \mu sec$ there are nonzero contributions to both the hot- and the cold-electron components. The solutions of the two rate equations now take the form

$$n_{h} = n_{ho} e^{-a_{h}t'}$$
$$a_{c} = c e^{-a_{h}t'} + \frac{\nu_{i}n_{ho}}{a_{c}-a_{h}} e^{-a_{h}t'},$$

where c is a constant to be determined, and t' = t - 235 × 10⁻⁶. In fitting the sum of these two terms to the measured total density it is assumed that $v_i = 2 \times 10^4$ (valid within a factor of 2 from 40 V to 40 keV at a hydrogen pressure of 2 × 10⁻⁵ Torr) and that the measured long-time decay of the density is characteristic of the hot-electron component. This last assumption yields $\tau_h = a_h^{-1} = 34 \times 10^{-3}$ sec. The fitting procedure fixes n_T at 235 µsec and at 950 µsec to the measured values at these times. The derivative of the density at t = 235 µsec was then varied to achieve a good fit over the entire time interval. Again, the results are shown in Fig. XIV-3. Note that following the instability the coldelectron density always exceeds the hot density. As in the first time interval, the predicted cold plasma loss rate is too slow to be consistent with space-charge effects. By assuming that the cold component can also ionize, an argument identical with that above yields an intrinsic cold-electron loss time constant of 41.7 µsec. This value is quite consistent with the time required for room temperature ions to leave the plasma.



Fig. XIV-4. Comparison of discharge characteristic before the instability.

3. Characteristics of the Discharge

Figure XIV-4 presents a comparison of the plasma diamagnetism and visible light with the total electron density before the time of the instability. In these figures the instability occurs at $t \cong 175 \ \mu \text{sec}$, causing approximately a 2- μ sec burst of radiation at 2498 MHz to be emitted by the plasma. Note that both the diamagnetism and light initially fall with a time constant of the order of 60 μ sec. Beyond approximately 50 μ sec, the diamagnetism continues to fall rapidly, while the light decays at a much slower rate. The total density also displays a two-exponential character. While its initial decay is much slower than either the light or the diamagnetism, its subsequent decay is roughly equal to that of the light. It cannot be said that they are equal, since the density measurements were obtained as an average over many plasmas, whereas the light signal is that of a single one.

Since the plasma diamagnetism is proportional to the energy density of the plasma



Fig. XIV-5. Comparison of discharge characteristics after the instability.

 $(n_h \langle T_h \rangle)$, a better comparison would be between the diamagnetism and the hot-electron decay. Although no direct measurement of $n_h(t)$ has been made, the model discussed in the previous section may be applied. There it was predicted that the hot-electron density does indeed decay at a rapid rate (see Fig. XIV-3). The time constant for this decay, however, is a factor of two smaller than the observed decay of the diamagnetism. This may result from the crudeness of the density model or from the averaging technique used to obtain the density curve.

Based on these observations, we believe that the plasma diamagnetism follows the decay of the hot-electron component up to the time of the instability. This is true of the visible light only up to approximately 50 μ sec. Beyond this time, the light follows the evolution of the cold density which subsequently far exceeds the density of the hot component. These conclusions are subject to the results of a direct measurement of the decay of the hot component. Such measurements are now being performed.

At the time of the instability, the diamagnetism is observed to drop and the light displays an increase. Both observations suggest that the instability causes a redistribution of the energy in the plasma. The very energetic electrons must be lost to account for the drop in the energy density. The light increase is probably due to an increase in the number of electrons with energies of the order of a few hundred electron volts where the excitation cross section is largest.

Figure XIV-5 displays the comparison between the long-time light and total density decays if the plasma is not reheated at $t = 1000 \ \mu sec$. No comparison is made with the diamagnetism, since this signal is too small to be extracted from the noise beyond the time of the instability. Note that the light follows the density decay. In terms of the model discussed in the previous section this decay rate is that of the hot-electron component. Based on the model for the decay of hot electrons from a magnetic mirror discussed by Fessenden,⁵ this decay rate corresponds to that of 13 keV electrons.

We are continuing to study this instability.

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References

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- T. J. Fessenden, "Pulsed Electron-Cyclotron Resonance Discharge Experiment," Technical Report 442, Research Laboratory of Electronics, M.I.T., Cambridge, Massachusetts, March 15, 1966, p. 48.

^{5. &}lt;u>Ibid</u>., p. 52.