VI. PHYSICAL ACOUSTICS*

Academic and Research Staff

Prof. K. U. Ingard
Dr. C. Krischer

Graduate Students

A. G. Galaitsis
G. F. Mazenko
P. A. Montgomery
R. M. Spitzberg

A. ULTRASONIC DISPERSION IN PIEZOELECTRIC SEMICONDUCTORS

We are continuing our investigation of the dependence of the phase velocity for piezoelectrically stiffened transverse waves in CdS on the electrical conductivity and applied drift field. In a previous progress report[1] we showed that our experimental results were not in good agreement with White's theory.[2] In his derivation, White takes into account the effect of electron trapping by defining a trapping factor f_o, which is a real number equal to the fraction of the acoustically bunched space charge which is mobile. Under this assumption, the relaxation time, τ, for equilibration between the trapped electrons and those in the conduction band is much smaller than the acoustic-wave period, and, therefore, the free and trapped bunched electrons oscillate in phase. For an arbitrary τ, the trapping factor has the complex form

$$f = \frac{f_o - j\omega\tau}{1 - j\omega\tau}. \tag{1}$$

This result can be derived either phenomenologically[3] or from a detailed treatment of the trapping kinetics.[4] By using this complex trapping factor, we obtain the following expression for the sonic velocity, v_s, which gives good agreement with our experimental results:

$$v_s(\omega, \sigma, E_d) = v_o \left\{ 1 + \frac{1}{2} k^2 \frac{\gamma(\gamma - a\omega_c/\omega) + (\omega/\omega_D + a)(\omega/\omega_D + \omega_c/\omega + a)}{(\gamma - a\omega_c/\omega)^2 + (\omega/\omega_D + \omega_c/\omega + a)^2} + \text{higher order terms in } k^2 \right\}, \tag{2}$$

where ω is the acoustic-wave angular frequency, σ is the electrical conductivity, E_d is the applied drift field, v_o is the "zero-field" unstiffened phase velocity, k^2 ($\ll 1$) is the square of the electromechanical coupling constant, $\gamma = 1 + \mu' E_d/v_s$ is the drift parameter, $\omega_c = \sigma/\epsilon$ is the conductivity relaxation frequency, $\omega_D = (e/kT)\, v_s^2/\mu'$ is the

*This work was supported by the U. S. Navy (Office of Naval Research) under Contract N00014-67-A-0204-0019.

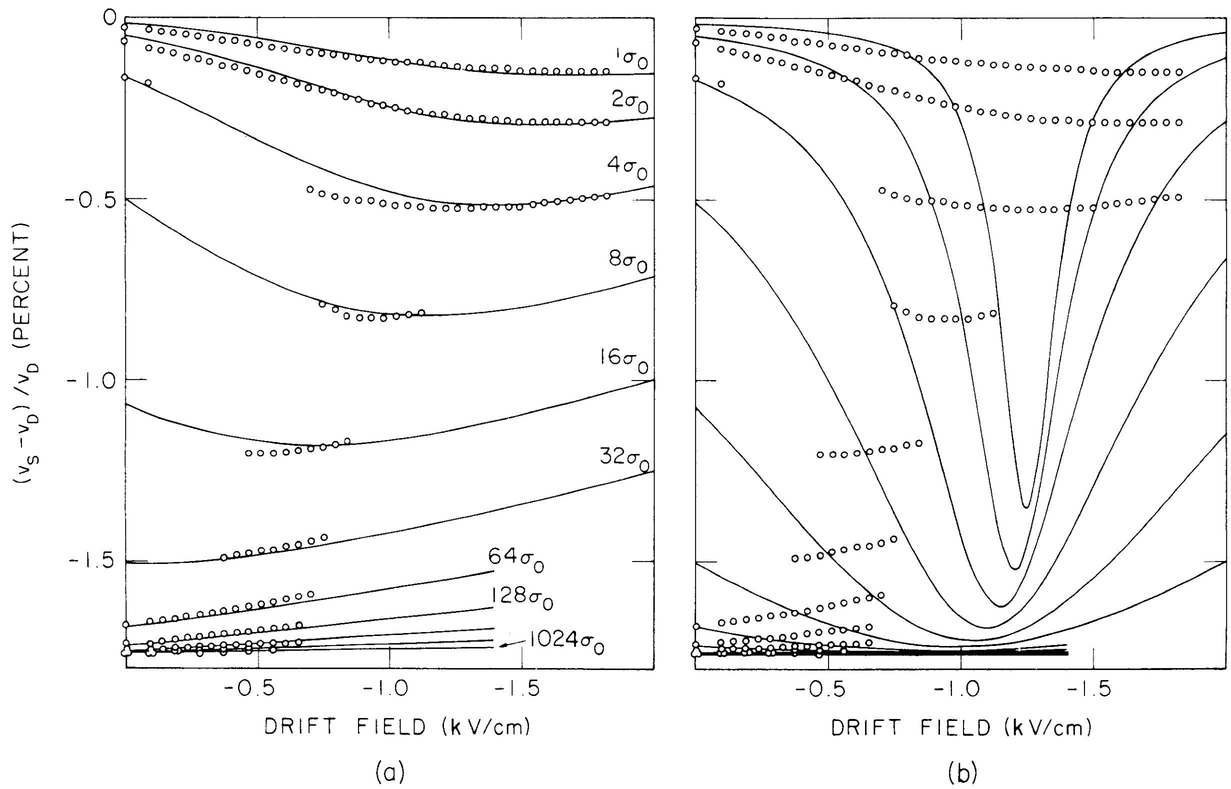

Fig. VI-1. Variation of phase velocity with applied drift field and electrical conductivity, for a 32-MHz transverse wave in CdS at 20.0°C. The value for σ_o is $1.3 \times 10^{-5}\ \Omega^{-1}\ \text{cm}^{-1}$.

(a) Theoretical curves calculated from Eq. 2, using $f_o = 0.18$, $\tau = 2.6$ ns, $k^2 = 0.036$, $v_o = 1.75$ km/s. Values for μ, obtained independently from ultrasonic amplification measurements at 406 MHz, vary from 206-332 $\text{cm}^2\ \text{V}^{-1}\ \text{s}^{-1}$ over the conductivity range 1 σ_o to 1024 σ_o.

(b) Theoretical curves calculated using $\tau = 0$ and $f_o = 0.68$ were adjusted to give the same effective drift mobility as in (a).

diffusion frequency, $\mu' = \left[\left(f_o^2+\omega^2\tau^2\right)/\left(f_o+\omega^2\tau^2\right)\right]\mu$ is the effective drift mobility, and $a = (1-f_o)\,\omega\tau/\left(f_o+\omega^2\tau^2\right)$.

Measurements of the dependence of the ultrasonic velocity on the applied drift field were carried out over the frequency range 32-345 MHz, and over a range of electrical conductivities from 10^{-5} to 10^{-2} $\Omega^{-1}\text{cm}^{-1}$. With the aid of a visual curve-fitting program on a computer with CRT output display, we found that Eq. 2 provided a good fit to the data over the entire range of measurement, with the values $f_o = 0.18$ and $\tau = 2.6$ ns used.

Figure VI-1a shows the experimental results obtained at 32 MHz, and the corresponding theoretical curves, calculated from Eq. 2, with the values for f_o and τ given above used. For comparison, the same data are shown in Fig. VI-1b, but with the theoretical curves calculated for $\tau = 0$ (White's theory).

We wish to express our gratitude to M. Beeler of Project MAC, an M. I. T. Research project sponsored by the Advanced Research Projects Agency, Department of Defense, under Office of Naval Research Contract Nonr-4102(02), for his kindness in providing the curve-fitting program.

C. Krischer

References

1. C. Krischer, Quarterly Progress Report No. 95, Research Laboratory of Electronics, M.I.T., October 15, 1969, p. 27.
2. D. L. White, J. Appl. Phys. 33, 2547 (1962).
3. I. Uchida, T. Ishiguro, Y. Sasaki, and T. Suzuki, J. Phys. Soc. Japan 19, 674 (1964).
4. C. A. A. J. Greebe, Philips Res. Rep. 21, 1 (1966).

B. NONLINEAR SOUND TRANSMISSION THROUGH AN ORIFICE

When sound of sufficiently high amplitude is transmitted through a sharp-edged orifice in a plate, flow separation will occur, and the velocity of the oscillatory flow through the orifice is no longer linearly related to the incident sound pressure. As a result, the transmitted sound will be distorted so that its frequency spectrum will be different from that of the incident sound.

This effect has been studied experimentally for the case in which the incident sound is a pure tone. In this experiment the orifice plate was set across a duct that was terminated by a 100% absorber.[1]

The experimental results are shown in Figs. VI-2 and VI-3. In Fig. VI-2 the drop in sound-pressure level across the orifice plate is shown as a function of the level of the driving sound pressure. This result refers to the frequency (150 Hz) of the incident sound. At sound-pressure levels about 130 dB we note that ΔL increases with the sound-pressure level, reaching a value of ~22 dB when the driving sound-pressure level is 162 dB. In other words, this nonlinear "transmission loss" is ~18 dB larger than the linear value.

In Fig. VI-3 the spectrum of the transmitted sound pressure when the driving pressure level is 162 dB at a frequency of 150 Hz is shown. We note that the transmitted spectrum contains a large number of harmonics and that the odd harmonics predominate.

In attempting to understand these results, we shall assume that the acoustic oscillations through the orifice can be treated quasi-statically. This means that the steady flow characteristic relating pressure drop and flow is assumed valid also in the acoustic case.

Thus, if the sound pressure on the "upstream" and "downstream" sides of the orifice are P_1 and P_3, the average velocity in the orifice is given by

$$u_o = \pm C_1 \sqrt{\frac{2}{\rho}|P_1 - P_3|}, \tag{1}$$

where C_1 is an orifice coefficient, $C_1 \approx 0.61$.

Since on the downstream side the sound-pressure field is an outgoing wave (no reflections), we have

$$P_3 \approx (A_o/A_1)\rho c u_o, \tag{2}$$

where A_o is the orifice area, A_1 is the duct area, ρ is the air density, and c is the speed of sound. If we also include an inertial component in the air flow, it follows from Eqs. 1 and 2 that we can relate P_1 and U_o through the equation

$$P_1 = \frac{\rho u_o |u_o|}{2C_1^2} + \frac{A_o}{A_1}\rho c u_o + \rho t_m \frac{du_o}{dt}, \tag{3}$$

where t_m is a characteristic thickness of the orifice (including mass end correction).

We now wish to determine the time dependence of the orifice velocity, u_o, that corresponds to a harmonic driving pressure $p_1 = p_{tot} \cos \omega t$. We shall not solve Eq. 3 in its complete form here, but consider only the high-intensity region in which the first term predominates. The time dependence of the orifice velocity is then simply

$$u_o \approx \pm C_1 \sqrt{\frac{2}{\rho} p_{tot}} \sqrt{|\cos \omega t|} \approx \pm u_{max} \sqrt{|\cos \omega t|}. \tag{4}$$

We expand this in a Fourier series

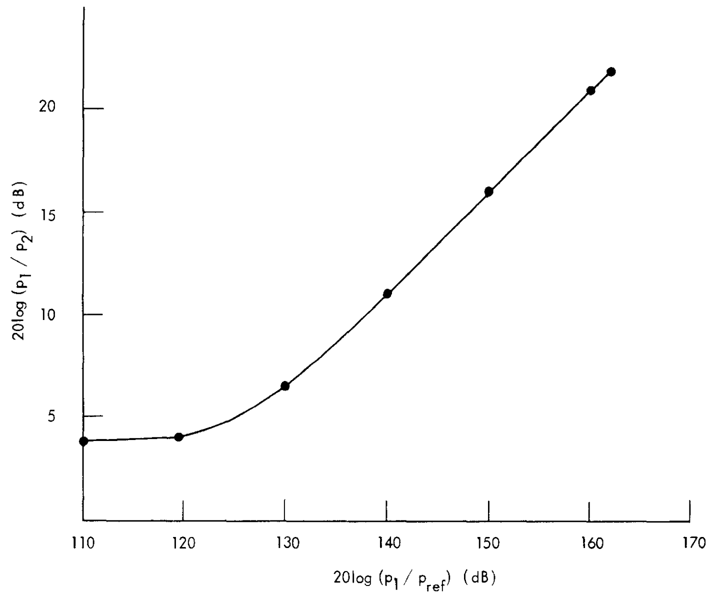

Fig. VI-2. Sound-pressure level difference 20 log (p_1/p_2) between the two sides of a thin orifice plate in a duct as a function of the sound-pressure level 20 log (p_1/p_{ref}) on the source side of the plate. Orifice diameter 0.7 cm; duct diameter 6.2 cm; frequency 150 Hz. p_{ref} = 0.0002 dyn/cm.

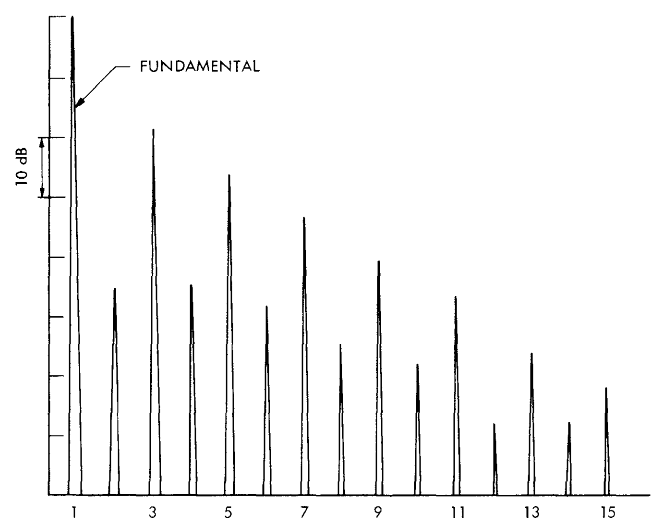

Fig. VI-3. Recorded spectrum of pressure wave transmitted through the orifice plate at 162 dB driving sound-pressure level.

$$u_o(t) = \Sigma\, U_n \cos n\omega t \tag{5}$$

and obtain

$$\begin{aligned} \frac{U_n}{U_{max}} &= \frac{4}{\pi}\int_0^{\pi/2} (\cos x)^{1/2} \cos nx\, dx \\ &= \frac{8\sqrt{2\pi}}{(1-4n^2)} \left[\Gamma\left(\frac{1+2n}{4}\right)\Gamma\left(\frac{1-2n}{4}\right)\right]^{-1} \qquad (n \text{ odd}) \\ U_n &= 0 \qquad (n \text{ even}) \end{aligned} \tag{6}$$

where Γ is the gamma function. The first few harmonic components of the velocity are

$$\begin{aligned} U_1 &= \frac{8\sqrt{2\pi}}{3.4\left[\Gamma\left(\frac{3}{4}\right)\right]^2} \approx \frac{2\sqrt{2\pi}}{3}\left[\Gamma\left(\frac{3}{4}\right)\right]^{-2} \approx 1.11 \\ U_3 &= \frac{U_1}{7} \\ U_5 &= \frac{5}{77} U_1 \\ U_7 &= \frac{3}{77} U_1. \end{aligned} \tag{7}$$

Having obtained the frequency spectrum of the velocity in the aperture, we can determine the corresponding spectrum of the transmitted sound pressure from

$$p_t = \Sigma\, p_n \cos \omega t \qquad p_n = \frac{A_o}{A_1} \rho c U_n. \tag{8}$$

It follows that the relative strength of the pressure amplitudes is obtained from the result in Eq. 7. First, we note that the even harmonics are absent. The odd harmonics decrease with the order n approximately as $1/n^2$ for large n. If the level of the first harmonic is chosen as reference, 0 dB, we see that the levels of the third, fifth, and seventh harmonics are -18 dB, -24.6 dB, and -29.2 dB. In other words, there is a large drop of 18 dB in level between the first and the third, whereas the level difference between the third and the fifth is only 6.6 dB. This is in excellent agreement with experimental results.

In order to determine the absolute level of the transmitted sound in terms of the driving pressure P_1, we have to solve for u_o in Eq. 3 and use it in Eq. 1. In the high-level regime such that $\rho u_o^2/2C_1^2 \gg (A_o/A_1)\ \rho c u_o$, that is, $u_o > (A_o/A_1)\ 2C_1^2 c$ and $\rho u_o^2/2C_1^2 \gg \rho t_m\ du_o/dt$, the calculation is simple and we get $u_o \approx C_1\sqrt{2p_1/\rho}$ and

$$p_3 \approx \frac{A_o}{A_1}\ \rho c C_1 \sqrt{\frac{2p_1}{\rho}}$$

$$p_3/\rho c^2 = C_1(A_o/A_1)\sqrt{p_1/\rho c^2}. \tag{9}$$

The corresponding pressure-level difference in this high-level region can be expressed as

$$20 \log (p_1/p_3) \approx \frac{1}{2} 20 \log (p_1/p_{ref}) + 20 \log (A_1/A_o C_1) - 98.5, \tag{10}$$

where $p_{ref} = 0.0002\ \text{dyn/cm}^2$. This expression shows that the pressure-level difference across the plate increases with the level of the driving pressure, an increase of 5 dB for every increase of 10 dB in the pressure level, in excellent agreement with experiments.

The absolute value of the predicted level difference is approximately 2 dB higher than the observed value in this high-level region. In view of several possible sources of error, this may be regarded as reasonably good agreement.

U. Ingard

References

1. U. Ingard and H. Ising, J. Acoust. Soc. Am. 42, 6-17 (1967).