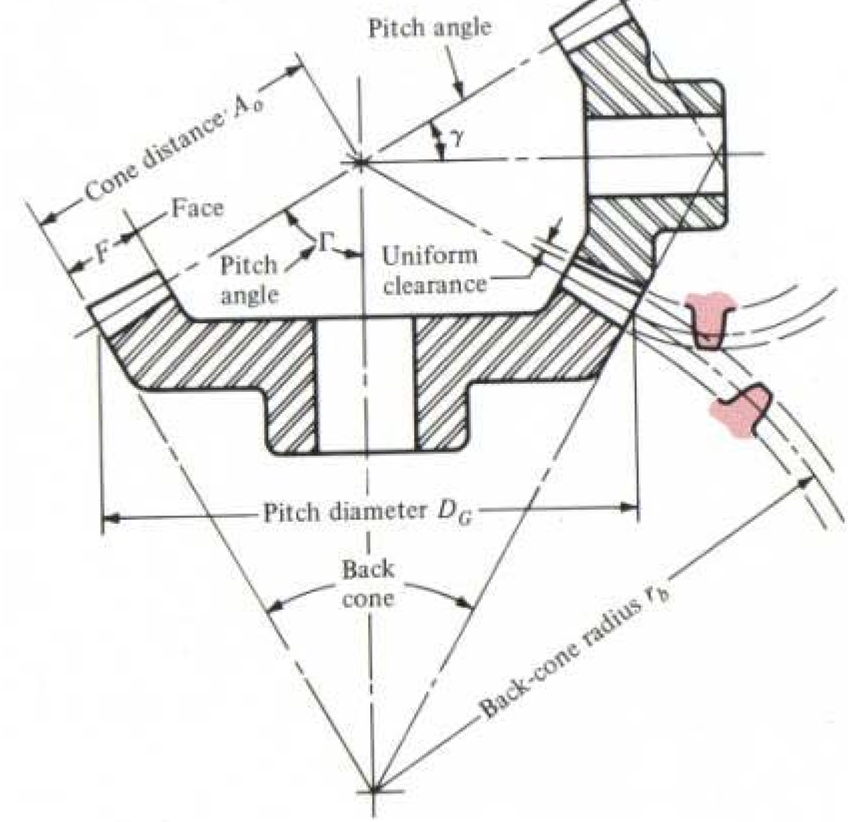
Bevel Gear Geometry

Figure 13-20 frim p.544 of "Mechanical Engineering Design" 5th edition by Shigley and Mischke



Drawings

List of symbols

N_P	Number of teeth on Pinion
N_G	Number of teeth on Gear
D_G	Pitch Diameter of Gear
D_P	Pitch Diameter of Pinion
F	Face Width (Length of one tooth)
γ	Pinion Pitch Angle (Radians)
Γ	Gear Pitch Angle (Radians)
A_o	Cone Distance (Distance from pitch circle to intersection of shaft axes)
r_b	Back-Cone Radius
P	Diametrical Pitch. Teeth per inch of Pitch Diameter (N/D)
p	Circular Pitch. Inches of circumference per tooth (Π/P)

Equations

$\text{Tan } \gamma = N_P / N_G$

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Tooth shape for bevel gears is determined by scaling spur gear tooth shapes along the face width. The further from the intersection of the gear and pinion axes, the bigger the tooth cross sections are. If the tooth face were to extend all the way to the axes intersection, the teeth would approach infinitesimal size there. The tooth cross-section at the largest part of the tooth is identical to the tooth cross-section of a tooth from a spur gear with Pitch Diameter of 2* r_b, or twice the Back-Cone Radius, and with an imaginary number of teeth (N') equal to 2*Π times the Back-Cone Radius (r_b) divided by the Circular Pitch of the bevel gear (p). This method of obtaining the dimensions and shape of the largest tooth profile is known at the "Tredgold" tooth-shape approximation. Refer to the profiles shown near the Back-cone radius dimension in the drawing above.

AGMA Method for Determining Stresses

The standard method for determining induced bending stresses in bevel gears comes from the American Gear Manufacturers Association and is based on the equation below.

$$\sigma = \frac{W_t K_a}{K_v} \frac{P}{F} \frac{K_s K_m}{J}$$

Symbol	**Name**	**Notes / How can we simplify these terms for static loading of normal gears**
σ	Max bending stress in tooth	Compare to gear material strength to determine if the gear will break
W_t	Transmitted tangential load	Taken at large end of tooth $\left(torque \times \left[\frac{pitch\_diameter}{2}\right]\right)$
K_a	Application factor	Accounts for probability of greater-than-design-load occurrences. This is not something we expect, so neglect this, and set K_a = 1
K_v	Dynamic factor	Accounts for dynamic effects and velocity of tooth contact. For static loading, which is an assumption we will make, since we are dealing with relatively low speeds, neglect this term,

		and set K_v = 1
P	Diametrical pitch	N / D
F	Face width	
K_s	Size factor	Accounts for unusually sized gears. Not applicable for normal gears. Set K_s = 1
K_m	Load-distribution factor	Accounts for shaft misalignment and shaft bending. Build your gearbox carefully, so you can neglect this term and set K_m = 1.
J	Geometry factor	Similar to Lewis form factor used for spur gears. Obtain from chart based on number of teeth on gear and pinion.

After setting those factors that do not apply to 1 (see notes above), we can come up with a simplified AGMA equation.

$$\sigma = W_t \frac{P}{F} \frac{1}{J}$$

This equation is highly simplified to allow for fast calculation with minimum table look-ups. It is not for use on critical applications, high speed gears, or gears of unusual materials.

Taken from p.618 of "Mechanical Engineering Design" by Shigley and Mischke

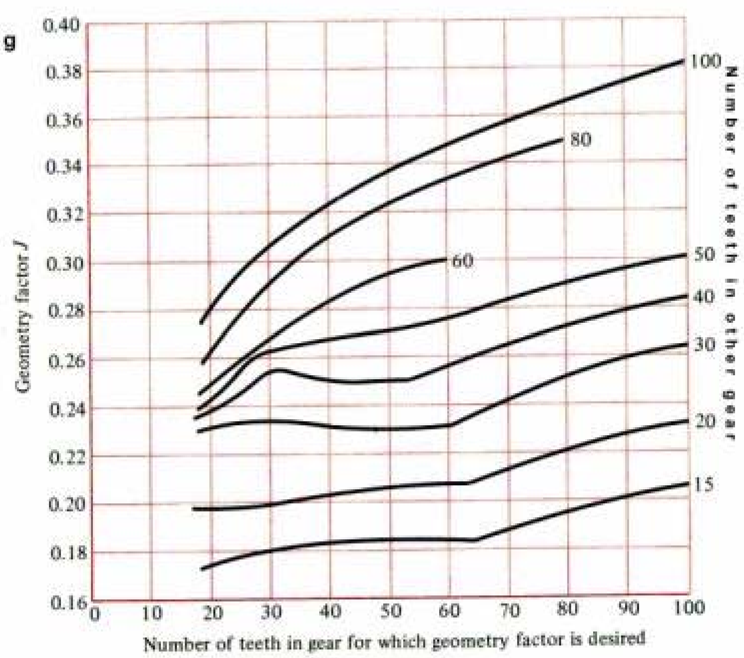
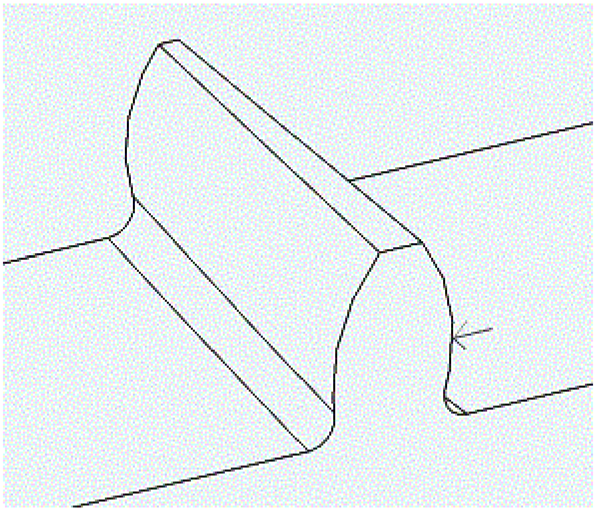


FIGURE 15-5

Geometry factors J for straight bevel gears; these are for a 90° shaft angle, a 20° pressure angle, and a clearance of $c = 0.240/P$ in. *(AGMA 225.1.)*

Beam Bending Method for Determining Stresses

To do this, first we simplify the shape of the tooth. We will assume that it is a beam with an isosceles trapezoidal cross section. If you look at a single tooth, you will see how this is applicable. We will also assume that the forces on this single tooth are equivalent to a resultant force applied at the pitch circle (about halfway up the tooth), at the large end of the tooth. We get this force by dividing the torque on the gear by the pitch radius. The length of our beam (from base to applied force) is the dedendum of the tooth section at the large end of the tooth face.



Knowing the applied force, the beam length, and the cross section of the beam (form which we can calculate the moment of inertia of the cross section), we can determine the maximum stresses experienced at the base of the beam through simple beam bending equations.

It is important to note that in practice, the location and direction of the force applied to each tooth changes as the gear rotates, and there are surface durability considerations, as well as many other factors that govern tooth strength that are not accounted for in this analysis. For design of production gears or critical applications, AGMA standards should be consulted. It is interesting to note though, that in many situations, the beam bending analysis and the simplified AGMA standards produce very similar results.