XVI. DIGITAL SIGNAL PROCESSING

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RESEARCH OBJECTIVES AND SUMMARY OF RESEARCH

1. Speed Transformation of Speech

U.S. Navy Office of Naval Research (Contract N00014-67-A-0204-0064)

Alan V. Oppenheim, Michael R. Portnoff

One aspect of our effort to apply digital signal processing techniques to speech is the development of a high-quality system to implement speed transformations on speech. Our goal is to devise a scheme whereby a high-fidelity speech signal is modified so that it is perceived as identical to the original signal in all aspects except that it appears that the rate of utterance has been changed.

A system to perform such high-quality speed transformations has numerous applications, such as increasing the rate of speech in "talking books" for the blind and generating speech stimuli for psychoacoustical perception experiments. In contrast, it is desirable to decrease the rate of speech in applications such as language learning, speech therapy, and the transcription of poor-quality recordings of speech. Furthermore, we sometimes wish to introduce adaptive changes in the rate of utterance. For example, in certain communication systems it is desirable to fit a given utterance into a time slot of given duration.

We are considering three approaches for implementing high-quality speed transformations. The first approach is based on a linear-predictive analysis of speech. The predictor coefficients are used to generate a linear time-variant least-squares inverse filter and an analogous time-variant acoustic tube synthesis filter for the speech waveform. The original speech signal is played through the inverse filter to produce an error signal. The error signal is appropriately modified and then played through the synthesis filter to produce the modified speech signal.

The second approach is based on short-time Fourier analysis. The short-time Fourier spectrum is estimated for the speech waveform as a function of frequency and time. The time rate of change of this spectrum is varied and the modified spectrum is used to synthesize the modified speech signal.

The third approach is a combination of the two previous schemes. Here the analysis and synthesis of the speech is performed by using the methods of linear prediction, and the modification of the error signal is accomplished by short-time Fourier analysis and synthesis.

2. Enhancement of Degraded Speech

U.S. Navy Office of Naval Research (Contract N00014-67-A-0204-0064)

Ronald H. Frazier, Alan V. Oppenheim, Siamak Samsam,

W. Hans Schuessler, Elliot Singer

We have started a new set of projects related to the enhancement of degraded speech in which the initial work is to apply variable comb filtering to track the pitch harmonics of the desired speech. An earlier investigation of this approach was carried out several years ago. Our present efforts are directed toward refining the technique and developing ways of extracting the pitch of the desired speaker automatically in a background of noise or multiple speakers. As one aspect of this work, an algorithm has been developed by which digital comb filters can be designed with passbands of arbitrary widths centered at arbitrary frequencies. The designs are obtained by converting a lowpass prototype filter by means of a general all-pass transformation. We have also developed techniques for designing the appropriate all-pass transfer function. A second aspect of this project is concerned with exploring and evaluating a class of nonlinear filtering techniques in which linear filtering is applied to a signal transformed to a function of its arc length.

3. Spectral Zeros in Linear Prediction

U.S. Navy Office of Naval Research (Contract N00014-67-A-0204-0064)

Gary E. Kopec, Alan V. Oppenheim

In recent years the technique of linear predictive coding (LPC) has received considerable attention as a basis for analysis-synthesis telephony, but no really high-quality vocoder has yet emerged. There are two possible explanations for this. Perhaps LPC, based on an all-pole model of speech production, suffers because it cannot efficiently represent the effects of spectral zeros. Alternatively, it may be that the problem arises from poor estimation of vocal tract excitation: the so-called voiced/unvoiced decision.

We aim to address the first possibility by performing series of experiments on artificially synthesized speech of known excitation and controlled spectrum. By eliminating all errors of pitch extraction, we can focus on the perceptual impact of explicit zero coding in a very "clean" environment. Basically, we shall compare all-pole LPC analysis with various schemes of pole-zero estimation. Subjective reproduction fidelity will be used as a performance measure.

4. Implementation of a Programmable Digital Filter

U.S. Navy Office of Naval Research (Contract N00014-67-A-0204-0064)

Anthony P. Holt, Alan V. Oppenheim, Ernest Vincent

We are completing the implementation of a nonrecursive programmable digital filter for use as a preprocessing filter for audio input to a digital computer. The filter is 64^{th} order in a nonlinear phase configuration and 128^{th} order for linear phase. During the coming year we plan to investigate the design and implementation of a digital filter for which the cutoff frequency is varied by changing a single parameter. The basic concept for implementation of variable cutoff digital filters of this type is closely related to the theory of digital frequency warping.

5. Seismic Data Analysis Using Homomorphic Filtering

U.S. Navy Office of Naval Research (Contract N00014-67-A-0204-0064)

Alan V. Oppenheim, José M. Tribolet

We have recently begun a research program to apply homomorphic deconvolution techniques to seismic data analysis. Our work has been concerned with efficient techniques for computation of the complex cepstrum, and to investigating the potential for combining homomorphic techniques with the techniques of predictive deconvolution. During the coming year we shall investigate some algorithms using both real and synthetic seismic data.

6. Two-Dimensional Digital Filter Design

National Science Foundation (Grant GK-31353)

David B. Harris, Russell M. Mersereau,

Alan V. Oppenheim, Thomas Quatieri

We have been exploring a number of techniques for the design of two-dimensional digital filters. Among these are use of transformations as suggested by McClellan, equiripple Finite Impulse Response (FIR) designs using the first algorithm of Remez, and design of Infinite Impulse Response (IIR) filters whose transfer functions have separable denominators.

The McClellan transformation technique converts a one-dimensional linear phase FIR filter into a two-dimensional linear phase FIR filter, by substituting a function of two frequency variables for the single frequency variable of the one-dimensional filter. The frequency response of the two-dimensional filter then depends in a rather simple way upon the frequency response of the one-dimensional filter and upon the specific transformation used. We have developed an algorithm for choosing the transformation parameters efficiently and are now comparing the performance of these filters with other two-dimensional design algorithms.

We have also developed an algorithm for finding the optimum equiripple two-dimensional linear phase approximation to an arbitrary frequency response. This uses the first algorithm of Remez which is modified to reduce the design time. The algorithm appears to be efficient, but further modifications can probably be made to increase the efficiency further. Convergence of the modified algorithm remains to be proved.

We have also been concerned with the design of IIR filters with transfer functions that have separable denominators. Unlike more general IIR filters, with such filters, it is straightforward to guarantee the stability and implementation. We have found an algorithm for designing these filters, but their performance remains to be compared with more general IIR and FIR filters.

7. Analysis and Design of Digital Filter Structures

National Science Foundation (Grant GK-31353)

Ronald E. Crochiere, Alan V. Oppenheim

Our research on digital filter structures involves the investigation of general digital network properties and the analysis of specific digital filter structures. During the past six months, a doctoral thesis, entitled "Digital Network Theory and Its

Application to the Analysis and Design of Digital Filters," was completed by Ronald Crochiere. This research has produced new major results, including a matrix formulation for digital networks, an approach to digital filter sensitivity analysis, the introduction of precedence relations for evaluating parallelism in the implementation of digital filters, and the development of a computer-aided network analysis program. During the coming year, our research in the analysis of digital filter structures will focus primarily on an attempt to understand the limit-cycle behavior of digital filters.

8. Small Signal Processor

U.S. Navy Office of Naval Research (Contract N00014-67-A-0204-0064)

Jonathan Allen

Design of a small signal processor (SSP) has been completed, and all parts obtained. Layout of the large, 24×24 multiplier (involving 160 arithmetic-logic unit packages) is complete, and the remaining layout will be facilitated by computer layout and wire-list programs. Construction should be complete during the first quarter of 1975.

The SSP is unique in that it is a "straight" 3-address design, using two 1024×24 data memories, and a separate 1024×54 program memory. The 24-bit data word length is intended for research applications for which high precision is required. The machine includes a $(X \cdot Y) + Z$ functional unit that can multiply two 24-bit numbers and add a third number (Z) to that result in 120 ns. Extensive shifting and rotating commands are provided, as well as a complete arithmetic-logic unit capability. Unlike many machines of this class, a block transfer DMA facility is provided, the status of which can be monitored at any time by the program. A timing clock, and direct D/A connections, are also included. The logic is all in ECL 10k.

We believe that this machine will provide many features that are missing in small signal-processing computers. The ease of programming, 24-bit word length, relatively large memory, and convenient I/O interface combine with high speed to provide a very useful machine. Additionally, the console provides the ability to display the contents of any specified location while the machine is running, or to halt whenever that location is addressed. We expect the SSP to provide an extremely fast, powerful, and convenient signal-processing capability for laboratory use.

A. COMPARISON OF DIGITAL FILTER STRUCTURES ON THE BASIS OF COEFFICIENT WORD LENGTH

National Science Foundation (Grant GK-31353)

Ronald E. Crochiere

1. Introduction

In this report we compare several different classes of recursive filter structures which have been proposed recently by numerous authors. Comparison is made primarily on the basis of coefficient word length and on the number of multiplies and adds required for various structures. Analyses of these structures were performed with the aid of the general-purpose computer-aided digital network analysis package (CADNAP).[1]

For the comparison[2] we chose an eighth-order elliptic bandpass filter (Fig. XVI-1).

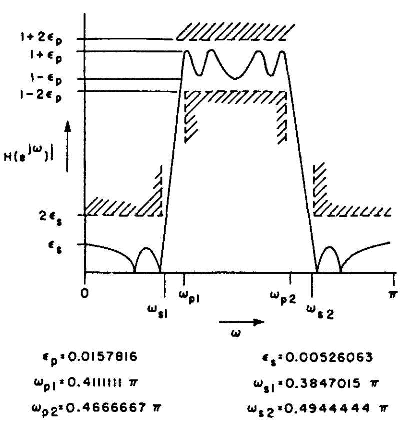


Fig. XVI-1.
Filter design specification for the eighth-order bandpass example.

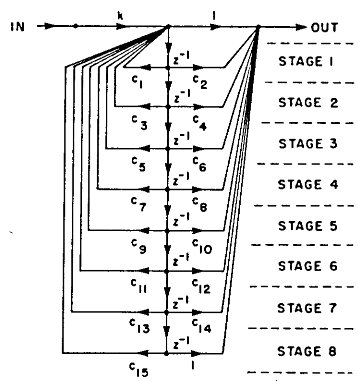
Elliptic filters are widely used, and hence we are able to show that for certain classes of filter designs, such as elliptic and bandpass transformed designs, the number of multiplies (in some classes of structures) can be reduced by taking advantage of the properties of these designs.

2. Filter Structure Examples

Thirteen different recursive filter structures were chosen for comparison (see Table XVI-1). Although these are not all of the known structures that are possible, they were chosen to give a typical cross section of the methods and philosophies for recursive filter structure synthesis. It is hoped that this comparison will provide further insight into promising methods for further research and thereby stimulate investigation in these areas.

We examined first the well-known direct form II structure (sometimes referred to as the "canonic form"), the cascade form structure, and the parallel form structure, where the cascade and parallel forms are implemented with first- and/or second-order direct form II sections. These three classes of structures, in various forms, were the first types of structures to be considered in digital signal processing. Their properties have been studied extensively and are well known.[3-6]

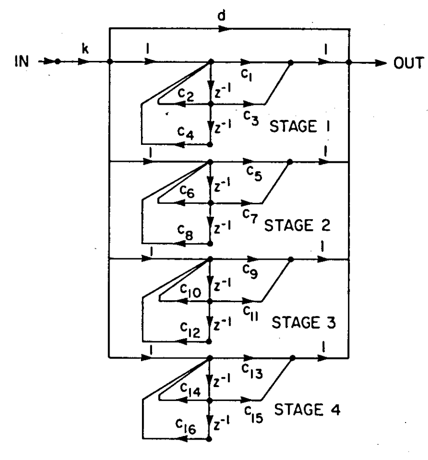
The direct form structures are synthesized from the ratio of polynomials comprising the system function and the coefficients in the structure take the values of the coefficients in these polynomials. Figure XVI-2 shows the direct form II structure for the



$k = 0.005656462366$

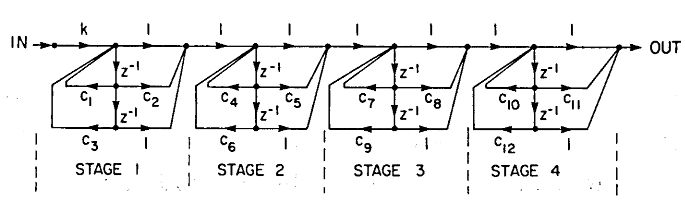
$c_1 = 1.479757145$ $c_2 = -1.387818861$
$c_3 = -4.548129731$ $c_4 = 3.990834175$
$c_5 = 4.352241828$ $c_6 = -3.801090558$
$c_7 = -6.802822262$ $c_8 = 5.978059098$
$c_9 = 4.094876357$ $c_{10} = -3.801090558$
$c_{11} = -4.026604318$ $c_{12} = 3.990834175$
$c_{13} = -0.7833447265$ $c_{14} = -1.387818861$
$c_{15} = 1.232007442$

Fig. XVI-2.
Direct form II structure (Example 1, Table XVI-1).



$k = 0.10000$ $d = 0.07220910762$
$c_1 = 0.2034508401$ $c_9 = -0.2052671178$
$c_2 = 0.2855823838$ $c_{10} = 0.196901665$
$c_3 = 0.9026252904$ $c_{11} = -0.2413054652$
$c_4 = -0.9116860616$ $c_{12} = -0.9695353354$
$c_5 = 0.182701196$ $c_{13} = -0.1965294023$
$c_6 = 0.4457342317$ $c_{14} = 0.5515388641$
$c_7 = -0.9457352763$ $c_{15} = 0.298888793$
$c_8 = -0.913078673$ $c_{16} = -0.9705899009$

Fig. XVI-3.
Parallel form structure (Example 2, Table XVI-1).



$k = 0.005656462366$
$c_1 = 0.2855823838$ $c_4 = 0.4457342317$ $c_7 = 0.196901665$ $c_{10} = 0.5515388641$
$c_2 = 0.4512591755$ $c_5 = -1.09869455$ $c_8 = -0.0099665157$ $c_{11} = -0.7304169706$
$c_3 = -0.9116860616$ $c_6 = -0.913078673$ $c_9 = -0.9695353354$ $c_{12} = -0.9705899009$

Fig. XVI-4. Cascade (direct form II sections) structure (Example 3, Table XVI-1).

eighth-order bandpass filter specified in Fig. XVI-1. Structure characteristics in this figure and many of those that follow are given in detail in the referenced examples in Table XVI-1. The parallel form structures are synthesized from a partial-fraction expansion of the system function, and the cascade form structures by factoring the numerator and denominator polynomials of the system function into first- and/or second-order polynomials. These structures are shown in Figs. XVI-3 and XVI-4 for the

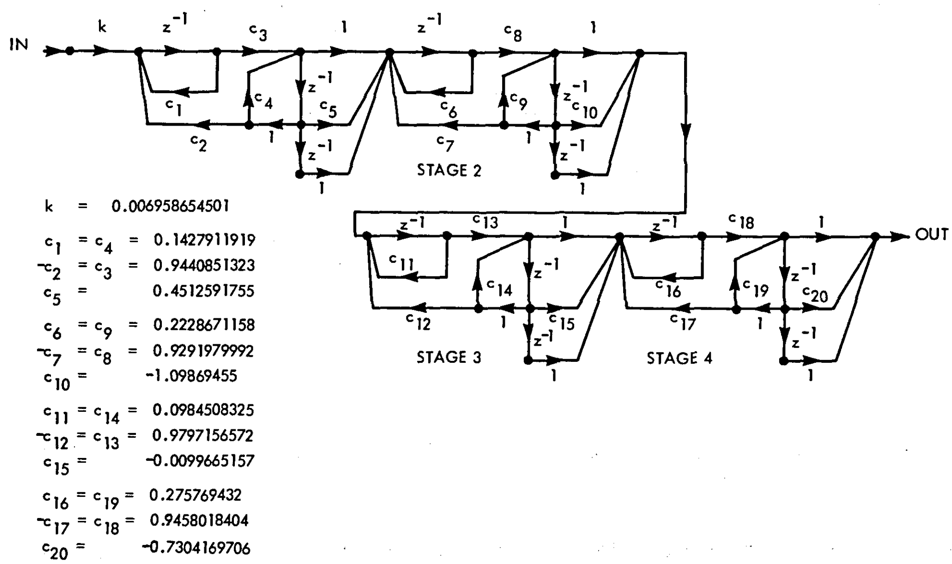


Fig. XVI-5. Cascade (coupled form) structure (Example 4, Table XVI-1).

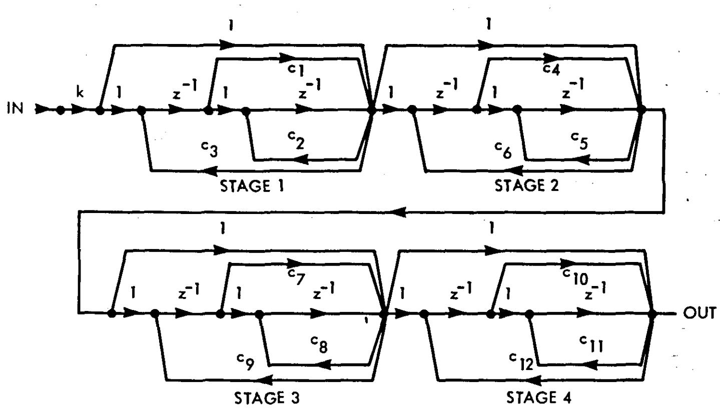
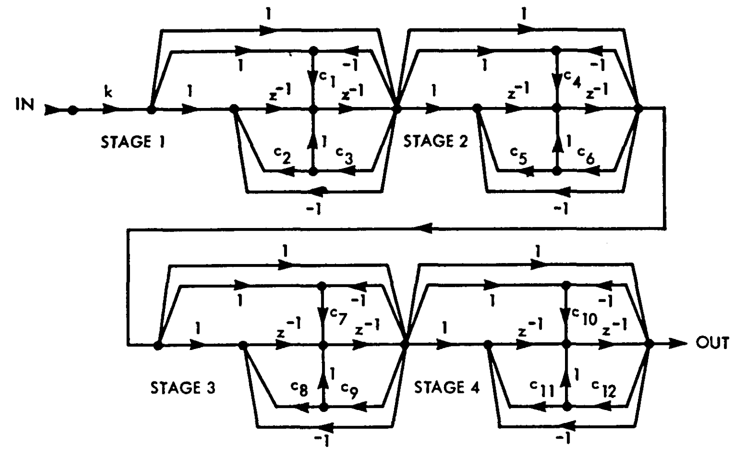


Fig. XVI-6.
Cascade structure with Avenhaus[10] type B sections (Example 5, Table XVI-1).

$k = 0.005656462366$

$c_1 = 0.4512591755$	$c_7 = -0.0099665157$
$c_2 = 0.6969890843$	$c_8 = 0.1872387759$
$c_3 = -0.9116860616$	$c_9 = -0.9695353354$
$c_4 = -1.09869455$	$c_{10} = -0.7304169706$
$c_5 = -0.5574603304$	$c_{11} = -0.157396471$
$c_6 = -0.913078673$	$c_{12} = -0.9705899009$



$k = 0.005656462366$

$c_1 = 0.4512591755$
$c_2 = 0.1198547195$
$c_3 = 0.7368415593$

$c_4 = -1.09869455$
$c_5 = -0.1331188504$
$c_6 = -0.6529603187$

$c_7 = -0.0099665157$
$c_8 = 0.1629691618$
$c_9 = 0.1869351493$

$c_{10} = -0.7304169706$
$c_{11} = -0.1644141906$
$c_{12} = -0.1788781065$

Fig. XVI-7. Cascade structure with Avenhaus[10] type F sections (Example 6, Table XVI-1).

eighth-order bandpass example. In addition to the direct form II sections for the cascade structures, many other possible network configurations for implementing these sections have been proposed.[4,7-10] Three different forms of second-order sections were selected from these possibilities for the eighth-order bandpass example. They are the coupled form sections[4,11] and the network configurations B and F proposed by Avenhaus[10] (see Figs. XVI-5, XVI-6, and XVI-7).

In the synthesis of bandpass filters such as the example that we are considering we can first synthesize a lowpass prototype of the filter and then transform this function to a bandpass function.[12] In this transformation we replace all functions z^{-1} with the all-pass mapping $g(z^{-1})$:

$$z^{-1} \underset{\text{replaced by}}{\Longrightarrow} g(z^{-1}) = -\frac{z^{-1}(z^{-1}-\alpha)}{1-\alpha z^{-1}}, \tag{1}$$

where α is the mapping parameter. This same process can be incorporated into the synthesis of filter structures by first synthesizing an appropriate lowpass filter and then replacing all delays in the structure with a circuit (Fig. XVI-8) that realizes the mapping in (1). The parameter α determines the bandpass center frequency ω_o to which the dc value of the lowpass filter function is mapped and can be expressed[1] as

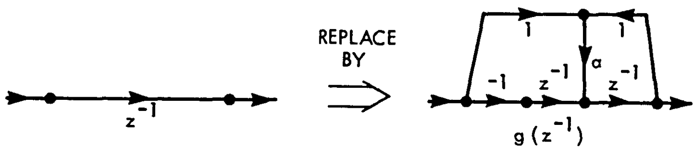


Fig. XVI-8. Lowpass to bandpass mapping.

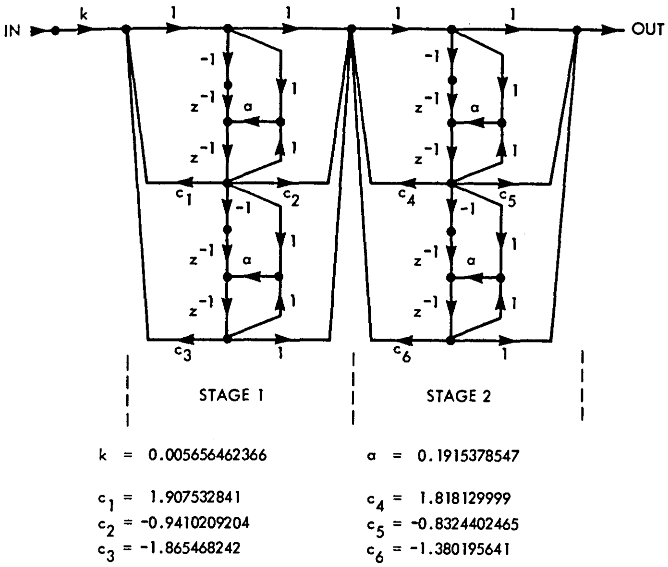


Fig. XVI-9.

Cascade (direct form II) structure synthesized with the bandpass mapping transformation (Example 7, Table XVI-1).

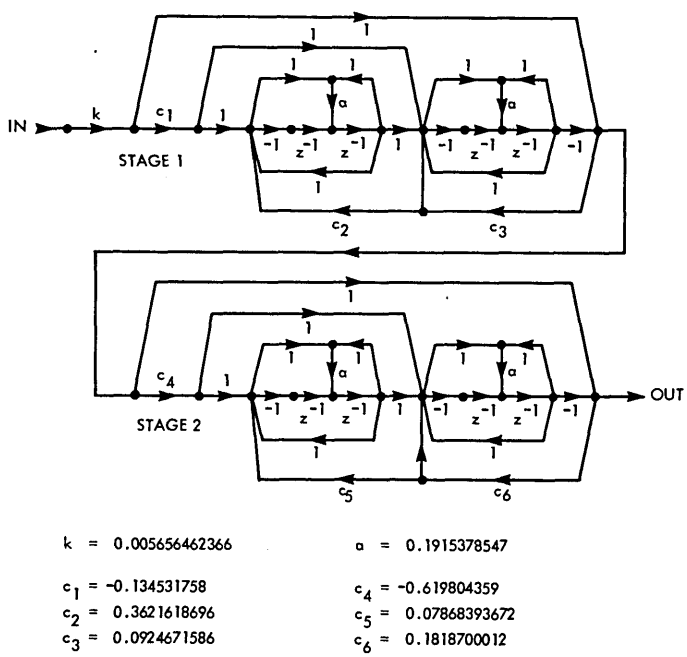


Fig. XVI-10. Cascade (Avenhaus[10] circuit E) structure synthesized with the bandpass mapping transformation (Example 8, Table XVI-1).

$$\cos \omega_0 = a. \tag{2}$$

This synthesis approach is illustrated in Figs. XVI-9 and XVI-10. The example in Fig. XVI-9 was synthesized from a fourth-order cascade structure with direct form II sections. The example in Fig. XVI-10 was synthesized from a fourth-order cascade structure with second-order sections in the form of circuit E proposed by Avenhaus.[10]

Another way of representing the system function of a filter is in terms of continued fraction expansions. Mitra and Sherwood[13] demonstrated that 4 basic forms of these expansions are possible and that from these expansions we can construct a class of structures. Two of these structures, type IA and type IB, were synthesized for the eighth-order bandpass example (Figs. XVI-11 and XVI-12). The continued fraction expansion of type IIA was not realizable for this example and the structure corresponding to type IIB is noncomputable.

Ladder or lattice structures have received considerable attention recently.[14-23]

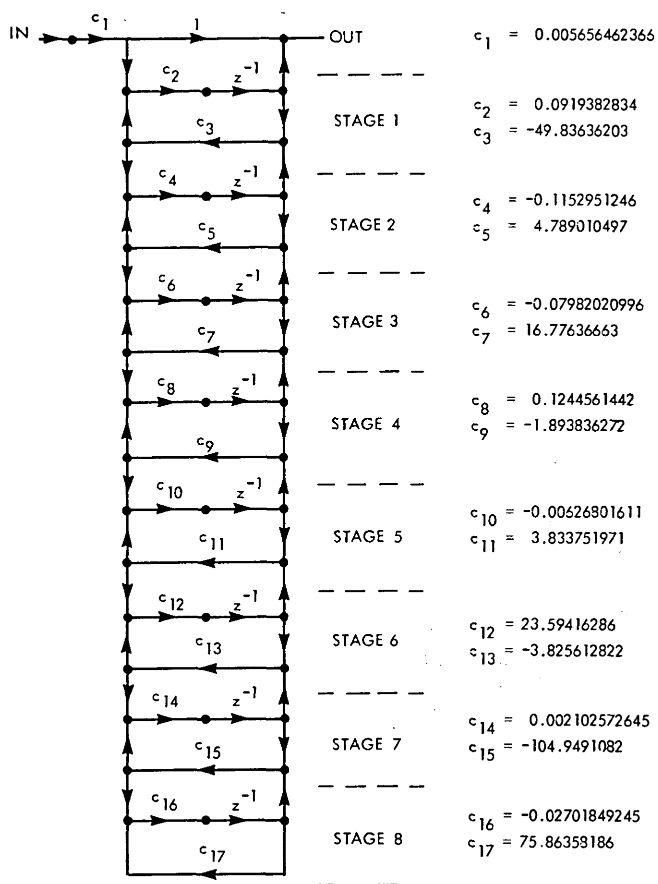


Fig. XVI-11.

Continued fraction expansion structure type IA (Example 9, Table XVI-1).

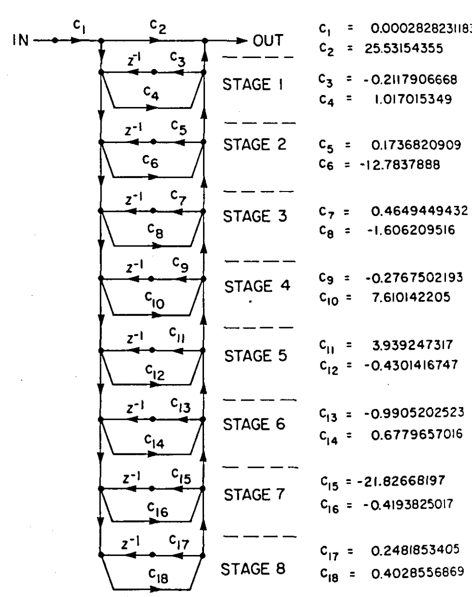
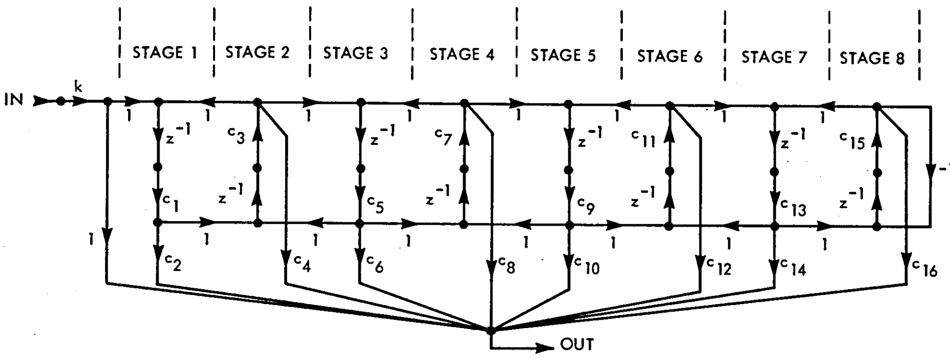


Fig. XVI-12.

Continued fraction expansion structure type IB (Example 10, Table XVI-1).

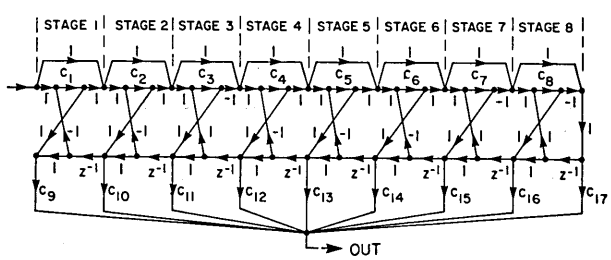
These structures are not synthesized by representing the system function in terms of some form of factorization or expansion. Instead, the mechanisms behind the synthesis procedures for these structures are closely related to the two-port network synthesis techniques associated with conventional analog RLC circuits.

The ladder structure proposed by Mitra and Sherwood[14] was synthesized and is shown in Fig. XVI-13. This structure has a ladder configuration only in terms of the implementation of the poles of the system function. The zeros are obtained as a linear combination of the internal variables or state variables in the ladder.



k = 0.005656462366
c_1 = 435.7362441
c_2 = 0.000210995263
c_3 = -0.00201410002
c_4 = 0.4799924366
c_5 = 12.16053691
c_6 = 0.03838502472
c_7 = -0.06547062948
c_8 = 0.5519655223
c_9 = 1.762511418
c_{10} = 0.3488742835
c_{11} = -0.3958370858
c_{12} = -0.8478865247
c_{13} = 1.085950607
c_{14} = -0.1967175203
c_{15} = -1.479757145
c_{16} = -0.1358646584

Fig. XVI-13. Ladder structure proposed by Mitra and Sherwood[14] (Example 11, Table XVI-1).



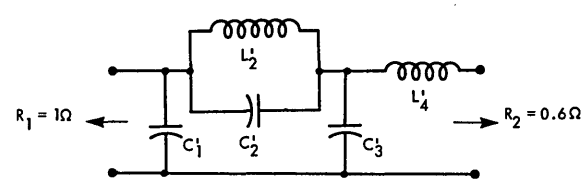
c_1 = 0.7833447265
c_2 = -0.1885428229
c_3 = 0.9843741951
c_4 = -0.1920962931
c_5 = 0.9936376827
c_6 = -0.1890252997
c_7 = 0.9914182038
c_8 = -0.1964790194
c_9 = 0.005656462366
c_{10} = 0.0002916124024
c_{11} = -0.001699657657
c_{12} = -0.06980454614
c_{13} = 0.2031623313
c_{14} = 0.01061747099
c_{15} = -0.01641846145
c_{16} = -1.031187912
c_{17} = 0.8446328267

Fig. XVI-14. Ladder structure proposed by Gray and Markel[15] (Example 12, Table XVI-1).

A second type of ladder structure, synthesized according to methods proposed by Gray and Markel,[15] is shown in Fig. XVI-14. This structure also has a ladder configuration only in terms of the implementation of the poles of the system function, and the zeros are obtained as a linear combination of the internal "state variables" in the structure.

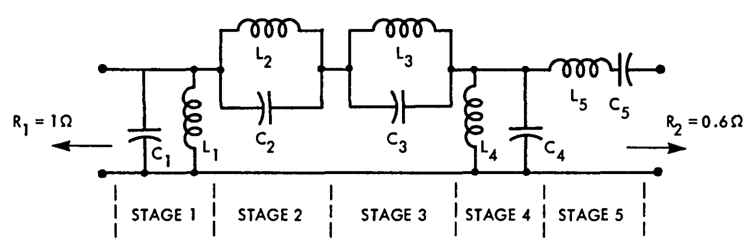
Another procedure for the synthesis of ladder structures was proposed by Fettweis,[16-19] and we have proposed[21] a modification for implementing the zeros in these structures. For this class of structure the synthesis procedure is not only similar to that for analog two-port circuits but, in fact, it is obtained by starting from an LC analog ladder design and appropriately transforming this design into a digital ladder design.

To perform the synthesis of the eighth-order bandpass example, we started with a fourth-order LC lowpass prototype (Fig. XVI-15a). This design was obtained by using a standard set of tables of Saal.[24] This structure was then transformed to the bandpass structure in Fig. XVI-15b, by using conventional transformations for LC ladder designs.[24] An important characteristic of this bandpass LC structure is that the gain of the structure is zero at dc and at infinity. This is in contradiction to the even-order optimal bandpass elliptic designs that require that the gain at these frequencies be finite.



$C'_1 = 1.258$
$C'_2 = 0.1937$
$L'_2 = 1.073$
$C'_3 = 1.965$
$L'_4 = 0.8605$

(a)



$C_1 = 7.05639$
$L_1 = 0.141716$
$C_2 = 1.82318$
$L_2 = 0.371889$
$C_3 = 2.68897$
$L_3 = 0.548493$
$C_4 = 11.02210$
$L_4 = 0.0907268$
$C_5 = 0.207180$
$L_5 = 4.82673$

(b)

Fig. XVI-15. (a) Normalized fourth-order LC lowpass prototype.
(b) Bandpass eighth-order LC ladder structure.

In fact, it is not possible to realize the optimal elliptic designs for even-order filters by using this class of LC structures (odd-order optimal elliptic designs present no problems). In practice, the optimal even-order elliptic designs are transformed to slightly less than optimal designs in a manner such that the largest zero in the frequency response is mapped to infinity and the smallest zero, for bandpass designs, is mapped to dc. This is illustrated in Fig. XVI-16. Because of this problem we could not synthesize an eighth-order digital ladder structure of this type for our specifications. Instead,

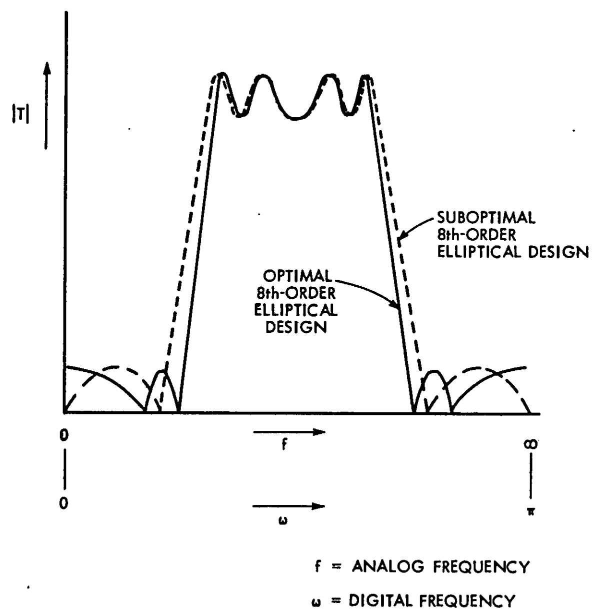
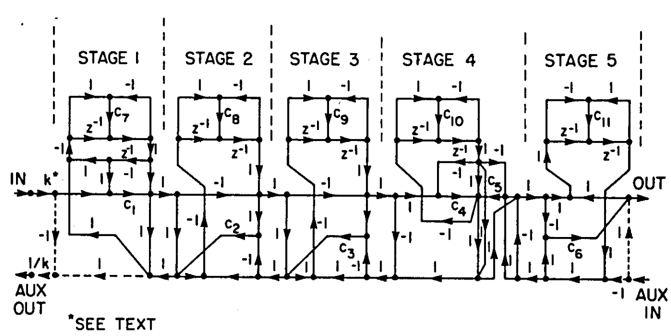


Fig. XVI-16. Comparison of optimal eighth-order elliptic bandpass filter and suboptimal design for LC ladder structure.



k = 1.31194961
c_1 = 0.06502370945
c_2 = 0.2235692269
c_3 = 0.5809284606
c_4 = 0.1626985222
c_5 = 0.007805575218
c_6 = 0.0574957168
c_7 = -0.1915378541
c_8 = 0.0003565321277
c_9 = -0.3698270573
c_{10} = -0.1915378541
c_{11} = 0.1915378541

Fig. XVI-17. Wave digital filter structure (Example 13, Table XVI-1).

we used the nearest suboptimal eighth-order design in the tables of Saal[24] which corresponded to the LC structures in Fig. XVI-15. The equivalent digital design specifications for this analog structure corresponded to

$$\epsilon_p = 0.01623 \qquad \epsilon_s = 0.0066705$$

$$\omega_{p1} = 0.4111111\ \pi \qquad \omega_{s1} = 0.3839368\ \pi$$

$$\omega_{p2} = 0.4666667\ \pi \qquad \omega_{s2} = 0.4952625\ \pi.$$

Using techniques suggested by Fettweis and Sedlmeyer[17,18] and our own modification for implementing zeros,[21] we transformed the analog ladder design of Fig. XVI-15b to the wave digital filter configuration shown in Fig. XVI-17.

3. Comparison of Recursive Filter Structure Characteristics

We shall now interpret our analyses of the recursive filter structures whose characteristics are summarized in Table XVI-1.

a. Coefficient Word Length

The coefficient word lengths of these structures are compared on the basis of the fixed-point statistical word length.[2] They are given for relative errors R = 1 (see Fig. XVI-1) and confidence factors y = 0.95 (i.e., x = 2). The statistical word length for each structure was determined separately for the passband and stop band in order to obtain a more general picture of the overall sensitivity properties of the structure.

For each structure the value i_M used in the statistical word length definition is given, where 2^{i_M} is the power of 2 represented by the most significant bit in the coefficient word length. This value i_M, in general, is different for different structures. It is chosen so that the largest value of the magnitudes of the coefficients in the structure is within the range 2^{i_M+1} and 2^{i_M} (exclusive of the gain constant).

In general, we found for this bandpass example that the direct form structure, the continued fraction form structures, and the ladder structure of Mitra and Sherwood required considerably larger coefficient word lengths. The structures with the lowest coefficient word lengths were the parallel form, the cascade (Avenhaus F) form, and the bandpass transformed structure designed from a cascade (Avenhaus E) lowpass prototype.

The statistical word length for the wave digital filter structure is essentially the same as that of the cascade (direct form II) structure in both the passband and the stop band. When we rounded the coefficients in fixed-point arithmetic, however, we found that in the passband the wave digital filter structure required approximately 3 bits less for a

Table XVI-1. Comparison of recursive filter structure characteristics.

Example	Network	Statistical Word Length (bits)†		i_M	No. Multiplies	No. Adds	Bit · Multiply Product
		Passband	Stop Band				
1	Direct Form II	20.86	10.85	2	16	16	334
2	Parallel Form	10.12	7.95	−1	18	16	182
3	Cascade (Direct Form II)	11.33	5.76	0	13	16	147
4	Cascade (Coupled Form)	11.70	5.78	0	21	20	246
5	Cascade (Avenhaus B)	11.22	5.96	0	13	16	146
6	Cascade (Avenhaus F)	10.57	5.95	0	13	24	137
7	BP Transformed (Cascade, Direct Form)	12.69	7.23	0	11	16	139
8	BP Transformed (Cascade, Avenhaus E)	9.99	6.21	−1	11	20	110
9	Continued Fraction 1A	29.64	21.69	6	17	16	504
10	Continued Fraction 1B	22.61	14.46	4	18	16	408
11	Ladder (Mitra and Sherwood)	28.72	20.97	8	17	16	488
12	Ladder (Gray and Markel)	13.97	10.81	0	17	32	238
13	Wave Digital Filter* (Fettweis)	11.35*	5.67	−1*	12/11*	31	136/125

*See text.

†See Crochiere.[2]

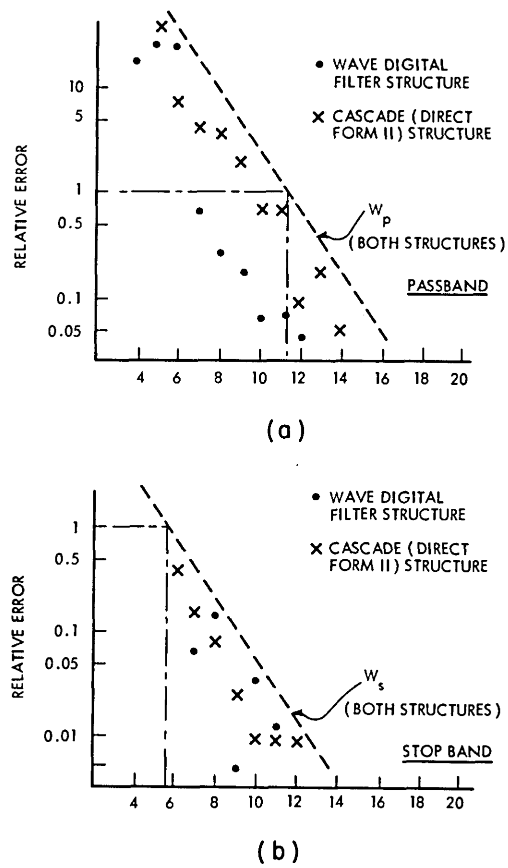


Fig. XVI-18. Comparison of the actual coefficient word length for the wave digital filter structure and the cascade (direct form II) structure: (a) passband, (b) stop band.

given relative error than the cascade structure, provided that the coefficients were not "grossly quantized" so that the relative errors exceeded approximately 1 or 2. This can be observed in Fig. XVI-18a. For the stop band, as seen in Fig. XVI-18b, the word lengths are approximately the same. Thus it appears that the statistical word length for the wave digital filter structure is somewhat overestimated in the passband.

b. Multiplies, Adds, and Bit·Multiplier Products

The number of multiplies and adds required for each structure is tabulated in Table XVI-1. The bandpass transformed structures and the wave digital filter structure could each be implemented with 11 multiplies. For the cascade structures, except the cascade (coupled form) structure, the number of multiplies was 13. The rest of the structures generally required approximately 17 multiplies, which is the number of degrees of freedom necessary to specify the poles and zeros of an arbitrary eighth-order system function with complex poles and zeros in conjugate pairs.

The number of adds in many of the structures was 16. For those structures where we could obtain lower coefficient word lengths than in the cascade (direct form II) structure (Examples 6, 8, and 13), we found that it was generally obtained at the expense of increasing the number of adds and the complexity of the structure.

To obtain a crude estimate of the total amount of computation involved in each structure, we evaluated the bit · multiplier products (see Table XVI-1). The number of bits was taken as the passband statistical word length.

c. Modularity and Complexity of Structures in Terms of Hardware

The modularity in a digital structure may be defined as the topological redundancy in the structure. Most structures exhibited a form of modularity that is identical from stage to stage. The only exception is the wave digital filter structure in which only some of the stages are identical. The bandpass transformed structures exhibited a dual form of modularity. They are identical from stage to stage and also in the bandpass mapping networks which replace the delays in the lowpass prototype.

4. Other Comments

The comparison of the structures made in this report has been primarily on the basis of coefficient word length and on the number of multiplies and adds required for the structures. These attributes are clearly important in the choice of a filter structure, but they are not the only factors to be considered. Of equal importance are the issues of round-off noise, dynamic range, and limit cycle effects. Considerable work has been done in these areas by Oppenheim, Weinstein, Kaiser, Jackson, Liu, and others.[5,6] Although these issues have not been considered in this report, it is important to mention them so that our results may be viewed in proper perspective to the overall issues of filter structure synthesis.

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