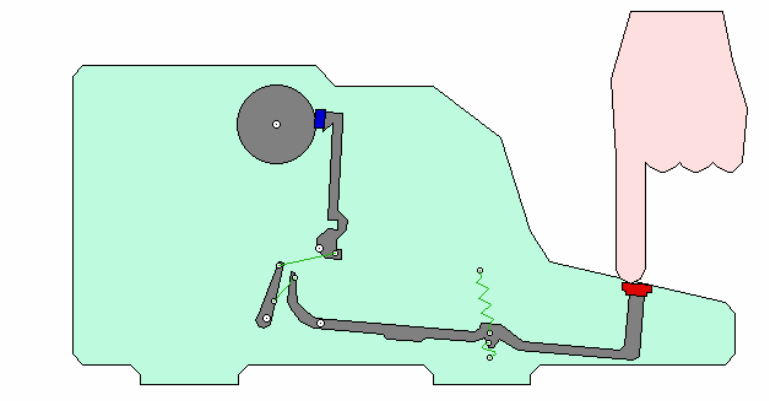
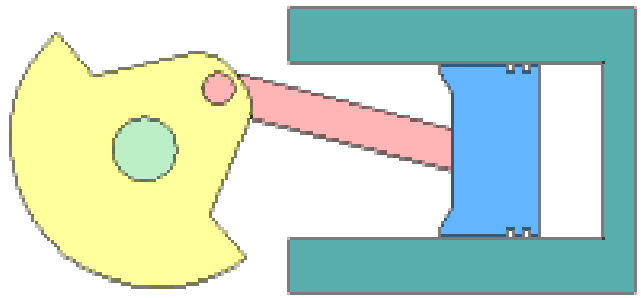
Session #5
2D Mechanisms: Mobility, Kinematic Analysis & Synthesis





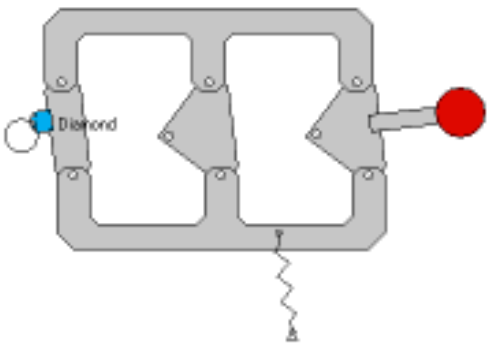
Dan Frey

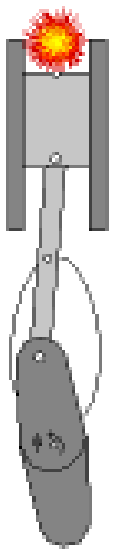
Today's Agenda

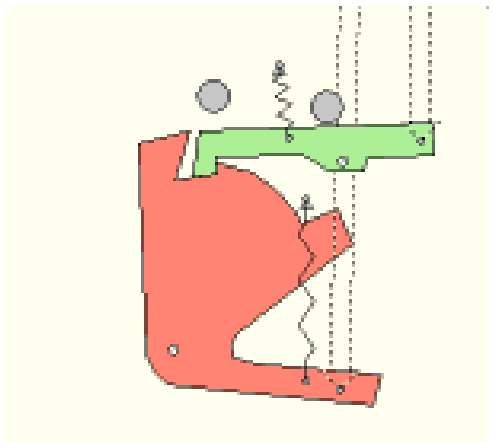
- Collect assignment #2
- Begin mechanisms
  - Mobility / degrees of freedom
  - Some basic mechanisms
  - Some basic synthesis and analysis techniques
  - MathCad preliminaries
  - Homogeneous Transformation Matrices
  - Solving systems of equations
  - Kinematic analysis
  - Mechanism synthesis
- Distribute & discuss assignment #3
- Distribute graded assignment #1

2D Mechanisms

- Where are they found?
- What do they do?







http://workingmodel.design-simulation.com/

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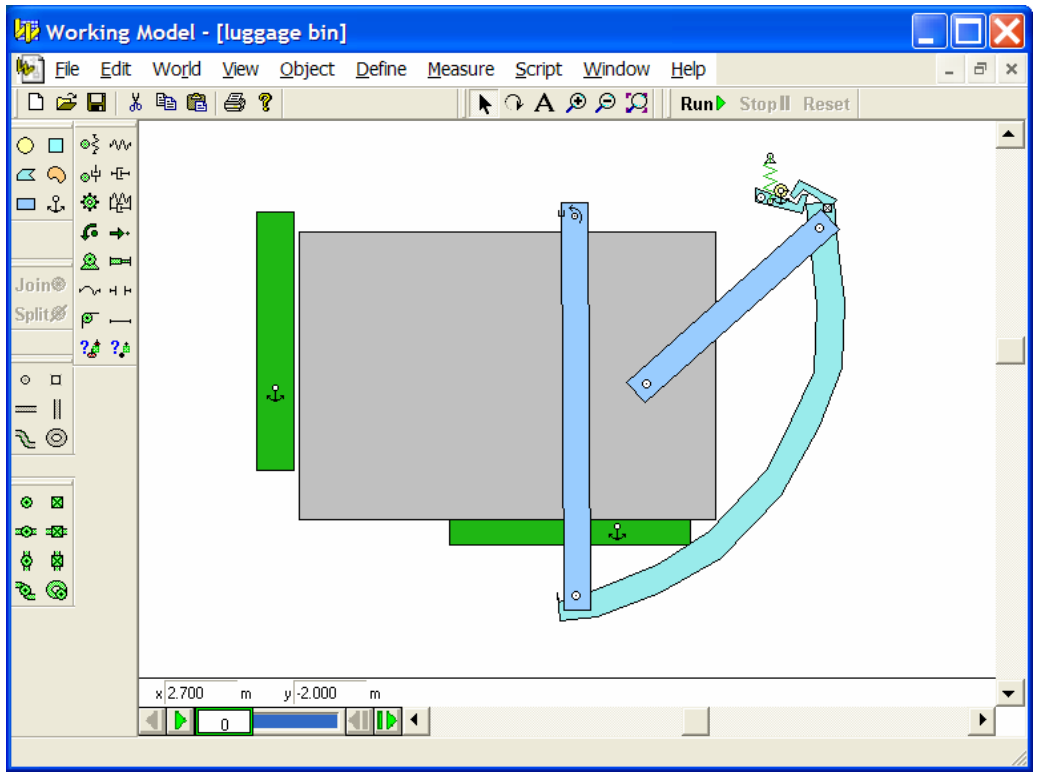

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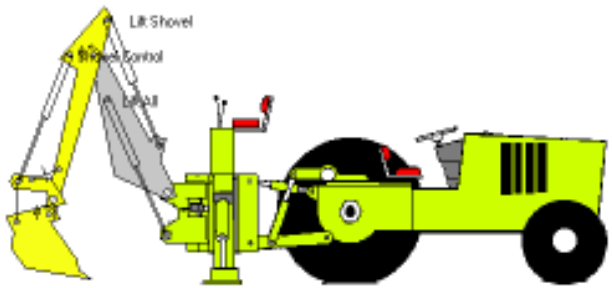

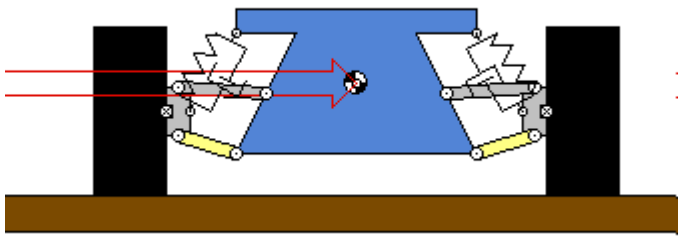

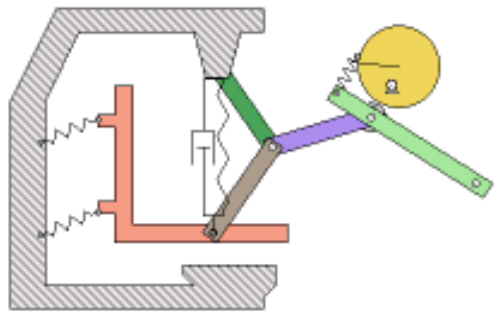

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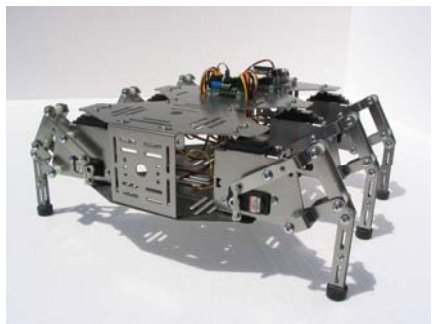
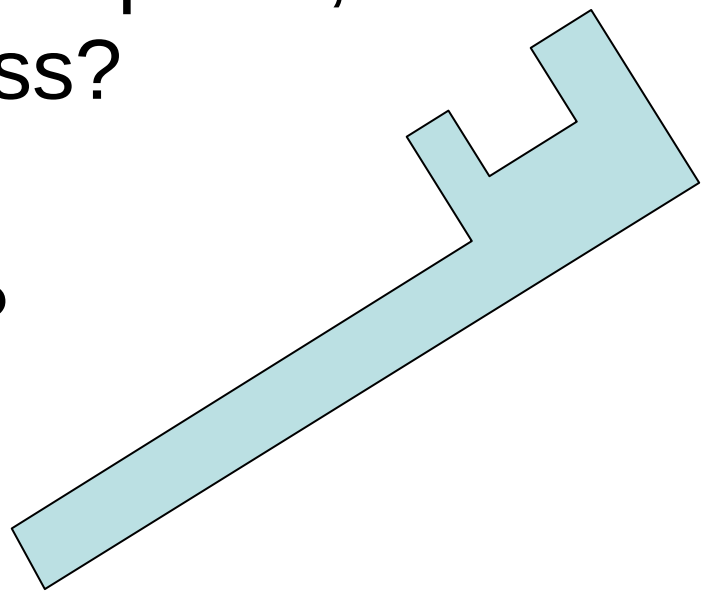
Two Goals in our Study of 2D Mechanisms

- Analysis
  - given a mechanism, predict its behavior
  - DOF, position, velocity, acceleration
- Synthesis
  - given a desired behavior, develop a mechanism



Degrees of Freedom

- DOF = the number of independent parameters required to completely define the entity's positioning
- If a body is constrained to a plane, how many DOF does it possess?
- If a pin joint is added?
- If a sliding joint is added?
- If it rolls without slipping?



Mobility Criteria

- Kutzbach criterion (to find the DOF)

$$F=3(n-1)-2j$$

DOF — F

# of bodies — n

# of joints — j

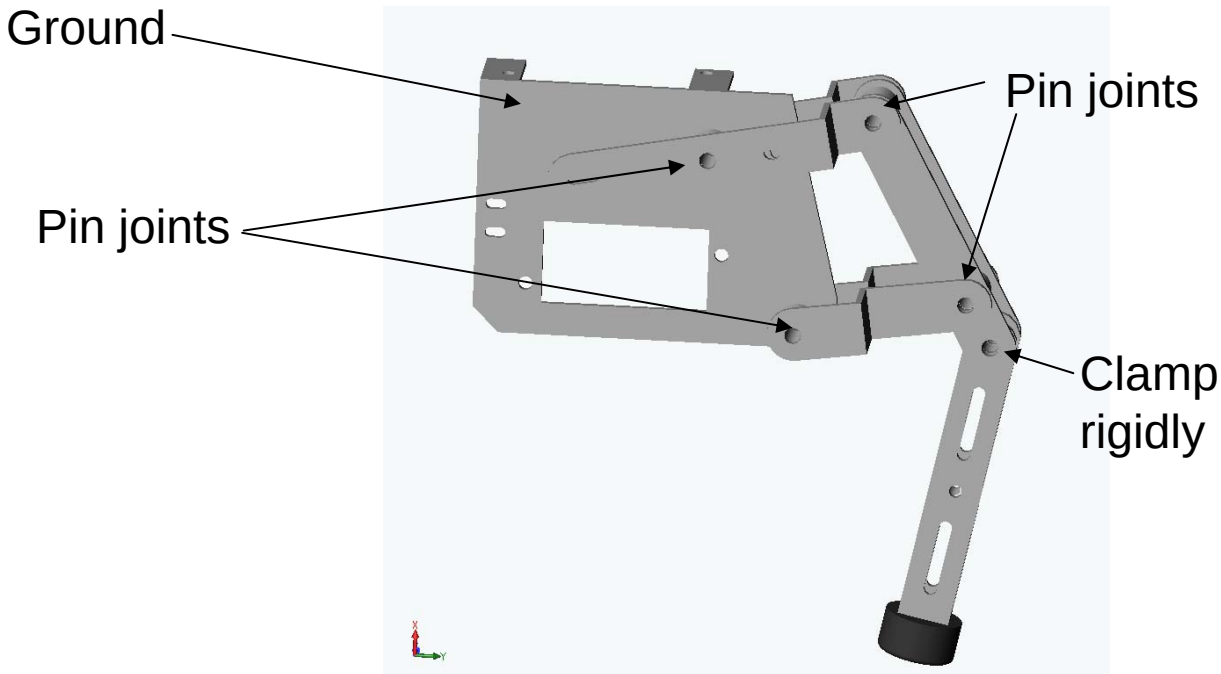
- Grübler criterion (to have a single DOF)

$$2j-3n+4=0$$

Concept Question

- How many degrees of freedom does this mechanism possess?

1)1 DOF
2)2 DOF
3)3 DOF
4)0 DOF



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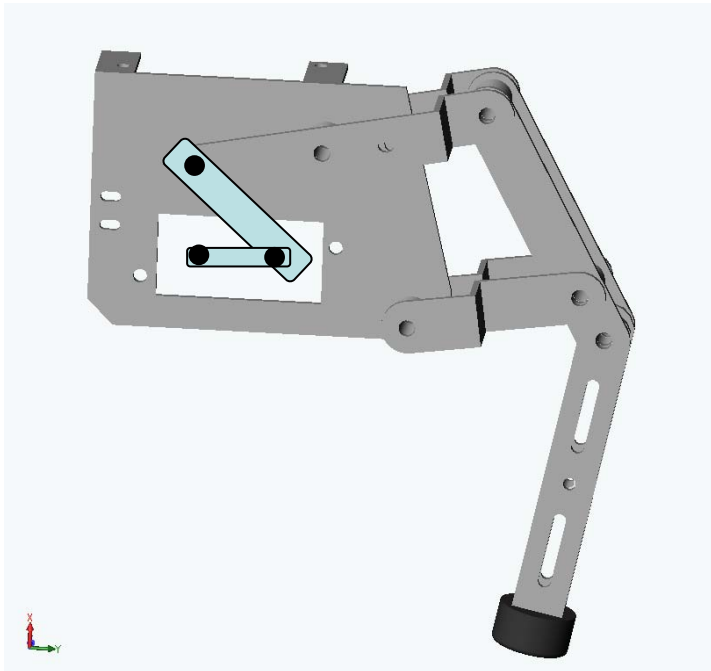


Degrees of Freedom

- How many degrees of freedom does this mechanism possess?

We'll cover
3D mechanisms later

Figure removed for copyright reasons.

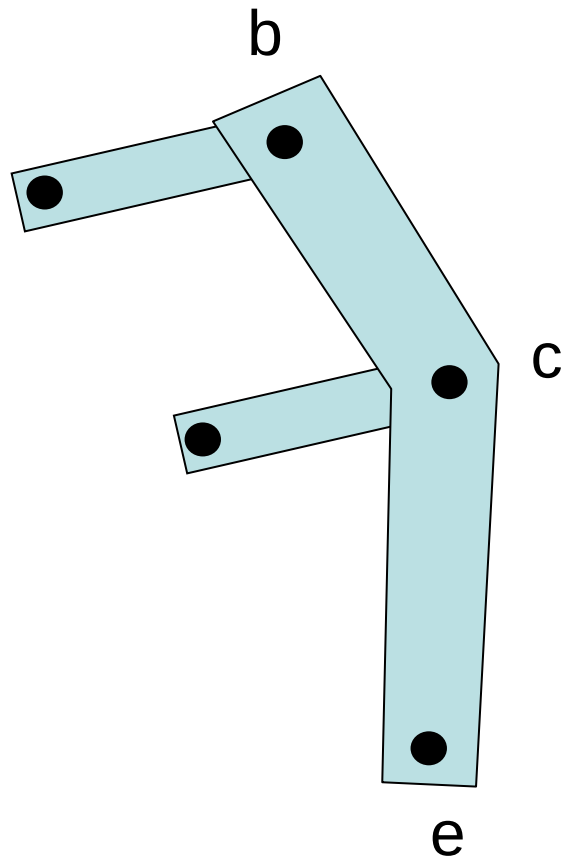
Rear suspension of a Honda Accord

The 4 Bar Mechanism

- Use working Model to arrange a mechanism in which neither link can rotate a full turn
- Use working Model to arrange a mechanism in which rotating the input link a full turn causes the output link to:
  - Oscillate
  - Rotate a full turn
    - Same direction
    - Opposite direction

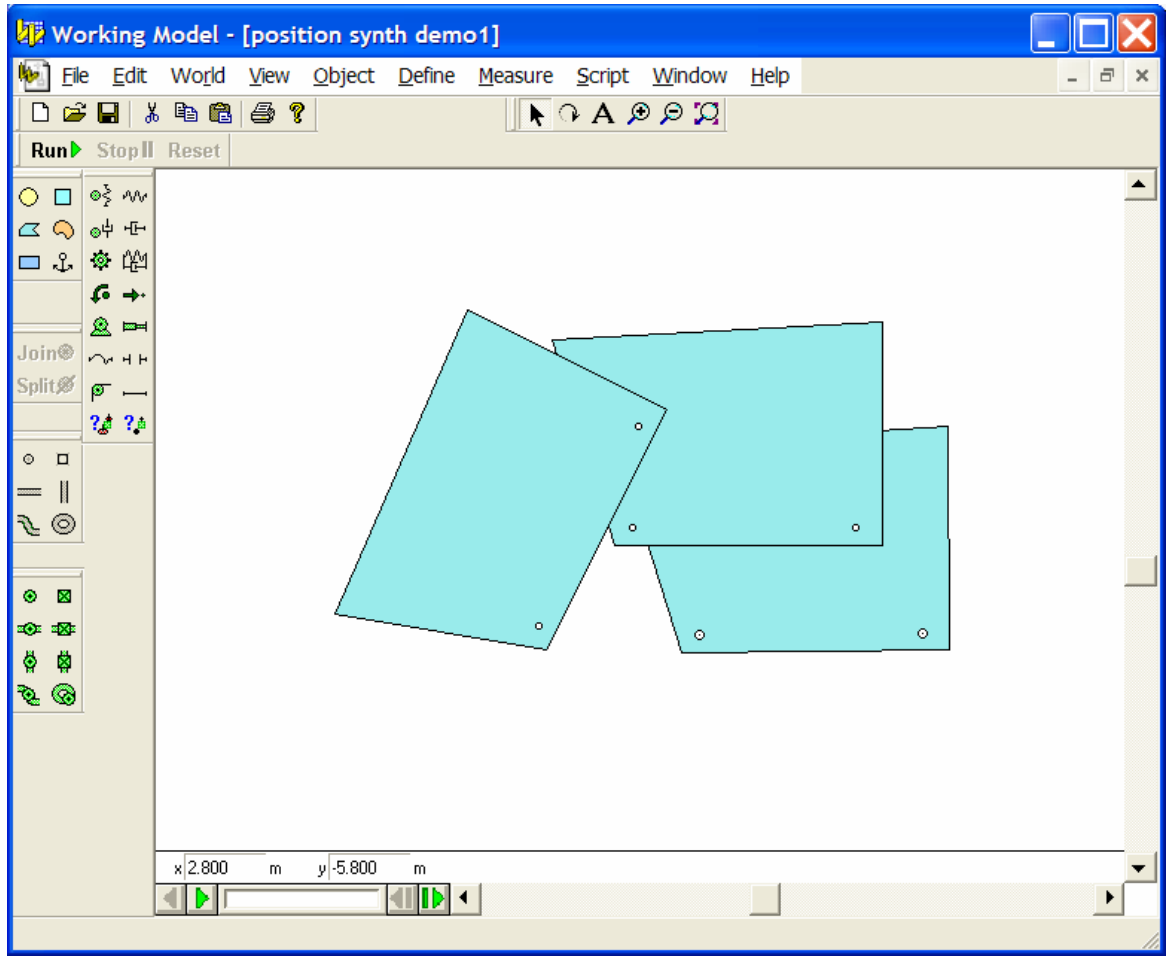
Parallelogram Linkage

- This object is connected to links that are parallel and of equal length
- What can we say (qualitatively) about the motion of the body (bce)?



3 Position Synthesis

- Say we want a mechanism to guide a body in a prescribed way
- Pick 3 positions
- Pick two attachment points
- The 4 bar mechanism can be constructed graphically



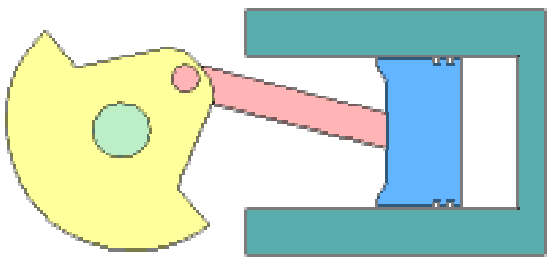
Concept Question

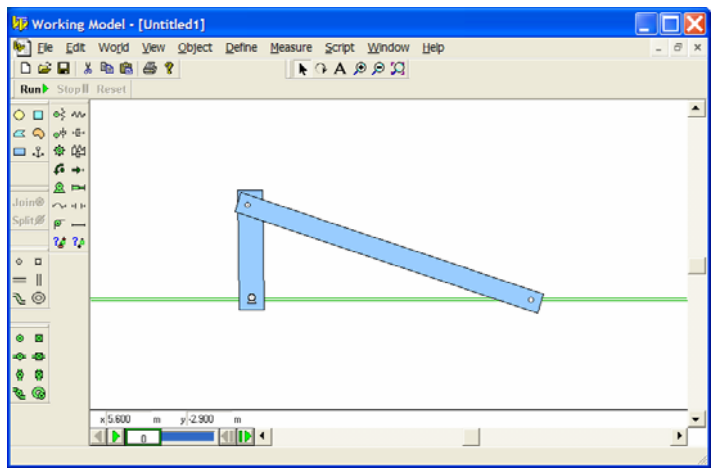
- If you do not specify the attachment point, how many positions can you specify and still generally retain the capability to synthesize a mechanism?

1)3
2)4
3)5
4)>5

Slider-Crank Mechanism

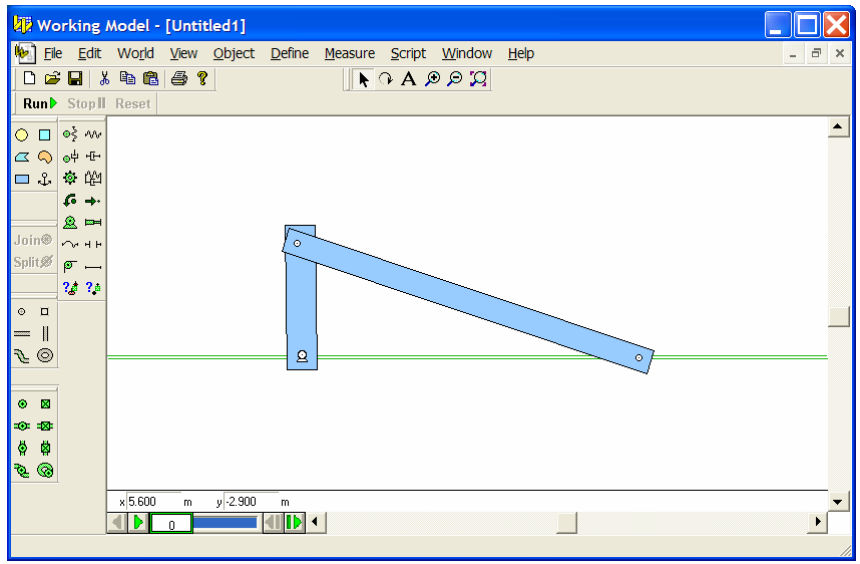
- Kinematically the same (in the limit) as a 4 bar mechanism with a very long output crank





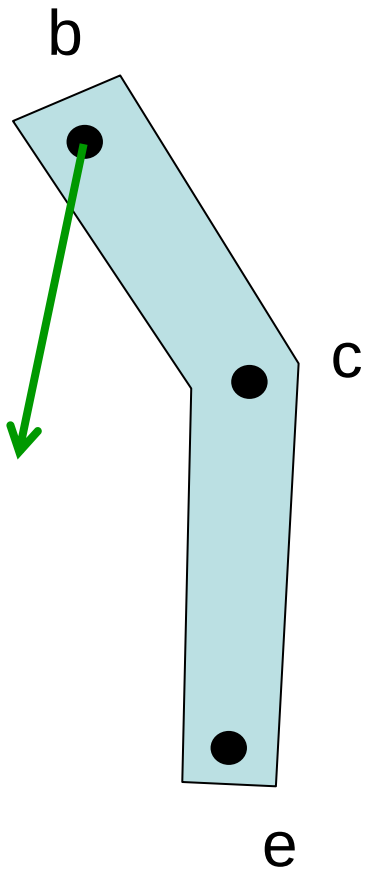
Inversion

- Take a mechanism
- Change which link is “ground”
- A new mechanism results



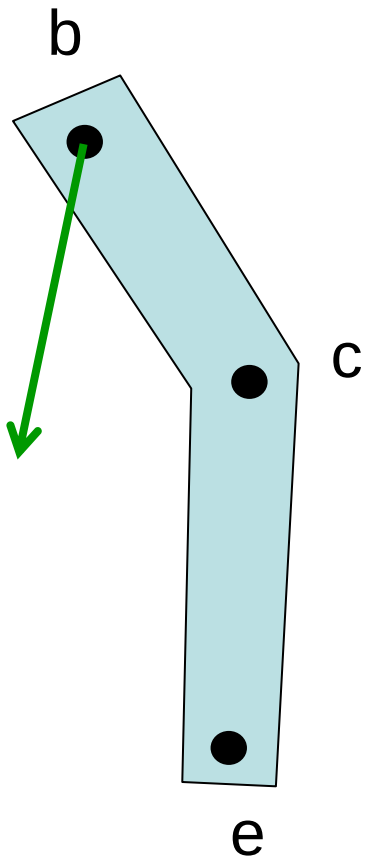
Velocity Analysis

- This object is translating and is NOT rotating
- The velocity if point “b” is indicated by a green arrow
- What do we know about the velocity of point “e”?



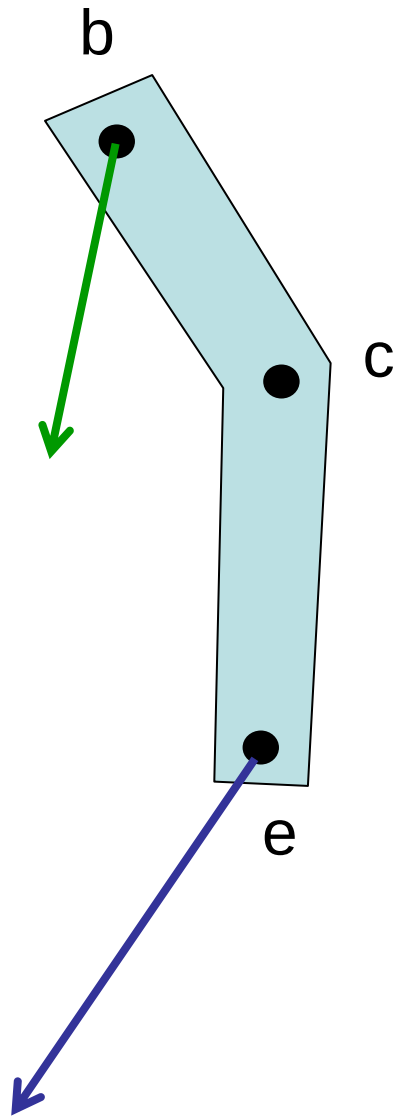
Velocity Analysis

- This object is translating and ALSO rotating about “b” at rate ω
- The velocity if point “b” is indicated by a green arrow
- Can we construct the velocity of vector of point “e”?



Velocity Analysis

- This object is a rigid body
- The velocity if point “b” is indicated by a green arrow
- The velocity of point “e” is indicated by a blue arrow
- Can the vector “e” take any value, or are there some restrictions on values it might have?



Homogeneous Transformation Matrices

- How can we mathematically express the motions of a rigid body?
  - Translations
  - Rotations
  - Both
- Matrix multiplication will do the trick

$$\text{body}' := \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 & \delta_x \\ \sin(\theta) & \cos(\theta) & 0 & \delta_y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \text{body}$$

MathCad: Preliminaries

- Assigning a value to a scalar variable
- Assigning a value to a vector or matrix
- Defining a function
- Displaying results
  - Numerically
  - Graphically

MathCad: Useful Capabilities

- Taking Derivatives
- Integration
- Symbolic manipulation
- Solving non-linear equations
- Solving blocks of non-linear equations including constraints
- Minimizing / maximizing under constraints
- Making animations
- Processing images

HTM MathCad Demonstrator

- See HTM_DEMO_2D v2.xmcd

A Challenge

- Can you modify the HTM Demonstrator to simulate the operation of a 4 Bar mechanism?
  - Specify input and output crank geometry
  - Specify an input crank angle
  - Solve for an HTM that satisfies the physical constraints created by the cranks

Assignment #3

- Do some problems from the handout
- Analyze the kinematics of a robot's leg mechanism
- Take some measurements of
  - Motion
  - Force
- Synthesize a mechanism (your choice)

Next Steps

- Start Assignment #3
- Lab tomorrow, Friday 24 FEB
- Next Class Tues 28 FEB