XV. INTERACTION OF LASER RADIATION WITH PLASMAS AND NONADIABATIC MOTION OF PARTICLES IN MAGNETIC FIELDS*

Academic and Research Staff

Prof. T. H. Dupree
Prof. L. M. Lidsky
Prof. N. L. Oleson

Graduate Students

K. R. S. Chen
H. Ching
J. N. Hamawi
P. Margosian
M. Murakami
A. A. Offenberger

A. STRONG TURBULENCE THEORY FOR A TRANSVERSE ELECTROMAGNETIC WAVE – COMPARISON WITH THE SINGLE-PARTICLE CALCULATION

In previous reports[1,2] we have discussed single charged-particle motion in a constant pitch and constant amplitude magnetic perturbation with relation to "corkscrew" injection and wave-particle interaction in magnetized plasma. We have worked out the motion of untrapped[1] and trapped[2] particles.

Here we apply Dupree's strong turbulence theory[3] to wave-particle interaction in a transverse electromagnetic wave propagating parallel to an external magnetic field. By using a transformation of the velocity coordinate,[4] which is essentially transformation to the wave frame used in studies of the single-particle motion,[2] we reduce the Vlasov equation to a one-dimensional diffusion-type equation. Then manipulation similar to that of the longitudinal wave case by Dupree[3] leads to nonlinear wave-particle interaction including "trapped" particle effects.

We compare the results in the strong field limit, where the trapping effect is important, with the results of the single-particle model. The results agree except for numerical factors. The single-particle calculation gives a proof of the correctness of the strong turbulence theory, which includes some crucial assumptions and provides some clearer physical intuitions.

1. Strong Turbulence Theory

We start with the Vlasov equation:

$$\frac{\partial f}{\partial t} + \underline{v} \cdot \frac{\partial}{\partial \underline{r}} f + \frac{q}{m} (\underline{E} + \underline{v} \times \underline{B}) \cdot \frac{\partial}{\partial \underline{v}} f = 0. \tag{1}$$

We consider a transverse electromagnetic wave propagating parallel to an external

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magnetic field. And for simplicity, we consider a spatially homogeneous plasma in the absence of zero-order electric field. We shall follow the discussion of Dupree's longitudinal wave case.

To solve Eq. 1, we expand the field in Fourier series

$$\left.\begin{aligned} \underline{E}(\underline{r},t) &= \sum_k \underline{E}_k(t)\, e^{i\underline{k}\cdot\underline{r}} \\ \underline{B}(\underline{r},t) &= B_o\hat{\underline{z}} + \sum_k \underline{B}_k(t)\, e^{i\underline{k}\cdot\underline{r}} \end{aligned}\right\} \tag{2}$$

Introducing (2) into Eq. 1, we obtain

$$\left(\frac{\partial}{\partial t} + \underline{v}\cdot\frac{\partial}{\partial \underline{r}} + \frac{q}{m}\underline{v}\times\underline{B}_o\cdot\frac{\partial}{\partial \underline{v}}\right)f + \frac{q}{m}\sum_k (\underline{E}_k + \underline{v}\times\underline{B}_k)\cdot\frac{\partial}{\partial \underline{v}} f = 0. \tag{3}$$

We divide the distribution function f into two parts:

$$f(\underline{r},\underline{v},t) = \left\langle f(v_{\|}, v_{\perp}, t)\right\rangle + \sum_k f_k(\underline{v},t)\, e^{i\underline{k}\cdot\underline{r}}, \tag{4}$$

where $\left\langle f(v_{\|}, v_{\perp}, t)\right\rangle$ is the distribution function averaged over a time much longer than a Larmor period so that it is independent of the gyration angle θ, which is shown in Fig. XV-1 with the definitions of $v_{\|}$, $v_{\perp}$, $\underline{B}_o$, and $\underline{k}$. In the time averaging we assume

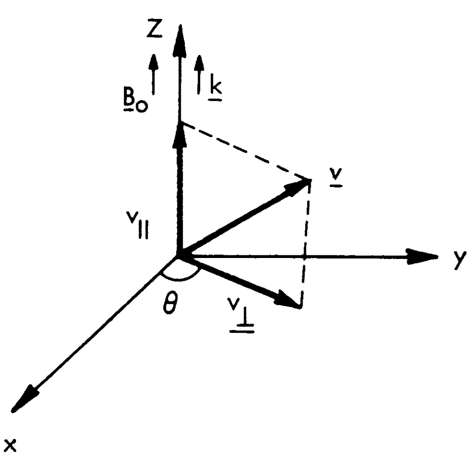

Fig. XV-1. Velocity coordinate system.

that fluctuation phases are uncorrelated; then we can use ensemble averaging for the correlation terms instead of time averaging. Thus

$$\left(\frac{\partial}{\partial t} + \underline{v} \cdot \frac{\partial}{\partial \underline{r}}\right) \left\langle f(v_{\|}, v_{\perp}, t)\right\rangle + \frac{q}{m} \sum_k \frac{\partial}{\partial \underline{v}} \cdot \left\langle (\underline{E}_k + \underline{v} \times \underline{B}_k) f_{-k}\right\rangle = 0, \tag{5}$$

where the $\frac{q}{m} \underline{v} \times \underline{B}_o \cdot \frac{\partial}{\partial \underline{v}}$ term drops out, because of the θ independence of $<f>$.

The fluctuation distribution function f_k is given correctly to first order in B_k (or E_k) by [The proof is parallel to the electrostatic case by Dupree[3].]

$$f_k(t)\, e^{i\underline{k} \cdot \underline{r}} = -\frac{q}{m} \int_{t_o}^{t} d\tau\, (\underline{E}_k(\tau) + \underline{v}(\tau) \times \underline{B}_k(\tau))\, U(t, \tau)$$

$$\cdot\, e^{i\underline{k} \cdot \underline{r}} \cdot \frac{\partial}{\partial \underline{v}} \left\langle U(\tau, t_o)\right\rangle f(t_o) + \left\langle U(t, t_o)\right\rangle f_k(t=0)\, e^{i\underline{k} \cdot \underline{r}}, \tag{6}$$

where the propagation operator $U(t, t_o)$ is the solution of

$$\left(\frac{\partial}{\partial t} + \underline{v} \cdot \frac{\partial}{\partial \underline{r}} + \frac{q}{m} \underline{v} \times \underline{B}_o \cdot \frac{\partial}{\partial v}\right) U + \frac{q}{m} \sum_k (\underline{E}_k + \underline{v} \times \underline{B}_k) \cdot \frac{\partial U}{\partial \underline{v}} = 0 \tag{7}$$

and the angular brackets indicate an ensemble average over wave phases.

Substituting Eq. 6 in Eq. 5, we obtain

$$\left(\frac{\partial}{\partial t} + \underline{v} \cdot \frac{\partial}{\partial \underline{r}}\right) \left\langle f(v_{\|}, v_{\perp}, t)\right\rangle = \frac{\partial}{\partial \underline{v}} \cdot \frac{q^2}{m^2} \sum_k \int_{t_o}^{t} d\tau\, (\underline{E}_k(t) + \underline{v}(t) \times \underline{B}_k(t))$$

$$\cdot\, (\underline{E}_{-k}(\tau) + \underline{v}(\tau) \times \underline{B}_{-k})\, e^{i\underline{k} \cdot r(t)} < U(t, \tau) > e^{i\underline{k} \cdot \underline{r}(\tau)} \cdot \frac{\partial}{\partial \underline{v}} < f(\tau) >, \tag{8}$$

where we have neglected the initial value term, since we are not interested in it.

The integrand of Eq. 8 is nonvanishing in only a small interval around $t = \tau$, and if the time dependence of it is sufficiently slow during this time interval, then we can replace $\frac{\partial}{\partial \underline{v}} < f(\tau) >$ with $\frac{\partial}{\partial \underline{v}} < f(t) >$. Thus, (7) is reduced to a diffusion equation:

$$\left(\frac{\partial}{\partial t} + \underline{v} \cdot \frac{\partial}{\partial \underline{r}} - \frac{\partial}{\partial \underline{v}} \cdot \underline{\underline{D}}(\underline{v}) \cdot \frac{\partial}{\partial \underline{v}}\right) \left\langle f(v_{\|}, v_{\perp}, t)\right\rangle = 0, \tag{9}$$

where

$$\underline{D}(v) = \frac{q^2}{m^2} \sum_k \int_{t_o}^{t} d\tau\, (\underline{E}_k(t) + \underline{v}(t) \times \underline{B}_k(t))\, (\underline{E}_{-k}(\tau) + \underline{v}(\tau) \times \underline{B}_{-k}(\tau))$$

$$\cdot\, e^{i\underline{k} \cdot \underline{r}(t)} < U(t, \tau) > e^{-i\underline{k} \cdot \underline{r}(\tau)}. \tag{10}$$

If the acceleration forces represent stationary time series, then the orbit does too; that is, $<U(t,\tau)> = <U(t-\tau)>$, and therefore

$$\underline{\underline{D}}(\underline{v}) = \frac{q^2}{m^2} \sum_k \int_0^{t-t_o} d\tau \, (\underline{E}_k(t) + \underline{v}(t) \times \underline{B}_k(t)) \, (\underline{E}_{-k}(t-\tau) + \underline{v}(t-\tau) \times \underline{B}_{-k}(t-\tau))$$

$$\cdot \, e^{i\underline{k} \cdot \underline{r}(t)} <U(\tau)> e^{-i\underline{k} \cdot \underline{r}(t-\tau)}. \tag{11}$$

Continuing to follow Dupree, we observe that

$$<U(\tau)> e^{-i\underline{k} \cdot \underline{r}(t-\tau)} \equiv \left\langle U(\tau) \, e^{-i\underline{k} \cdot \underline{r}(t-\tau)} \right\rangle = \left\langle e^{-i\underline{k} \cdot r_c(-\tau)} \right\rangle, \tag{12}$$

where $r_c(-\tau)$ is a trajectory at time $-\tau$ with initial values $\underline{r}(t)$ and $\underline{v}(t)$. For sufficiently large $t - t_o$ (compared with the nonlinear correlation time which will be defined in Eq. 25), we can replace the upper limit of the integral in Eq. 11 by ∞. Then

$$\underline{\underline{D}}(\underline{v}) = \frac{q^2}{m^2} \sum_k \int_0^{\infty} d\tau \, (\underline{E}_k(t) + \underline{v}(t) \times \underline{B}_k(t)) \, (\underline{E}_{-k}(t-\tau) + \underline{v}(t-\tau) \times \underline{B}_{-k}(t-\tau))$$

$$\cdot \, e^{i\underline{k} \cdot \underline{r}(t)} \left\langle e^{-i\underline{k} \cdot \underline{r}_c(-\tau)} \right\rangle. \tag{13}$$

The quantity $\left\langle e^{-i\underline{k} \cdot r_c(-t)} \right\rangle$ is a function only of the orbit; therefore, it must satisfy the same equation as $<f>$,

$$\left(\frac{\partial}{\partial t} + \underline{v} \cdot \frac{\partial}{\partial \underline{r}} - \frac{\partial}{\partial \underline{v}} \cdot \underline{\underline{D}} \cdot \frac{\partial}{\partial \underline{v}}\right) \left\langle e^{-i\underline{k} \cdot r_c(-\tau)} \right\rangle = 0. \tag{14}$$

We must solve Eqs. 13 and 14 simultaneously. To facilitate this solution, we use the following coordinate transform,[4]

$$u = v_\perp^2 + v_\parallel^2 - 2 \int^{v_\parallel} \frac{\omega_k(v_\parallel')}{k} \, dv_\parallel' \tag{15}$$

$$v = v_\parallel$$

This transform is a transform in which one of the coordinates is along the diffusion path of the particles (see sec. 2). $\underline{E}_k$ and $\underline{B}_k$ are related through the Maxwell equations. For a transverse wave propagating parallel to the external magnetic field, the fluctuating electric field is

$$\underline{E}_k = -\frac{\omega_k}{k^2} \, \underline{k} \times \underline{B}_k. \tag{16}$$

Therefore, noting again that $\frac{\partial}{\partial \underline{v}} \left\langle f(v_\parallel, v_\perp) \right\rangle$ does not include a $\hat{\theta}$ component, we obtain

$$(\underline{E}_{-k}+\underline{v}\times\underline{B}_{-k})\cdot\frac{\partial}{\partial\underline{v}}=(\hat{\underline{\theta}}\cdot\underline{B}_{-k})\left\{v_{\perp}\frac{\partial}{\partial v_{\parallel}}-\left(v_{\parallel}-\frac{\omega_k}{k}\right)\frac{\partial}{\partial v_{\parallel}}\right\}$$

$$=(\hat{\theta}\cdot\underline{B}_{-k})\,v_{\perp}\frac{\partial}{\partial v} \tag{17}$$

$$\frac{\partial}{\partial\underline{v}}\cdot\underline{\underline{D}}(\underline{v})\cdot\frac{\partial}{\partial\underline{v}}=\frac{q^2}{m^2}\sum_k\left\{\frac{\partial}{\partial\underline{v}}\cdot(\underline{E}_k(t)+\underline{v}(t)\times\underline{B}_k(t))\,e^{ikz(t)}\left\langle e^{-ikz_c(-\tau)}\right\rangle\right.$$

$$\left.\cdot\,(\hat{\theta}(t-\tau)\cdot\underline{B}_{-k}(t-\tau))\,v_{\perp}\frac{\partial}{\partial v}\right\}$$

$$=\frac{q^2}{m^2}\sum_k\frac{\partial}{\partial v}\int_0^\infty(\hat{\underline{\theta}}(t)\cdot\underline{B}_k(t))\,(\hat{\underline{\theta}}(t-\tau)\cdot\underline{B}_{-k}(t-\tau))\,v_{\perp}(t)\,v_{\perp}(t-\tau)$$

$$\cdot\,e^{ikz(t)}\left\langle e^{-ikz_c(-\tau)}\right\rangle\frac{\partial}{\partial v}$$

$$=\frac{\partial}{\partial v}D(v,u)\frac{\partial}{\partial v}, \tag{18}$$

where

$$D(v,u)=\frac{q^2}{m^2}\sum_k\int_0^\infty d\tau\,(\hat{\underline{\theta}}(t)\cdot\underline{B}_k(t))\,(\hat{\underline{\theta}}(t-\tau)\cdot\underline{B}_{-k}(t-\tau))\,v_{\perp}(t)\,v_{\perp}(t-\tau)$$

$$\cdot\,e^{ikz(t)}\left\langle e^{-ikz_c(-\tau)}\right\rangle. \tag{19}$$

Therefore Eq. 13 is converted to the following one-dimensional equation

$$\left(\frac{\partial}{\partial t}+v\frac{\partial}{\partial z}-\frac{\partial}{\partial v}D(v,u)\frac{\partial}{\partial v}\right)\left\langle e^{-ikz_c(-t)}\right\rangle=0. \tag{20}$$

In general, the diffusion coefficient is a function of v and u. If the dependence on v is retained, Eq. 20 is very difficult to solve. If we ignore the v dependence of D, however, the solution is simple

$$\left\langle e^{-ikz_c(-t)}\right\rangle=e^{-ikz+ikvt-\frac{1}{3}k^2D(u)t^3}. \tag{21}$$

We shall assume that this result is also approximately true for the diffusion coefficient $D(v,u)$. This is one of the crucial approximations in this theory. The agreement of the results in this theory with those of the single-particle model (which will be described later) implies that the assumption does not hide any important gross features of the theory.

To evaluate the diffusion coefficient, Eq. 19, we substitute the unperturbed orbit for $v_\perp(t)$, namely

$$v_\perp(t-\tau)\,\hat{\underline{\theta}}(t-\tau) = v_\perp(t)\,\hat{\underline{\theta}}(t)\, e^{i\omega_c\tau} \tag{22}$$

(which will be correct to "first order"), where ω_c is the cyclotron frequency including the sign of the charge; that is,

$$\omega_c = \frac{q}{m} B_o. \tag{23}$$

Also, the complex conjugate of the magnetic field is

$$\underline{B}_{-k}(t-\tau) = \underline{B}_{-k}(t)\, e^{i\omega_{-k}\tau} = \underline{B}_{-k}(t)\, e^{-i\omega_k\tau} \tag{24}$$

Substituting these in Eq. 19, we obtain an integral equation for $D(v,u)$,

$$D(v,u) = \frac{q^2}{m^2} \sum_k |\underline{B}_k|^2\, v_\perp^2(v,u) \int_0^\infty d\tau\, e^{i(kv-\omega_k+\omega_c)\tau - \frac{1}{3}k^2\tau^3 D(v,u)} \tag{25}$$

Noticing that $\frac{q}{m} B_k v_\perp$ is an acceleration that produces the random process, we define nonlinear correlation time[5] $\tau_{C,NL}$

$$\tau_{C,NL} = \frac{\sum_k |\underline{B}_k|^2 \int_0^\infty d\tau\, e^{i(kv-\omega_k+\omega_c)\tau - \frac{1}{3}k^2\tau^3 D(v,u)}}{\sum_k |\underline{B}_k|^2} \tag{26}$$

If we neglect the diffusion coefficient in the integrand, $\tau_{C,NL}$ reduces to the normal autocorrelation time τ_{AC}

$$\tau_{AC} = \frac{\sum_k |\underline{B}_k|^2 \int_0^\infty d\tau\, e^{i(kv-\omega_k+\omega_c)\tau}}{\sum_k |\underline{B}_k|^2}. \tag{27}$$

The autocorrelation time is approximately evaluated from the difference in phase velocity of the wave packet $\left|\frac{\omega_1}{k_1} - \frac{\omega_2}{k_2}\right|$,

$$\tau_{AC} \cong \left\{ k_o \left| \frac{\omega_1}{k_1} - \frac{\omega_2}{k_2} \right| \right\}^{-1}, \tag{28}$$

where k_o is the average wave number of the spectrum.

As shown by Dupree,[3] the resonant function

$$R[kv-\omega_k+\omega_c, D(v,u)] \equiv \text{Real} \int_0^\infty d\tau \, e^{i(kv-\omega_k+\omega_c)\tau - \frac{1}{3}k^2\tau^3 D(v,u)} \tag{29}$$

for a real frequency ω_k has a maximum at $v = (\omega_k-\omega_c)/k$, at which $R = (1/3)!\,(1/3k^2D)^{-1/3}$, and R goes to zero for $|kv-\omega_k+\omega_c| \gtrsim (1/3k^2D)^{1/3}$, and the area under R is π. For simplicity, we replace this bell shape by a square shape with the same area and approximately the same height and width. For the complex frequency, $\omega_k + i\gamma_k$, the nonresonant interaction attributable to the small imaginary part, γ_k, is ineffective for the resonant region, while it is only the remaining contribution in the nonresonant region. Therefore R is approximated by the following function:

$$R[kv-\omega_k+\omega_c, D(v,u)] = \begin{cases} \dfrac{\pi}{2kw} & |kv-\omega_k+\omega_c| < kw \\[2ex] \dfrac{\gamma_k}{(kv-\omega_k+\omega_c)^2} & |kv-\omega_k+\omega_c| > kw \end{cases} \tag{30}$$

where $w = (D/3k)^{1/3}$ is k^{-1} times the half-width of the resonance function $R[kv-\omega_k+\omega_c, D(v,u)]$. The resonance function for the average wave number k_o is one important time scale, the trapping time, τ_{TR}:

$$\tau_{TR} = \frac{\pi}{2} \frac{1}{k_o w} = \frac{\pi}{2} \left(\frac{3}{k_o^2 D} \right)^{1/3}. \tag{31}$$

Now we consider the two limiting cases.

1. $\tau_{TR} \gg \tau_{AC}$. This case corresponds to the case in which the spectrum of the waves is so broad and/or the wave amplitude is so small that the trapping effect is not important. Because of the resonance width $w \ll \left| \frac{\omega_1}{k_1} - \frac{\omega_2}{k_2} \right|$, the resonant part of the resonance function becomes delta function. Therefore, the diffusion coefficient in this is given by

$$D(v,u) = \frac{q^2}{m^2} \sum_k |\hat{\underline{\theta}} \cdot \underline{B}_k|^2 v_\perp^2(v,u)\, \delta(kv-\omega_k+\omega_c) + \frac{q^2}{m^2} \sum_k |\hat{\underline{\theta}} \cdot \underline{B}_k|^2 v_\perp^2(v,u) \frac{\gamma_k}{(kv-\omega_k+\omega_c)^2}. \tag{32}$$

This is the diffusion coefficient predicted by the conventional quasi-linear theory. The first term represents the resonant interaction of wave and particle, and the second represents the additional "long-range" term, which describes the nonresonant or adiabatic interaction of wave and particle.

2. $\tau_{TR} \ll \tau_{AC}$. This case corresponds to the case in which the spectrum of the wave is so sharp around the resonance condition and/or the wave amplitude is so large that trapping phenomena are important. The nonlinear correlation time $\tau_{C,NL}$ is equal to the trapping time τ_{TR}.

Using the simple resonance function with Eq. 25, we obtain an approximate diffusion coefficient for a finite bandwidth of the spectrum $\left|\frac{\omega_1}{k_1} - \frac{\omega_2}{k_2}\right|$:

$$D(v,u) = \frac{q^2}{m^2} \sum_k |\hat{\underline{\theta}} \cdot \underline{B}_k|^2 v_\perp^2(v,u) \frac{\pi}{2k} \left(\frac{3k}{D(v,u)}\right)^{1/3}$$

$$= \left[\frac{\sqrt[3]{3}}{2} \pi \frac{q^2}{m^2} \sum_k |\hat{\underline{\theta}} \cdot \underline{B}_k|^2 v_\perp^2(v,u) k^{-2/3}\right]^{3/4} \quad \text{for } \frac{\omega_1 - \omega_c}{k_1} - w \lesssim v \lesssim \frac{\omega_2 - \omega_c}{k_2} + w \tag{33}$$

$$D(v,u) = \frac{q^2}{m^2} \sum_k |\hat{\underline{\theta}} \cdot \underline{B}_k|^2 v_\perp^2(v,u) \frac{\gamma_k}{(kv - \omega_k + \omega_c)^2} \quad \text{otherwise.} \tag{34}$$

Note the 3/2 power dependence of the resonant diffusion coefficient on the amplitude of the wave, and also that the nonresonant diffusion coefficient has the same form as in the $\tau_{TR} \gg \tau_{AC}$ case. We shall now compare these results with those of the single-particle model.

2. Comparisons of the Strong Turbulence Theory with the Single-Particle Model

We have already solved the problem of determining particle orbits in resonant and nonresonant constant-pitch, constant-amplitude, magnetic perturbation.[1,2] Here we compare the results of the single-particle model (SPM) with the strong turbulence theory (STT) derived above. We shall show that the two solutions are identical within the accuracy of the assumptions made in their derivations. Thus, in the future, we shall be able to use the strong physical insight into the dynamics of the interaction gained in the SPM to elucidate and justify results gained by use of the mathematically more powerful STT.

We first note some differences between them. In the STT, an infinitely large number of incoherent waves allows us to ensemble-average with respect to wave phases, while in the SPM we discuss the particle motion in a single coherent wave. In the very last stage of the SPM, however, we disregard the fine structure of the particle orbit to obtain the diffusion coefficients. In effect, we study diffusion, resonant or nonresonant, resulting from the sequential interaction of the particle with many uncorrelated waves. Thus, in the SPM we perform a time average, and in the STT an ensemble average. As expected, the two models give the same results if the corresponding correlation times are properly defined.

In the STT, we have used the transform, Eq. 15. Here we discuss the physical meaning of this coordinate transform. For simplicity, we consider a case of single wave ω and k. Then, (15) can be reduced to

$$u = v_{\parallel}^2 + v_{\perp}^2 - 2v_{\parallel} \frac{\omega}{k} = \left(v_{\parallel} - \frac{\omega}{k}\right)^2 + v_{\perp}^2 - \left(\frac{\omega}{k}\right)^2. \tag{35}$$

Since the diffusion equation (18) does not include derivatives with respect to u, particles diffuse on the constant u-surface, as described above. We shall explain this fact on the basis of energy and momentum conservation in the wave-particle system. If we denote wave total energy (including wave electromagnetic energy and nonresonant particle kinetic energy) by I_k, it can be shown[6] that the total wave momentum density is kI_k/ω. (Notice the analogy with quantum mechanics.) Therefore the conservations of the energy density and momentum density parallel to the propagation (and $\underline{B}_o$) in the wave-particle system are

$$\frac{d}{dt}\left[\frac{n}{2} m\left(v_{\parallel}^2+v_{\perp}^2\right)\right] + \frac{d}{dt} I_k = 0 \tag{36}$$

$$\frac{d}{dt}\left(nmv_{\parallel}\right) + \frac{d}{dt}\left(\frac{k}{\omega} I_k\right) = 0, \tag{37}$$

where n is particle density. (In passing, the perpendicular momentum is automatically conserved in Larmor motion.) These equations lead us to the following relationship:

$$\left(v_{\parallel} - \frac{\omega}{k}\right) dv_{\parallel} + v_{\perp} dv_{\perp} = 0 \tag{38}$$

or

$$\left(v_{\parallel} - \frac{\omega}{k}\right)^2 + v_{\perp}^2 = \text{constant}. \tag{39}$$

This shows that particles diffuse on a constant u-surface upon which the energy and momentum densities of the wave-particle system are conserved.

Now we return to comparison of the results of both theories. Let us first consider the resonant interaction. In the STT, $\tau_{TR} \ll \tau_{AC}$, the diffusion of the particle broadens the δ-function resonance interaction. The SPM shows more clearly what is going on with each particle. In the electrostatic (longitudinal) wave O'Neil[7] shows that the linear theory breaks down when the resonant particle becomes trapped inside the trough of the wave electrostatic potential. A very similar situation occurs in the transverse-wave case,[1] namely, the particle becomes trapped inside the trough of some "pseudo potential." Once the particle becomes trapped, the simple perturbation theory fails. Also, since this situation is more complicated than the electrostatic case, the exact orbit calculation,[8] does not help in understanding the gross structure. Instead we introduce a singular perturbation technique that treats the trapping phenomena (including reflection points)[2] properly. The important consequences are the following: In the expanded scale with a scaling factor $1/\sqrt{\beta}$ (β is the ratio of the wave magnetic field amplitude to the external magnetic field amplitude), there exist two kinds of "pseudo potentials," which are always symmetric with respect to $(\omega_k-\omega_c)/k$ (see Fig. XV-2).

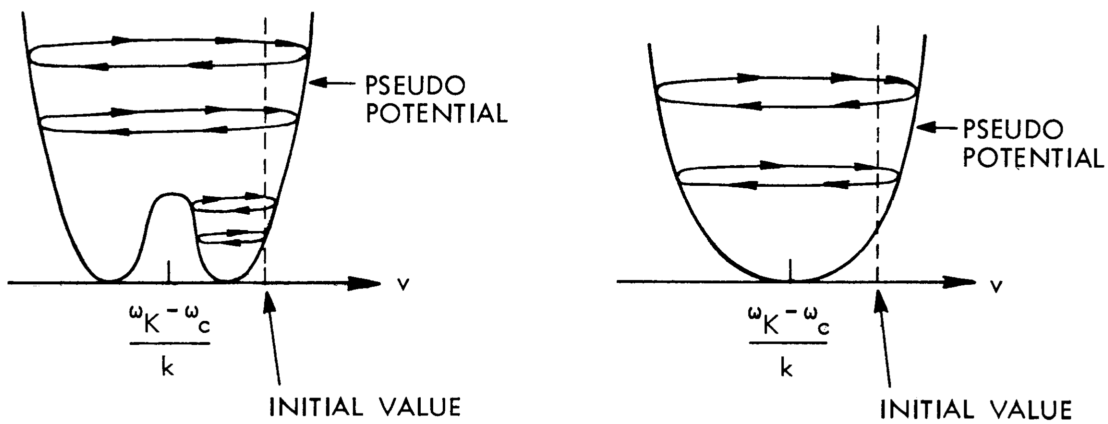

Fig. XV-2. Two kinds of "pseudo potential" inside which the particle is trapped.

The initial condition determines the detailed structures (forbidden region) of the particle motion. But we need use only the gross features to compare with the STT which assumes random wave phases. As shown in our last report,[2] the trapping time, which is evaluated from the spatial oscillation period, is given by

$$(\tau_{TR})_{SPM} = c_1\sqrt{\frac{m}{q}\frac{1}{kv_\perp B_k}}, \tag{40}$$

where c_1 is a numerical factor of the order of unity. In the STT, the nonlinear correlation time $T_{C,NL}$ is evaluated as

$$\tau_{C,NL} = \frac{\pi}{2}\left(\frac{1}{3}k^2D\right)^{-1/3} = \frac{\pi}{2}\sqrt{\frac{2\sqrt[3]{3}}{\pi}\frac{m}{q}}\sqrt{\frac{1}{kv_\perp B_k}}; \tag{41}$$

therefore, this correlation time can be properly interpreted as the trapping time.

From the symmetrical point of the "pseudo potentials," we observe that the average velocity of the trapped particle is $(\omega_k-\omega_c)/k$, which is the phase velocity seen by a gyrating particle.

The diffusion coefficient is estimated by the width of the phase plane trajectories and the trapping time in the SPM:

$$(D_{TR})_{SPM} = c_2 k \left(\frac{qv_\perp B_k}{mk}\right)^{3/2}, \tag{42}$$

where c_2 is a numerical factor of the order of unity. In the STT, Eq. 33 shows

$$(D_{TR})_{STT} = \left(\frac{\sqrt[3]{3}\,\pi}{2}\right)^{3/4} k \left(\frac{qv_\perp B_k}{mk}\right)^{3/2}, \tag{43}$$

which agrees with (42), except for a numerical factor.

In the STT, we define the resonance width w, which is the maximum difference between particle velocity and wave-phase velocity for which a particle can still be diffused by the wave and the value is

$$(w)_{STT} = \left(\frac{D}{3k}\right)^{1/3} = \left(\frac{\sqrt[3]{3}\,\pi}{2}\right)^{1/4} \frac{1}{\sqrt[3]{3}} \left(\frac{qv_\perp B_k}{mk}\right)^{1/2}. \tag{44}$$

On the other hand, in the SPM, the resonance width is decided by the boundary between trapping and nontrapping, which gives

$$(w)_{SPM} = \frac{qv_\perp B_k}{mk}^{1/2}. \tag{45}$$

Again both are in agreement.

A particle whose velocity is beyond the resonance width around $(\omega_k-\omega_c)/k$ interacts nonresonantly (or adiabatically) with the waves. It oscillates in the electromagnetic field of the wave with an amplitude proportional to the field strength. The diffusion coefficient of the untrapped particles is given both in the SPM and STT in the form

$$(D_{UTR})_{STT} = (D_{UTR})_{SPM} = \frac{q^2}{m^2} B_k^2 v_\perp^2 \frac{\gamma_k}{(kv-\omega_k+\omega_c)^2}. \tag{46}$$

We may conclude that the results of the strong turbulence theory agree with those of the single-particle model. The strong turbulence theory includes some crucial assumptions, but the agreement implies that none of them hides physically important gross features. Therefore they give a more rigid basis to the strong turbulence theory. Furthermore, the single-particle model provides clearer insight to what is going on physically.

M. Murakami, L. M. Lidsky

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