A. EFFECT OF SURFACE ROUGHNESS ON EMISSIVITY

California Institute of Technology (Contract 953524)

Leung Tsang, Jin-Au Kong

With a radiometer looking directly downward at a rough surface, after integration over $d\theta_s$, the emissivity of the surface is given by

$$e = 1 - \frac{1}{4\pi} \int \gamma(0, \bar{k}_s) \sin \theta_s d\theta_s,$$

where

$$\gamma(0, \bar{k}_s) = \frac{k^2 |f|^2}{2 I}$$

is the scattering cross section for a wave at normal incidence, and $\bar{k}_s$ is the scattering $\bar{k}$-vector. In Eq. 2, $|f|^2 = 2\pi(2\sin^2 \theta_s + 2|R_{01}|^2 - |R_{01}|^2 \sin^2 \theta_s - 2\cos \theta_s + 2|R_{01}|^2 $.

The solution to the integral $I$ in (3) has been a controversial subject. In this report, we use the double saddle-point method to evaluate $I$. The integral is extended to the complex plane.
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\[ I = \frac{1}{2} \int_{-\infty}^{\infty} \xi H_o^{(1)}(k_{dp} \xi) e^{-\kappa(1-\rho(\xi))} d\xi \]
\[ = \frac{2}{\pi} \int_{-\infty}^{\infty} \xi e^{-\kappa(1-\rho(\xi))} d\xi \int_{0}^{\infty} \left( i4k_{dp} \xi^2 y^2 \right)^{-1/2} e^{-y^2/2} dy. \] \hspace{1cm} (4)

Assuming a Gaussian correlation function

\[ \rho(\xi) = \exp(-\xi^2/L^2), \] \hspace{1cm} (5)

where \( L \) is the correlation length, we determine the saddle point \( \xi_s \) from

\[ \xi_s \exp\left(-\xi_s^2/L^2\right) = ik_{dp}L^2/2\kappa \] \hspace{1cm} (6)

and it is found to be on the imaginary axis. Applying the transformation

\[ ik_{dp} \xi - \kappa(1-\rho(\xi)) - [ik_{dp} \xi_s \kappa(1-\rho(\xi_s))] = -s^2/2, \]

we can expand the integrand into a double power series of \( s \) and \( y \). The result of the integration is an asymptotic series, which to second order is

\[ I \approx \left( \frac{\xi_s}{ik_{dp}} \right)^{1/2} \exp(-\kappa(1-\rho(\xi_s)) + ik_{dp} \xi_s) \left( \frac{L^2 \exp(\xi_s^2/L^2)}{2\kappa(1-2\xi_s^2/L^2)} \right)^{1/2} \]
\[ \left\{ 1 + \frac{\exp(\xi_s^2/L^2)}{4\kappa \left( 1 - \frac{2\xi_s^2}{L^2} \right)^3} \left[ 1 - \frac{12\xi_s^2}{L^2} + \frac{8\xi_s^4}{L^4} - \frac{8\xi_s^6}{L^6} \right] \right\}. \]

(7)

In the limit \( \xi_s \ll L \), Eq. 6 becomes

\[ \xi_s \approx ik_{dp}L^2/2\kappa, \] \hspace{1cm} (8)

and Eq. 7 becomes

\[ I \approx \frac{L^2}{2\kappa} \frac{-k^2_{dp} L^2/4\kappa}{1 + \left( \frac{3k^2_{dp} L^2}{8\kappa^2} + \frac{k^4_{dp} L^4}{32\kappa^3} \right)}. \] \hspace{1cm} (9)

The first term in (9) is the solution from geometrical optics.\(^5\)
In order to calculate emissivity, another integration must be performed. We can apply the saddle-point method with the saddle point at $\theta_s = 0$ and obtain

$$e = 1 - |R_{01}|^2 \left( 1 - \frac{1}{3} \frac{\sigma^2}{L^2} - \frac{1}{16k^2\sigma^2} \right)$$

(10)

The approximation $\xi_s \ll L$ leading to (8), in view of (6), implies $1 > \sigma^2/L^2 > 1/k^2\sigma^2$. Thus in Eq. 10 the last term is negligible. The scattering effect reduces the reflectivity and consequently increases the emissivity. In view of the last term, the emissivity decreases as frequency increases.

In the limit $k\sigma \ll 1 \ll kL$, this method breaks down and our results are invalid. In this limit we can follow Stogryn's small-perturbation approach, which utilizes the bistatic scattering cross-section results of Rice and Valenzuela, and obtain

$$e = 1 - |R_{01}|^2 \left( 1 + 4k^2\sigma^2 + 2k^2\sigma^2(n-1)(1+R_{01})/|R_{01}| \right).$$

(11)

Thus the scattering effect increases the reflectivity and consequently decreases the emissivity. The emissivity decreases as frequency increases.

References


B. COMPOSITE MODEL FOR MICROWAVE REMOTE SENSING

California Institute of Technology (Contract 953524)

Jin-Au Kong

In microwave remote sensing of the Earth, the factors that affect emission properties include absorption, layering, surface roughness, anisotropy, inhomogeneities, and scattering. No model that takes into account only individual factors can interpret actual field data because the data are affected by all of the factors. In this report, we attempt to develop a composite model that accommodates all of these effects.

We use as our starting point a stratified model, which has been developed to account for absorption, layering, and anisotropy. The solution is exact and in closed form. We propose that the reflectivity at each interface in the solution be modified to include rough surface effects as discussed in Section IX-A. The effects attributable to buried scattering centers can be treated separately and subtracted from the resultant emissivity.

As an example to illustrate the procedure, we consider a two-layer model such as a slab of ice with depth d and relative complex permittivity \( \varepsilon' + i\varepsilon'' \) on top of water or land. We calculate emissivity as a function of frequency. The reflection coefficient is

\[
R = \frac{R_{01} + R_{12} \exp(i2k_xd)}{1 + R_{01}R_{12} \exp(i2k_xd)},
\]

where \( R_{01} \) is the reflection coefficient at the air-ice boundary, \( R_{12} \) is the reflection coefficient at the ice-water or the ice-land boundary, and \( k_x = k_0(\varepsilon' + i\varepsilon'' - \sin^2 \theta)^{1/2} = k_0 + ik_x', \) where \( \theta \) is the angle of observation measured from the nadir.

The reflectivity \( r = |R|^2 \) is calculated as

\[
r \approx \frac{A^2 + |R_{01}|^2}{1 + A^2|R_{01}|^2},
\]

where

\[
A = |R_{12}| \exp(-2k_x''d).
\]

In arriving at (2), we have neglected the oscillatory behavior of \( r \) with frequency because scattering and inhomogeneity will render the wave incoherent. We observe that absorption is accounted for by \( A \), which becomes small for large layer depth or highly lossy.
Fig. IX-1. Emissivity with and without scattering.

Fig. IX-2. Emissivities.
ice, and \( r \approx |R_{01}|^2 \). When the interfaces are not flat, we take into account rough surface effects by modifying \( |R_{01}|^2 \) and \( |R_{12}|^2 \) according to Eqs. 10 and 11 in Section IX-A.

We treat scattering effects arising from medium inhomogeneity by extending Gurvich's result\(^2\) for an unbounded half space. Assuming scattering is caused only by ice because the penetration depth for water or land is vanishingly small, we obtain

\[
s = \frac{n}{Z_1} \sigma^2 \frac{n k_{0} z_{0}^2}{1 + 4(n k_{0} z_{0}^2)} \left(1 - |R_{01}|^2 \right) \left(1 - \exp(-4n_{i} k_{0} d)\right),
\]

where \( n + i n_{1} \) is the complex refractive index for ice, \( z_{0} \) is the characteristic correlation depth for the refractive index, and \( \sigma^2 \) characterizes the variance of the refractive index. Equation 4 is valid only for observation from the nadir and the last factor accounts for the finite depth of ice.

The emissivity with all effects present is given by

\[
e = 1 - r - s.
\]

Neglecting the rough surface effect and observing from the nadir, we find that \( e \) is governed by \( n + i n_{1} = (\epsilon_{1} + i \epsilon_{1})^{1/2} \), \( d \), \( z_{0} \), and \( p = \sigma^2 / n_{1} \). In Fig. IX-1, we present a numerical result for \( s \), \( 1-r \), and \( e \). The Lorentzian-shaped \( s \) has its maximum point shifted toward the higher frequency side when \( z_{0} \) decreases, and the magnitude of the maximum point increases as \( p \) increases. In Fig. IX-2, we show emissivities for two sets of different parameters.

I am indebted to Dr. Joe W. Waters for bringing to my attention Gurvich's paper and to Leung Tsang for his perceptive views toward this work and many illuminating discussions.

References


C. PROBING DEPTH FOR MICROWAVE REMOTE SENSING OF ICE AND SNOW CALCULATED BY A SEMINUMERICAL APPROACH USING MACSYMA

California Institute of Technology (Contract 953524)

Jin-Au Kong

In microwave remote sensing of snow and ice fields, we are especially interested in the penetration depth. A simple layer model was used and with the aid of MACSYMA we quickly obtained a closed-form solution for the probing depth (see the appendix). The radiometer sensitivity is assumed to be accurate within 1%.

For a layer of ice with relative dielectric constant $\varepsilon = 3.2 + i\varepsilon''$ on top of water, the probing depths are as follows.

For lake ice with $\varepsilon'' = 0.004$, $dp = 255\lambda$.

For sea ice with $\varepsilon'' = 0.2$, $dp = 5.1\lambda$.

For glacier ice with $\varepsilon'' = 6.14 \times 10^{-4}/\lambda$, $dp = 1660\lambda$.

($\lambda$ is the free-space wavelength in meters.)

I wish to thank Professors J. Moses and A. Bers for giving me access to their facilities, and Dr. David Yun, Charles Karney, and John Kulp for teaching me MACSYMA.
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Appendix

THIS IS MACSYMA 244

FIX 244 DSK MACSYM BEING LOADED
LOADING DONE

(C1) batch(probing,depth);

(C2) T:2$

(C3) LINEL:73$

(C4) D[0]:0$

(C5) D[T]:0$

(C6) REF[T]:0$

(C7) FOR L:T-1 THRU 0 STEP -1
/(EXP(-%I*2*KX[L+1]*(D[L+1]-D[L]))/R[L,L+1]+REF[L+1])$

(C8) R2:RATSIMP(REF[0]);

(C9) MATCHDECLARE(A,TRUE)$

IIATCOM FASL DSK MACSYM BEING LOADED
LOADING DONE

(C10) TELLSIMP(SIN(A)**2,1-COS(A)**2); $\text{RULE PLACED ON} **$

(C11) KX[1]:KP+%I*KPP$

(C12) R2:EV(R2);
(C13) PN: PART(R2, 1)
(C14) PD: PART(R2, 2)
(C15) PK: \frac{\text{REALPART}(PN)^2 + \text{IMAGPART}(PN)^2}{\text{REALPART}(PD)^2 + \text{IMAGPART}(PD)^2}
(C16) KP: \sqrt{3.2}
(C17) KPP: \sqrt{.0032}
(C18) R[0, 1]: (1 - \sqrt{3.2}) / (1 + \sqrt{3.2})
(C19) K[1, 2]: (1 - \sqrt{80/3.2}) / (1 + \sqrt{80/3.2})
(C20) PLOT([1-R[0,1]**2, 1-PK], U[1, 0, 10*%PI, [%]);

GRAPH FASL DSK MACSYM BEING LOADED
LOADING DONE

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(C21) KP:0$

(C22) PLOT([R[0,1]**2,PR],D[1],C,10*PI,[%]);

(C23) KILL(R[0,1],R[1,2],KPP);

(C24) P1r:EV(PR);

(D24) - 2 U KPP

\[-2 U \left( R_{1,2} E_{0,1} + R_{0,1} \right)\]

(D25) EQ:RATSUBST(X,EXP(-2*D[1]*KPP),PR-R[C,1]**2=DEL);

\[-R_{1,2} \left( R_{0,1} X - X \right) + R_{2,1} \left( 2 R_{0,1} X - 2 R_{0,1} + 1 \right)\]

(D25) \[-2 \left( R_{0,1} + 1 \right) \]

\[-2 \left( R_{0,1} + 1 \right) \]

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(C26) SOL: SOLVE(EQ, X);

SOLUTION

\[ R^{2} - 1 \sqrt{DEL + R^{2}} + R^{3} \frac{DEL + R}{0, 1} 0, 1 - \frac{1}{0, 1} 0, 1 0, 1 \]

(E26) \( X = \frac{-R}{2} \)

\[ R^{2} - 1 \sqrt{DEL + R^{2}} + R^{3} \frac{DEL + R}{0, 1} 0, 1 - \frac{1}{0, 1} 0, 1 0, 1 \]

(E27) \( X = \frac{-R}{2} \)

(D2)

(C28) SOL: EV(SOL)$

(C29) \( R[0, 1] \): ABS((1 - SQRT(3.2)) / (1 + SQRT(3.2)));

(D29) 0.28285965

(C30) \( R[1, 2] \) : ABS((1 - SQRT(80/3.2)) / (1 + SQRT(80/3.2)));

(D30) 0.66666666

(C31) DEL: .01$

(C32) SOL: EV(SOL);

(D32) [X = -0.80592103, X = 0.02812089235]

(C33) LOG(X), EVAL, SOL[2];

(D33) -3.5712425

(C34) DP: -%/(2*KPP)$

(C35) KPP: (EPP/SQRT(EP)) * %PI/LANDA$

(C36) DP: EV(DP, NUMER);

(D36) \[ \frac{1.76562124 \text{ EP}}{\text{LANDA}} \frac{0.5}{\%\text{PI EPP}} \]

(C37) EP: 3.2$

(C38) DP1: EV(DP, EPP = .01*EP, NUMER);

(D38) 31.7734566 LANDA

(C39) DP2: EV(DP, EPP = .0004*EP, NUMER);

(D39) 794.33646 LANDA

(C40) DP3: EV(DP, EPP = .000192*EP/LANDA, NUMER);

(D40) 1654.86766 LANDA

(D41) BATCH DONE

QPR No. 114 85
D. MACSYMA STUDIES OF WAVES IN UNIAXIAL MEDIA

Joint Services Electronics Program (Contract DAAB07-71-C-0300)

Zemen Lebne-Dengel, Eni G. Njoku, Jin-Au Kong

By using the symbolic manipulation program MACSYMA, waves in uniaxial media have been studied analytically and numerically. In this report we summarize results that have not been reported elsewhere.

Case 1: We studied waves in a uniaxial medium that moves in a direction perpendicular to the optical axis. The dispersion relation is derived by using a Lorentz transformation or the kDB system. For extraordinary waves, the result is

\[ k^2 + \alpha k_x^2 + \frac{1 - \alpha \beta^2}{1 - \beta^2} \left[ k_x + \beta \frac{\alpha - 1}{1 - \alpha \beta^2} \frac{\omega}{c} \right]^2 - \frac{\alpha^2}{1 - \alpha \beta^2} \frac{\omega^2}{c^2} = 0, \]

where \( k_x, k_y, \) and \( k_z \) are the three components of the wave vector, \( \beta = v/c \) determines the velocity of the medium, \( \omega \) is the angular frequency, \( n \) is the refractive index in the rest frame of the medium, and \( \alpha \) characterizes the anisotropy (for a positive uniaxial medium \( \alpha > 1 \), for a negative uniaxial medium \( \alpha < 1 \)). In Fig. IX-3, we plot the wave surfaces in the \( k_x \) and \( k_z \) planes at different velocities. Since \( \beta = 0 \), we obtain the usual ellipse. The ellipse is shifted to the left as the velocity increases, which indicates that the wave velocity decreases in the direction of motion of the medium. Since the velocity of the medium exceeds the Cherenkov velocity \( \beta = 1/n \), the wave surfaces become hyperbolas.

Case 2: We studied the reflections of a plane wave incident upon a stationary
uniaxial medium. The optical axis of the uniaxial medium is oriented in an arbitrary direction. In Fig. IX-4 we present the reflectivity for both TE and TM incident waves as a function of incident angle $\theta_i$. The optical axis in this case is oriented $45^\circ$ with respect to normal and lies in the plane of incidence. The crossover of the two curves occurs because for the incident TE wave only the ordinary wave is transmitted, while for the incident TM wave only the extraordinary wave is transmitted.

E. APPLICATION OF THE RADIO-FREQUENCY INTERFEROMETRY METHOD TO STRATIFIED ANISOTROPIC EARTH

Joint Services Electronics Program (Contract DAAB07-71-C-0300)

Leung Tsang, Jin-Au Kong

In previous publications,\textsuperscript{1-3} we determined interference patterns produced by a horizontal electric dipole laid on the surface of a stratified isotropic medium. Other work in the past has been reported for stratified earth with anisotropic conductivity.\textsuperscript{4-7} In this report, we apply the same techniques to calculate electromagnetic fields by using an anisotropic model. We assume that both permeability and permittivity are uniaxial with the optical axis in the $z$ direction, perpendicular to the planes of stratification.

First, we calculate the radiation patterns produced by a horizontal electric dipole pointing in the $\hat{x}$ direction lying on the surface of a half-space uniaxial medium. The results indicate that in the broadside direction ($\hat{y}$ direction), the anisotropy in permittivity does not affect the radiation patterns because the radiated fields are ordinary waves with electric field vector perpendicular to $\hat{z}$. By the same argument, radiation...
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Fig. IX-5. Interference patterns for a two-layer medium with the geometric optics approach.

Fig. IX-6. Interference pattern for a four-layer medium using FFT.

Patterns in the end-fire direction are identical to the isotropic case but not affected by the anisotropy in permeability. Using the parameters $a$ and $b$ to characterize uniaxial anisotropy in permittivity and permeability, we see that power couples more into the half-space medium when the medium is positive uniaxial ($a > 1$, $b > 1$). The opposite is true when the medium is negative uniaxial ($a < 1$, $b < 1$).

To illustrate the results with geometric optic and mode approaches, we treat the
case with uniaxial permittivity tensors only. We consider a slab with thickness d on
top of a half space. The electric-field component in the end-fire direction, \(|E_x|\), is
plotted as a function of distance. In the geometric optics approach, the asymptotic field
solution is calculated as

\[
E_\rho = \frac{i \omega_\mu}{4\pi} \left\{ \frac{2}{\omega_\rho^2} e^{ik_\rho} + \frac{ak_1^2}{k_\rho^2} \frac{\epsilon}{\epsilon_1} \exp(i\sqrt{\alpha} k_1 \rho) \right. \\
\left. - \frac{1}{k^2} \sum_{m=1}^{\infty} \sqrt{\alpha} k \kappa_{lz} Y_0 \exp(\frac{ik_\rho^m}{\epsilon_1} \exp(i\frac{\sqrt{\alpha} R_m}{\epsilon_1} \exp(i\frac{\sqrt{\alpha} R_m}{\epsilon_1}) \right) \}
\]

In Fig. IX-5, we show interference patterns for \(|E_\rho|\) for \(a = 0.8\) as compared with the
isotropic case for \(a = 1\).

For thinner layers, the residue method is applied. In the case of very thin layers
where no modes are excited, the contributions came from the two branch cuts and the
result is

\[
E_\rho = \frac{i \omega_\mu}{4\pi} \left\{ \frac{2}{\omega_\rho^2} e^{ik_\rho} + \frac{2k^2}{\omega_\rho^2} \left( \frac{k_\rho Y_0}{1 + S_{01} \exp(2ik_\rho d)} \right) \frac{\epsilon}{\epsilon_2} \exp(i\sqrt{\alpha} k_2 \rho) \right. \\
\left. \frac{1}{k_\rho} \sum_{m=1}^{\infty} \sqrt{\alpha} k \kappa_{lz} Y_0 \exp(\frac{ik_\rho^m}{\epsilon_1} \exp(i\frac{\sqrt{\alpha} R_m}{\epsilon_1} \exp(i\frac{\sqrt{\alpha} R_m}{\epsilon_1}) \right) \}
\]

For a multilayer medium, we use the fast Fourier transform (FFT) method to find
the interference patterns. Figure IX-6 illustrates a four-layer stratified medium. We
plot the \(H_z\) components. The permittivities are assumed to be isotropic. The layered
media are all assumed to have the same anisotropic factor \(b\). To assume fast conver-
gence, the receiving antenna is at a height of 0.05 wavelength above ground at a fre-
quency of 8 MHz. The result is compared with the isotropic case.

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