COMMUNICATION SCIENCES

AND

ENGINEERING

XIII. OPTICAL PROPAGATION AND COMMUNICATION

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The broad objectives of our programs are (i) to formulate propagation models for important optical communication channels from the underlying physical processes, (ii) to determine the fundamental limits on the performance that can be realized with these channels, (iii) to develop techniques that achieve or approach these limits, and (iv) to establish the validity and guide the evolution of the theoretical results through experiment.

1. QUANTUM COMMUNICATION THEORY

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Robert S. Kennedy

The issue of central concern in this work is the extent to which commonly accepted conclusions concerning the limitations imposed by quantum effects upon system performance are correct. The importance of the issue stems from the fact that such effects often contribute the dominant "noise" in the optical communication systems that are now contemplated. We wish to determine whether the limits encountered thus far are fundamental in nature, or are merely associated with the specific systems that have been considered. If they are not fundamental, there may be opportunities for substantial improvement in the performance of optical systems.

To determine the possibility and extent of substantial improvement, we shall continue to address three major questions:

(i) Given that we are limited by the laws of physics rather than by present technology, in which situations does the performance of systems now contemplated fall substantially short of what could be realized by an optimum quantum system?

(ii) What is the structure of the optimum quantum system for any given application, expressed in the abstract language of quantum physics?

(iii) How can a desired optimum quantum system be realized, or approximated at least, in situations wherein it offers a significant performance advantage?

2. IMPROVED LOW-VISIBILITY COMMUNICATION

National Science Foundation (Grant ENG74-00131-A01)

Robert S. Kennedy, Jeffrey H. Shapiro

This investigation, which is carried out jointly with the M.I.T. Center for Materials Science and Engineering, is concerned with the performance of terrestrial line-of-sight communication systems under conditions of low visibility. Its objective is to determine the extent to which performance can be improved through appropriate system design, and to develop the devices for achieving this improvement. The potential for improvement resides in the energy and information contained in the scattered component of the received field.

During the past year the major tasks have been developing an operational 2 μm laser for use with the propagation facility, determining figures of merit for electro-optic materials that could be used in an electron-gun addressed phase plate and selecting candidate materials, upgrading the capabilities of our propagation facility, and identifying and analyzing structures for spatial phase equalization.

In the coming year we shall continue the regular collection of propagation data to establish the variability, frequency, and regularity of channel parameters. Our test facility will be enlarged and will also be improved by replacing an obsolete ruby laser with either a 1.06 μm YAG:Nd laser or a 0.85 μm YLF:Er laser, modifying the holmium laser for Q-switched operation, and adding semiautomatic data acquisition equipment.

We shall continue development of monolithic phase-compensating devices employing electro-optic materials addressed by electron beams.

We shall also carry out preliminary exploration of space-distributed measurement and spatial phase control systems whose complexity does not increase linearly with the number of resolution elements.

3. OPTICAL PROPAGATION AND COMMUNICATION THROUGH ATMOSPHERIC TURBULENCE

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Jeffrey H. Shapiro

Under clear-weather conditions, the major impediment to atmospheric optical communication results from the random spatiotemporal variations in refractive index caused by turbulent mixing in the atmosphere. Our research is aimed at solving the dual problems of optical wave propagation and communication through turbulence by developing accurate spatiotemporal propagation models for the atmosphere, and applying them to the study of adaptive imaging and communication systems in which channel estimates are made and used to compensate for the effects of turbulence. During the past year we have made significant progress in several problem areas, and we are now pursuing some promising investigations.

Focused-Beam Propagation

In a Master's thesis, Gerard P. Massa[1] used the extended Huygens-Fresnel principle to calculate the irradiance variance at the free-space focus of a large transmitter pupil. His results show that the normalized irradiance variance saturates at a value that is consistent with Gaussian field statistics and, contrary to an earlier evaluation,[2] point out that there are two scale parameters that determine the normalized variance. By means of the reciprocity principle,[3] we have converted this focused-beam calculation into a signal-to-noise ratio calculation for a single-element heterodyne-detection communication receiver[4] and found that it disagrees with the previously accepted performance evaluation by Fried.[5] We have verified the correctness of our result and identified the error made by Fried.[5] We are now pursuing the communication implications of our findings.[6]

Adaptive Imaging Systems

In a previous study,[7] we found that the optimum known channel atmospheric imaging system uses a channel-matched filter. We have since shown that a transmitted reference phase-compensated imaging system achieves substantially equivalent performance, within the limitations set by isoplanatism.[8,9] By analogy with radio-frequency scatter channels, we have developed a wide-sense stationary uncorrelated scatter (WSSUS) propagation model for an atmosphere-lens system.[10] Using this model, we have shown how to circumvent in principle the isoplanatic limitations encountered in previous work,[8,9] through parallel processing. The WSSUS model offers valuable insight into the propagation process. Further applications of this model are under consideration.

Adaptive Communication Systems

We have begun the synthesis of propagation theory and communication theory results for adaptive systems. In a Master's thesis, Pierre A. Humblet[11] has investigated the problem of simultaneous channel estimation and signal detection in a two-way adaptive atmospheric communication system. In this work he includes some practical suboptimum systems and provides quantitative results for the degradation of error probability caused by estimation errors. At present, we are studying robust spatial-diversity receivers[6,12] and phase-estimation systems[13] for communication through turbulence. This last work is also relevant to the compensated imaging problem.

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COMMUNICATION SCIENCES

AND

ENGINEERING

A. QUANTUM SYSTEM THEORY FOR TWO-PHOTON LASERS

National Aeronautics and Space Administration (Grant NGL 22-009-013)

Horace P. Yuen, Jeffrey H. Shapiro

The quantum-mechanical theory of optical communications seeks to delineate the ultimate physical constraints imposed on the transmission of information. This theory is usually formulated in terms of abstract quantum states and measurement operators.[1] Moreover, it is generally assumed that the radiation sources emit electromagnetic waves which are in Glauber coherent states or their classically random mixtures. Recently, Yuen has shown the mathematical existence of "generalized" coherent states of the radiation field, whose novel quantum noise characteristics offer the potential for greatly improved optical communications in a state-generating receiver.[2] These initial results were derived from an abstract unitary transformation of photon annihilation and creation operators. It has since been shown that it is precisely such a unitary transformation that characterizes the radiation quantum state that would be generated, through a quadratic Hamiltonian, in a two-photon laser or degenerate parametric amplifier.[3,4] Given this possibility for generating two-photon coherent states (TCS), we have sought to clarify a number of issues relating to the propagation and detection of TCS electromagnetic waves.

1. Propagation of Quantized Electromagnetic Fields

In the usual formulation of quantum communication theory, the electromagnetic field is quantized by representing the positive-frequency field within a receiver cavity as a sum of orthogonal space-time modes, and imposing the photon annihilation operator commutation rule on the coefficients in this expansion.[5] This approach suppresses the effects of wave propagation from the source to the receiver. Moreover, because the state-generating receiver achieves its performance improvement by the beneficial spatial coupling between a signal plus local-oscillator mode and the receiver measurement operator,[6] the cavity quantization method is not suited for examining the performance constraints that may result from spatial-mode mismatching or finite receiver diameter. In order to analyze such problems, it is necessary to have a quantum-mechanical diffraction theory, analogous to the classical Huygens-Fresnel-Kirchhoff theory, in which quantum field propagation in vacuum can be rigorously treated as a boundary-value problem.

We have shown that the desired diffraction theory may be developed via space-time mode propagation, in a manner similar to the classical mode theory of diffraction,[7] if proper care is taken to ensure conservation of energy and mode orthogonality at all planes transverse to the (nominal) direction of wave propagation. Two logical choices for the mode set are the set of spatially bandlimited modes associated with the propagating waves in a rigorous scalar diffraction theory, and the set of space-limited modes associated with Fresnel (paraxial) diffraction theory. We propose to choose between these mode sets by formulating the propagation problems explicitly for the state-generating receiver. In particular, this investigation will include evaluation of the divergent-beam antenna performance in this system.

2. Superposition of Two-Photon Radiation Fields

In the analysis of quantum optical systems involving two-photon laser radiation, such as the state-generating receiver,[6] it is important to be able to describe mathematically how two-photon radiation fields are superposed. We have solved this problem by the method of equivalent sources, using an approach closely related to the one we have just described. Our method also gives the correct derivation of coherent-state field superpositions. It includes one- and two-photon field generation as well as two-photon amplification, but does not include one-photon amplification.

We have also considered the influence of an ideal dielectric beam splitter on two incoming quantized electromagnetic fields. It was found that the action of the beam splitter can be described by a unitary transformation, if we take into account all of the relevant fields. A more detailed description of the relations between this beam splitter model for field combining and realistic devices will be determined.

3. Photoemissive Detection of Two-Photon Radiation

In the structured approach to optical communication theory, wherein we optimize the parameters in a system model whose components represent reasonable idealizations of available devices, an ideal photon detector produces a classical photocurrent that is characterized, statistically, as a Poisson random process conditioned on knowledge of the photon flux (optical power) incident on the detector. This model, which is most nearly achieved by high-quality photomultiplier tubes, follows from the theoretical analysis of photoemissive detectors by Kelley and Kleiner,[8] and is well accepted by the quantum-electronics community. However, a strictly conditional Poisson model results only when the radiation field is in a coherent state or a classically random mixture of such states. Thus, a new statistical model for photoemissive detection of two-photon coherent states must be developed.

By applying the photon-counting probabilities for two-photon coherent states[4] to the general photoemissive detector model of Kelley and Kleiner, we have derived the joint event-time density and hence completely characterized the classical photocurrent obtained from the detector under TCS illumination. From this event-time representation, it is apparent that the photocurrent is not a conditional Poisson process, nor does it possess (conditionally) statistically independent increments. We have used the event-time statistics to study the shot noise resulting from linear filtering of the photocurrent, and obtained a modified version of Campbell's theorem. This result indicates that the conventional homodyne configuration will not suffice as the field-quadrature measurement in the state-generating receiver described elsewhere.[2] We have also used the event-time statistics to show that the counting process associated with the photocurrent is a pure-birth process, and hence the usual likelihood-ratio results[9] for regular point processes apply to photoemissive detection of TCS radiation. The required transition rates for implementation of the likelihood ratio have been derived, and are given by appropriate derivatives of the z-transform of the TCS counting probabilities. Finally, we have proven that results of a similar nature can be derived for the photocurrent that is generated by photoemissive detection of an arbitrary pure state of a single-mode radiation field. Work is continuing on the case of multimode radiation.

4. Simultaneous Measurement of Two-Photon Field Quadratures

We have found an abstract operator-valued measurement that yields a Gaussian joint density on the quadratures of a two-photon field, with the properties that the mean values of the density are the average quadrature fields and the noise observed in the two quadratures can be exchanged, subject to the uncertainty principle, by varying the parameters of the two-photon coherent state of the field. This measurement is not the usual simultaneous measurement described by coherent states. It provides the basic mathematical description of the model employed in the following sensitivity analysis.

5. Phase Tracking in Asymmetric Gaussian Noise

It is easily shown that in the state-generating receiver[6] it is necessary to maintain optical phase difference between the signal and local-oscillator fields to a fraction of 2π radians that is comparable to the desired quantum noise reduction, in order to avoid severely compromising system performance. When noise reduction of one or two orders of magnitude is sought, it seems clear that the required phase stability can only be realized in a closed-loop (tracking) system. Moreover, the appropriate classical phase-tracking problem appears to be that of tracking the phase of the low-noise quadrature in an asymmetric passband Gaussian process.

By using standard techniques,[10] we have obtained preliminary results for the structure of the MAP phase tracker for the asymmetric noise problem. A performance analysis based on the waveform Cramer-Rao inequality has been carried out, but seems to be insufficient to characterize true system behavior, i.e., the threshold effect is critical in the problem of interest. We intend to develop threshold performance results in future studies. There are some indications that the desired results may be obtained from the Barankin bound.[11]

6. Predetection Quantum Processors

We propose to develop a proper mathematical quantum description of all the predetection processing that we can employ realistically to explore possible system structures with two-photon lasers such as the state-generating receiver. One example is the beam splitter field-combiner discussed in this report. Another important example is the one-photon linear amplifier acting upon TCS radiation.

7. Quantum Analysis and Synthesis of Field Quadrature Measurements

A quantum-mechanical analysis of the physical operations involved in ordinary heterodyne detection is to be carried out to provide the basis for extrapolation to the two-photon case. A realistic physical scheme for simultaneous measurement of field quadratures such as the one we have described is needed for practical realization of the state-generating receiver. We shall study this specific measurement realization problem by analyzing the effect of predetection processing and the conditions for the validity of the multidimensional central limit theorem.

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B. LOWER BOUND OF M-ary PURE-STATE DETECTION ERROR

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Nam-Soo Myung

We have developed a lower bound to the minimum probability of error of an ideal digital communication system in which there is no other randomness in the channel nor any additive noise. The problem of interest can be stated precisely as follows. One of possible M messages, say message i, is transmitted with a priori (nonzero) probability p_i. The transmission of message i generates a field at the receiver described by the density operator ρ_i. The objective is to decide which message is transmitted in a way that minimizes P_e, the (average) probability of being incorrect. The problem may be reduced to that of minimizing

$$P_e = \sum_{i=1}^{M} p_i \operatorname{tr}\left(\sum_{\substack{j=1\\ j\neq i}}^{M} Q_j\rho_i\right) \tag{1}$$

over the M self-adjoint operator Q_i, $i = 1, \dots, M$ subject to the constraints

$$Q_i \geq 0, \qquad \text{for all } i = 1, \dots, M \tag{2a}$$

$$\sum_{i=1}^{M} Q_i = I. \tag{2b}$$

For binary detection (M = 2), optimum performance and detection operators are known.[1] For M greater than two, the necessary and sufficient conditions for optimality are available.[2-4]

It is very difficult to solve the M-ary detection problem except in a few well-structured situations.[5] Thus it is of interest to develop bounds to the error probability.

1. Results for Binary Signaling

In the derivation of the lower bound of probability of error, we shall use the following results of binary detection.[1]

Suppose ρ_o and ρ_1 are density operators for message "0" and message "1" where messages "0" and "1" occur with probabilities p_o and "1" and $p_1 = 1 - p_o$, respectively. We seek the minimum probability of detection error

$$\min P_e = \min\left[p_o \operatorname{tr}[Q_1\rho_o] + p_1 \operatorname{tr}[Q_o\rho_i]\right] \tag{3a}$$

subject to the constraints

$$Q_i \geq 0, \quad \text{for } i = 0,1 \tag{3b}$$

$$Q_o + Q_i = I. \tag{3c}$$

The minimization (3a) is equal to

$$P_e^* = p_o + (1-p_o) \min_{I \geq Q_o \geq 0} \left[\operatorname{tr} \left\{ Q_o \left[\rho_1 - \frac{p_o}{1-p_o} \rho_o \right] \right\} \right]. \tag{4}$$

P_e^*, the minimum of P_e, is attained when $\operatorname{tr} \left\{ Q_o \left[\rho_1 - \frac{p_o}{1-p_o} \rho_o \right] \right\}$ is minimized over $0 \leq Q_o \leq I$. Furthermore, the minimum value for the last expression is

$$\min_{0 \leq Q_o \leq I} \operatorname{tr} \left[Q_o \left\{ \rho_1 - \frac{p_o}{1-p_o} \rho_o \right\} \right] = \sum_{k: \xi^{(k)} < 0} \xi^{(k)}, \tag{5}$$

where the $\xi^{(k)}$ are the negative eigenvalues of Hermitian operator[1]

$$\rho_1 - \frac{p_o}{1-p_o} \rho_o. \tag{6}$$

Therefore the minimum probability of error is

$$P_e^* = p_o + (1-p_o) \sum_{k: \xi^{(k)} < 0} \xi^{(k)} \tag{7}$$

and the optimum decision operator Q_o^* is

$$Q_o^* = \sum_{k: \xi^{(k)} < 0} |\xi^{(k)}\rangle\langle\xi^{(k)}| \tag{8}$$

where the summation is over the eigenkets of operator (6) that belong to the negative eigenvalues. In conclusion, the binary detection problem is equivalent to an operator eigenvalue problem

$$\left(\rho_1 - \frac{p_o}{1-p_o} \rho_o \right) |\xi\rangle = \xi |\xi\rangle. \tag{9}$$

In particular, if the states are pure, i.e.,

$$\rho_i = |S_i\rangle\langle S_i| \quad i = 0,1$$

where $|S_o\rangle$ and $|S_1\rangle$ are linearly independent, then the eigenvalues of (9) are

$$\xi_{\pm} = \frac{1}{2(1-p_o)} \left[(1-2p_o) \pm \left(1 - 4(1-p_o)\, p_o |\langle S_o | S_1 \rangle|^2\right)^{1/2}\right] \tag{10}$$

and

$$P_e^* = \frac{1}{2}\left[1 - \left(1 - 4(1-p_o)\, p_o |\langle S_o | S_1\rangle|^2\right)^{1/2}\right]. \tag{11}$$

2. Lower Bound for M-ary Signaling

For any set of M self-adjoint operators satisfying (2a) and (2b) the probability of error is given by Eq. 1.

We shall now determine a lower bound to the attainable value of P_e. Consider a set of errors E_i such that when the message i is transmitted it is decided that some other

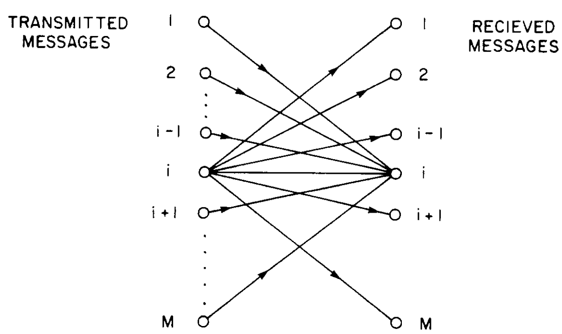

Fig. XIII-1. The set of errors E_i.

message was transmitted and that when the message j, other than i, was transmitted, it is decided that message i was transmitted (see Fig. XIII-1).

The probability of the error set E_i is

$$\Pr[E_i] = \sum_{\substack{j=1 \\ j\neq i}}^{M} \operatorname{tr}[Q_j p_i \rho_i] + \operatorname{tr}\left[Q_i \sum_{\substack{j=1 \\ j\neq i}}^{M} p_j \rho_j\right]. \tag{12}$$

The sum of $\Pr[E_i]$ over i is exactly twice the probability of error; i.e.,

$$P_e = \frac{1}{2} \sum_{i=1}^{M} \Pr[E_i]. \tag{13}$$

Introducing (12) in (13) yields

$$P_e = \frac{1}{2}\sum_{i=1}^{M}\left[\mathrm{tr}\left(\sum_{\substack{j=1\\j\neq i}}^{M} Q_j p_i \rho_i\right) + \mathrm{tr}\left(Q_i \sum_{\substack{j=1\\j\neq i}}^{M} p_j \rho_j\right)\right]$$

or

$$= \frac{1}{2}\sum_{i=1}^{M}\left\{\mathrm{tr}\left[(I-Q_i)\,p_i\rho_i\right] + \mathrm{tr}\left(Q_i \sum_{\substack{j=1\\j\neq i}}^{M} p_j\rho_j\right)\right\}. \tag{14}$$

The minimum error probability P_e^o can be obtained by minimizing (14) with respect to the Q_i subject to (2). Since the minimum of a sum is not less than the sum of the term-by-term minima,

$$P_e^o \geq \frac{1}{2}\sum_{i=1}^{M} \min\left\{\mathrm{tr}\left[(I-Q_i)\,p_i\rho_i\right] + \mathrm{tr}\, Q_i \sum_{\substack{j=1\\j\neq 1}}^{M} p_j\rho_j\right\}. \tag{15}$$

The i^{th} term in (15) may be restated as

$$\min_{0\leq Q_i\leq I}\left\{\mathrm{tr}\left[(I-Q_i)\,p_i\rho_i\right] + \mathrm{tr}\left[Q_i \sum_{\substack{j=1\\j\neq i}}^{M} p_j\rho_j\right]\right\}$$

$$= p_i + (1-p_i) \min_{0\leq Q_i\leq I} \mathrm{tr}\left[Q_i\left\{\sum_{\substack{j=1\\j\neq i}}^{M} \frac{p_j}{1-p_i}\rho_j - \frac{p_i}{1-p_i}\rho_i\right\}\right]. \tag{16}$$

A comparison of (16) with (4) reveals that the minimization in (16) is equivalent to a binary detection problem between the hypotheses "i" or "not i". In this equivalent problem hypothesis "i" occurs with probability p_i and has a state described by density operator ρ_i, while the hypothesis "not i" occurs with probability $(1-p_i)$ and has the density operator

$$\rho_i' = \sum_{\substack{j=1\\j\neq i}}^{M} \frac{p_j}{1-p_i}\rho_j. \tag{17}$$

Consequently, from previous results,

$$\min \operatorname{tr}\left\{Q_i\left[\rho_i' - \frac{p_i}{1-p_i}\rho_i\right]\right\} = \sum_{k:\xi_i^{(k)}<0} \xi_i^{(k)}, \tag{17}$$

where the $\xi_i^{(k)}$ are the negative eigenvalues of

$$\rho_i' - \frac{p_i}{1-p_i}\rho_i. \tag{18}$$

It follows from (15), (16), and (17) that

$$P_e^o \geq \frac{1}{2}\sum_{i=1}^{M}\left[p_i + (1-p_i)\sum_k \xi_i^{(k)}\right]. \tag{19}$$

Equation (19) is the bound that we seek. Note that its evaluation requires the solution of the M eigenvalue problems

$$\left[\rho_i' - \frac{p_i}{1-p_i}\rho_i\right]|\xi^i\rangle = \xi^i|\xi^i\rangle, \tag{20}$$

where superscript i has been added to the i^{th} term in (19) to index the eigenvalue problem.

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C. IMPROVED LOW-VISIBILITY COMMUNICATION

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Stanley R. Robinson, Robert S. Kennedy

An investigation of the use of phase-compensation receivers to improve the reliability of line-of-sight communication has been completed.[1] The optimum phase control, which spatially concentrates the signal power in the focal plane, is the minimum-mean-square error estimate of the phase of the aperture field when the error is "small enough." Thus phase measurement-estimator structures are an integral part of the receiver structure.

The most useful structures that we examined were the local reference method, which requires a local oscillator, and the phase contrast method, which is a direct detection scheme that depends explicitly upon the closed-loop nature of the receiver. Estimator structures and their average estimation performance were both determined. The homodyne measurement-estimation system requires the least amount of signal power to stay in phase lock, at the expense of additional hardware and complexity to keep the local oscillator locked in frequency to the signal field. The direct detection system is much simpler, but requires more signal power to stay in lock when the system is below the quantum threshold. Both approaches are appealing in that they require simple baseband processing at the output of each detector. The results will be used to direct the development of the phase plates and the detector array that will be required for eventual implementation of a phase-compensation receiver.

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