Section 2 Quantum-Effect Devices

Chapter 1. Statistical Mechanics of Quantum Dots

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1.1 Project Description

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In 1995, the main objective of our research was the nonuniversal properties of quantum dots. Typical behavior of these closed systems of large but finite number of electrons is usually called quantum chaos.[1] This means something opposite to behavior of quantum integrable systems where spectra can be characterized by a set of quantum numbers. Quantum chaos can be caused either by disorder or by geometry of a quantum dot or by interactions between electrons. It was intensively studied for several years.[2] Probably the most popular characteristics of *chaotic* quantum systems are the statistics of energy spectra. The statistics that describe distribution of eigenstates in energy and parameter space were found for systems with well developed chaos to be universal with a pretty high accuracy. This means, e.g., that the correlation functions at small energies, after proper rescaling, become independent from the particular features of the quantum system and particular perturbation. Earlier, we evaluated these functions explicitly.[3]

The global properties of the spectra (i.e., correlation functions at higher energies), are different for different systems. We have solved the long time problem in the field of quantum chaos by expressing these features through the properties of the underlying classical system.

First, we considered disordered quantum dots.[4] Using a nonperturbative approach, we have evaluated the large frequency ω asymptotics of the two-point correlation function

$$R(\omega) = < \Sigma_{ij}\ \delta(\varepsilon - \varepsilon_i)\ \delta(\varepsilon + \omega - \varepsilon_j) > \qquad (1)$$

where ε_i and ε_j denote the eigenstates of our system (quantum dot) measured in units of the mean level spacing and <...> stands for the averaging. We found that this function for systems with broken T-invariance (quantum dot in a magnetic field) can be presented as

$$R(\omega) = \delta(\omega) - R_{smooth}(\omega) + R_{osc}(\omega)\cos(2\pi\omega). \qquad (2)$$

The functions $R_{smooth}(\omega)$ and $R_{osc}(\omega)$ are not universal: they depend on the shape of the quantum dot, its conductance, etc. In the universal limit,

$$R_{smooth}(\omega) = R_{osc}(\omega) 1/(4\pi^2\omega^2) \qquad (3)$$

is very important for the understanding of the connection between classical and quantum problems; these functions are completely determined by the properties of classical diffusion in the system. Namely, $R_{osc}(\omega)$ is the spectral determinant of the diffusion operator and $R_{smooth}(\omega)$ is the Green's function of the same operator. As a result they are connected as

$$R_{smooth}(\omega) = -(1/4\pi^2)\partial\{\ln [R_{osc}(\omega)]\} \ / \partial\omega^2. \qquad (4)$$

1 M.C. Gutzwiller, *Chaos in Classical and Quantum Mechanics* (New York: Springer-Verlag, 1990).

2 M.V. Berry, in *Chaos in Quantum Physics,* eds. M.-J. Gianonni, A. Voros, and J. Zinn-Justin, Les Houches, Session LII, 1989 (Amsterdam: North Holland, 1991), p. 251.

3 For complete discussion see B.L. Altshuler and B.D. Simons, "Universalities: from Anderson Localization to Quantum Chaos," in *Mesoscopic Quantum Physics*, eds. E. Akkermans, G. Montambaux, J-L. Pichard, and J. Zinn-Justin, Les Houches, Session LXI, 1994 (Amsterdam: North Holland, 1996), p. 1.

4 A.V. Andreev and B.L. Altshuler, "Spectral Statistics beyond Random Matrix Theory," *Phys. Rev. Lett.* 75: 902-905 (1995).

This relation is universal even for quantum dots without any disorder.[5] We have been able to show that, in a very general case, there is a classical operator whose spectrum completely determines the spectral statistics of the quantum dot. This operator describes the time evolution of the classical counterpart of our quantum dot. It is crucial for the whole theory of quantum chaos that this operator corresponds to the *time irreversible* classical dynamics. This understanding allowed us to prove a conjecture proposed long ago by Bohigas, Giannoni and Schmit[6] for a certain class of quantum systems. By making use of numerical studies, these authors found that the spectral statistics even for *simple* chaotic systems are very close to universal Wigner-Dyson statistics. From our calculations,[5] it follows that this is true for systems with exponential relaxation. By making use of the nonperturbative approach, we can also evaluate nonuniversal corrections to this universal behavior, given the set of the relaxation times. This connection between quantal behavior of the quantum dot and irreversible classical dynamics of its classical counterpart is of a particular fundamental importance for the theory of both classical and quantum complex systems.

To complete our study of the universal properties of quantum dots, we evaluated the statistics of the oscillator strengths, starting from the Wigner-Dyson hypothesis.[7] These statistics provide a characterization of quantum chaos which complements the usual energy level statistics. This theory can be applied to quantum dots and also to systems like the hydrogen atom in a strong magnetic field. Without additional assumptions, we have evaluated exactly the correlation function of oscillator strengths at different frequencies and different magnetic fields. We have also discovered an unexpected differential relation between the density of states statistics and those of the oscillator strengths.

Using the mathematical similarity between quantum particles in one dimension and energy levels, we have obtained important results that can be applied for better understanding of the quantum dots with interactions between electrons. In particular, we have discovered a novel relationship between equal time current and density correlations in the model known as Calogero-Sutherland model, where the interaction between one dimensional fermions is inverse proportional to the distance between them.[8] This relationship exists in addition to the usual Ward identities.

1.2 Publications

Agam, O., B.L. Altshuler, and A.V. Andreev. "Spectral Statistics: From Disordered to Chaotic Systems." *Phys. Rev. Lett.* 75: 4389 (1995).

Andreev, A.V., and B.L. Altshuler. "Spectral Statistics beyond Random Matrix Theory." *Phys. Rev. Lett.* 75: 902-905 (1995).

Taniguchi, N., A.V. Andreev, and B.L. Altshuler. "Statistics of Oscillator Strength in Chaotic Systems." *Europhysics Lett.* 29: 515 (1995).

Taniguchi, N., B.S. Shastry, and B.L. Altshuler. "Random Matrix Model and Calogero-Sutherland Model: A Novel Current-Density Mapping." *Phys. Rev. Lett.* 75: 37 (1995).

[5] O. Aham, B. Altshuler, and A.V. Andreev, "Spectral Statistics: From Disordered to Chaotic Systems," *Phys. Rev. Lett.* 75: 4389 (1995).

[6] O. Bohigas, M.J. Giannoni, and C. Schmidt, *Phys. Rev. Lett.* 52: 1 (1984).

[7] N. Taniguchi, A.V. Andreev, and B.L. Altshuler, "Statistics of Oscillator Strength in Chaotic Systems," *Europhysics Lett.* 29: 515 (1995).

[8] N. Taniguchi, B.S. Shastry, and B.L. Altshuler, "Random Matrix Model and Calogero-Sutherland Model: A Novel Current-Density Mapping," *Phys. Rev. Lett.* 75: 37 (1995).