VII. ELECTRODYNAMICS OF MEDIA*

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A. EXAMINATION OF SELF-SUSTAINED OPTICAL PULSES

There has been considerable discussion about distortionless (steady-state) optical pulse propagation.[1-3] A related topic of interest is laser self-pulsing, in which a pulse shuttles back and forth in a laser cavity, thereby causing an output train of pulses. Self-pulsing refers to the fact that no mode-locking device is used to force this pulsing behavior. The purpose of this report is to prove that steady-state self-pulsing is not possible in the rate-equation approximation with a lumped active medium. (The active medium is assumed to be much shorter than the pulses.)

We place the medium in a traveling-wave cavity, which insures that the medium will encounter only traveling-wave intensities.

Steady-state pulsing can occur only if there is some pulse shape that will undergo only amplification and delay in a single pass through the material. Loss is introduced through mirror transmissivity, the loss at the mirror being the same as the gain in the medium. Thus the medium experiences the same pulse shape and amplitude over many pulse round trips in the cavity.

Using the rate-equation approximation, we have

$$\frac{dn}{dt} = -\alpha In + \frac{n_o - n}{T_1}, \tag{1}$$

where

n = population inversion

n_o = equilibrium inversion

$\frac{1}{T_1}$ = relaxation rate of n toward its equilibrium value

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I = intensity of the pulse

α = cross section for stimulated emission.

We permit the pulse to be amplified and either delayed or advanced. After passage through the slab, I changes by ΔI.

$$\Delta I = \lambda I + \mu \frac{dI}{dt} = \beta I n, \tag{2}$$

where, by conservation of energy,

$$\beta = h\nu\alpha \sqrt{\frac{\mu_o}{\epsilon_o}},$$

where ν is the frequency of the pulse.

Solving Eq. 2 for n and substituting the expression in Eq. 1 gives an equation in I alone.

$$\mu \frac{d^2I}{dt^2} = \frac{\mu}{I}\left(\frac{dI}{dt}\right)^2 + \frac{1}{T_1}(\beta n_o - \lambda)I - \alpha\lambda I^2 - \left[\alpha\mu I + \frac{\mu}{T_1}\right]\frac{dI}{dt}. \tag{3}$$

This equation can be normalized by setting

$$R = \frac{\beta n_o T_1}{\mu}$$

$$S = \frac{\lambda}{\mu} T_1$$

and defining

$$A = \alpha I$$

$$\tau = \frac{t}{T_1}.$$

Then (3) becomes

$$\frac{d^2A}{d\tau^2} = \frac{(\mathring{A})^2}{A} + RA - (1+A)\left(SA + \frac{dA}{d\tau}\right). \tag{4}$$

This equation is analogous to a potential-well problem in which A is the position of a particle, τ is time, and the quantities on the right side of (4) are potential and positive or negative frictionlike terms.

The test of whether steady-state pulsing can occur then reduces to determining whether the particle can descend from an initial position A, and return to it with no loss or gain in energy, analogous to $\left(\frac{dA}{dt}\right)^2$.

$$\int_{A_{max}}^{A_{min}} dt \frac{d}{dt}\left[\left(\frac{dA}{dt}\right)^2\right] + \int_{A_{min}}^{A_{max}} dt \frac{d}{dt}\left[\left(\frac{dA}{dt}\right)^2\right] = 0$$

or

$$\int_{A_{max}}^{A_{min}} dA \left(\frac{d^2A}{dt^2}\right) + \int_{A_{min}}^{A_{max}} dA \left(\frac{d^2A}{dt^2}\right) = 0$$

or

$$\int_{A_{min}}^{A_{max}} dA \left(\frac{d^2A}{dt^2}\right)_{\text{reversed time}} = \int_{A_{min}}^{A_{max}} dA \left(\frac{d^2A}{dt^2}\right). \tag{5}$$

I shall show that, over every interval dA, the integrand on the right is strictly less than the integrand on the left, and hence pulsing is impossible.

Let A_r refer to the position in the right integrand in Eq. 5, and A_ℓ refer to the time-reversed position on the left side.

Equation 4 for these variables becomes

$$\frac{d^2A_r}{d\tau^2} = \frac{(\mathring{A}_r)^2}{A_r} + RA_r - (1+A_r)\left(SA_r + \frac{dA_r}{d\tau}\right) \tag{6a}$$

$$\frac{d^2A_\ell}{d\tau_-^2} = \frac{(\mathring{A}_\ell)^2}{A_\ell} + RA_\ell - (1+A_\ell)\left(SA_\ell - \frac{dA_\ell}{d\tau_-}\right), \tag{6b}$$

where τ_- refers to reversed time.

Starting from a minimum, at which $\mathring{A}_{r\&\ell} = 0$, $A_r = A_\ell = A_{min}$, we see that $d^2A_r/d\tau^2 < d^2A_\ell/d\tau_-^2$ after the first short interval of time, because of the difference in sign in the last terms of (6a) and (6b). We now compare the change in $\mathring{A}_r$ and $\mathring{A}_\ell$ over each interval of A to prove that $\mathring{A}_r$ remains less than $\mathring{A}_\ell$.

$$\Delta\mathring{A}_r = \overset{\circ\circ}{A}_r\Delta t, \qquad \text{but} \quad \Delta t = \frac{\Delta A}{\mathring{A}_r},$$

where ΔA is the length of the interval. Using (6a) and (6b) yields

$$\Delta\mathring{A}_r = \frac{\overset{\circ\circ}{A}_r\Delta A}{\mathring{A}_r} = \Delta A\left[\frac{\mathring{A}_r}{A_r} + \frac{RA_r}{\mathring{A}_r} - (1+A_r)\left(\frac{SA_r}{\mathring{A}_r} + 1\right)\right] \tag{7a}$$

$$\Delta\mathring{A}_\ell = \Delta A\left[\frac{\mathring{A}_\ell}{A_\ell} + \frac{RA_\ell}{\mathring{A}_\ell} - (1+A_\ell)\left(\frac{SA_\ell}{\mathring{A}_\ell} - 1\right)\right], \tag{7b}$$

where $A \leqslant A_r$, $A_\ell \leqslant A + \Delta A$.

If $\mathring{A}_r$ were to approach $\mathring{A}_\ell$ in size over successive intervals, then all but the last terms in (7a) and (7b) would be equal. But the last term in (7b) will be strictly greater than that in (7a), and so $\mathring{A}_\ell$ will remain greater than $\mathring{A}_r$.

It can be seen in (6a) and (6b) that if $\mathring{A}_\ell > \mathring{A}_r$ over every interval $A \leqslant A_r \leqslant A + dA$, $A \leqslant A_\ell \leqslant A + dA$, then $d^2A_\ell/d\tau^2 > d^2A_r/d\tau^2$ over every such interval, and thus the integrand on the left side of (5) is greater than that on the right. Therefore energy is strictly lost on each oscillation, and hence steady-state pulsing is impossible.

Note that this treatment also rules out the possibility of local minima, followed by increase to the maximum (see Fig. VII-1).

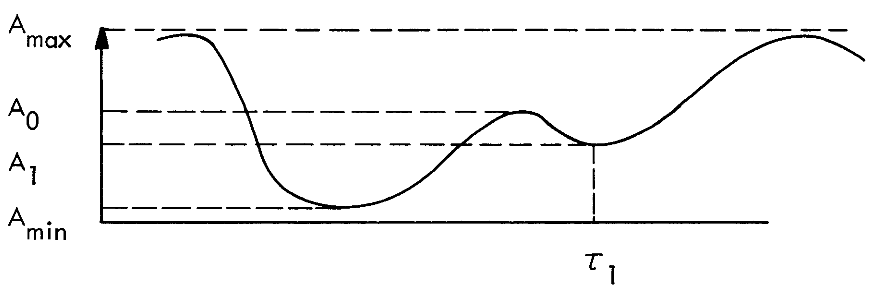

Fig. VII-1. Occurrence of relative maximum.

By treating A_1 as a new minimum, the proof above shows that A cannot increase to A_o after τ with as much energy as it had in its first encounter with A_o.

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References

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