X. DETECTION AND ESTIMATION THEORY*

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A. DETECTION OF DEEP SEISMIC REFLECTORS ABOARD THE R/V ATLANTIS II

Multiple reflections in the water column often make the detection of deep seismic reflectors in oceanographic exploration very difficult. In this report we describe a novel method of modeling the unwanted feedback generating these multiples. This model formulates the removal of the multiples as a system-identification problem, and uses the maximum-likelihood method of statistics to derive an algorithm for estimating the model parameters. This algorithm can be implemented on a small onboard computer and yields an enhanced performance over comparable methods previously used. Experimental results of applying the algorithm using data taken aboard the R/V Atlantis II during Woods Hole Oceanographic Institute's International Decade of Oceanographic Exploration cruise to the coast of Southwest Africa are presented.

In seismic profiling, a short pulse of acoustic energy is generated by the sudden release of compressed air (2000 psi) or by the discharge of a high-voltage capacitor (15-20 kV) through a submerged electrode. This energy is reflected at the sea floor and beneath it at interfaces of seismic layers (reflectors), where there is a finite change in the acoustic impedance of the medium. Over a sequence of returns, the geologic structure is indicated by the horizons generated by these reflectors. In many situations, particularly in shallow water, this reflected energy reverberates in the water column via multiple reflections off the sea floor and sea surface. This energy often appears with the same travel time as reflections from deeper layers in the primary return so as to create a very low signal-to-reverberation level. The net result is that detection of the deeper interfaces is made very difficult by these "multiples."

Numerous studies have been directed toward eliminating this reverberation. Generally, the methods can be classed as those which exploit the temporal structure of the signals and those which use the spatial structure by combining outputs of elements in a trailing streamer array.[1,2] With the temporal methods it is recognized that the multiple

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stems from unwanted feedback. The closed-loop transfer function is calculated, and an inverse filter is designed to cancel the unwanted feedback. The technique that we shall discuss is in this category. In the spatial methods, the streamer element outputs are combined with a delay and array shading so that they focus on a common point. This is done as a function of depth to produce the output signal. Since the water column multiples do not add coherently, the desired signal is extracted by use of the array gain attainable with the streamer.

The disadvantage of spatial methods is the expense of streamer arrays, the required equipment for recording many outputs simultaneously, and the subsequent digital computer. With our technique we suppress the water-column multiples. We use only a single-array segment and recording channel, and the processing is done by a small onboard digital computer. As a result, the processing can be done inexpensively and in approximately real time.

In previous approaches that exploit the temporal structure of the signals, the signals of the multiple reflections have been modeled as delayed and attenuated versions of the incident signal. In this approach we assume that the source and receiver generate narrow beams and that the reflecting horizons are smooth compared with quarter-wavelength spacings within these beams; that is, a ray-path analysis is appropriate. In routine oceanographic work these assumptions are questionable. The source has virtually no directivity, the streamer segment is short (~50 m), and variations in the sea/air boundary from swells of 3-4 m are common. The net effect is that reflections at the boundaries introduce a smearing of the signals in addition to delay and attenuation. Consequently, we model the generation of the unwanted feedback using a more general filter structure than the perfect impulsive reflector. This is indicated by $\rho(s)$ in Fig. X-1. The incorporation of this more general structure has led to a significant improvement in our experimental results.

The closed-loop response of this system is

$$R(s) = \frac{H_b(s)\, e^{-2sT_w}}{1 - \rho(s)\, H_b(s)\, e^{-2sT_w}} S_i(s),$$

where

T_w = travel time in the water column

$H_b(s)$ = transfer function of the seismic structure

$\rho(s)$ = smearing filter for modeling the multiple feedback

$s_i(t)$ = input signal source [$S_i(s)$ is the frequency-domain representation]

$r(t)$ = observed signal [$R(s)$ is the frequency-domain representation].

To remove the multiple we need only find the inverse filter $1 - \rho(s)\, H_b(s)\, e^{-2sT_W}$ and process the observed signal r(t) through it to cancel the multiple. In the rest of our discussion we focus on (i) specifying a parametrized model for the inverse filter appropriate for implementation on a small onboard computer, (ii) applying the maximum-likelihood method of statistics to estimate the parameters, and (iii) demonstrating some experimental results with these methods.

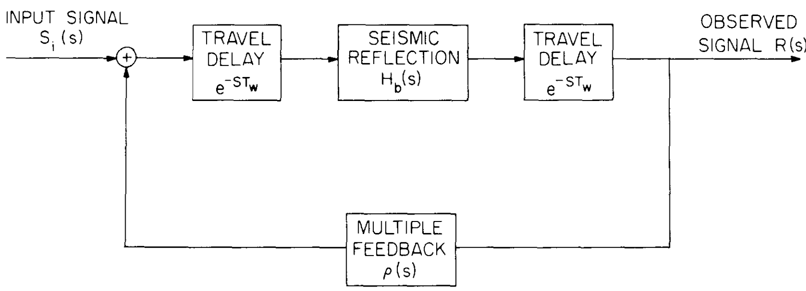


Fig. X-1. Feedback model of multiple generation.

In most of the situations encountered, most of the energy is reflected from the sea floor boundary; only a comparatively small fraction of the energy penetrates to the deeper layers. This situation does not apply where the sea floor is acoustically "soft"; there we would need to modify our subsequent results. In designing the inverse filter we model the term $H_b(s)\, \rho(s)$ to be a wideband short-duration filter $\rho_o(s{:}\underline{c})$,

$$H_b(s)\, \rho(s) = -\rho_o(s{:}\underline{c}).$$

Thus the boundaries of the sea/air and sea floor for the water-column multiple are modeled as having a nonimpulsive response with a short, but finite, duration. The $\underline{c}$ notation specifies that there is a parameter set upon which this filter is dependent. The particular parametrized model structure that we use to model the filtering operation $\rho_o(s{:}\underline{c}) \exp(-2sT_W)$ is illustrated in Fig. X-2 in the context of structuring the inverse filter for the denominator of

$$H_o(s{:}T_W, \underline{c}) = \frac{H_b(s)\, e^{-2sT_W}}{1 + \rho_o(s{:}\underline{c})\, e^{-2sT_W}}.$$

In this model the pole locations $\{p_i\}$ are chosen to be larger than the bandwidth of the seismic signals, while the zeros of $\rho_o(s)$ are determined by the coefficients $\{c_i\}$, which are to be estimated from the return. It can be demonstrated that as the number of elements, N, tends to infinity, an arbitrary impulse response can be represented. We do not want this to occur, however, as excess degrees of freedom lead to filter structures that remove real signal as well as reverberation. The essence of our procedure is to find the filter that represents the generation of B2 from B1, B3 from B2, and so forth.

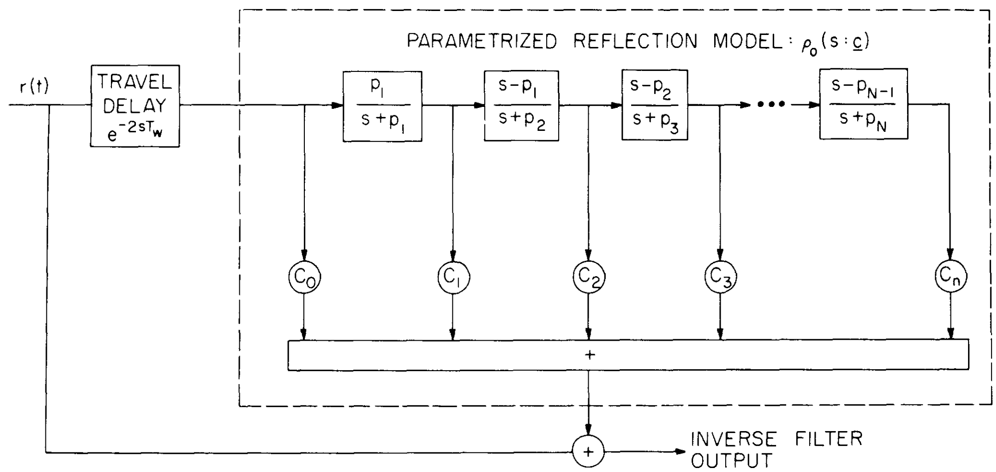


Fig. X-2. Parametrized model for maximum-likelihood multiple removal.

To estimate the parameters T_w and $\underline{c}$, we use the maximum-likelihood procedure of statistics (see Van Trees[3] for a general discussion of this). The essential concept is to maximize the log-likelihood function $\ell(r(t):T_w, \underline{c}) = \ln [p(r(t):T_w, \underline{c})/p(r(t):\text{white noise})]$ vs the parameters $T_w, \underline{c}$. Under the assumptions of a high signal-to-noise ratio, Gaussian process statistics, and a broadband source and response,

$$\ell(r(t):T_w, \underline{c}) = -\frac{1}{2}\int_{-\infty}^{\infty} [r(t) + \rho_o(t:c) * r(t-2T_w)]^2\, dt$$

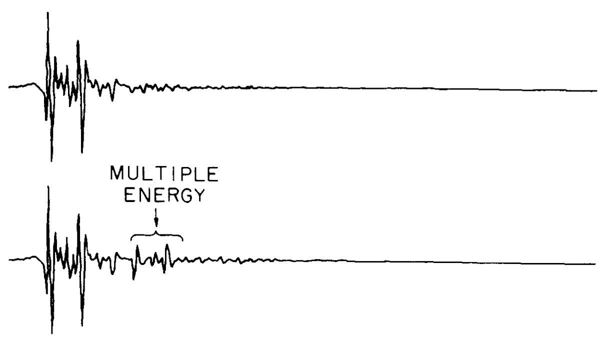
$$-\frac{1}{2}\int_{-2\pi W}^{2\pi W} \ln\left[1 + \frac{2}{N_o}\left(\frac{|H_b(\omega)|^2}{|1 + \rho_o(\omega:\underline{c})\exp(-j\omega 2T_w)|^2}\right)\right]\frac{d\omega}{2\pi},$$

where the star denotes the convolution operation for representing the output of a linear time-invariant system, and W is the bandwidth of the signal (typically 150 Hz). Higher frequencies are present, but the deep reflectors produce only low-frequency responses. Therefore we confine our attention to this region. The second term is independent of the observation; it is simply a bias. Within the range of acceptable values of the parameter $T_w, \underline{c}$ this term is nearly constant; consequently, we ignore it in the maximization operation. Maximizing the log-likelihood function is accomplished by minimizing

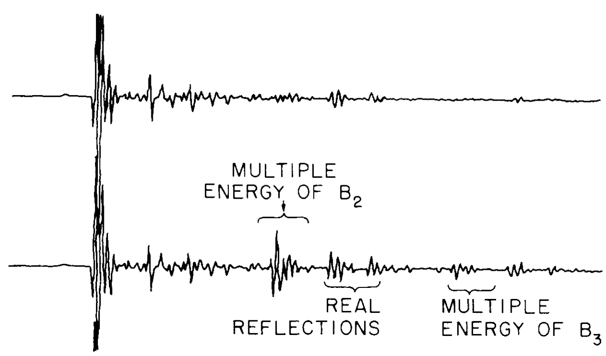
$$q(r(t):T_w, \underline{c}) = \int_{-\infty}^{\infty} r^2(t)\, dt + 2 \int_{-\infty}^{\infty} \rho_o(\tau:\underline{c}) \int_{-\infty}^{\infty} r(t)\, r(t-\tau+2T_w)\, dtd\tau$$

$$+ \int_{-\infty}^{\infty} \rho_o(\tau_1:\underline{c}) \int_{-\infty}^{\infty} \rho_o(\tau_2:\underline{c}) \int_{-\infty}^{\infty} r(t-\tau_1+2T_w)\, r(t-\tau_2+2T_w)\, dtd\tau_1 d\tau_2.$$

The first and third terms are independent of T_w. Since the response $\rho_o(\tau:\underline{c})$ is nearly impulsive, the middle term can be minimized by finding the largest negative value of the autocorrelation of the received signal r(t) in the local of the travel time. We have for our estimate of T_w

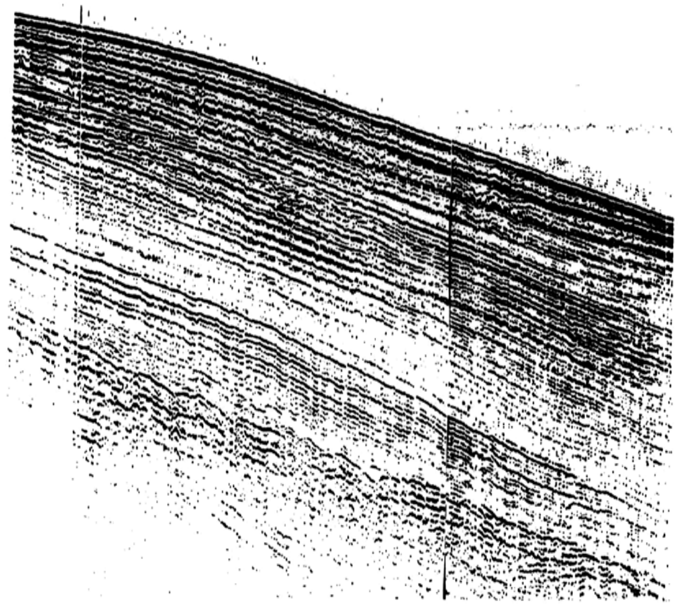


(a)

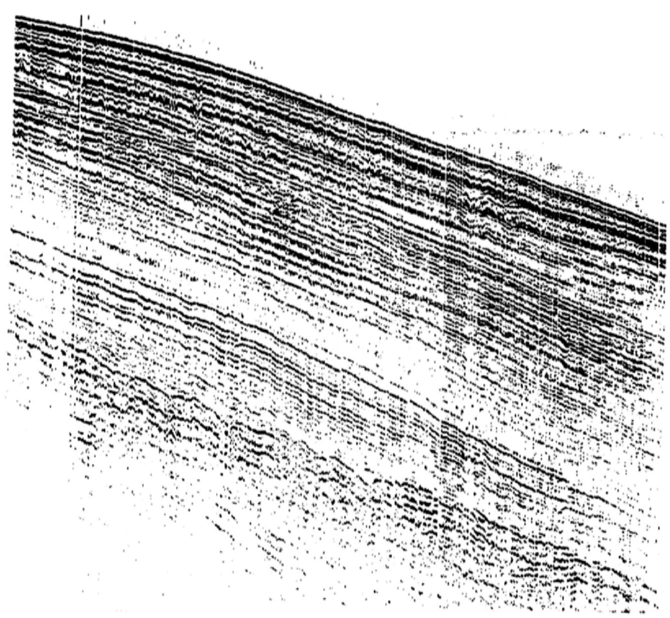


(b)

Fig. X-3. Comparison of signals before and after processing for data recorded at two points on a line northwest of Capetown. (a) Upper: Signal No. 1 after processing. Lower: Signal No. 1 before processing. (b) Upper: Signal No. 2 after processing. Lower: Signal No. 2 before processing.



(a)



(b)

Fig. X-4. Seismic record (a) before and (b) after processing for a line northwest of Capetown.

$$\hat{T}_W: \min_{\tau \approx T_W} \int_{-\infty}^{\infty} r(t)\, r(t-2\tau)\, dt. \tag{1}$$

If we assume that our selection of T_W is approximately correct, we can observe that the parameters $\underline{c}$ enter the term

$$q(r(t):\hat{T}_W, \underline{c}) = \int_{-\infty}^{\infty} [r(t) + \rho_o(t:c) * r(t-2\hat{T}_W)]^2\, dt$$

quadratically, since $\rho_o(t:\underline{c})$ is a linear function of $\underline{c}$. Therefore we are led to a least-squares procedure for estimating the parameters $\underline{c}$. We then have

$$\hat{\underline{c}}: \min_{\underline{c}} \int_{-\infty}^{\infty} [r(t) + \rho_o(t:c) * r(t-2T_W)]^2\, dt. \tag{2}$$

Equations 1 and 2 specify the estimation of the parameters T_W, $\underline{c}$. We estimate these parameters and then design the inverse filter $1 + \rho_o(s:\underline{c}) \exp(-2sT_W)$ to remove the reverberation caused by the multiples.

This procedure was applied, on an onboard HP-2116C computer, to data taken aboard R/V Atlantis II during her recent cruise to the Eastern Atlantic Continental Margin off the coast of South Africa. Two examples of seismic returns before and after processing are indicated in Fig. X-3. The dominant energy in the multiple has been suppressed very effectively. A comparison using a complete line, that is, several hours of returns in an intensity modulated display vs depth, is indicated in Fig. X-4. This line is on a line running down the continental slope northwest of Capetown. While residual multiple energy remains, approximately 14-18 dB of reverberation energy has been removed. It is possible in many more instances to distinguish deeper real seismic layers from those fictitiously introduced by the multiple return, and this is considerably easier than using unprocessed records. The processing time required is approximately four times real time. With some minor provisions for fixed-point arithmetic this time can be reduced significantly.

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References

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