CONVECTION AND THE DYNAMICS OF LARGE-SCALE HORIZONTAL FLOW

by

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ABSTRACT

An observational and statistical approach is taken to investigate certain implications which follow from a simple mathematical consideration of the interaction between convection and rotation in a fluid body. The mathematical treatment leads to the suggestion that when convection is present to a proper extent, a large-scale horizontal eddy flux of momentum equatorward is likely to be generated as a result of the rotation's influence on the induced field of convergence. Such a phenomenon, it is advanced, provides a basic explanation for the observed field of differential rotation on the sun, and also perhaps for that on Jupiter.

The major part of this study, however, deals with the implications of the theory as it might be expected to apply to terrestrial atmospheric motions in mid-latitudes. The hypothesis that strong convective activity tends also to be associated with large-scale equatorward fluxes of momentum on the earth is investigated by considering four years of convective season data for a fixed region in the central United States. An index measuring the extent of convective activity is devised based on station reports of thunderstorm occurrence in the region. A measure of the direction of the large-scale eddy momentum flux over the region is obtained from observations of the trough tilt in the synoptic scale wave patterns which pass through the region at the 500 mb level.

Enough data has been compiled here to permit meaningful statistical tests to be performed. Results of two such tests indicate that a significant relationship between cases of high convective activity and troughs with a NW-SE orientation (representative of an equatorward momentum flux) does exist in the data, in accordance with what the theory presented suggests.

An index providing a rough estimate of the amount of convergence over the region also is tested in conjunction with the same trough tilt data. Again, an association between instances of strong convergence and troughs tilting NW-SE is found to exist, at a high level of statistical significance.

The conclusion is drawn that a tendency does exist in our atmosphere for outbreaks of strong convective activity to be related to an orientation in the synoptic scale trough system which implies an equatorward flux of momentum, at least for the region of our study.

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CHAPTER I.

INTRODUCTION

Ia. Background and Statement of Problem

When a comparison is made between the solar and terrestrial mean fields of differential rotation, it is observed that the maximum zonally averaged zonal wind lies at the equator in the former case, but is to be found in mid-latitudes in the latter instance. Whether this difference is a consequence of still more basic differences between the two bodies or is just happenstance is a question not easily disposed of. The viewpoint held here is that the solar equatorial acceleration (as it is often referred to) is a result of eddy heating, and hence eddy motions, arising from Rayleigh-type convection in a vertically unstable system. Our planet, of course, represents in the main a baroclinically unstable system, driven essentially by pole-to-equator temperature contrasts. It is suggested that this, the form of thermal instability predominant in a system (either vertical or baroclinic), is what may help determine whether the system possesses an equatorial acceleration or not. It is interesting in this regard to mention also the observation that the planet Jupiter appears to exhibit an equatorial acceleration, and it is widely believed to possess an internal heat source as well (see, for example, Aumann et. al., 1969).

The main concern of this work will be with showing a connection between convective instability in a system and the direction in which large-scale horizontal fluxes of angular momentum proceed. The solar and Jovian systems, however, will not be considered in much detail as cases in point, primarily because of the lack of sufficient relevant observational data. The situation is such that we are really quite uncertain as to the exact nature of the complex processes actually going on in these bodies. Indeed, it is not surprising that Durney (1974) is forced to note that a commonly accepted theory of the sun's differential rotation simply does not exist at present. So rather than contend with the difficulties caused by a lack of data for these extra-terrestrial bodies, our attention will be focused largely on conditions here on earth.

Although, as stated earlier, the terrestrial atmosphere is for the most part subjected to baroclinic forcing, the interplay between vertical convective activity and the larger-scale circulation is not entirely an unimportant process. Since the introduction of the so-called CISK theory by Charney and Eliassen (1964), this interplay has been the subject of much attention, particularly as regards motions in the tropics. Less work has been done, however, concerning the interaction between vertical convective activity and the large-scale circulation in the extra-tropical regions, though Tracton (1973) has recently given evidence of the importance of convection in the initiation of cyclogenesis. Thus, Tracton's work seems to show that even in mid-latitudes, convection, if it is intense enough, can be organized in such a way as to have an effect on synoptic scale motions. In light of this and our earlier suggestion that vertical convection on the sun is what gives rise to its equatorial acceleration, we pose the following question: If there is sufficiently intense and extensive convection over a region of the earth, is there also an associated equatorward flux of momentum on the synoptic scale? In the mean, the mid-latitudes are not characterized by such a flux in the troposphere. However, does the situation change when convective processes become important enough so that, in some loose sense, the earth resembles the solar case? It is towards seeking answers to these questions that the present study is primarily directed.

Ib. Basic Approach

The hypothesis under consideration then is that there is an association between the degree of convective activity over a region of the earth and the direction of momentum flux in the synoptic scale over the region. When the convection is intense enough, an equatorward flux is anticipated. Several approaches to investigating this hypothesis are possible, but the one chosen here is observational in nature. However, having decided to rely on observational data still leaves open any number of possible ways to proceed. Our task is made complicated by the fact that other processes besides the one which we are interested in are going on in the atmosphere simultaneously and may be acting to obscure the picture. Then too, it is also possible that an equatorward flux of momentum in mid-latitudes may sometimes result from processes not connected with convection, though this is not normally to be expected, since these latitudes are after all characterized in the mean by strong poleward fluxes. What we should perhaps be looking for is, therefore, a statistical trend in the data, showing in the long term a tendency for intense convection and equatorward fluxes to be statistically related. This is the approach taken here. Thus, statistics relating to convective activity and to the direction of momentum flux have been accumulated by the writer for a lengthy time period over a selected region of the United States. Individual synoptic case histories have not been studied in favor of gathering a large enough data sample to allow meaningful statistical conclusions to be drawn.

Ic. Outline of the Presentation of Results

The main body of this thesis is concerned with the matters discussed above. Chapter 2 contains the mathematical argument on which our contention that there is a link between convective activity and an equatorial acceleration is based. A simple physical picture may be inferred from the mathematics, and its application to the earth is discussed. Some evidence already in existence which may be taken to show a relationship between convection and equatorward fluxes on the earth is briefly mentioned.

Chapter 3 presents in detail one portion of the observational study performed. This portion of the study, following the basic approach discussed in the previous section, uses thunderstorm statistics as the measure of convective activity in the region of interest. The data is described, and its weaknesses for certain purposes are pointed out. The manner in which the data is organized for statistical testing and the statistical procedures used and results obtained are discussed.

Chapter 4 describes a second portion of the observational study, this one employing computations of vertical velocity to serve as a measure of horizontal convergence over the region. The technique for calculating the vertical velocities is reviewed, and statistical tests similar to those performed in Chapter 3 are made.

Chapter 5 contains a summary of all the results obtained here concerning the relationship between convection and large-scale fluxes. Suggestions for future research are also included.

An appendix to this work is included which makes further use of the vertical velocities discussed in

Chapter 4 to investigate a question of some current interest which, however, is otherwise largely independent of the study conducted and reported in the main body of this work. This question deals with the role of large-scale vertical eddy fluxes of momentum in the general circulation. No attempt has been made to link this study with the earlier one. Even so, it is felt that the results of Appendix A are of sufficient interest to warrant their brief inclusion here.

CHAPTER 2.

THEORETICAL CONSIDERATIONS

2a. A Mathematical Treatment

Starting from the dynamical equation governing the absolute angular momentum of a fluid body, it is possible to derive a simple system of equations describing the balance between the square of its zonal average and its variance along the length of closed latitude circles. It is this system of balance equations from which the suggestion made earlier was inferred, that thermal forcing of the pure eddy type (perhaps like Rayleigh convection) tends to establish an equatorial acceleration in a fluid body, through negative eddy viscous effects. The equations on a sphere are presented below. A derivation by this writer is given in Appendix B.

$$\frac{d}{dt}\int \rho \frac{[M]^2}{2} dT = -\int [M]\, \nabla\cdot\rho M \underset{\sim}{v}\, dT \qquad - \int [M]\, \nabla\cdot \underset{\sim}{\tau}_m\, dT \tag{1}$$

$$\frac{d}{dt}\int \rho \frac{[(M')^2]}{2} dT = +\int [M]\, \nabla\cdot\rho M \underset{\sim}{v}\, dT - \int M' \left(\frac{\partial p}{\partial \lambda}\right)' dT - \int M'\, \nabla\cdot \underset{\sim}{\tau}_m\, dT \tag{2}$$

where the following symbols have been employed:

t = time
ρ = density
M = absolute angular momentum per unit mass = ru, where r is perpendicular distance to the polar axis and u is the absolute linear particle velocity eastward.
T = volume
$\underset{\sim}{V}$ = vector velocity
$\underset{\sim}{\tau}_m$ = two dimensional, purely frictional, transport of angular momentum vector lying in the meridional plane.
p = pressure
λ = longitude
$[(\ )]$ = zonal average around a complete latitude circle.
$(\ )'$ = deviation from zonal average

Any explicit variation of density with λ and t has been neglected in writing (1) and (2), for the sake of simplicity. The more general case is given in Appendix B. As pointed out by Starr and Rosen (1972) in discussing the above system of equations, any number of steady states exist which would satisfy it. However, following accustomed procedure, we will assume that if initial conditions exist which favor the growth of a particular state from a condition of rest and that such a state may become steady according to (1)-(2), then indeed this is the state likely to be found dominant in the fluid body as observed.

We wish to demonstrate that if the thermal forcing in a sphere of fluid is such as to make a convective regime dominant, then an equatorward flux of momentum is likely to ensue. Consider therefore a fluid mass initially in solid rotation. The solid rotation could of course persist as a steady state, but we imagine now that an eddy disturbance is introduced by the appearance of a purely eddy heating and cooling. Such an eddy component to the thermal field will give rise to non-zero values of $\left(\frac{\partial p}{\partial \lambda}\right)'$. Such pressure forces will in turn give rise to non-zero eddy velocities.

If the eddy motions are now supposed to persist so that the final steady state approached differs from the original solid body rotation, then the pressure force term in (2) must be positive in its effect, bearing in mind that the friction term in (2) is undoubtedly dissipative in its effect. In other words, there must be a negative correlation between M' and $\left(\frac{\partial p}{\partial \lambda}\right)'$ in order that there should take place an initial building up of the eddy motions by the pressure term. If the disturbances are on a large enough scale or otherwise arranged so that the effect of the Coriolis force is not insignificant, then this would imply a negative correlation (N.H.) between u' and v' and hence a sorting mechanism whereby particles having high momenta move selectively toward the equator. Such a convergence further implies a negative value for the conversion term,

$\int [M] \nabla \cdot \rho M \underset{\sim}{v} \, dT$, since the convergence is taking place in a region where $[M]$ is largest. Thus a balance is achieved between, on the one hand, the eddy pressure term tending to build up the eddy motions and, on the other hand, the conversion and, presumably, the friction terms tending to limit the eddy growth. The conversion term is also of a proper sign so that the $[M]^2$ quantity in (1) is increased at the expense of $[(M')^2]$ in (2). The general result, then, is that of generating an equatorial acceleration of some type depending upon further particulars. A more complete picture would no doubt depend also upon the latitudes and levels at which the eddy heating is introduced and other such matters.

What the preceding considerations suggest, therefore, is that an equatorial jet could develop as a consequence of eddy heating introduced by Rayleigh-type convection arising from a vertical instability. It is felt that the arguments presented in the preceding paragraphs provide a basic explanation for conditions observed on the sun. Certainly, however, the picture on the sun is in reality more complicated than the simple one we have drawn here. If we were to consider the solar case in more detail, we would probably want to investigate the properties of an ensemble of convective cells which range over several scales of motion and are in different stages of their lifetimes. Under such circumstances, more care would have to be taken in simply concluding, as we did earlier, that the sign of the meridional component of motion may simply replace the sign of $\left(\frac{\partial p}{\partial \lambda}\right)'$ because of Coriolis turning. Rather, statistical considerations such as those discussed by Starr (1973) should be entered into before deciding whether a positive value for the integral $-\int M'\left(\frac{\partial p}{\partial \lambda}\right)' d\tau$ in (2) necessarily implies an equatorward momentum flux. Let the coefficient of linear correlation between M' (or equivalently u') and $\left(\frac{\partial p}{\partial \lambda}\right)'$ along the length of a complete latitude circle be denoted by A , the correlation coefficient between v' and $\left(\frac{\partial p}{\partial \lambda}\right)'$ by B , and the correlation coefficient between u' and v' by C . It can then be shown that a sufficient condition for C to be negative in the N.H., thus implying an equatorward flux of momentum, is that $A^2+B^2 > 1$ (so long as it is still true that $A<0$ and $B>0$). Hence, even for a complicated ensemble of cells, so long as this inequality remains satisfied, then our arguments concerning the convection and an equatorial acceleration will still follow, by and large. Of course, for the larger-scales of motion on a rotating fluid system, we might expect B to be quite large, and the inequality normally satisfied.

Numerical experiments have been performed by several investigators attempting to model the role of convection in maintaining the solar differential rotation. The models tend to be somewhat complex, and so a clear interpretation of their results is made difficult. A recent model by Gilman (1972), for example, seems to confirm the possibility that convective cells can induce an equatorward flux of momentum. However, while equatorial regions do generally rotate faster than the mid-latitudes in the model, a local equatorial deceleration is sometimes produced as well. The reason for this is not clear, but it may be related to the artificiality imposed by boundary conditions or the choice of wave modes to be studied. There are other deficiencies in the model as well, but a major problem seems to be the lack of proper solar data, requiring use of non-dimensional parameters in the model whose true values are not well known. It would seem that the problems involved in modelling the solar motions numerically are formidable ones.

Before closing this section dealing with the analysis of equations (1)-(2), mention should be made of what can be said concerning a fluid driven primarily by baroclinic forcing. Unfortunately, it seems little can be said, for the system (1)-(2) does not seem to lend itself to easy interpretation in the case of motions arising initially from a baroclinic type of instability, involving as it does a species of convection due to meridional temperature gradients between the equator and poles. Unlike the horizontal eddy motions induced by convection, baroclinic wave motions cannot be so simply claimed to be the result of zonal eddy pressure gradient forces. Therefore, the growth of an eddy regime from an initial resting state in the case of baroclinic forcing cannot be treated by our simple system of equations in a manner analogous to that already given. This does not mean that we have shown that an equatorial acceleration cannot result in a purely baroclinic fluid; merely, the point is that the equations considered here are just not capable of dealing with this case. It is true though that once baroclinic motions have ensued, they are characterized by a field of differential thermal advection. Such thermal contrasts may induce zonal eddy pressure gradients as a secondary effect, and this in turn may create a secondary tendency to transport momentum equatorward (perhaps in opposition to the basic wave). The thermal contrasts would, of course, have to be strong enough to be effective in the sense of creating a positive value for the generation term, $-\int M'\left(\frac{\partial p}{\partial x}\right)' d\tau$, in (2). Whether this happens we cannot say. However, although it is possible for a secondary effect of this sort to transport momentum equatorward, we reiterate our point that the basic thermal field responsible for creating the original baroclinic wave cannot be shown by our analysis to induce an equatorial acceleration. Actually, one might suspect otherwise, since our experience is that the mid-latitudes of the earth are characterized by both baroclinic forcing and poleward momentum fluxes in the mean.

2b. Application of Theory to the Terrestrial Atmosphere

Much of what has been written in the previous section was initially motivated by a concern to explain conditions thought to exist on the sun and Jupiter. The interaction between convection and rotation which (1)-(2) seek to describe is felt to be the basic mechanism responsible for the equatorial westerly jets observed on these bodies. However, as mentioned before, lack of available data for these systems would probably make unambiguous results from further study difficult to come by. For this reason, among others, our attention is now to be shifted away from further consideration of the sun or Jupiter to matters dealing with terrestrial conditions. By comparison, observations of terrestrial motions are plentiful. Moreover, of course, we live here, and so the role of convection in our own atmosphere is of more than academic interest. As was discussed in the Introduction, it is now generally recognized that convective activity on the earth can be organized in such a way as to have an effect on larger scales of motion, even in mid-latitudes, mainly through the mechanism of latent heat release. The question we seek to answer here is whether one such effect might be an attempt to induce an equatorward flux of momentum in mid-latitudes.

Our theoretical considerations thus far have supposed a fluid whose prime thermal drive is a vertical instability. This, of course, is not the mean situation to be found in middle latitudes on the earth. However, there are times when convective activity can become quite strong over a portion of a mid-latitude belt. Now even though strictly speaking the system (1)-(2) has been derived on the basis of considering conditions around complete latitude circles, it is felt that essentially similar conclusions can be drawn when attention is restricted more locally to a segment of a latitude circle whose central portion is affected by convection. That is to say, it is our belief that based on the theory developed in the previous section, one might expect to see an equatorward flux of momentum characterize the region of a mid-latitude belt in which there is sufficiently intense convective activity occurring, provided that the region influenced by the convection is large enough.

To better visualize the situation, it is possible to construct a very simple physical picture suggested by our analysis of equations (1)-(2). Fig. 1 depicts a segment of a latitude circle drawn for an arbitrary height in the atmosphere, the central portion of which is assumed to be the subject of intense convective activity, as denoted by the shading. Actually, ϕ in the picture can be viewed more as being representative of a band of latitudes over which the convection is assumed to extend, in line with the observational studies described in the next two chapters. An eddy component to the thermal field along ϕ will result,

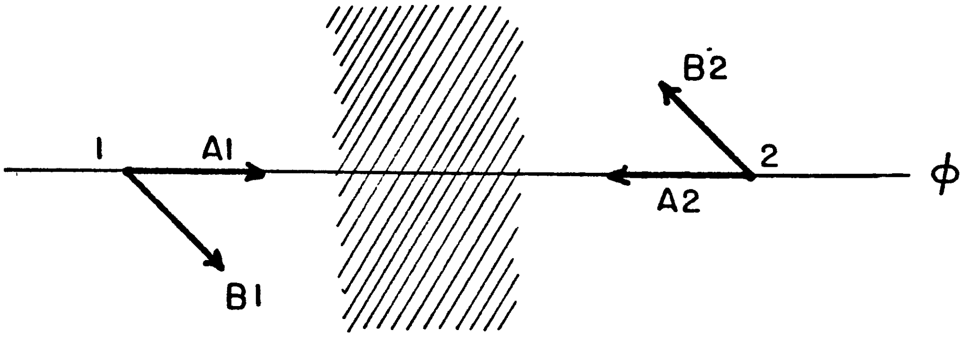


Fig. I

An idealized picture of the proposed effect of convection on the horizontal motion field over a portion of a representative latitude circle. Convection, indicated by the shaded region in the figure, induces a pressure gradient at points I and 2 indicated by vectors $\vec{A1}$ and $\vec{A2}$. The final flow field is given by vectors $\vec{B1}$ and $\vec{B2}$.

most effectively because of the release of latent heat. If the latent heat release is strong enough, and Tracton's work certainly indicates that this is possible, an eddy pressure gradient force is created along the latitude circle. Its direction at the two chosen points in Fig. 1 is indicated by the vectors $\overrightarrow{A1}$ and $\overrightarrow{A2}$, these vectors also giving the direction in which zonal eddy components of motion begin to respond to the forcing. Thus, a non-zero value to the generation term in (2), $-\int M'\left(\frac{\partial p}{\partial x}\right)' dT$, has been created.

If we now assume, however, that the motions are on a large enough scale to feel the effects of the Coriolis force, then the actual motions will be deflected toward the right (N.H.) of the pressure gradient force. Of course, some component of flow radially inward will remain to allow for a release of the convective instability. The vectors $\overrightarrow{B1}$ and $\overrightarrow{B2}$ indicate the final flow resulting from the action of the pressure gradient and Coriolis forces. Assuming that the mean of the components of the velocity field around the latitude circle vanish, what vectors $\overrightarrow{B1}$ and $\overrightarrow{B2}$ imply is a negative value for the correlation $[u'v']$ and so an equatorward flux of momentum. It might be noted that if the flow were perfectly geostrophic, so that no component of inflow existed, there would be no net momentum flux across ϕ , since the velocity vectors $\vec{B}$ would now lie perpendicular to the latitude circle.

This simple picture, following as it does the mathematical analysis which suggested it, shows the manner in which a field of convergence, induced by convection, might be expected to lead to an equatorward flux of momentum over a region of a rotating fluid system. The same result holds, by the way, if we had considered a region influenced by strong divergence. Of course, in our atmosphere many other complex processes would be occurring at the same time, and all sorts of interactions might serve to mask the effect with which we are concerned. Still, it is believed that if the convective activity is of sufficient magnitude, a tendency towards developing an equatorward flux of momentum in the larger scales will make itself shown, in accordance with the basic physics contained in Fig. 1 and our mathematical analysis. One cautionary note about the figure though: if one wishes to consider the effect of a random ensemble of large convective units on the horizontal motion field over a whole sphere, such as may be desired in applications to the sun, then simple pictures like Fig. 1 will no longer suffice, and attention should be returned to equations (1)-(2) for such a case, as discussed in the previous section. In other words, Fig. 1 should be regarded as suggestive of, but not containing entirely, the whole story told by equations (1)-(2). For our present purposes though, the picture contained in Fig. 1 will be useful to keep in mind.

2c. Some Indirect Observational Evidence on the Earth

There is already in existence some indirect observational evidence that convection may be associated with an equatorward momentum flux on the earth. An analysis of southern hemisphere data by Starr and Rosenstein (1973) shows an equatorward eddy transport of angular momentum in the zonal mean for low levels at all latitudes in the five-year average. While there is no specific evidence that this low level equatorward transport is related to convective processes, it seems plausible to suggest that the presence of vast expanses of relatively warm ocean waters in the southern hemisphere is conducive to the generation of large areas of convective activity.

A similar eddy flux of momentum equatorward at low levels can also be found in the northern hemisphere, but only on the mean meridional cross-section for the winter months, as shown for example by Starr, Peixoto and Gaut (1970). Here, there is displayed an equatorward flux in the zonal average for all latitudes south of 60°N, and again convection could be a factor, since it is observed that large areas of cumulus clouds develop over the oceans downwind from continents when outbreaks of polar air take place in winter.

Such evidence of course is quite indirect, as has been noted. Still, it is an indication that the process we have envisioned may indeed be operating in our atmosphere. In the next chapter, an observational study is described which seeks to show more directly an association between convection and equatorward fluxes of momentum in the mid-latitudes of the northern hemisphere.

CHAPTER 3.

OBSERVATIONAL STUDY I

THUNDERSTORM ACTIVITY AS THE PARAMETER OF CONVECTION

3a. Selection of Data

As previously discussed in the Introduction, the basic philosophy behind the data studies performed here has been a desire to accumulate a fairly large sample of observations on which to base conclusions. The reason for this stems from an appreciation of the fact that the theory outlined in the foregoing chapter, while highly suggestive of what we might expect to find, is limited in the sense that effects of the various other processes (baroclinic or otherwise) which also are occurring in the atmosphere have not been included in it. In short, our atmosphere exhibits a wide range of interacting processes, and in individual cases the picture may be quite complicated and the effect we are looking for easily masked. However, it is strongly believed that if proper statistics are gathered, a relationship between outbreaks of strong convective activity and the appearance of equatorward fluxes of momentum will be found, even on customary meteorological maps. The use of a large enough sample of data is required for this purpose, so that we may have a measure of confidence in our statistics. Unfortunately, it is true that the use of such bulk procedures will involve the loss of much information about the details of the process being considered. On the other hand, the accumulation of such details from the study of individual synoptic case histories, for example, would be frought with difficulties, as observed above. We have chosen what we believe to be the most fruitful way of initially proceeding into largely unexplored territory. If it can be reasonably demonstrated, through even fairly gross procedures, that convection is related to the large-scale flow field in the way we have proposed, it is believed this much would represent a substantial accomplishment.

Having decided then to make use of a large data sample in our investigations, we are further constrained by the necessity of using data which is easily accessible and handled. Additional compromises are thus required in order that a simple nature to the study be maintained. In this manner perhaps, a clear, even though limited, interpretation of our results might be hoped for. Accordingly, it was felt that attention should be restricted to a fixed region of the middle latitudes, one which is known to exhibit convective activity, is fairly well observed, and is not tremendously influenced by orography. For these reasons, following the guide provided by a pilot study into these matters by Macdonald and Ward (1974), a region of the midwestern United States was selected. Specifically, the region is the box bounded by the latitude circles at 45°N and 35°N, and by the meridians at 97.5°W and 85°W (See Fig. 2).

Now that a choice of fixed region has been made, the next problem to be confronted is the manner in which fluxes over the region are to be identified. A difficulty arises in this matter because less than a complete latitude circle is being considered, making the definition of horizontal eddy components of motion somewhat ambiguous. However, resort can be made to general considerations of the types of wave patterns commonly found in our atmosphere and the momentum fluxes they generally imply. Longwave troughs in the N.H. motion field which are oriented basically in a N-S direction may be considered in a sense to contribute no eddy flux of momentum to the total amount crossing a latitude circle, whereas those with a NE-SW orientation contribute a poleward flux and those with a NW-SE orientation, an equatorward flux. It was felt, therefore, that an acceptable method of determining the sense of the direction in which momentum is being transported over our region would be to simply identify the type of wave pattern characterizing the region. For this purpose, use was made of a daily series of upper-level weather maps. More details about this source of data will be provided in the next section.

In keeping with our desire to study a fairly large amount of data, it was decided to consider the late spring and summer seasons for a period of four years. The choice of season was motivated, of course, by the knowledge that this is when convective activity is most pronounced in the selected region. A period of four years was required in order that there would be enough cases of troughs with a NW-SE tilt (indicative of equatorward momentum fluxes) to make our chosen statistical procedures valid. More precisely, then, the period of our study consists of the months May, June, July and August for the years 1959 through 1962, a total of 16 months. It is interesting to note, by the way, that a cursory examination of the fall and winter seasons for these four years revealed no more than a handful of cases of troughs with a NW-SE tilt. It appeared however that all these cases were associated with greater than normal convective activity for these seasons. If the hypothesis under investigation is valid, of course, then this observation should not be very surprising.

The final process that needs to be dealt with in some manner is the convective one. Many parameters for convective activity are available and would probably be suitable. The one we have selected for use in this first study is an obvious one -- thunderstorm (⛈) occurrence. Thus, a gross measure of the thunderstorm activity in our region for each day of the period was decided upon, in a way elaborated upon in the next section. It is recognized that strong convection can occur without thunderstorm development, but the occurrence of a thunderstorm near a station does serve as a generally reliable, and very convenient, measure of intense convective activity, particularly when the collection of a great deal of data is desired. In any case, for this first study, it is thunderstorm activity which has been used as the parameter for convection. In light of this choice, and the manner in which equatorward fluxes of momentum over the region are to be identified, our basic hypothesis may be reformulated thus: instances of widespread thunderstorm activity in the region considered should be most related to the existence of synoptic scale troughs over the region which have a NW-SE orientation.

3b. Further Details Regarding the Data

Data concerning the orientation of troughs passing through the region was obtained in part from the Daily Weather Map series published by the United States Weather Bureau. This series contains once daily (00 Z) maps of the field of geopotential height observed at the 500 mb level over the United States. For the year 1959 only, the Daily Series of Synoptic Weather Maps, Part I (Northern Hemisphere Sea Level and 500 Millibar Charts) are available to provide similar upper-level maps of better quality. The procedure followed was to identify from these map series the type of trough, if any, located over our selected region for the day of the map. To qualify for identification, a trough was required to have a wave amplitude, defined as the distance between the northern and southernmost portions of a given contour line, greater than 10° in latitude. If any portion of such a trough was located over our region on the map for a particular day, then that day became labeled as a "trough day". No attempt was made to keep track of troughs on an individual basis. The single determination regarding the existence of some trough over our fixed region at the time of the map was the only information used in making the "trough day" identification.

Once a trough day has been labeled as such, it then becomes necessary to characterize the nature of the momentum flux implied by the wave pattern for that day. Three categories of wave tilt were recorded: negative or NW-SE tilt (equatorward flux), zero or N-S tilt (no net flux), and positive or NE-SW tilt (poleward flux). Although there was a degree of subjectivity involved in deciding into which category a particular trough might be placed, a rough criterion was adhered to. If the trough axis intersected the relevant meridian at an angle of greater than 10°, then the trough was placed in the appropriate category, negative or positive. Otherwise, the trough was labeled as having a zero tilt. For the 16 convective season months in our study, there were 28 negative, 69 zero, and 55 positive trough days, in all, a total of 152 trough days out of the complete record of 492 calendar days in the sample.

At this point, perhaps, mention should be made of some of the shortcomings of the trough data used as they affect our study. For one thing, the Daily Weather Map series was produced on a more or less real-time basis, and so not much error checking was performed in preparing the maps. Moreover, maps for only the 500 mb level are available, and while this level is certainly within the limits of the atmospheric heights likely to be affected by convection, other levels would possibly show stronger relationships. It would have been interesting in any event to investigate other levels, but maps for them are not easily obtained. Finally, of course, there is the point that the maps are drawn in terms of the contours of the height field, and not of the wind field (although some wind data is included on the maps and these were looked at). It is, after all, the wind field which physically is responsible for a momentum flux. However, for the scale of motions indicated on the maps, the tilt given by the height field should be a fair enough approximation to the tilt in the actual wind field, particularly if only the sign of the tilt is the desired quantity, which is the case here. These, then, are some of the shortcomings in the trough data which we have chosen to accept, in return for the accessibility the data provides. It is believed, however, that these, and other deficiencies in the trough data which may exist are neither very serious nor very limiting, considering the nature of what we hope to show.

Within our midwest region, there existed continuously during the period forty (40) surface observing stations which reported on thunderstorm occurrence. These reports have been collected by the National Weather Records Center in Asheville, N.C. and are part of the information available on the Local Climatological Data Sheets series. It was from this series that our thunderstorm data was collected. Data on thunderstorms is recorded for each station only on a daily basis in this series. If at any time during a day at least one thunderstorm occurs at or near the observing station, then that day is simply labelled as a "thunderstorm day". No information as to the hour of occurrence is reported in this series, nor is it reported if more than one thunderstorm occurred that day. So again, as the case for the trough data, we will be using a somewhat gross measure of conditions.

For each station then, a given date will be labelled as a "thunderstorm day" if at least one thunderstorm occurred sometime between 12:01 a.m. and midnight local time, without further regard to the intensity of activity. Our procedure was to then form for a given calendar date the sum of the "thunderstorm day" reports for all the stations in the region. It is this statistic, hereafter to be referred to as the daily "area ☈ number", which serves as our basic measure of the convective activity in the region on each day of the period studied. If, for example, on a certain day twelve of the stations reported that day to be a "thunderstorm day" (i.e., they each reported the occurrence of at least one thunderstorm near them sometime during the day), then the "area ☈ number" for that day is 12. If all the observing stations individually reported a day to be a "thunderstorm day", then the number 40 is used to characterize the thunderstorm activity over the region for that day and is the assigned "area ☈ number". At the other extreme, if no station reported the day to be a "thunderstorm day", then the "area ☈ number" is zero.

It is recognized that the area β number, as it is defined here, can only be a rough estimate of the convective activity in a region. Nevertheless, this number does satisfy physical intuition. When it has a relatively large value, this does correspond to a situation in the region as a whole which we *would* tend to think of as deserving the label "high or widespread convective activity". When its value is low, then the absence of widespread thunderstorm activity which that indicates does make the situation a logical candidate for the label "low convective activity". It is true that the procedure of defining an area β number means that any effects which may arise from the convection's being localized on a scale smaller than that of the entire area itself cannot be studied. However, the use of our area β number will enable us to study whether convective activity on the scale it does measure has an effect on large-scale momentum fluxes. For the present, we would be quite pleased enough were we able to show that such a connection does exist.

This, then, concludes our more detailed description of the manner in which the two elements basic to our study, momentum flux and convective activity, have been measured.

To summarize, the former has been obtained from a consideration of the wave pattern seen to exist over the region on the 500 mb synoptic map made once each day. If no trough is present on the daily map, then the day is so labelled. If a trough is present, then its tilt is noted and the day is labelled appropriately. Convective activity has been observed independently, with the area ☈ number defined earlier obtained as the measure of activity for each day. We thus have daily time series for both trough tilt and thunderstorm data over the period, and in a following section we subject these series to statistical testing to investigate our main hypothesis that there should be a connection between negative trough tilts and large values of the area ☈ number.

One limitation imposed by the data used here and our method of treating it should be discussed first, however. The theory delineated in Chapter 2 indicates that a causal relationship between convection and equatorward momentum fluxes should exist. Ideally, we would like to have obtained data capable of showing such a causal relationship, but this would not have been a very simple task. The data we do have, and which has been described here, restricts attention to troughs as they pass through a fixed region, the other history of the troughs being ignored. Under such conditions, an ambiguity in testing the hypothesis that intense convection should precede a negative tilting trough in time arises from the fact that convection would normally be expected to be found ahead of an advancing trough in space as well. Moreover, the troughs are viewed once a day only, while the thunderstorm data has a 24 hour period of resolution. Therefore, determining cause-and-effect relationships on the basis of our data alone is a tricky matter, and we must be careful of what we claim for our results. Along these lines, it might be mentioned that there does exist the possibility that, once established, a negative tilting trough might produce conditions which are particularly conducive to further convective activity, perhaps because of a favorable advection of its associated vorticity field. In other words, a feedback mechanism between convection and negative tilting troughs may exist. The possible existence of such an instability should be explored theoretically, but this is not in the purview of our present study. Certainly though, _if_ such an instability is operating in the atmosphere, our data lacks the preciseness necessary to identify it and to establish beyond doubt just which element, the convection or the trough tilt, is "causing" the other. In light of all this, therefore, perhaps the best view of what our data is capable of showing at present is simply that an association between convection and trough tilt does exist, with further details left unresolved.

3c. A Statistical Test Relating The Area ☈ Number to The Presence of Troughs

Before proceeding to the testing of our main hypothesis, it seems reasonable to consider the behavior of our index of convection, the area ☈ number, as it relates in general to the presence, or absence, of troughs over the region, leaving the matter of trough tilt momentarily aside. Thus, the question to be explored now is whether convection is associated more with trough passages than otherwise in our data. Our synoptic experience tells us that for the region considered, we would indeed expect more convective activity when there is a trough in the region than when no trough is present. Therefore, we might expect to find in our data that larger values of the area ☈ number are associated with trough days (as we defined these earlier) than with non-trough days, and it is this expectation which we seek to test statistically.

Unfortunately, the matter is not so straightforward. Two problems exist. First, not surprisingly, the time series of area ☈ numbers is serially correlated, i.e. there is persistence in this data. In fact, the autocorrelation at lag 1 day has been computed to be 0.347. Reliable methods for taking this type of dependence in the data into account are not handy. A usual, though rough, procedure is to simply reduce the number of degrees of freedom assumed by an appropriate amount. Our second problem deals with the difference in time spans measured by the thunderstorm and trough data, the former dealing only in quanta of 24 hour periods, the latter viewing conditions at only one time of the day for a fixed region. Under such circumstances, can it be said, for example, that if there is a high area ☈ number on a particular date and that date is not a "trough day", but the next date is, then the convection is not associated with a trough passage? In other words, given the factors involved, the question of associating convection with the presence of troughs in our data involves some ambiguity. We will take the simplest approach for now though by allowing an area β number for a given calendar date to be associated with a trough if and only if that very same date is also recorded as a trough day. Otherwise, the area β number will be placed in the "no trough" sample.

In brief then, the statistical test we have chosen here involves creating two samples out of our entire data set, one consisting of trough days (regardless of tilt) and the other composed of non-trough days. The area β number for each day will then fall into one or the other of the two samples, in the manner described at the end of the last paragraph. The means of the area β numbers for each sample may then be computed and compared for the significance of the difference between them. Let the mean of the series of area β numbers in the sample consisting of trough days be denoted by $\overline{X}_1$, their variance by S_1^2 , and the total number in the sample by N_1 . Define similarly $\overline{X}_2$, S_2^2 , and N_2 to apply to the sample of non-trough days. One then forms the null hypothesis that these two sample means, $\overline{X}_1$ and $\overline{X}_2$, have been drawn from populations in which an equality between the means of these sample statistics exists, i.e. $\overline{\overline{X}}_1 = \overline{\overline{X}}_2$. The null hypothesis is normally rejected if it can be shown that obtaining the observed difference in the sample means by chance is less than 5%.

With the aid of the central limit theorem, it may be shown that the distribution of

$$y = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\frac{S_1^2}{N_1-1} + \frac{S_2^2}{N_2-1}}} \qquad (3)$$

is normal, with mean 0 (under the null hypothesis) and standard deviation 1. Now in our case, assuming that the data were independent, we have the following statistics:

$$\begin{array}{llll} \overline{X}_1 = 9.7 & S_1^2 = 60.4 & N_1 = 152 & \text{(trough days)} \\ \overline{X}_2 = 7.3 & S_2^2 = 40.1 & N_2 = 340 & \text{(other days)} \end{array} \qquad (4)$$

Substituting (4) into (3) gives a value for y = 3.33, which in the normal distribution has a probability of occurrence of less than 0.1%. We feel justified, therefore, in rejecting the null hypothesis. In other words, the difference between $\overline{X}_1$ and $\overline{X}_2$ in our data is significant, and we may conclude that our index of convection, the area ↯ number, does behave as anticipated. On the average, larger values of the index do seem to be associated with trough conditions than with non-trough conditions.

If we wish to take the lack of independence in the data into account by recognizing that we actually have fewer degrees of freedom than implied by the previous calculation, we may accomplish this crudely by noting that probably every third day in our series does from an independent subset. Ideally, the values of N_1 and N_2 used in (3) should represent the amount of independent information available, and therefore we divide the numbers N_1 and N_2 in (4) each by 3, keeping the other numbers in (4) the same. The new value for y obtained in this case is 1.92. Note that this is smaller than the result of our previous calculation, but the null hypothesis can probably still be rejected, since even this new value will occur only about 5% of the time by chance.

It seems we may conclude then that the area β number is related to the presence of troughs, though we also recognize that the statistical testing procedure employed here is not without its weaknesses. One way we may view this result is to see it engendering additional confidence in our belief that the area β number is a suitable index of convection. However, there is one additional thing the statistical test just performed tells us. When we next investigate a statistical connection between convection and equatorward fluxes, we should probably restrict attention to the vicinity of trough days only, and not consider the whole time series of 492 days. Since we have just indicated that the area ☈ number is significantly smaller on non-trough days, the inclusion of these days in our further testing might unduly bias results in our favor. In other words, since we anticipate finding that the highest values of the area ☈ number will be associated with negative tilting troughs as opposed to other types of trough tilt, then including data for the non-trough days in the comparisons will only serve to make an association seem that much more significant. This seems somehow unfair, and so in the next section, we center attention around trough days alone, for these form a sample, we are saying, significantly different from the aggregate of non-trough days. The question is whether there are significant differences among the three types of trough tilt days themselves with regard to convective activity. We hope, of course, the answer will be in the affirmative.

3d. Tests Relating Convection to Trough Tilt

We now proceed in this section with the testing of our main hypothesis, that the more intense cases of convection and the larger-scale equatorward fluxes of momentum tend to be related. Use will be made of the area β number and the trough tilt data, the bulk of non-trough days being excluded as per the discussion above. This exclusion has the added benefit of mitigating the problem of serial correlation in our testing, since the area β numbers associated with each of the trough tilt types are not taken from successive days as a rule. With regard to the other problem discussed in connection with the testing performed in the previous section, namely the lack of complete homogeneity between the time spans sampled by our thunderstorm data and by our trough data, an attempt has been made to deal with this in some manner here. In the following tests, for each trough day, regardless of tilt, the area β number assigned will be the average of the area β numbers observed for the actual date of the trough and the date immediately preceeding. This is done uniformly for all the trough days and so should not introduce any particular systematic bias in our results. What this limited running mean process creates, in effect, is a statistic which is broader than the original area R number, in the sense that it allows any effects of convection that might be occurring on the calendar date prior to the one on which a trough is actually identified as being over the region to be included to some extent in our analysis. This seems to represent a reasonable attitude, and so for the remainder of this chapter, it is this averaging technique which has been used in connection with our convection parameter.

Two statistical tests have been used to investigate the connection between convection and trough tilt. Since there does exist a tendency to view the application of statistical methods with some degree of suspicion, we have taken here what are believed to be the simplest and most straightforward approaches. The first of these is the so-called "chi-square" (χ^2) test. It is probably the simplest of all statistical tests available for use when the data can be handled in terms of being able to assign a certain number of cases to each of a certain number of categories, yet it is also one of the most general tests available because of the minimum number of restrictions it requires the data to satisfy. A full description of it may be found in Panofsky and Brier (1958). Its main weakness lies in the fact that some information contained by the data can be lost when the procedure of placing the data merely into broad categories is followed. Thus, it is often felt that if significance can be shown from the χ^2 test alone, then such significance is probably real. On the other hand, if significance cannot be shown, other tests may be resorted to.

The particular procedure followed in applying the χ^2 test to our data is as follows. To each trough day there is assigned the revised area β number obtained by forming the two-day average discussed earlier. This series of revised area β numbers, 152 in all, is then inspected independently and divided into categories having roughly the same number of members in each. For our purposes, three categories were chosen, corresponding to high (H), moderate (M) and low (L) convective activity. Requiring, for statistical purposes, the categories to be fairly equally populated meant the following ranges of values in the area β number had to be associated with each category: H, area β numbers from 14 through 28 (the maximum in our data); M, numbers from 7 through 14; and L, numbers from 0 through 6. Next, attention was returned to the trough-day data so that the tilt for each day could be noted and placed in the category of convection appropriate to the value of its associated area β number. The resulting contingency table is given in Table IA.

Table IB gives the hypothetical distribution of the number of cases in each category which can be expected on the basis of pure chance. The expected number in each box is given by the product of the probability of reaching that box, and the total number of cases. This may be computed by forming the product of row total times column total and dividing by the grand total. Note that column and row totals are kept the same as in Table IA. This restriction means that there are only four degrees of freedom in the table, for after numbers have been inserted in the four boxes to the upper left, the numbers in the other five must follow from the conditions imposed by the sums of rows and columns. An important condition on the propriety of using the χ^2 procedure is that for no category should the expected number in the hypothetical distribution be less than 5. This is why it was found necessary to collect a minimum of four years of convective season data, so that enough cases

TABLE I

Contingency tables used in conjunction with the chi-square test performed in the text. The rows in each table are for different ranges of the area $\mathcal{R}$ number (indicated in parentheses) corresponding to High, Moderate and Low convection. The columns in each table are for the three types of trough tilt: negative, zero and positive. The entries within each table are the number of cases corresponding to each category in A) the observed distribution and in B) the hypothetical distribution.

		TROUGHS			
		NEGATIVE	ZERO	POSITIVE	ROW TOTAL
CONVECTION	H (14-28)	16	26	10	52
	M (7-13)	9	22	21	52
	L (0-6)	3	21	24	48
	COLUMN TOTAL	28	69	55	152

TABLE IA: OBSERVED DISTRIBUTION

		TROUGHS			
		NEGATIVE	ZERO	POSITIVE	ROW TOTAL
CONVECTION	H (14-28)	9.6	23.6	18.8	52
	M (7-13)	9.6	23.6	18.8	52
	L (0-6)	8.8	21.8	17.4	48
	COLUMN TOTAL	28	69	55	152

TABLE IB: HYPOTHETICAL DISTRIBUTION

of negative tilting troughs would be provided to avoid such an occurrence.

A comparison between Tables IA and IB reveals that, indeed, high convective activity and negative tilting troughs do seem to be strongly related. Certainly much more than the expected number of cases of high convection are associated with negative tilts. Positive tilts have almost half their expected number of such cases. Zero tilts have about the same number of high convection cases as might be expected from chance, perhaps revealing that they combine characteristics of the other two types of tilts. The relatively small number of cases of low convection associated with negative tilts is striking. The entries in the "M" row and the "0" column in Table IA do not differ markedly from their counterparts in Table IB. It appears the major contrast between the two tables lies mainly in viewing the four corner boxes: negative tilts and positive tilts certainly do appear to be different species with regard to the degree of convective activity associated with them.

Our view that Tables IA and IB reveal a strong association between high convection and negative tilting troughs can be confirmed statistically. To do so, we may form the quantity, χ^2 , defined by

$$\chi^2 = \sum_{i=1}^{m} \frac{(O_i - H_i)^2}{H_i} \tag{5}$$

where O_i refers to the number of individuals in the i^{th} category of the observed distribution, H_i refers to the number of individuals in the same i^{th} category of the hypothetical distribution, and m is the number of categories. In our case, m = 9.

The theoretical behavior of χ^2 is well known. From (5) it is clear that if the hypothetical distribution is far different from the observed one, then χ^2 will be large. When χ^2 is larger than certain limits, the null hypothesis that the observed distribution could have been drawn by chance from the hypothetical population is rejected, and the observed distribution is deemed "significantly" different from the hypothetical one. For the four degrees of freedom present in our tables, rejection at the 1% level (meaning that the probability of obtaining the sample value of χ^2 from the hypothetical population is less than 1%) occurs for any value of $\chi^2 \geqslant 13.28$. For our data, we compute from (5) a value of $\chi^2 = 15.39$. Thus, we feel reasonably confident in rejecting the null hypothesis and concluding that this analysis has shown that an association between high convective activity and negative tilting troughs does indeed exist to at least a 99% level of confidence. We might also view this analysis in less specific terms as providing some support for our contention, made purely on theoretical grounds in Chapter 2, that a convective rotating fluid regime is likely to feature equatorward fluxes of momentum as well.

Though we might well be content to subject our trough and thunderstorm data to no further testing, there is one additional test which would be of some interest. As we remarked earlier, the χ^2 test does require, in general, some information contained in the data to be lost. In our particular study, this may be seen from the fact, for example, that all values of the area ☈ number greater than 14 have been placed in the same category (H), with no further distinctions made. Also, values of the area ☈ number equal to 14 and to 13 find themselves somewhat arbitrarily being placed in different categories, though the difference between them is obviously small. Given this somewhat gross treatment of the data by the χ^2 test, the positive result derived from it appears even the more significant. Still, it seems desirable to perhaps consider a somewhat more precise treatment of the data, which is what the next statistical testing procedure is designed to do.

The test to be described now is widely referred to as an analysis of variance procedure. First, the average area β number associated with each of the three trough tilt types is computed by merely adding up the numbers that have been associated with each tilt and dividing by the number of cases in each tilt group. The resulting mean values of the area β number for each group are: negative tilts, 13.6; zero tilts, 10.5; positive tilts, 8.0. Considering our previous results, it is not surprising to find that high convection is associated with negative tilts now in this average sense. Of course, we may again ask the question of whether the differences among these means is significant or if this distribution could have been reasonably expected to occur by chance. This is where the analysis of variance procedure comes directly into play. Essentially the procedure takes into account the amount of variance contained in the area β numbers forming each of the three groups before deciding if the mean values derived from them, and given above, can be said to really be significantly different.

The variances computed for each group of area $\Gamma\zeta$ numbers turn out to be 43.5, 38.6 and 30.8 for the series associated respectively with negative, zero and positive tilting troughs. Given this amount of variance <u>within</u> each group forming the means, can we say that the amount of variance <u>between</u> the means of the groups represents a significant spread? This question may be answered by forming a ratio of numbers measuring, in essence, the variance between the groups in the numerator and the variance within the groups in the denominator. If this number exceeds a certain limit, then we may view the between-groups variance as being significant. The actual ratio formed also incorporates the number of degrees of freedom associated with each variance and is called an F ratio. The distribution of F is well known, and its limiting values for significance at different levels is tabulated as functions of the degrees of freedom of numerator and denominator.

There are two restrictions on the appropriateness of using the tables for F, however, which were not imposed by the chi-square test. The first of these is that the data used be normally distributed. This cannot really be said to be true of the distribution of our area β numbers, since quite a few of its values do fall in the wings. The sensitivity of the F test to various degrees of non-normality is not really well known however, so the best we can do is to present our results keeping this weakness in mind. Secondly, testing for the significance of differences in the group means requires that there be no significant differences among the group variances. There are tests, of course, to determine whether significance between sample variances exists, but they are all approximate in nature. The test used here is described by Wadsworth and Bryan (1960, p. 266), but we omit the details. It turns out that the three numbers referred to earlier (43.5, 38.6 and 30.8) are not significantly different, there being at least a 20% probability of observing such a distribution in the data. Thus, our data would seem to permit meaningful testing of the difference in means (13.6, 10.5, 8.0) associated with each tilt to be accomplished by the F test, analysis of variance procedure.

For our data, the value of the F ratio is 8.02: the numerator, measuring the between-groups variance, having 2 degrees of freedom and the denominator, measuring the within-group variance, having 149. For these values of the degrees of freedom, a value of $F \geq 4.75$ would indicate significance at the 1% level, meaning only a 1% probability exists of obtaining such a large value of F from data chosen at random. Indeed, the value $F = 8.02$ is still significant at the 0.1% level. We seem reasonably assured in concluding, therefore, that the greater amounts of convection apparent in our data associated with negative tilting troughs indicate the workings of a real phenomenon. Our confidence is increased, of course, by there existing a theoretical framework which has suggested the likelihood of finding such a relationship all along. Some theoretical grounds for performing a statistical study is an important factor, for otherwise one leaves oneself open to the criticism of having "gone on a fishing expedition". Such criticism, we trust, is not justified in our case, having laid beforehand a theoretical foundation for anticipating that a connection between high convective activity and equatorward fluxes of momentum would be found. Since high area $\mathcal{R}$ numbers and negative trough tilts serve as some measure of these quantities over a fixed region, we feel able to conclude that the positive statistical results relating them do provide some observational verification for our theory. Of course, several other measures of convective activity and momentum fluxes do exist and should be investigated. However, of particular interest to us now would be an independent measure of the degree of horizontal convergence over our region, and its relation to the trough tilt data we have. To a limited extent this is what is pursued in the next chapter.

CHAPTER 4.

OBSERVATIONAL STUDY II

AREAL CONVERGENCE AS A PARAMETER

4a. An Index of Convergence And Its Suitability

Within the region chosen for our study, there exist 7 upper-air sounding stations for which twice daily (00 Z and 12 Z) measurements of horizontal velocities at various levels have been made during the period commensurate with our trough data. These stations, enclosed by the box in Fig. 2, are numbered 48, 50, 60, 62, 71, 72 and 81 in that figure. At each of these stations, twice daily values of $\omega = \frac{dp}{dt}$ have been computed from making use of the observed horizontal winds in the equation of mass continuity written in pressure coordinates. This quantity is often taken as a measure of vertical motion, since it is directly related to the vertical component of motion $w = \frac{dz}{dt}$ when the hydrostatic assumption is made. Strictly, it is a measure of the amount of horizontal convergence in the column below the level for which it is computed. It is for this reason that we find ω a desirable basis for an index, because it is the interaction between the field of convergence and the

Coriolis force that is supposed to give rise, in our theory, to equatorward fluxes of momentum.

The actual index used here is formed by averaging the 7 station values of ω found at 500 mb for each time of the observations. Such area averaged ω's, to be denoted as $\{\omega\}$, form a series of twice daily values, and represent the index used in this chapter in conjunction with the trough tilt data already in hand. The $\{\omega\}$ index is relevant to our investigation in the following sense. If it is to have the effect on large-scale horizontal fluxes which we claim, then convective activity (and so latent heat release) must influence a large enough region to a great enough extent so as to induce convergence on a proper scale. The area ☈ number used in the previous chapter was intended to measure the extent of convective activity, in that it tried to provide some accounting of the area covered by convection and dealt with its most intense form. The $\{\omega\}$ index, on the other hand, is intended to measure the actual process presumed to be associated with the convection which, if it is strong enough, is then supposed to give rise to an identifiable equatorward flux of momentum. We would like to demonstrate that when convergence over a large enough region is unusually strong, an equatorward flux of momentum, viewed as we have chosen to do here in terms of a negative tilting trough, also tends to be observed. This is what our theory suggests should be the case and is the motivation behind using the $\{\omega\}$ index.

Of course, there are many shortcomings involved in relying upon $\{\omega\}$, as it has been defined here, as a measure of areal convergence. Most obviously, there is the question of the accuracy with which ω at each station has been determined. It is a notoriously difficult quantity to evaluate, although the technique described in the next section seems more reliable than most. Then too, since our major concern will be with comparing values of $\{\omega\}$ for different situations, it is the information contained in relative differences that will interest us most, and the possible presence of random, and even some systematic errors may not be of much worry. Another obvious shortcoming of the $\{\omega\}$ index is the procedure used in defining the area average. The 7 stations are not uniformly spaced within the box, nor may they really constitute a large enough sample for the area. However, these biases should be independent of the trough tilt type, and their effect on our study may be mitigated somewhat by having considered a large enough sample of trough tilts.

As with the area β number, the $\{\omega\}$ index assigns to the whole region a single number, and so does not allow any finer resolution in space. Restriction to computing the $\{\omega\}$ index at a single level was necessary because of the effort involved, and even though the 500 mb level was a logical choice (it is the level of our trough data), it may not have been the one best suited to demonstrate the effect we are interested in. Also, values of ω at only 00 Z and 12 Z are available. This, coupled with the shortcomings discussed earlier concerning the use of trough data over a fixed region, would seem to preclude the possibility of really saying anything concrete about a causal relationship from the data.

Thus, there are admittedly several weaknesses in the $\{\omega\}$ index. However, our main concern for now is to simply accomplish a demonstration that there does exist some connection between the degree of convergence over our region and the tilt of the trough which is observed. For purposes of showing so broad a relationship, the $\{\omega\}$ index would seem to be satisfactory. In any case, if such a gross index can reveal the relationship we seek, we might tend to view the phenomenon engendering it as being fairly basic in nature, to be so captured. Before looking to the results of the statistical tests, however, the manner in which the station values of ω were computed should be mentioned further.

4b. Technique For Computing ω

The values of ω used here have been derived from schemes advanced by Kung (1972, 1973). His method is a kinematic one, based on the vertical integration of the mass continuity equation in the pressure coordinate system, and so makes no assumptions about the nature of the circulation except the hydrostatic relationship used to transform into pressure coordinates. Because of this generality, the kinematic estimate is an important one and seems the most appropriate for use here. Unfortunately, the kinematic estimate is also very sensitive, often leading to vertical profiles of ω which have unacceptably large values near the top of the atmosphere. Kung's technique, however, is based on the presumption that the vertical profile of ω should converge to a near-zero value at the top of the atmosphere without being so forced when a proper balance between the analysis scheme and the data disposition is achieved. To this end, the scheme provides a range of options with regard to the type of horizontal analysis performed around each station in the network. The analysis option for each station and time which yields the best ω profile is the one chosen, basically according to a criterion of how close to zero at the top the profile can be made to converge. As it turns out, most of the profiles computed, but not selected, for a station are clearly erroneous in nature with unacceptably large ω at upper levels. However, normally at least one profile will display an acceptable form, thus showing that when the right choice as to fitting an analysis scheme to the data disposition is made, errors in computing the divergence can be minimized. An attractive aspect of the scheme is that it is objective in nature, all steps being accomplished through automatic computation. The scheme has been applied to the network of North American stations contained in the M.I.T. General Circulation Library for the period May 1958 through April 1963. Data taken at 00 Z and 12 Z for these stations have been reported for every pressure level between 1000 and 50 mb at 50 mb intervals, and at the 70 mb level as well.

The stations for which ω has been computed are indicated in Fig. 2 by the dots. The stars indicate stations which were used only to help further in the analysis. The box which has been drawn on the figure encloses the region that has been singled out for our convection study.

The details of Kung's approach will not be given here, inasmuch as they are provided in his papers referenced earlier. Essentially, the procedure involves computing the divergence $\nabla\cdot\mathbf{v}$ at each sounding station and level, with wind data taken from that station and surrounding ones, by describing the horizontal variation of the wind field in terms of a polynomial surface representation. The options referred to before involve the use of fitting the data at each level with either a quadratic or a plane surface for the purposes of evaluating the horizontal derivatives of the wind components, interpolating missing wind data, and performing in some cases, if required, a space smoothing of the computed divergence. Combinations of these procedures permit the computation of up to 14 vertical profiles of ω for each station and time. The profile having the best shape and convergence properties is the one selected, so long as certain prescribed rejection criteria are also met.

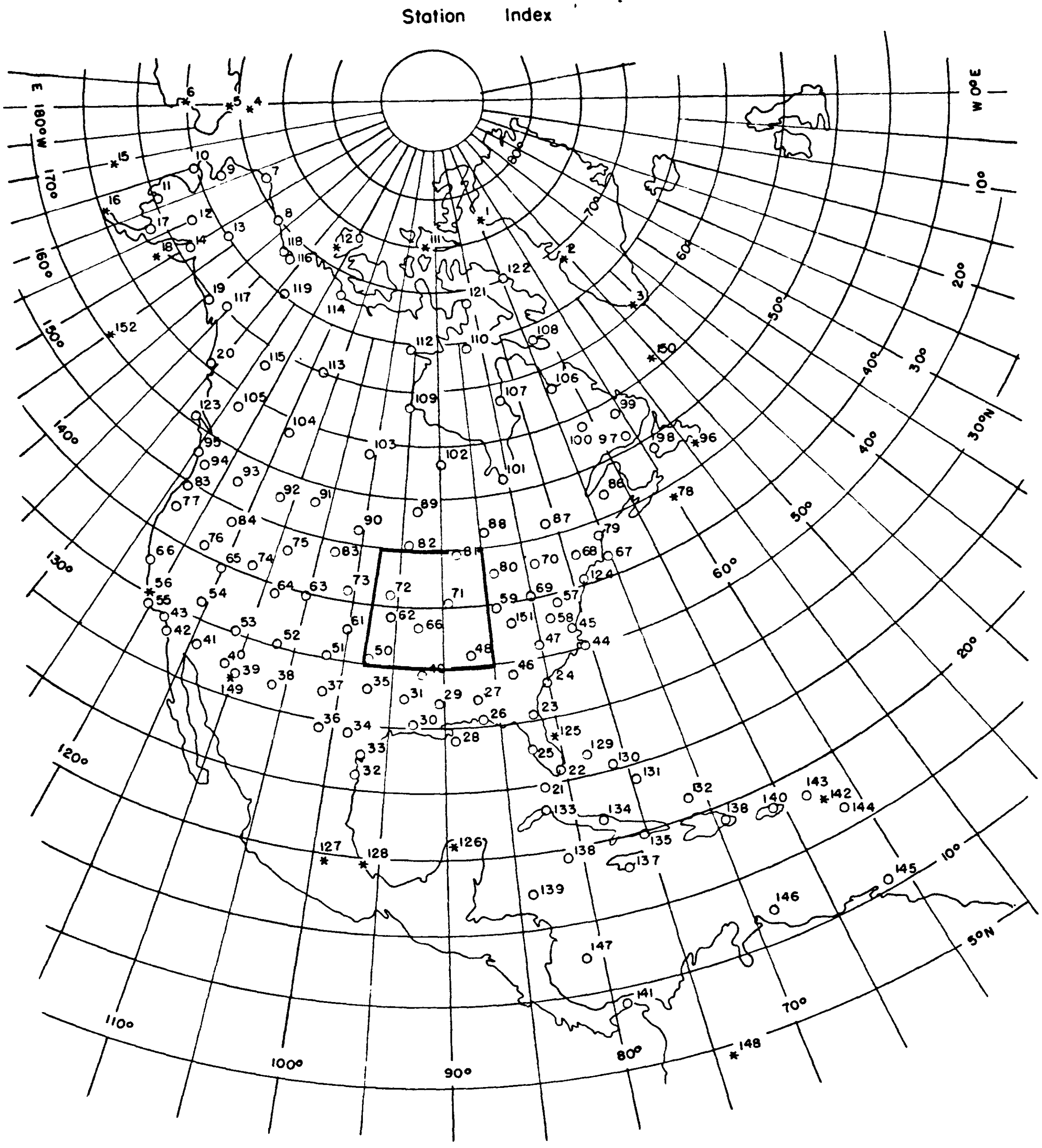


Fig. 2

Distribution of upper-air sounding stations used in the computations of ω. Values of ω were obtained at stations indicated by dots, stations indicated by stars being used solely to aid in the analysis. The region enclosed by the box represents the area focused upon for our study.

In the process of the vertical integration, $\omega = 0$ is prescribed at the surface.

Kung's works referenced earlier present examples of the scheme's results. Also, we have had the opportunity of evaluating the results for ourselves. For the most part, the ω profiles obtained do yield meaningful spatial distributions of the vertical motion field through the depth of the atmosphere, and do seem to correspond well with synoptic features on the map. Given further the general considerations of the previous section, the ω profiles computed by this scheme would seem to be satisfactory for the purposes of what we desire to demonstrate here.

4c. Statistical Test Results

Two time series of $\{\omega\}$ have been independently created for the purposes of statistical testing. The first series, referred to as the contemporary $\{\omega\}$ series, consists of those values of $\{\omega\}$ which have been obtained for our region corresponding to the same dates and times as the trough data compiled in the previous chapter. The second series, referred to as the 12-hour lead $\{\omega\}$ series, consists of those values of $\{\omega\}$ computed from data taken 12 hours prior to the time the troughs are observed on the once daily maps. Each of these time series has occasional gaps, caused whenever more than 3 of the stations simultaneously displayed missing values of ω , thus making an estimate of $\{\omega\}$ unreliable. When 3 or fewer stations had missing values of ω , $\{\omega\}$ was calculated as the mean of those stations that did report. Most often, however, all 7 stations did supply ω values. When $\{\omega\}$ had to be treated as missing in one of the time series, information about the corresponding trough was also viewed as missing. This explains the reason for numbers less than the true amounts of trough and non-trough days appearing in the following results.

The statistical tests used in this section are the same as those used previously in conjunction with the area ß numbers, but now with either the contemporary $\{\omega\}$ or the 12-hour lead $\{\omega\}$ values independently replacing that statistic. Therefore, discussions of the tests themselves, their strengths and weaknesses, will not be repeated here. Following the order of testing established in the previous chapter, the first test which we describe now deals with comparing values of $\{\omega\}$ for trough and non-trough days. In order for them to seem of much value, we would expect that the values of $\{\omega\}$, in each series, which are associated with troughs will show greater convergence than those values of the series not associated with troughs. Letting the subscript 1 refer to trough days, and subscript 2 to non-trough days, we have for the contemporary $\{\omega\}$ series (symbols as defined in Chapter 3):

$$\begin{array}{llll} \overline{X}_1 = -0.36 & S_1^2 = 0.49 & N_1 = 147 & \text{(trough days)} \\ \overline{X}_2 = 0.34 & S_2^2 = 0.62 & N_2 = 333 & \text{(other days)} \end{array} \qquad (6)$$

Negative values for $\{\omega\}$, of course, imply convergence, so (6) would certainly indicate that greater amounts of convergence occur on trough days than otherwise, in the mean. To test for the significance of the difference between these means, (6) is substituted into (3), with the values of N reduced by 1/3 in an attempt to account for any lack of independence in the data. The resulting absolute value of y is 5.56, which is certainly significant at the 1% level. For the 12-hour lead $\{\omega\}$ series, the following statistics have been independently computed:

$$\begin{array}{llll} \bar{X}_1 = -0.30 & S_1^2 = 0.43 & N_1 = 149 & \text{(trough days)} \\ \bar{X}_2 = 0.15 & S_2^2 = 0.70 & N_2 = 336 & \text{(other days)} \end{array} \qquad (7)$$

In this case, the absolute value of y from (3) is 3.62, which, again, is significant at the 1% level. Thus, in the case of each series, the difference between the mean value of $\{\omega\}$ associated with trough days and that associated with non-trough days is a significant one. We therefore feel somewhat comfortable about investigating the question of convergence as it is related specifically to trough tilt with these two $\{\omega\}$ series, and about restricting attention to the trough day data alone. It may be mentioned here in passing that tests with $\{\omega\}$ series taken at 24-hour lead and 12-hour lag with respect to the trough data did not yield significant results.

Coming now to the relationship between convergence, as measured by $\{\omega\}$, and the tended direction of momentum flux, as determined by trough tilt, the first statistical test to be presented (as it was in Chapter 3 in this connection) is the χ^2 contingency table test. The observed and the hypothetical (chance) distributions of $\{\omega\}$ with respect to trough tilt type are given in Tables 2A and 2B for the contemporary $\{\omega\}$ series. Tables 3A and 3B give similar results for the 12-hour lead $\{\omega\}$ series. The ranges of $\{\omega\}$ (given in μb sec^{-1}) used to characterize the rows in both sets of tables have been chosen so as to make nearly equal the total number of cases for each of the rows in a table. A mere glance at the tables reveals that negative tilts do seem to be associated with much more than their expected share of cases of strong convergence, both on a contemporary and 12-hour lead basis. Resort to the equation for χ^2 given in (5) is necessary, of course, to determine the level of statistical significance contained in our results. A value of $\chi^2 \geq 13.28$ is required for significance at the 1% level with four degrees of freedom. For the contemporary $\{\omega\}$ series data in Tables 2, $\chi^2 = 15.35$. For the 12-hour lead $\{\omega\}$ series data in Tables 3, $\chi^2 = 13.66$. Thus, we would seem to be able to conclude, within the limitations imposed by our data, that indeed an equatorward flux of momentum particularly tends to be associated with strong convergence measured over a region, whether this convergence is determined at the same time as, or 12 hours prior to, the observing of the trough tilt.

TABLE 2

Contingency tables relating contemporary values of $\{\omega\}$ and trough tilt. The rows in each table are for different ranges of $\{\omega\}$, which has been measured at the same time that the trough tilt is observed. Values of $\{\omega\}$ are in units of μb sec^{-1}. The columns in each table list the trough tilt types. The entries within each table are the number of cases corresponding to each category in A) the observed distribution and in B) the hypothetical distribution.

TROUGH TILT

DEGREE OF CONVERGENCE

	NEGATIVE	ZERO	POSITIVE	ROW TOTAL
$\{\omega\} \leq -.55$	16	19	11	46
$-.55 < \{\omega\} < 0$	7	25	23	55
$0 < \{\omega\}$	3	25	18	46
COLUMN TOTAL	26	69	52	147

TABLE 2A: OBSERVED DISTRIBUTION

TROUGH TILT

DEGREE OF CONVERGENCE

	NEGATIVE	ZERO	POSITIVE	ROW TOTAL
$\{\omega\} \leq -.55$	8.1	21.6	16.3	46
$-.55 < \{\omega\} < 0$	9.8	25.8	19.4	55
$0 \leq \{\omega\}$	8.1	21.6	16.3	46
COLUMN TOTAL	26	69	52	147

TABLE 2B: HYPOTHETICAL DISTRIBUTION

TABLE 3

Same as Table 2, but for values of $\{\omega\}$ measured 12 hours prior to the time the trough tilt is observed.

	TROUGH TILT			
DEGREE OF CONVERGENCE	NEGATIVE	ZERO	POSITIVE	ROW TOTAL
$\{\omega\} \leq -.55$	16	21	11	48
$-.55 < \{\omega\} < 0$	7	20	19	46
$0 \leq \{\omega\}$	4	27	24	55
COLUMN TOTAL	27	68	54	149

TABLE 3A: OBSERVED DISTRIBUTION

	TROUGH TILT			
DEGREE OF CONVERGENCE	NEGATIVE	ZERO	POSITIVE	ROW TOTAL
$\{\omega\} \leq -.55$	8.7	21.9	17.4	48
$-.55 < \{\omega\} < 0$	8.3	21.0	16.7	46
$0 \leq \{\omega\}$	10.0	25.1	19.9	55
COLUMN TOTAL	27	68	54	149

TABLE 3B: HYPOTHETICAL DISTRIBUTION

The mean values of $\{\omega\}$ associated with each type of trough tilt may be compared for the significance of the differences among them by use of the F ratio test. For the contemporary $\{\omega\}$ series, the mean $\{\omega\}$ associated with negative, zero and positive tilts respectively is -0.84, -0.30 and -0.20 μb sec^{-1}. The variance between these group means apparently easily outweighs in importance the total variance within the groups, for the value of the F ratio arising from the analysis of variance procedure applied to this data is $F = 7.63$ (The three variances associated with each of the preceding means do not differ significantly.). Since a value of $F \geq 4.75$ is all that is required for significance at the 1% level, it would certainly seem that the large value of mean convergence associated with negative tilts in the contemporary series is statistically meaningful. Significance also exists, though to a lesser extent, in the data for the 12-hour lead $\{\omega\}$ series, for which each of the mean values associated with the troughs of negative, zero and positive tilts is -0.63, -0.29 and -0.15 μb sec^{-1}, respectively. The value of F for this data is 4.82, still indicating significance at the 1% level for the spread in these means.

The results of the statistical procedures outlined in this section all seem to point to the same conclusion. Trough tilts which are negative seem to form the group associated the most often with strong convergence over our region. The discovery of this general relationship in our data would seem to lend some measure of support to the theory expressed in Chapter 2. Apparently, the gross presence of an interaction between the field of convergence presumably associated with organized convective processes and rotation can be found to exist on the earth, at least when each of these components is strong enough so as not to be masked by other competing processes. Of course, we have not been able to discern details about further conditions required for the interaction to proceed, nor details about the evolution of the process in time or space. The results of this section do show somewhat greater significance in connection with the contemporary, rather than 12-hour lead, $\{\omega\}$ series, but caution should be taken in trying to deduce further implications from this. The fact that significance can be shown to exist at all for either series, however, is important. This, and the results of the previous chapter, should be viewed as offering some new evidence for the existence of a further role of convection in the atmosphere.

4d. A Test Relating The Area $\mathcal{R}$ Number And $\{\omega\}$ Indices

Before concluding our series of statistical test procedures, there is one additional test which would prove to be of some interest, namely a test designed to investigate specifically the relation between our separate indices of convection and convergence. The tests performed in Chapter 3 were intended to demonstrate a connection between widespread convective activity and negative tilting troughs. The presumed relevant physical action of the convection is envisioned to be the creation of horizontal pressure gradients favoring the enhancement of large-scale convergence. The tests performed in this chapter sought to determine more directly if, in fact, strong large-scale convergence and negative tilting troughs are related. Apparently, the answer is that they are. Combined with the results of the previous chapter, this would seem to imply of our data that it must be satisfying the expectation that widespread convection and strong convergent motion over our region are related. This expectation, of course, has been derived in part from the results of Tracton's work. Still, it would seem worthwhile to evaluate directly from our data the correlation between the two independently obtained indices, $\{\omega\}$ and the area ζ number.

Unfortunately, because of the presence of auto-correlation in each of the complete series for $\{\omega\}$ and the area ζ number, testing for the significance of the cross-correlation between them is not without uncertainties. In any event, the 486 daily values of $\{\omega\}$ at 12 Z were correlated with the value of the area ζ number for the same date. The resulting linear correlation coefficient is -0.391. To test the significance of this value, one may hypothesize that our samples were chosen at random from populations having zero correlation. If we assume that independent information is contained in only every third pair of values in the data series, then it may be shown (see Panofsky and Brier, p. 92) that a cross-correlation coefficient for our data exceeding 0.205 in absolute value has a probability of originating from uncorrelated populations of less than 1%. Hence our computed value of -0.391 may be viewed as being significant, and a real relationship between $\{\omega\}$ and the area ζ number may, therefore, be supposed to exist. (An analysis of variance procedure aimed independently at testing the significance of the cross-correlation also yields significance at the 1%

level.)

Of course, tests aimed at showing significance for correlation coefficients must be viewed cautiously. Correlation coefficients themselves tend to show unpredictable trends and oscillations in meteorological data. Also, if a correlation coefficient is "significant" in the statistical sense, the relation implied may still have little predictive value. Moreover, the existence of auto-correlation in each time series remains a problem, although we have attempted somewhat to correct for this here. More elaborate techniques of cross spectrum analysis are available to handle this problem, but these seem somehow too elaborate for our present purposes. After all, our indices are fairly gross, the area ☈ number encompassing a 24-hour range, while $\{\omega\}$ is computed only for instantaneous conditions within that period. Therefore, no more than the simple cross-correlation procedure performed here would seem advisable. The test does seem sufficient enough to show that intense, widespread convection over an area does tend to be related generally to convergence over the region at 500 mb. Our indices are not precise, but given the plausibility of there existing a physical mechanism (such as latent heat release) which relates the two processes, the significance of the linear correlation coefficient between $\{\omega\}$ and the area $\mathfrak{R}$ number is probably real.

CHAPTER 5.

FINAL COMMENTS

5a. Summary of Results

This study has been aimed at investigating one possible effect of convection on the dynamics of large-scale flows. Our initial considerations were theoretical, an attempt being made to demonstrate mathematically and physically how, under proper conditions and isolated from other processes, convection might tend to induce horizontal eddy motions which favor a transport of momentum equatorward. The mechanism proceeds through the action of the Coriolis force upon the large-scale convergent motion field which the convection tends to enhance. We noted earlier that attempts by some investigators at numerical modelling of the sun have included consideration of this interaction between convection and rotation, but the results have been mixed, mainly because, it is suspected, of the various simplifying assumptions required by these models. The approach taken in this present study has been to investigate the implications of our theory for motions over a region of our atmosphere by turning to available observations concerning convective activity and the large-scale horizontal flow.

The notion is now generally accepted that on the earth the cumulus scales of motion can cooperate, in a sense, with the larger scales by supplying energy for them in the form of latent heat release. This is the principle behind the CISK theory mentioned earlier, and behind Tracton's study of mid-latitude cyclogenesis. What our present theory further suggests, in essence, is that the convective supply of latent heat to the large-scale motions also tends, in some instances, to induce a preferential tilt to those motions. Such a tendency would likely be observed most often when the convection is of a great extent, since the simultaneous presence of other, possibly competing, processes might otherwise tend to mask the effect. Thus, we have defined an index of convective activity, the area ☈ number, to provide some measure of both areal extent and intensity of the convective activity over the portion of the central United States chosen for our study.

Because of the complicated nature of the mixture of atmospheric processes which exist in mid-latitudes, it was decided that initial attempts at finding evidence of the behavior we seek should be based on a fairly large sample of data. If the particular effect of convection on the large-scale flow we are concerned with cannot first be shown to have a general presence in a large data sample, then we might have good reason to doubt its reality. We are, after all, interested in a fairly gross effect, one which should be capable of being noticed in a statistical sense, if it is of any importance. Thus, our approach has been a statistical one, with resort made to a fairly lengthy series of data. This has precluded the possibility of performing detailed synoptic studies of individual case histories here, but it is not obvious that, given the uncertainties involved, this would be the best way to proceed initially anyway. Relying on enough data to provide statistically meaningful results has at least had the advantage of providing us with some measure of confidence in those results we do obtain.

Results of comparing values of the area $\mathcal{R}$ number (actually, its two-day running mean) for 152 cases of trough passage which were observed during four years of warm season months over our region in the midwest U.S. were presented in Chapter 3. The tilt of the trough was taken as indicating the direction of the eddy momentum flux, and it was found that negative tilts (equatorward fluxes) and high values of the area ☈ number were strongly related. Viewing the area ☈ number largely as just a means of providing some measure of the amount of overall convective activity in the region, we therefore concluded that our data supported our theoretical expectations. There are, of course, some instances of high area ☈ numbers being associated with a tilt other than a negative one, as well as there being a few cases of negative tilts associated with low area ☈ numbers. Many of the possible explanations for such occurrences must be based in part on the acknowledged frailties inherent in using such a gross index as the area ☈ number. This index, after all, does not serve to isolate phenomena very precisely in time or space. Furthermore, concerning the cases of high area ☈ numbers which are not associated with negative tilts per se, it may still be that they do represent convective activity which is tending to reduce the amount of non-negative tilt, but for one reason or another is not completely succeeding. Some of our results for zero tilting troughs may be capable of being viewed in this light. On the other hand, concerning the cases of negative tilts associated with low area ☈ numbers, it should be recognized that mid-latitude processes other than convection may sometimes cause a negative tilt, though this is probably infrequent. Such questions aside, however, it may be reiterated that our study with the area $\mathcal{R}$ numbers does seem to show a clear tendency for there to be a relationship between high convective activity and negative tilting troughs in the long term mean. This is just the behavior that our general theory had predicted.

A second index, this one attempting to provide a measure of the convergent component of the wind field, was tested in Chapter 4. Again, the results were encouraging. Negative tilting troughs and strong mean convergence over the region were shown to be very much related. This is quite in accord with our theoretical notions, for it is the effect of convection in enhancing large-scale convergence which is what then, in our view, leads to an equatorward momentum flux. It is true that our index of convergence is rather crude, but the manner in which it still tends to be related to trough tilt despite this fact is, we believe, an assertion of the relationship's being real.

Unfortunately, our data studies have not permitted a direct confirmation of the cause-and-effect relationship implied in our theory. Some presence of a causal relationship is suggested in our data results, since our index of convection and the 12-hour lead $\{\omega\}$ series have been weighted towards viewing conditions in the region which exist prior to the time the trough tilt is noted. However, as we remarked earlier, since we do not follow a trough's history, but instead focus attention on a fixed region, we must be careful about claiming too much for the results of our data studies with regard to cause-and-effect. Of course, a theory suggesting that a negative tilt to a trough would cause a high degree of convection to follow does not, to our knowledge, presently exist. Such a theory would not be unwelcome though, for combined with our present considerations, it would suggest the existence of an instability in fluid motions not previously suspected. In any event, the observational evidence we have offered here, albeit limited in some respects, demonstrating that convection and trough tilt are related pursuant to theory would seem to be in the nature of breaking new ground, and in that respect important.

5b. Suggestions For Future Research

Having chosen to regard the work reported here as representing an important beginning in exploring the existence of a previously largely unrecognized phenomenon carries with it the recognition that many questions still remain. The suggestions for future research we could offer are numerous, but we will mention only some of the more obvious. With regard to the type of broad study performed here, other indices of convection might well be tested. For that matter also, schemes for actually computing the momentum flux, rather than simply relying on a trough tilt index, should be considered. Also, it would be interesting to perhaps consider other regions in the U.S. to see if they can also be shown to come under the influence of the phenomenon. The southeastern U.S., in particular, has been proposed as an attractive candidate for study. In searching for other areas of the globe to consider, our earlier mention of there apparently being equatorward momentum fluxes over the oceans in the N.H. winter and S.H. yearly conditions comes to mind. Winds over ocean areas can now be reasonably estimated from satellite observations, and surface weather observations made by ships at sea are also available. Both sources could be used to analyze the situation which results around the time of outbreaks of cold air over warm waters, in the hope of finding systematic equatorward momentum fluxes.

Besides fairly broad based data studies, further attention is due more detailed considerations. Individual synoptic case studies of developing negative trough tilts should be attempted, with attention not necessarily focused on any fixed region. The greater detail in time and space such studies would permit might now provide more definitive answers to the questions concerning cause-and-effect we raised earlier. In general, such detailed studies could permit greater insight into the more exact nature of the time and spatial requirements needed for the interaction we are investigating to proceed. In particular, the role of such configurations as squall lines may be made more plain. It is recognized that such studies will not be easily performed.

In combination with the synoptic studies discussed above, an approach used to good advantage by Tracton may be of some utility for our purposes as well. The procedure involves making reference to forecasts of the wind field made by large-scale numerical models of the atmosphere, which are in routine operation at the National Meteorological Center. Such models either ignore convection or parameterize it very crudely, and should these models display systematic errors in the momentum flux on occasions when convection was prominent, one might be able to surmise that convection was responsible. It was proceeding in a manner similar to this that allowed Tracton to infer the role of convection in extratropical cyclogenesis. Our problem is not quite so straightforward as his in certain respects though, and some caution will be required in our case about trying to form conclusions on the basis of comparisons with the numerical prognoses. For one thing, the numerical forecasts proceed by advancing an observed initial state, and it should be made certain that the observations forming the initial state have not been affected by convection, regardless of how convection is accounted for by the model in advancing that state. Also, one should check to see, in cases when negative trough tilts are forecast, whether the model has also taken the environment to be saturated and allowed a parameterized convection to occur.

A more serious problem with the "model-comparison" approach would seem to be that it is based somewhat on the assumption that the models portray the large-scale dynamic processes fairly accurately. Since it is observed on more than a few occasions that the numerical forecasts will suggest greater intensification of a storm than actually happens, this assumption may not be fully justified. In terms of our problem, the appearance of a negative tilting trough or lack thereof in a particular numerical forecast may be due to errors in treating the large-scale dynamics and not entirely because of failure to deal with convection properly. In any event, the implications of this sort of possibility should be reckoned with, if such a study is performed.

Besides these opportunities for future research of an observational nature, some further theoretical work can be suggested as well. Among them would be seeking a clear answer to the question of why it is that synoptic-scale baroclinic processes in the earth's atmosphere apparently are not prone to give rise to equatorward momentum fluxes in the mean. As we mentioned before, the mathematical analysis performed in Chapter 2 is not immediately seen applicable to the case of a baroclinic instability. Perhaps, however, a similar analysis applied to the meridional component of motion and its variance would be more revealing with regard to baroclinic motions. The problem is not likely to be easily solved, however.

Also, theoretical approaches less global in nature than that performed in Chapter 2 would be desirable in considering the interaction between meso- and synoptic scales over a limited region. Numerical experiments aimed at investigating the process might also be devised. Though highly suggestive of the basic phenomenon involved, the fairly broad analysis based on (1)-(2) certainly leaves room for further explication, which such more detailed analytical or numerical procedures might supply.

Finally, of course, there are the implications of what we have done as regards future solar or extra-terrestrial planetary studies. The evidence we have gathered on the earth in support of our theory would certainly suggest that the interaction between convection and rotation could display an even more dominant effect in a largely convective atmosphere such as, say, the sun's. In fact, it is mainly this process which we feel accounts for the solar equatorial acceleration. There are, of course, many complications to deal with in the solar case, and little conclusive data on which to base hypotheses exists. Indeed, one recommendation would be to direct the analysis of that data which can be collected for motions on the sun towards investigating this particular phenomenon. With regard to theoretical considerations applicable to the sun, further study should perhaps be made concerning the statistics to be expected from a random ensemble of convective units, all of which are in various stages of growth. A step towards dealing with this question was made earlier in offering a sufficient condition for which the increase of $[(M')^2]$ in (2) also implies an equatorward momentum flux by such an ensemble of motions. A necessary condition should also be sought, if one exists. The shape of the convective units may be an important factor. Starr (1974) has recently suggested that the solar supergranulation cells have a preferred N-S orientation, and this helps render these convective units capable of contributing to the equatorward flux. This matter deserves more attention.

In conclusion, it is believed that the basic process responsible in large measure for the solar differential rotation, as well as for certain dynamical behavior in our own atmosphere, has been grossly delineated by our present efforts, although several questions do remain. It has been the basic intent of this study to make further appreciated the contention that the interaction between convection and rotation represents a phenomenon which should no longer be overlooked. The evidence presented here certainly seems to weigh much in favor of this viewpoint. The role of convection in the dynamics of mid-latitude larger-scale motions has been a field of growing interest of late, and it is felt that the results of this work should serve as an impetus for increasing efforts in this direction.

APPENDIX A

LARGE-SCALE VERTICAL FLUXES OF MOMENTUM

The vertical velocity fields computed for use in Chapter 4 can find application in a wide variety of other studies as well. Because of the manner of their derivation, they seem particularly well suited for correlation with other fields of meteorological variables for study of vertical transport phenomena in the general circulation. The role of one such transport phenomenon has been of increasing interest in recent years, namely the vertical eddy flux of momentum. Starr, Peixoto & Sims (1970) and Peixoto et. al. (1973) have presented results indicating that such fluxes may be of some significance in the dynamics of the general circulation. However, the technique for determining the vertical eddy flux used by these authors was an indirect one, making use of hemispheric upper-air horizontal wind observations and the concepts of conservation of mass and absolute angular momentum in a mean meridional plane (details may be found in Starr, Peixoto & Sims). The existence of directly computed vertical velocities available through the technique described in Chapter 4 should be able to provide somewhat independent information concerning the vertical eddy flux of momentum.

Values of the kinematic estimates of ω have been obtained at each of the stations indicated in Fig. 2 for 00 Z and 12 Z, in the manner described in Chapter 4 for the period May 1, 1958 through April 30, 1963. These values were then correlated with the corresponding observed station values of u , and so for each station and level, the quantity $\frac{1}{g}\overline{u'\omega'}$ was obtained for each of four 15-month composite seasons (winter consisting of the 15 January, February and March months in the data sample, and so on for the other seasons). It should be realized, of course, that the fluxes obtained from these kinematic ω's pertain only to conditions over North America and only to transient eddies of the scale deducible from twice daily observations, whereas the conclusions drawn by Peixoto et. al. regarding the role of vertical eddy fluxes of momentum are based on deductions of the zonally averaged flux around complete latitude circles. Moreover, their results include effects from eddies of all types, with separation as to scale not being possible. However, their results indicate that the latitude belt corresponding to that which covers a major portion of the

North American data network is characterized in the zonal average for most seasons by strong upward, countergradient momentum fluxes. Since the time periods treated by them are identical to those dealt with here, it seems reasonable to investigate whether the vertical momentum fluxes obtainable from the kinematic ω's over North America show a similar character.

The actual results obtained here for $\frac{1}{g}\overline{u'\omega'}$ over North America for each of the composite seasons reveal patterns of the stress field at various heights that do show good spatial coherence. Widespread regions of negative stress in the mid and upper troposphere are visible, revealing the presence of interesting negative viscous actions.(An example of the fields obtained may be found in Fig. 3, which gives the stress field computed from the twice daily data for the 15-month winter season at 300 mb.). However, certain basic features characterizing the cross-sections of the stress given by Peixoto et. al. have not been duplicated by these new results. For one thing, the maximum values of the negative stresses computed from the kinematic ω's tend to center around the 300 mb level, still below the level of the mean jet, but not at the 600 mb height which the results

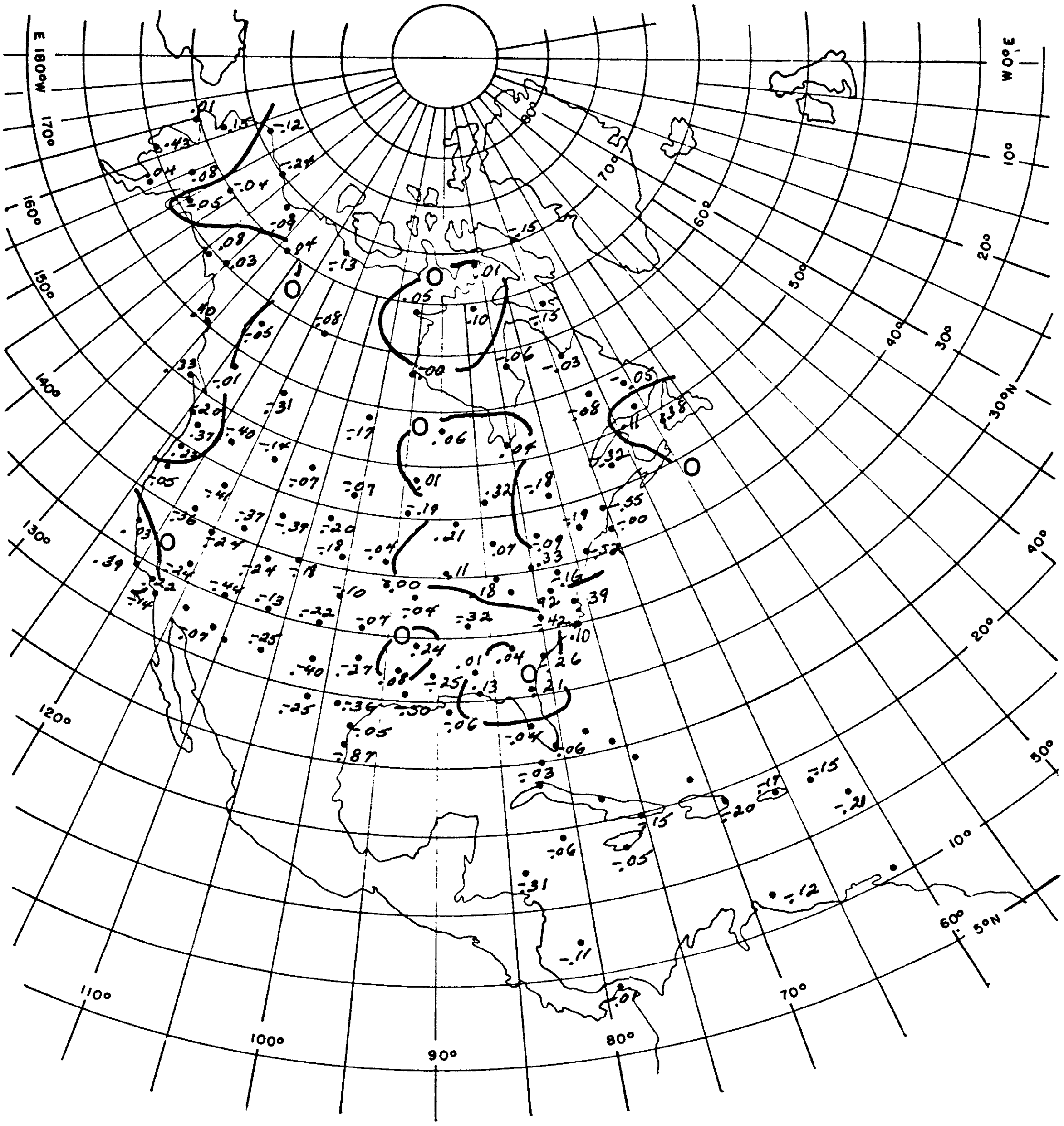


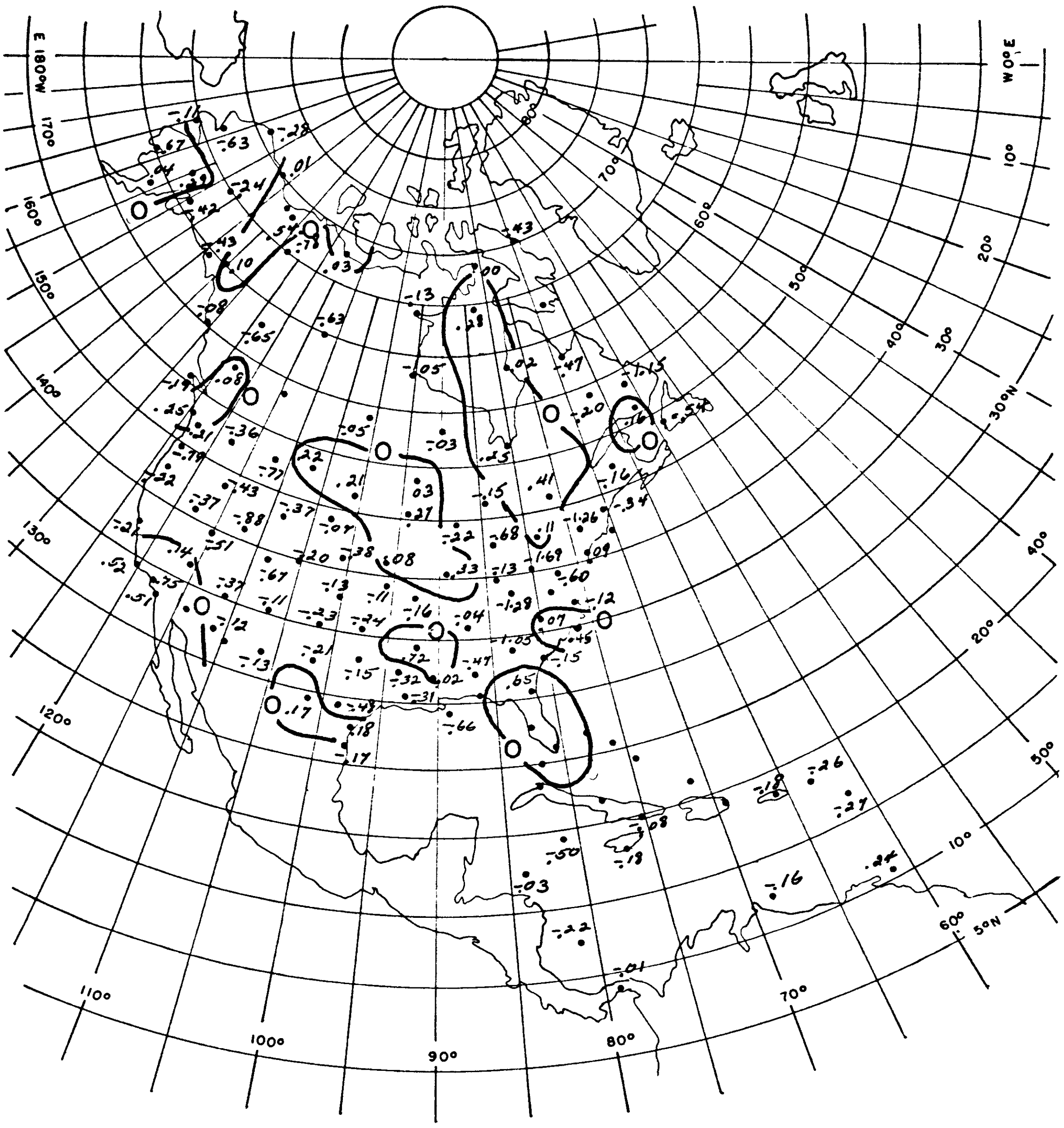
Fig. 3

Reynolds' stress values associated with large-scale vertical eddies for winter at 300 mb. Units are dyne cm^{-2}, and negative values correspond to upward momentum fluxes.

of these earlier authors would suggest. Also their finding of noticeable seasonal fluctuations in the stress has not been recaptured here. Perhaps the greatest discrepancy between the two sets of results, however, lies in the generally small magnitude of the computed stress values at the N.A. stations when compared with the zonally averaged values obtained by Peixoto et. al. for the same periods. In the region from approximately 40°N to 55°N, for example, which is characterized by Peixoto et. al. as exhibiting predominantly negative values of the zonally averaged total vertical eddy stress sometimes exceeding -1.0 dyne cm^{-2}, no value of $\frac{1}{g}\overline{u'\omega'}$ for any N.A. station, level or season ever exceeds in magnitude -0.7 dyne cm^{-2}. Values, in fact, are generally much smaller than this, and the average value of the stress along the portions of these latitudes over N.A. do not at all correspond to what we might have anticipated on the basis of their reported zonal values. Of course, the stress could be larger along other portions of the latitude belt, but such a viewpoint is not encouraged by having found such generally small values over N.A.. We must conclude that pictures such as are contained in Fig. 3 do not compare well with the earlier results of Peixoto et. al. However, the N.A. stress field computations do show patterns which seem of some inherent interest, though it is not our intent to study them further now.

There are, of course, several possible explanations for the lack of better correspondence between the two sets of results for the vertical eddy stresses. The most obvious arises from the fact that the computations involving the kinematic ω's result in stress values associated with only transient eddies of a certain scale. Quite possibly, eddies on scales not resolvable by this technique are the major contributors to the zonal means of the total eddy stress given by Peixoto et. al. Also, the procedure for computing the kinematic ω's does make some assumptions which might introduce a bias in the results for the stress associated with them. Setting the bottom boundary condition to be $\omega = 0$ and restricting the selections of ω profile to those which converge "best" at the top may both contribute to an underestimate of tropospheric vertical motions, perhaps resulting in an underestimate of the eddy stresses. On the other hand, it is simply possible too that the previous findings about the vertical eddy stresses were in error or did not measure what they were believed to.

Interestingly, results for $\frac{1}{g}\overline{u'\omega'}$ over N.A. computed on an individual monthly basis have sometimes shown a character far different from the results discussed thus far for the composite seasons. An example is shown in Fig. 4, which depicts the stress field acting on the 600 mb surface over N.A. for the month of January 1963 determined from once daily (00 Z) measurements of the wind field. The appearance of some very strong negative values in Fig. 4 is in striking contrast with what was found when the time averaging extended over the 15-month seasonal periods. Indeed, the stresses are of such a magnitude in Fig. 4, that if they are real, it would seem unlikely that they could be neglected should the energy balance over N.A. for the month of January 1963 be investigated. Studies by Hess and Clarke (1973) and by Tucker (1973) have also dealt with the role of vertical eddy fluxes of momentum for periods on the order of a month, but only for very limited regions. Their methods of deducing vertical velocities differ from ours, but they also draw the conclusion that the eddy flux of momentum in the vertical (particularly those associated with synoptic scale waves) can be of importance in considerations of atmospheric energetics, at least for such time periods as a month. Caution apparently is required though in attempting to extend this conclusion to longer time periods,



$\frac{1}{g}\overline{u'\omega'}$ 600mb Jan 1963

Fig. 4

Same as Fig. 3, but for the month of January 1963 at 600 mb.

considering the nature of our long-term results presented earlier. However, it should be remarked again that those results may well have suffered from the infrequency of the soundings used, only twice a day. It still remains quite possible that standing eddies, or transient ones which our direct technique could not adequately capture, may make significant contributions to the vertical eddy flux. The entire problem is deserving of much more study than the brief mention undertaken here, and it would appear that indeed such an attitude is becoming more widespread.

APPENDIX B

DERIVATION OF THE MOMENTUM BALANCE EQUATIONS

The purpose of this appendix is to provide details regarding the derivation of the system of equations (1)-(2), which govern the square of the zonal average of the absolute angular momentum and its variance along the length of closed latitude circles. For the sake of generality, we will make no special assumptions regarding the spatial or temporal distribution of density to begin with. That is to say, in what follows we shall treat

$$\rho = \rho(R, \lambda, \phi, t) \tag{8}$$

The notation to be used in this appendix is standard, with most of the symbols already having been defined in the main text.

Our starting point in the derivation of both (1) and (2) is the dynamical equation governing M, written on the assumption that the only real forces acting are pressure and frictional forces. Thus,

$$\rho \frac{DM}{Dt} = -\frac{\partial p}{\partial \lambda} + rF \tag{9}$$

where F is the zonal frictional force per unit volume assumed positive in the λ direction. By virtue of the theorem that

$$\rho\frac{D(\,)}{Dt} = \frac{\partial\rho(\,)}{\partial t} + \nabla\cdot\rho(\,)\underset{\sim}{v} \tag{10}$$

which may be derived from the equation of continuity and Euler's relation between substantial and partial time derivatives, we may reformulate (9) as

$$\frac{\partial\rho M}{\partial t} + \nabla\cdot\rho M\underset{\sim}{v} = -\frac{\partial p}{\partial\lambda} + rF \tag{11}$$

Before proceeding further, however, we pause to note the following useful identity

$$\frac{\partial}{\partial t}\left\{\rho\frac{(\,)^2}{2}\right\} \equiv (\,)\frac{\partial\rho(\,)}{\partial t} - \frac{(\,)^2}{2}\frac{\partial\rho}{\partial t} \tag{12}$$

where, for that matter, ρ could have been replaced by any function. The proof of (12) is straightforward calculus and so is omitted here. Continuing now, our immediate objective is to obtain a pair of differential equations governing $[M]^2$ and $(M')^2$. An equation governing the first may be gained by letting $M=[M]+M'$ in (11) and then multiplying by $[M]$. The result is

$$[M]\frac{\partial}{\partial t}(\rho[M]) + [M]\frac{\partial}{\partial t}(\rho M') + [M]\nabla\cdot\rho[M]\underset{\sim}{v} + [M]\nabla\cdot\rho M'\underset{\sim}{v} = -[M]\left(\frac{\partial p}{\partial\lambda} - rF\right) \tag{13}$$

Use is next made of the theorem

$$(\,)\nabla\cdot\rho(\,)\underset{\sim}{v} = \nabla\cdot\rho\frac{(\,)^2}{2}\underset{\sim}{v} - \frac{(\,)^2}{2}\frac{\partial\rho}{\partial t} \tag{14}$$

which the interested reader may easily derive for himself from the equation of continuity, to rewrite the third term on the left hand side of (13), so that it becomes

$$[M]\frac{\partial}{\partial t}(\rho[M]) + \nabla\cdot\rho\frac{[M]^2}{2}\underset{\sim}{v} - \frac{[M]^2}{2}\frac{\partial\rho}{\partial t} + [M]\frac{\partial}{\partial t}(\rho M') + [M]\nabla\cdot\rho M'\underset{\sim}{v} = -[M]\left(\frac{\partial p}{\partial\lambda} - rF\right) \qquad (15)$$

The identity (12) may be applied to combine the first and third terms on the left hand side of (15), and the relationship (10) may be used to combine the fourth and fifth terms. The final result is

$$\frac{\partial}{\partial t}\left\{\rho\frac{[M]^2}{2}\right\} + \nabla\cdot\rho\frac{[M]^2}{2}\underset{\sim}{v} + \rho[M]\frac{DM'}{Dt} = -[M]\left(\frac{\partial p}{\partial\lambda} - rF\right) \qquad (16)$$

We next seek a differential equation governing the variance of angular momentum. The procedure is identical to that used in arriving at (16), except that now (11) is multiplied by M' after having again let $M=[M]+M'$, so that we have

$$M'\frac{\partial}{\partial t}(\rho[M]) + M'\frac{\partial}{\partial t}(\rho M') + M'\nabla\cdot\rho[M]\underset{\sim}{v} + M'\nabla\cdot\rho M'\underset{\sim}{v} = -M'\left(\frac{\partial p}{\partial\lambda} - rF\right) \qquad (17)$$

As before, theorem (14) is again used but now to rewrite the fourth term on the left hand side of (17). Both (12) and

(10) are again used to simplify the resulting equation, so that we may finally write

$$\frac{\partial}{\partial t}\left\{\rho\frac{(M')^2}{2}\right\}+\nabla\cdot\rho\frac{(M')^2}{2}\underset{\sim}{v}+\rho M'\frac{D[M]}{Dt}=-M'\left(\frac{\partial p}{\partial\lambda}-rF\right) \qquad (18)$$

Thus, (16) and (18) represent the set of differential equations which we shall integrate over a closed volume T. Before doing so, however, we take note of the following relationships:

$$[ab]=[a][b]+[a'b'] \qquad (a, b \text{ arbitrary}) \qquad (19)$$

$$[(M')^2]=[M^2]-[M]^2 \qquad (20)$$

$$(M^2)'=\{(M')^2\}'+2M'[M] \qquad (21)$$

$$\int_T\rho M'\frac{D[M]}{Dt}\,dT+\int_T\rho[M]\frac{DM'}{Dt}\,dT=\int_T\frac{\partial}{\partial t}(\rho[M]M')\,dT \qquad (22)$$

The proof of (19) is obvious. Equation (20) follows in part from the definition of the variance of a quantity. Equation (21) is also easily gotten if it is noticed that $M^2\equiv[M^2]+(M^2)'$ by virtue of the definition of the bracket and prime operators, but also that $M^2=\{[M]+M'\}^2$. Hence, one has that $(M^2)'=(M')^2+2M'[M]-[(M')^2]$, through the additional use of (20). Since $[(\ )']\equiv 0\equiv[(\ )]'$ and $(\ )''\equiv(\ )'$, (21) is reproduced.

Lastly, to obtain (22) we note that its left hand side may be rewritten as

$$\int_T \rho \frac{D[M]M'}{Dt} dT$$

which may in turn be reformulated using (10), as

$$\int_T \frac{\partial}{\partial t}(\rho[M]M')dT + \int_T \nabla\cdot\rho[M]M'\underset{\sim}{v}\, dT$$

Since T represents a closed system for our purposes, the volume integral of a divergence vanishes, and therefore (22) is reproduced.

Let us now proceed to get the desired integral equations. We perform an integration over T upon (16). The divergence term vanishes, and by (19),

$$\left[\rho\frac{[M]^2}{2}\right] = [\rho]\frac{[M]^2}{2} \qquad \text{since} \qquad \{[M]^2\}' = 0$$

Hence, we have simply that

$$\frac{d}{dt}\int_T \rho\frac{[M]^2}{2}dT = -\int_T \rho[M]\frac{DM'}{Dt}dT - \int_T [M]\,\nabla\cdot\underset{\sim}{\tau}_m\, dT \qquad (23)$$

where we have also reformulated friction in terms of the divergence of an angular momentum transport vector lying in the meridional plane, in the manner of Starr and Rosen (1972). Finally, we integrate (18) over T . We note that

$$\left[\rho\frac{(M')^2}{2}\right] = [\rho]\frac{[(M')^2]}{2} + \left[\rho'\frac{\{(M')^2\}'}{2}\right]$$

by virtue of (19), and that on further use of (21), we have

$$\left[\rho\frac{(M')^2}{2}\right] = [\rho]\frac{[(M')^2]}{2} + \left[\rho'\frac{(M^2)'}{2}\right] - \left[\rho' M'[M]\right]$$

This last relationship may be applied in connection with the integral of the first term on the left hand side of (18). The second term on the left hand side of (18) vanishes upon integration over T. Finally, the integral of the last term on the left hand side of (18) may be written, by virtue of (22), as

$$-\int_T \rho[M]\frac{DM'}{Dt}\,dT + \int_T \frac{\partial}{\partial t}(\rho[M]M')\,dT$$

Thus, treating the friction term on the right hand side of (18) in the same manner as before, we may write at last

$$\frac{d}{dt}\int_T \rho\frac{[(M')^2]}{2}\,dT = \int_T \rho[M]\frac{DM'}{Dt}\,dT - \int_T \frac{\partial}{\partial t}\left[\rho'\frac{(M^2)'}{2}\right]dT \qquad (24)$$
$$- \int_T M'\left(\frac{\partial p}{\partial x}\right)' dT - \int_T M'\,\nabla\cdot\underset{\sim}{\tau}_m\,dT$$

Equations (23) and (24) represent the generalized balance equations we have sought, derived with no formal restrictions being placed on the distribution of density in T. This generality has resulted in the appearance of additional terms in (23)-(24) which were not included in (1)-(2), terms which do not appear to be readily amenable to physical interpretation. It may be easily shown though that (23)-(24) reduce to (1)-(2) under the condition that ρ be treated as independent of λ and t. For in such a case, the second term on the right hand side of (23) vanishes. Furthermore, by (10), we have

$$\int_T \rho[M]\frac{DM'}{Dt}\,d\tau = \int_T [M]\frac{\partial \rho M'}{\partial t}\,d\tau + \int_T [M]\,\nabla\cdot\rho M'\underset{\sim}{v}\,d\tau \qquad (25)$$

But since we now assume $\rho=[\rho]$, the first term on the right hand side of (25) vanishes on integrating around a closed latitude circle. As for the second term on the right hand side of (25), we may make use of the fact that $M'=M-[M]$ so that

$$\int_T [M]\,\nabla\cdot\rho M'\underset{\sim}{v}\,d\tau = \int_T [M]\,\nabla\cdot\rho M\underset{\sim}{v}\,d\tau - \int_T [M]\,\nabla\cdot\rho[M]\underset{\sim}{v}\,d\tau \qquad (26)$$

But the second term on the right hand side of (26) vanishes by virtue of theorem (14) and our present assumption that $\frac{\partial \rho}{\partial t}=0$. Hence, equations (1)-(2) are recovered from the more general set (23)-(24). It, of course, is possible to derive (1)-(2) more directly by assuming $\frac{\partial \rho}{\partial \lambda}=\frac{\partial \rho}{\partial t}=0$ from the beginning, but the presentation as well of the more general system seems desirable.

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BIOGRAPHICAL NOTE

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1971. Vertical transport of mean zonal kinetic energy from five years of hemispheric data. Tellus, 23, 302-309.

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