XXII. PROCESSING AND TRANSMISSION OF INFORMATION*

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RESEARCH OBJECTIVES AND SUMMARY OF RESEARCH

1. Optical Communication

The fundamental limitations and efficient utilization of optical communication channels are the concern of these investigations. Our interests include the turbulent atmospheric channel, quantum channels, the cloud channel, and over-the-horizon scatter channels. Our activities now focus on deepening and broadening the understanding of quantum communication theory and on practical, or at least feasible, methods of exploiting the attributes of optical channels.

The atmospheric optical channel continues to be a major subject of investigation. Our earlier results,[1] which were based upon an oversimplified model and ignored basic quantum issues, have been extended and refined in a doctoral thesis.[2] A principal result of this work is that, subject to some reasonable assumptions, the optimum (quantum) receiver for orthogonal signals should employ energy detection. Substantial progress has also been made in obtaining useful analytical bounds to some functions that are fundamental to the performance of atmospheric optical systems.[3] Furthermore, the processing and performance of array receivers that employ direct detection has been studied for the atmospheric channel.[4]

We have begun to explore the possibility of improving communication performance by utilizing knowledge of the instantaneous state of the atmosphere. Four doctoral level investigations of this possibility are now in progress and should be completed during the coming year. The first of these investigations seeks to determine the performance and structure of receivers which track, or adapt to, the instantaneous channel state. Broadly stated, the objective of such systems is to process the spatial diversity in the received aperture coherently rather than incoherently. Two other investigations are concerned with transmitter, rather than receiver, adaptation. One of these investigations is concerned with variable rate, or burstlike, communication schemes which use the channel only when it is good. This is an appealing idea, because of the large variability in the gain of the atmospheric channel. The other investigation is addressed to the possibility of predistorting the transmitted wavefront to compensate for, and hence prevent, the beam spreading caused by the atmosphere.[5] Such a technique yields much larger transmitting

*This work was supported principally by the National Aeronautics and Space Administration (Grant NGL 22-009-013), and in part by the Joint Services Electronics Program (U.S. Army, U.S. Navy, and U.S. Air Force) under Contract DA 28-043-AMC-02536(E), and the U.S. Army Research Office – Durham (Contract DAHCO4-69-C-0042).

antenna gains than can now be realized. The fourth investigation will explore the feasibility of implementing wavefront compensating systems.[6] It is encouraging that all of these schemes involve only limited knowledge of the channel state – knowledge which one may realistically hope to obtain.

An investigation into the structure and performance of analog receivers that employ direct detection is starting. This investigation will be concerned with both the free-space and atmospheric channels and will emphasize receiver structures that appear to be relatively simple to implement.

Two doctoral level investigations of quantum communication theory are nearing completion. The objective of one of them is to determine the structure and performance of the optimum quantum receiver for analog communication through the free-space and atmospheric channels.[7,8] It is based upon the common assumption that the "channel" is specified in terms of the density operators at the receiving aperture. Preliminary results suggest that optimum AM and FM receivers and their performance can be determined. The second investigation is concerned with the transition from a classical channel model to a quantum model and with the interactions between the channel and idealized measurements. This involves some new approaches to quantum communication and may circumvent some of the difficulties encountered in previous investigations.[9]

R. S. Kennedy, E. V. Hoversten

2. Coding for Noisy Channels and for Sources

The investigation of convolutional codes and decoding algorithms for convolutional codes (in particular, sequential decoding and the Viterbi algorithm) will be continued in the coming year. A doctoral research project by John L. Ramsey is under way investigating the potentialities of a concatenated coding scheme in which there are inner and outer levels of convolutional coding separated by scrambling.[10,11] Such techniques have practical promise and appear to have some advantages over straight sequential decoding in terms of transmission rate and ability to cope with burst noise. Research will also continue on a number of theoretical issues concerning convolutional coding and sequential decoding.

Research in the processing of information sources subject to a fidelity criterion will also be continued in the coming year. There is a well-developed mathematical theory (see, for example, Gallager[12]) concerned with determining the minimum number of binary digits required to represent the output of a given source within a given fidelity criterion. There are also numerous practical problems involving the conversion of raw data to a binary sequence that must be transmitted and then processed. There is, unfortunately, a very wide gap between the theory and the practical problems. Part of this gap is due to the difficulty in finding reasonable stochastic models for the raw data and measures for the distortion. Another part of the gap is due to the failure of the theory to suggest practical processing techniques. One of our research goals is to narrow this gap by looking for reasonable classes of processing techniques. Another goal, under way as a doctoral research project by David L. Cohn, is to find the minimum mean-square distortion that can be achieved when a Gaussian random variable is processed and transmitted over a white Gaussian noise channel subject to an energy constraint. This problem is closely related to the analog modulation problem of transmitting a bandlimited source over a wideband additive Gaussian noise channel and, if successful, it will provide ultimate performance limitations on a broad class of modulation techniques.

R. G. Gallager

3. Simple Encoding Techniques for Analog Signals

Work on the behavior of optimum quantizers in the one-dimensional case (quantizing one signal sample at a time) has been accepted for publication.[13] The multidimensional quantization problem has been explored, and the principal one-dimensional result has been extended to arbitrary n-dimensional probability distributions, applicable to the grouping of statistical data, as well as to communications problems.

A doctoral candidate has started to explore the use of feedback in transmitting analog signals in the presence of additive non-white Gaussian noise.

P. Elias

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A. THEORY OF QUANTUM SIGNAL DETECTION

We shall discuss briefly some results in the theory of quantum signal detection.[1,2] A detailed treatment will be given elsewhere.

1. Formulations

Suppose we have an M-ary equiprobable message alphabet $\{j=0, \ldots\ldots, M-1\}$ with the corresponding channel output for message j described by the density operator ρ_j. The most general and meaningful formulation of our problem follows.

Given a set of positive semidefinite self-adjoint linear operators of unit trace $\{\rho_j\}$ on a separable Hilbert space $\mathcal{H}$, find the maximum of

$$\sum_j \operatorname{tr} \pi_j \rho_j, \tag{1}$$

where each π_j is a self-adjoint bounded linear operator on $\mathcal{H}$ and has to obey the constraint of being simultaneously diagonal in a complete or overcomplete set of vectors in $\mathcal{H}$. That is,

$$\begin{aligned} \pi_j &= \int \pi_j(x) |x\rangle \langle x| \, dx \\ &\int |x\rangle \langle x| \, dx = 1 \end{aligned} \tag{2}$$

with

$$\sum_j \pi_j = 1 \qquad \pi_j(x) \geq 0. \tag{3}$$

This problem, which we call I, is unfortunately rather untractable because the constraint that the π_j be simultaneously expressible in diagonal form in the same representation is hard to handle. It makes the domain of optimization nonconvex and it cannot be expressed as an explicit equality constraint. We are then led to consider some variants of the problem.

A more general problem, which we call II, is the following. Given the set of density operators $\{\rho_j\}$ maximize (1) by choosing the set of self-adjoint positive semidefinite bounded operators $\{\pi_j\}$ subject to (3). This problem differs from the first one, in that the π_j are not guaranteed to be simultaneously expressible in diagonal form in the same set of vectors. Even if they are simultaneously diagonal in an overcomplete representation the $\pi_j(x)$ of (2) are not necessarily positive for all x. This formulation has the advantage, however, of allowing exact solutions. If one finds all of the solutions to problem II he might find one that satisfies the constraints of problem I.

There is a third possibility of interest. In this formulation we restrict our consideration to measurements of complete orthonormal sets only. We can then use to advantage the proposition that we can consider only nonrandom strategies for optimal error performance. It follows that in such cases

$$\pi_i\pi_j = 0 \qquad i \neq j \tag{4}$$

which is a set of additional constraints on our original problem. They imply that the π_j become commuting projection operators so that there always exists an orthonormal set of simultaneous eigenvectors for them. Thus the problem, which we call III, is to maximize (1) by choosing positive semidefinite self-adjoint operators subject to conditions (3) and (4).

2. Results

The basic methods that we use in the solutions of problems II and III are the general duality theorem and Lagrange multiplier theorems,[3] together with operator derivative techniques.[4] We shall not give a discussion of these methods here, and will also limit ourselves to a statement of the results.

Problem II is completely solved by the following theorem in principle, apart from uniqueness which probably does not hold for an arbitrary set of $\{\rho_j\}$.

<u>Theorem 1</u>

There exists a set of $\{\pi_j\}$ which solves problem II. The necessary and sufficient conditions for this set π_j are, in addition to the constraints, that for every j

$$(\lambda-\rho_j)\pi_j = 0 \tag{5}$$

for some self-adjoint bounded linear operator λ over $\mathcal{H}$ such that for each ρ_j, $\lambda - \rho_j$ is positive semidefinite.

For problem III we can state a necessary condition and a separate set of sufficient conditions. Existence has not yet been established.

Theorem 2

A necessary condition for the solution set $\{\pi_j\}$ of problem III is, apart from constraints,

$$\sum_j \pi_j\rho_j = \sum_j \rho_j\pi_j.$$

Theorem 3

A sufficient set of conditions for $\{\pi_j\}$ to solve problem III is, apart from constraints,

$$\sum_j \pi_j\rho_j = \sum_j \rho_j\pi_j$$

and that for every j,

$$\sum_i \pi_i\rho_i - \rho_j$$

is positive semidefinite.

There are actually some more restrictive necessary conditions than Theorem 2. We do not list them here because, at present, they are in a more complicated form, and we think that in a final analysis the sufficient conditions of Theorem 3 are also necessary. It is then important and interesting to establish existence for problem III explicitly. Note that the known solution of problem I in the special case, where

$$[\rho_i-\rho_j, \rho_k-\rho_l] = 0$$

for every (i, j, k, l), falls under Theorem 3.

3. Conclusions

It should be possible by the same kinds of techniques to generalize considerably our results summarized above to yield a more complete solution for the original problem I. We have already found the appropriate methods and language to treat quantum detection theory, and have at least some handle on the problem. The solution of problem II provides an upper bound on problem I, and there is in fact a good chance that we can solve the original problem once we have all of the solutions to problem II. The task of solving the resulting set of operator equations and inequalities still remains. For the M pure states case we have further simplifications. The methods that we have used can also be applied to parameter estimation problems.

H. P. H. Yuen

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