Essays on Liquidity and Information

by

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Abstract

This dissertation studies the interaction of liquidity and incomplete or asymmetric information.

In Chapter 1, I study a dynamic economy with illiquidity due to adverse selection in financial markets. Investment is undertaken by borrowing-constrained entrepreneurs. They sell their past projects to finance new ones, but asymmetric information about project quality creates a lemons problem. The magnitude of this friction responds to aggregate shocks, amplifying the responses of asset prices and investment. Indeed, negative shocks can lead to a complete shutdown in financial markets. I then introduce learning from past transactions. This makes the degree of informational asymmetry endogenous and makes the liquidity of assets depend on the experience of market participants. Market downturns lead to less learning, worsening the future adverse selection problem. As a result, transitory shocks can create highly persistent responses in investment and output.

In Chapter 2, I study why firms can choose to be illiquid. Optimal incentive schemes for managers may involve liquidating a firm following bad news. Fragile financial structures, vulnerable to runs, have been proposed as a way to implement these schemes despite their ex-post inefficiency. I show that in general these arrangements result in multiple equilibria and, even allowing arbitrary equilibrium selection, they do not necessarily replicate optimal allocations. However, if output follows a continuous distribution and creditors receive sufficiently precise individual early signals, then there exists a fragile financial structure such that global games techniques select a unique equilibrium which reproduces the optimal allocation.

In Chapter 3, I study speculative attacks against illiquid firms. When faced with a speculative attack, banks and governments often hesitate, attempting to withstand the attack but giving up after some time, suggesting they have some ex-ante uncertainty about the magnitude of the attack they will face. I model that uncertainty as arising from incomplete information about speculators' payoffs and find conditions such that unsuccessful partial defences are possible equilibrium outcomes. There exist priors over the distribution of speculators' payoffs that can justify any possible partial defence strategy. With Normal uncertainty, partial resistance is more likely when there is more aggregate uncertainty regarding agents' payoffs and less heterogeneity among them.

Thesis Supervisor: Iván Werning
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For Pam

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Contents

Chapter 1

Lemons, Market Shutdowns and Learning

1.1 Introduction

Financial markets are fragile, volatile and occasionally shut down entirely. The recent financial crisis has intensified economists' interest in understanding the causes of financial instability and its effects on real economic variables such as investment, output and productivity. In this paper I develop a model of financial imperfections to explain how instability in general and market shutdowns in particular can result from macroeconomic shocks and in turn amplify and propagate them.

I focus on one specific type of financial market imperfection: asymmetric information about the quality of assets. There are several reasons why it is worth studying this particular imperfection. First, the ability of creditors to seize a debtor's assets, either as a possible equilibrium outcome or as an off-equilibrium threat, is crucial for enabling financial transactions to take place, both in theory (Hart and Moore, 1994; Kiyotaki and Moore, 1997) and in everyday practice. If there is asymmetric information about asset qualities, which is a natural assumption, this has the potential to interfere with a large subset of financial transactions. Second, asymmetric information is a central concern in corporate finance. Following Myers and Majluf (1984), asymmetry of information between firm managers and their outside investors is seen as a key determinant of firms' capital structure. Third, sometimes financial markets simply cease to function, as documented for instance by Gorton and Metrick (2009) for the repo market in 2007-2009. Since Akerlof (1970), it is well known that the complete breakdown of trade is a theoretical possibility in economies with asymmetric information. This means that asymmetric information at least has the potential to explain extreme crises and may shed light on less extreme phenomena as well.

I embed imperfect financial markets in a simple dynamic macroeconomic model. In the model, entrepreneurs hold the economy's stock of capital. Every period, they receive random idiosyncratic investment opportunities. The only way to obtain financing is by borrowing against existing assets or, equivalently, selling them. Assets are bought by entrepreneurs who in the current period have poor investment opportunities but nevertheless wish to save part of their dividends. Unfortunately, some fraction of existing assets are useless *lemons* and buyers can't tell them apart from high quality assets (*nonlemons*), creating a classic lemons problem.

I show that the lemons problem introduces a wedge between the return on saving and the cost of funding, persuading some entrepreneurs to stay out of the market. This is formally equivalent to introducing a tax on financial transactions, with revenues rebated lump-sum to entrepreneurs. This defines a notion of *liquidity* where the degree of illiquidity of assets is the size of the implicit tax. The tax lowers asset prices, the rate of return obtained by uninformed investors and the rate of capital accumulation. Furthermore, the implicit tax depends on the proportions of lemons and nonlemons sold, which respond to aggregate shocks. Standard productivity shocks increase current dividends, which increases the supply of savings and raises asset prices. This persuades more entrepreneurs to sell their nonlemons, improving the overall mix of projects that get sold and lowering the implicit tax on financial transactions. Shocks to the productivity of investment have similar effects because they increase entrepreneurs' desire to invest and thus their willingness to sell nonlemons. The endogenous response of the size of the tax implies that asymmetric information can be a source of amplification of the effects of shocks on both capital accumulation and asset prices. Large negative shocks may lead financial markets to shut down entirely.

The model predicts that capital becomes more liquid in economic expansions. This prediction is consistent with empirical research by Eisfeldt and Rampini (2006), who find that the costs of reallocating capital across firms are countercyclical. It is also consistent with the evidence in Choe, Masulis, and Nanda (1993), who find that the negative price reaction to an offering of seasoned equity is smaller and the number of firms issuing equity is larger in the expansionary phase of the business cycle, suggestive of countercyclical adverse selection costs.

In reality, asymmetric information does not mean that relatively uninformed parties do not know anything. Instead, it can be a matter of degree. In order to investigate how the degree of asymmetry could vary endogenously, I extend the baseline model by introducing public information about the quality of individual assets. Each asset issues a signal which is correlated with its true quality. The correlation is imperfectly known and changes over time.

The precision of entrepreneurs' estimates of the correlation determines how informative they find the signals. They learn about the current value of the correlation by observing samples of past transactions. More transactions lead to larger sample sizes, more precise estimates, more informativeness of future signals and lower future informational asymmetry. Conversely, market shutdowns lead to smaller sample sizes, less certainty about the correlation between signals and quality and more severe informational asymmetry in the future. This can be a powerful propagation channel by which temporary negative shocks can lead to financial crises followed by long recessions, in a manner consistent with the evidence in Cecchetti, Kohler, and Upper (2009), Claessens, Kose, and Terrones (2009) and Cerra and Saxena (2008).

The learning mechanism in the model formalizes the notion that assets will be more liquid if they are more familiar and familiarity depends on experience. Accumulated financial experience is a form of intangible social capital which increases liquidity and reduces frictions in the investment process. Learning-by-doing in financial markets plays the important role of building up that capital.

Because investment opportunities are heterogeneous, the distribution of physical investment across entrepreneurs matters for capital accumulation. Asymmetric information lowers the level of investment of entrepreneurs with relatively good opportunities, who may decide not to sell their existing nonlemons due to depressed prices or receive lower prices if they do sell them. At the same time, it increases the level of investment of entrepreneurs with relatively poor investment opportunities, who might decide to undertake them anyway rather than buy assets from others for fear of receiving lemons. These effects lower the average rate of transformation of consumption goods into capital goods and thus can be seen as a determinants of endogenous investment-sector-specific productivity. Shocks to this type of productivity have been found to be an important driver of output fluctuations in estimated quantitative models by Greenwood, Hercowitz, and Krusell (2000), Fisher (2006) and Justiniano, Primiceri, and Tambalotti (2008a). Furthermore, Justiniano, Primiceri, and Tambalotti (2008b) find that movements in investment-sector productivity are correlated with measures of the smooth functioning of financial markets, as would be predicted by the model.

Measurement of investment sector productivity depends on accurate measures of capital formation. If these fail to take into account changes in the the efficiency of investment due to changes in the degree of informational asymmetry, information effects in one period would show up as measured Solow residuals in future periods. Thus the movement of implicit tax wedges in the model can be a source of changes in (measured) TFP or, in the terminology of Chari, Kehoe, and McGrattan (2007), movements in efficiency wedges.

In common with Kiyotaki and Moore (1997), Bernanke and Gertler (1989), Bernanke, Gertler, and Gilchrist (1999) and Carlstrom and Fuerst (1997) among others, financial frictions in my model are sensitive to wealth effects. However, what governs their severity in my model is not the wealth of financially-constrained investors, since the margin that determines whether to sell or keep nonlemons is independent of wealth. Instead, the wealth of those who finance them matters because it governs the demand for assets.

The structure of the model is close to that developed by Kiyotaki and Moore (2005, 2008), which also features random arrival of investment opportunities, borrowing constraints and partially illiquid assets. Those papers use a reduced-form model of the limitations on selling capital and investigate whether this may explain why easier-to-sell assets command a premium. In contrast, I develop an explicit model of what the sources of these limitations are, which allows me to investigate how they respond to aggregate shocks. My model also shares some of the simplifying assumptions of the Kiyotaki-Moore framework, in particular that entrepreneurs have no labour income and log preferences. One additional simplification that I make is that physical capital is the only asset. Thanks to this assumption, entrepreneurs' policy functions can be found in closed form despite having an infinite dimensional state vector (due to the continuum of possible signals) and a nonlinear budget set. This makes it possible to derive most of the qualitative results analytically and to simulate the model at little computational cost.

Following (Stiglitz and Weiss, 1981), adverse selection played an important early role in the theory of credit markets, although the emphasis was on the riskiness of projects rather than the quality of assets. Financial market imperfections that arise specifically due to a lemons problem in asset quality have recently been studied by Bolton, Santos, and Scheinkman (2009) and Malherbe (2009). These papers model games where dynamic strategic complementarities can give rise to different types of equilibria, with more or less severe adverse selection. Instead in my model the equilibrium is unique so the severity of the lemons problem responds to aggregate shocks in a predictable way.

The lemons problem remains relatively unexplored in macroeconomic settings. One exception is Eisfeldt (2004). In her model, entrepreneurs hold different vintages of projects and cannot diversify risks. The reason financial transactions are desirable is that they enable entrepreneurs to smooth consumption when they suffer poor realizations of income from previous vintages of risky projects. Thus her paper is about how asymmetric information interferes with risk-sharing whereas mine is about how it interferes with the financing of investment. On a more technical side, one limitation of her approach is that it requires keeping track of of the distribution of portfolio holdings across different vintages of projects, for all entrepreneurs, which makes it necessary to limit attention to numerical simulations of steady states or simple deterministic cycles, since stochastic simulations are computationally infeasible.

The idea that economic recessions are associated with reduced learning is explored by Veldkamp (2005), van Nieuwerburgh and Veldkamp (2006) and Ordoñez (2009). In these models, what agents need to learn about is the state of aggregate productivity. The speed of learning governs how fast output and prices align themselves with fundamentals but the direction of this alignment is just as likely to be towards higher or lower output. In contrast, in my model agents learn about parameters of the information structure. More learning alleviates informational asymmetries, which helps the functioning of financial markets for any given level of productivity. Another difference is that in my model the activity which generates information is selling projects rather undertaking them. Since the volume of financial transactions can be very volatile, this opens the door to strong learning effects.

The rest of the paper is organized as follows. Section 1.2 introduces the model and section 1.3 and describes frictionless benchmarks. Section 1.4 describes the equilibrium conditions under asymmetric information and contains the results for the model without signals. Section 1.5 contains the extension of the model with signals and learning. Section 1.6 offers some brief final remarks. Proofs are collected in Appendix 1.7.4.

1.2 The environment

Households. There are two kinds of agents in the economy, workers and entrepreneurs. There is a continuum of mass L of identical workers, each of whom supplies one unit of labour inelastically; they have no access to financial markets, so they just consume their wage. In addition, there is a continuum of mass one of entrepreneurs, indexed by j, who have preferences

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t u(c_t^j)$$

with $u(c_t^j) = \log(c_t^j)$. They do not work.

Technology. Final output (coconuts) is produced combining capital and labour. The capital stock consists of *projects* owned by entrepreneurs. Entrepreneur j's holdings of projects are denoted k_t^j so the aggregate capital stock is $K_t = \int k_t^j dj$. Every period a fraction λ of projects becomes useless or "lemons". Each entrepreneur's holdings of projects is sufficiently well diversified that the proportion λ applies at the level of the individual entrepreneur as well. Each of the $(1-\lambda)K_t$ projects that do not become lemons is used for production, so that output is $Y_t = Y\left((1-\lambda)K_t, L; Z_t\right)$, where Y is a constant-returns-to-scale production function that satisfies Inada conditions and Z_t is exogenous productivity. The marginal product of capital and labour are denoted Y_K and Y_L respectively.

The aggregate resource constraint is

$$Lc_t^w + \int \left(c_t^j + i_t^j\right) dj \leq Y\left(\left(1-\lambda\right) K_t, L; Z_t\right) \tag{1.1}$$

where c_t^w denotes consumption per worker, c_t^j is consumption by entrepreneur j and i_t^j represents physical investment by entrepreneur j.

Physical investment is undertaken in order to convert coconuts into projects for period $t+1$. Each entrepreneur can transform coconuts into projects using an idiosyncratic linear technology with a stochastic marginal rate of transformation A_t^j. In addition, each nonlemon project turns into γ projects at $t+1$, so it is possible to interpret $1-\gamma\left(1-\lambda\right)$ as an average rate of depreciation. Aggregate capital accumulation is given by

$$K_{t+1} = \gamma\left(1-\lambda\right) K_t + \int i_t^j A_t^j dj \tag{1.2}$$

A_t^j is *iid* across entrepreneurs and across periods and follows distribution F with mean $\mathbb{E}\left(A\right) < \infty$.

Allocations. The exogenous state of the economy is $z_t \equiv \left\{Z_t, \bar{A}_t\right\}$. It includes productivity Z_t and the function $\bar{A}_t$, which maps each entrepreneur to a realization of A_t^j. An *allocation* specifies consumption and investment for each agent in the economy and aggregate capital after every history: $\left\{c^w\left(z^t\right), c^j\left(z^t\right), i^j\left(z^t\right), K\left(z^t\right)\right\}$. An allocation is *feasible* if it satisfies constraints (1.1) and (1.2) for every history given some K_0.

Information. At time t each entrepreneur knows which of his own projects have become lemons in the current period, but the rest of the agents in the economy do not. In section 1.5, I augment the model by allowing for publicly observable signals about individual projects. Informational asymmetry lasts only one period. At $t+1$, everyone is able to identify the projects that became lemons at t, so they effectively disappear from the economy, as illustrated in figure 1.1. This assumption is made for simplicity as it eliminates the need to keep track of projects of different vintages. Daley and Green (2009) study the strategic issues that arise when informational asymmetries dissipate gradually over time.

The investment opportunity A_t^j is and remains private information to entrepreneur j, so entrepreneurs know their individual state $z_t^j \equiv \left\{Z_t, A_t^j\right\}$ but not the aggregate state $z_t \equiv \left\{Z_t, \bar{A}_t\right\}$.

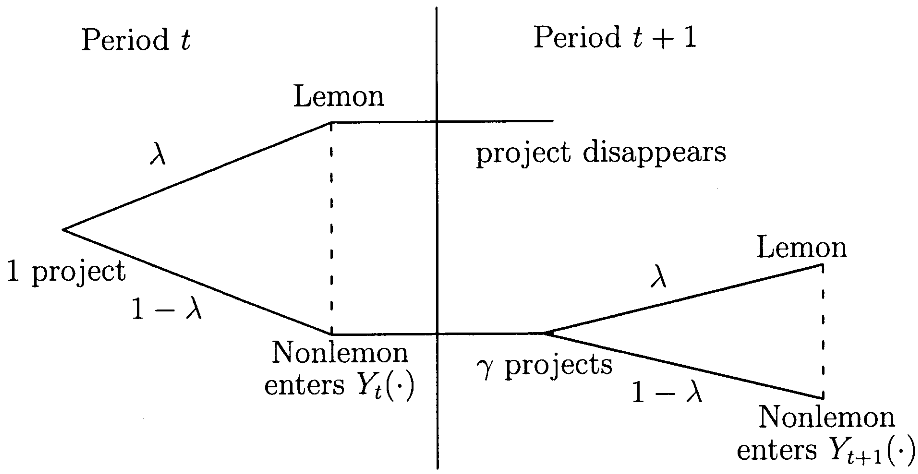


Figure 1.1: Information about a project over time

1.3 Symmetric information benchmarks

1.3.1 Complete Arrow-Debreu markets

Suppose z_t and the quality of individual projects were public information and there were complete competitive markets. The price of lemons will be zero so we can just focus on factor markets and trades of coconuts for nonlemons and state-contingent claims.

Factor markets are competitive. Entrepreneurs hire workers at a wage of $w(z^t) = Y_L(z^t)$ coconuts and obtain dividends of $r(z^t) = Y_K(z^t)$ coconuts for each nonlemon project.[1] Coconuts are traded for nonlemon projects, ex-dividend, at a spot price of $p_{NL}(z^t)$ coconuts per nonlemon project. State-contingent claims are traded one period ahead: it requires $q(z^t, z_{t+1})$ coconuts at history z^t to obtain a coconut in history (z^t, z_{t+1}) and $\rho(z^t, z_{t+1})$ is the associated state-price density.

An entrepreneur who starts with k_0^j projects solves the following program:

[1] As is standard, this could be the result of competitive firms renting capital from entrepreneurs or of entrepreneurs operating the productive technology themselves. With asymmetric information, the latter interpretation avoids the need to analyze adverse selection in the rental market.

$$\max_{\{c(z^t),k(z^t),d_{NL}(z^t),b(z^t,z_{t+1}),i(z^t)\}} \mathbb{E}\sum_{t=0}^{\infty}\beta^t u\left(c\left(z^t\right)\right) \tag{1.3}$$

$$s.t.$$

$$c\left(z^t\right)+i\left(z^t\right)+p_{NL}(z^t)d_{NL}\left(z^t\right)+\mathbb{E}\left[\rho\left(z^t,z_{t+1}\right)b\left(z^t,z_{t+1}\right)\right] \le r\left(z^t\right)(1-\lambda)\,k\left(z^t\right)+b\left(z^{t-1},z_t\right) \tag{1.4}$$

$$k\left(z^t,z_{t+1}\right)=\gamma\left[(1-\lambda)\,k\left(z^t\right)+d_{NL}\left(z^t\right)\right]+A^j\left(z_t\right)i\left(z^t\right) \tag{1.5}$$

$$i\left(z^t\right)\ge 0,\ d_{NL}(z^t)\ge -(1-\lambda)k(z^t) \tag{1.6}$$

$$\lim_{t\to\infty}\mathbb{E}\left[b(z^t)\prod_{s=0}^{t-1}\rho(z^s,z_{s+1})\right]\ge 0 \tag{1.7}$$

Constraint (1.4) is the entrepreneur's budget constraint in terms of coconuts. The entrepreneur's available coconuts are equal to the dividends from his nonlemons $r\left(z^t\right)(1-\lambda)$ $k\left(z^t\right)$ plus net state-contingent coconuts bought the previous period $b\left(z^{t-1},z_t\right)$. These are used for consumption plus physical investment plus net purchases of nonlemons $d_{NL}\left(z^t\right)$ plus purchases of state-contingent coconuts for period $t+1$. Constraint (1.5) keeps track of the entrepreneur's holdings of projects. $k\left(z^t,z_{t+1}\right)$, the total number of projects the entrepreneur has in history $\left(z^t,z_{t+1}\right)$, is equal to the nonlemon projects he owned at the end of period z^t, which were $(1-\lambda)\,k\left(z^t\right)+d_{NL}\left(z^t\right)$, and have grown at rate γ plus the projects that result from his physical investment in the previous period, $A^j\left(z_t\right)i\left(z^t\right)$. Constraint (1.6) states that investment must be nonnegative and sales of nonlemons are limited by the number of nonlemons the entrepreneur owns. Constraint (1.7) is a no-Ponzi condition.

The first order conditions with respect to $i\left(z^t\right)$ and $d_{NL}\left(z^t\right)$ imply

$$A^j\left(z_t\right)\le\frac{\gamma}{p_{NL}\left(z^t\right)},\ \text{with equality if } i\left(z^t\right)>0 \tag{1.8}$$

Let $A^{\max}$ be the highest possible value of A. By the law of large numbers, at each history there will be an entrepreneur (the *best entrepreneur*) with $A^j=A^{\max}$ who can transform each coconut into $A^{\max}$ projects at $t+1$.[2] Equation (1.8) then implies that $p_{NL}\left(z^t\right)=\frac{\gamma}{A^{\max}}$ for all z^t. At each history, the best entrepreneur is the only one to undertake physical investment. He finances this investment by selling claims to coconuts one period ahead (i.e. borrowing) which he then satisfies with the dividends plus proceeds of selling the newly

[2]With a continuous F there will be a zero measure of best entrepreneurs, but a positive measure of entrepreneurs with $A^j\in(A^{\max}-\delta,A^{\max}]$ for any positive δ. The results below follow from taking the limit as $\delta\to 0$.

created projects in the spot market. Since $r(z^t)$ is stochastic, capital is a risky asset, and the best entrepreneur will use state-contingent securities to share this risk with the rest of the entrepreneurs. Complete markets imply that risk-sharing will be perfect.

There is an alternative market structure, not requiring state-contingent securities, which will also result in the complete markets allocation. Suppose the only market that exists is for newly created projects. An entrepreneur can create and sell a project simultaneously, selling it for a price $p_{NEW}(z^t)$ in the same instant it is created, at which time no one knows whether it will become a lemon in the following period. There is no borrowing, no state-contingent securities and no spot market for existing assets. The constraints on the entrepreneur's problem simplify to

$$c(z^t) + i(z^t) + p_{NEW}(z^t) d_{NEW}(z^t) \leq r(z^t)(1-\lambda) k(z^t)$$
$$k(z^t, z_{t+1}) = \gamma (1-\lambda) k(z^t) + d_{NEW}(z^t) + A^j(z_t) i(z^t)$$
$$i(z^t) \geq 0, \; d_{NEW}(z^t) \geq -A^j(z_t) i(z^t)$$

The first order conditions for $i(z^t)$ and $d_{NEW}(z^t)$ imply

$$A^j(z_t) \leq \frac{1}{p_{NEW}(z^t)}, \text{ with equality if } i(z^t) > 0$$

which implies that new projects will sell for $p_{NEW}(z^t) = \frac{1}{A^{\max}}$ and the best entrepreneur will be the only one to invest, just as in the complete markets allocation.[3] Furthermore, entrepreneurs' only asset in any given period consists of projects, so they automatically share aggregate risk in proportion to their wealth. Since they have identical homothetic preferences, this coincides with what they would do with complete markets.

Proposition 1. *If there are complete markets, all the physical investment is undertaken by the entrepreneur with $A^j = A^{\max}$; all entrepreneurs obtain a return of $A^{\max}$ projects per coconut saved and they bear no idiosyncratic risk. The same allocation is obtained if the only market that exists is for newly-created projects.*

The aggregate economy behaves just like an economy where the rate of transformation of coconuts into projects is fixed at $A^{\max}$, there is a representative entrepreneur and workers are constrained to live hand-to-mouth. Due to log preferences, it is straightforward to compute the entrepreneur's consumption choice, which will be given by

$$c^j(z^t) = (1-\beta)(1-\lambda)\left[Y_K(z^t) + \frac{\gamma}{A^{\max}}\right] k^j(z^t) \tag{1.9}$$

[3] The difference between p_{NEW} and p_{NL} is due to the fact that old nonlemons grow by a factor γ and new projects don't.

and hence aggregate capital accumulation will be:[4]

$$K\left(Z^{t},Z_{t+1}\right)=\beta\left(1-\lambda\right)\left[A^{\max}Y_{K}\left(Z^{t}\right)+\gamma\right]K\left(Z^{t}\right) \tag{1.10}$$

1.3.2 Borrowing constraints with symmetric information

For various reasons, it may be difficult for an entrepreneur to borrow against his future wealth, i.e. to choose negative values of $b\left(z^{t},z_{t+1}\right)$. For instance, the he may be able to run away with his wealth rather than honouring his debts.[5] Creditors' main means of enforcing their claims is the threat to seize the entrepreneur's assets. In other words, the entrepreneur's assets serve as *collateral* for any obligations he undertakes. Kiyotaki and Moore (2008) point out that it is important to distinguish between assets that are already in place at the time the financial transaction is initiated and those that are not, since the latter are harder for creditors to keep track of and subject to more severe moral hazard problems. In what follows I make the extreme assumption that entrepreneurs can costlessly run away with coconuts and hide new projects from their creditors, which makes them useless as collateral. However, they cannot hide projects that already exist a the time the transaction is initiated. These constitute the only form of collateral.

Collateralized financial transactions could take many forms. However, in this model the future payoffs of current nonlemon projects are binary: either they become a lemon at $t+1$ or they do not. Any financing transaction must therefore have zero repayment if the project becomes a lemon and positive repayment otherwise. If there is no aggregate risk, this makes selling the asset and using it as collateral for borrowing exactly equivalent.[6] Selling is simpler to model, so I assume that the only kind of transaction is ex-dividend sales of existing projects. This is intended to represent the wider range of transactions that use existing assets as collateral.[7]

The entrepreneur will solve program (1.3), with the added constraint:

$$b\left(z^{t},z_{t+1}\right)\geq 0 \tag{1.11}$$

Constraint (1.11) will bind for the best entrepreneur. As a result, he will not be able

[4]Equations (1.9) and (1.10) assume that the nonnegativity of aggregate investment is not binding. Otherwise $c^{j}(z^{t})=Y_{K}(z^{t})k^{j}(z^{t})$ and $K\left(Z^{t},Z_{t+1}\right)=\gamma(1-\lambda)K(Z^{t})$.

[5]Alternatively, he could refuse to exert effort if he has pledged the output to someone else, as in Holmström and Tirole (1998).

[6]If there is aggregate risk, selling the asset is equivalent to state-contingent borrowing proportional to the value that the asset would have in each state of the world if it does not become a lemon.

[7]In Kurlat (2009) I study the case of a general joint distribution of asset qualities and investment opportunities and allow for arbitrary contracts.

to undertake all the investment in the economy. Instead, there will be a cutoff $A^*(z^t) = \frac{\gamma}{p_{NL}(z^t)}$ such that entrepreneurs with $A^j(z^t) < A^*(z^t)$ will not invest and entrepreneurs with $A^j(z^t) > A^*(z^t)$ will sell all their existing nonlemons in order to obtain coconuts for investment.

This equilibrium is inefficient in two related ways. First, the economy does not exclusively use the most efficient technology ($A^{\max}$) for converting coconuts into projects. The best entrepreneur is financially constrained and thus unable to invest all the coconuts the economy saves, so others with $A^j \in (A^*(z^t), A^{\max})$ also undertake physical investment. Secondly, entrepreneurs are exposed to idiosyncratic risk. If they draw a low value of A^j, they must convert their coconuts into projects through the market, which only provides a return $A^*(z^t)$, whereas if they draw a higher value they convert them at a rate $A^j(z^t)$.

1.4 Asymmetric information

Assume, as in section 1.3.2, that the only transactions in financial markets are sales of existing projects. However, now there is asymmetric information: only the owner of a project knows whether it is a lemon, and each entrepreneur observes only his own A^j. Those who purchase projects have rational expectations about $\lambda^M(z^t)$, the proportion of lemons among the projects that are actually sold in the market.[8]

Since A^j is private information, decisions must be conditioned on individual histories $z^{j,t}$ rather than full histories z^t. However, it is easy to verify that the aggregate variables r, p and λ^M that are relevant for the entrepreneur's decision depend only on the history of productivity Z^t, which is part of the entrepreneur's information set. An entrepreneur who starts with k_0^j projects solves the following program:

$$\max_{\left\{c(z^{j,t}),k\left(z^{j,t},z^j_{t+1}\right),i(z^{j,t}),s_L(z^{j,t}),s_{NL}(z^{j,t}),d(z^{j,t})\right\}} \mathbb{E}\sum_{t=0}^{\infty}\beta^t u\left(c\left(z^{j,t}\right)\right) \tag{1.12}$$

$$s.t.$$

$$c\left(z^{j,t}\right)+i\left(z^{j,t}\right)+p\left(Z^t\right)\left[d\left(z^{j,t}\right)-s_L\left(z^{j,t}\right)-s_{NL}\left(z^{j,t}\right)\right]\le(1-\lambda)\,r\left(Z^t\right)k\left(z^{j,t}\right)$$

$$k\left(z^{j,t},z^j_{t+1}\right)=\gamma\left[(1-\lambda)\,k\left(z^{j,t}\right)+\left(1-\lambda^M\left(Z^t\right)\right)d\left(z^{j,t}\right)-s_{NL}\left(z^{j,t}\right)\right]+A^j\left(z^j_t\right)i\left(z^{j,t}\right)$$

$$i\left(z^{j,t}\right)\ge 0,\; s_L\left(z^{j,t}\right)\in\left[0,\lambda k\left(z^{j,t}\right)\right],\; s_{NL}\left(z^{j,t}\right)\in\left[0,(1-\lambda)\,k\left(z^{j,t}\right)\right],\; d\left(z^{j,t}\right)\ge 0$$

[8] One might still ask why an entrepreneur cannot sell claims against his entire portfolio of projects (by the law of large numbers, he is not asymmetrically informed about it) instead of selling them individually. Kiyotaki and Moore (2003) assume that it is possible to credibly bundle all of one's projects by paying some cost. I assume this cost is prohibitively large.

Program (1.12) incorporates the borrowing constraint (1.11) and the fact that the price $p(Z^t)$ applies equally for sales of lemons $s_L(z^{j,t})$, sales of nonlemons $s_{NL}(z^{j,t})$ and purchases of projects of unknown quality $d(z^{j,t})$, a proportion $\lambda^M(Z^t)$ of which turn out to be lemons.

I will look for a recursive competitive equilibrium with $X \equiv \{Z, \Gamma\}$ as a state variable, where $\Gamma(k_t, A_t)$ is the cumulative distribution of entrepreneurs over holdings of capital and investment opportunities.[9] The relevant state variable for entrepreneur j's problem is $\{k^j, A^j, X\}$ so (dropping the j superscript) he solves the following recursive version of program (1.12):

$$V(k, A, X) = \max_{c, k', i, s_L, s_{NL}, d} \left[u(c) + \beta \mathbb{E}\left[V(k', A', X') \,|X\right]\right] \tag{1.13}$$
$$s.t.$$
$$c + i + p(X)\left[d - s_L - s_{NL}\right] \leq (1-\lambda)\, r(X)\, k$$
$$k' = \gamma\left[(1-\lambda)\, k + \left(1 - \lambda^M(X)\right) d - s_{NL}\right] + Ai$$
$$i \geq 0,\; d \geq 0$$
$$s_L \in [0, \lambda k],\; s_{NL} \in [0, (1-\lambda)\, k]$$

Denote the solution to this program by $\{c(k, A, X),\, k'(k, A, X),\, i(k, A, X),\, s_L(k, A, X),\, s_{NL}(k, A, X),\, d(k, A, X)\}$ and define the supply of lemons and nonlemons, total supply of projects and demand of projects respectively as

$$\begin{aligned} S_L(X) &\equiv \int s_L(k, A, X)\, d\Gamma(k, A) \\ S_{NL}(X) &\equiv \int s_{NL}(k, A, X)\, d\Gamma(k, A) \\ S(X) &\equiv S_L(X) + S_{NL}(X) \\ D(X) &\equiv \int d(k, A, X)\, d\Gamma(k, A) \end{aligned}$$

Definition 1. *A recursive competitive equilibrium consists of prices* $\{p(X), r(X), w(X)\}$*; market proportions of lemons* $\lambda^M(X)$*; a law of motion* $\Gamma'(X)$ *and associated transition density* $\Pi(X'|X)$*; a value function* $V(k, A, X)$ *and decision rules* $\{c^w(X),\, c(k, A, X),\, k'(k, A, X),\, i(k, A, X),\, s_L(k, A, X),\, s_{NL}(k, A, X),\, d(k, A, X)\}$ *such that (i) factor prices equal marginal products:* $w(X) = Y_L(X)$, $r(X) = Y_K(X)$*; (ii) workers consume their*

[9] Since A^j is *iid*, then it is independent of k^j and Γ is just the product of F and the distribution of k. The more general formulation could easily accommodate the case where an entrepreneur's individual A^j has some persistence, which would create some correlation between k^j and A^j.

wage $c^w(X) = w(X)$; *(iii)* $\{c(k,A,X),\ k'(k,A,X),\ i(k,A,X),\ s_L(k,A,X),\ s_{NL}(k,A,X),\ d(k,A,X)\}$ *and* $V(k,A,X)$ *solve program (1.13) taking* $p(X)$, $r(X)$, $\lambda^M(X)$ *and* $\Pi(X'|X)$ *as given; (iv) the market for projects clears:* $S(X) \geq D(X)$, *with equality whenever* $p(X) > 0$; *(v) the market proportion of lemons is consistent with individual selling decisions:* $\lambda^M(X) = \frac{S_L(X)}{S(X)}$ *and (vi) the law of motion of* Γ *is consistent with individual decisions:* $\Gamma'(k,A)(X) = \int_{k'(\tilde{k},\tilde{A},X)\leq k} d\Gamma(\tilde{k},\tilde{A})\ F(A)$

1.4.1 Solution of the entrepreneur's problem and equilibrium conditions

I solve the entrepreneur's problem and find equilibrium conditions in steps. First I show that all the policy functions are linear in k, which implies an aggregation result. Second I show that, given choice of c and k', the choices of d, s_L, s_{NL} and i reduce to a simple arbitrage condition. Third I solve a relaxed problem, converting the entrepreneur's nonlinear budget set into a weakly larger linear one and show that there is a simple static characterization of the consumption-savings decision. Based on the solution to the relaxed problem it is possible to derive supply, demand and a market clearing condition. Finally I show that the equilibrium price must satisfy the market-clearing condition whether or not the solutions to the two programs coincide. In either case the rest of the equilibrium objects follow immediately.

Linearity of policy functions. The constraint set in program (1.13) is linear in k and the utility function is homothetic. Hence the policy functions $c(k,A,X)$, $k'(k,A,X)$, $i(k,A,X)$, $s_L(k,A,X)$, $s_{NL}(k,A,X)$ and $d(k,A,X)$ are all linear in k. This implies the following aggregation result:

Lemma 1. *Prices and aggregate quantities do not depend on the distribution of capital holdings, only on total capital* K.

By Lemma 1, $\{Z, K\}$ is a sufficient state variable; in order to compute aggregate quantities and prices it is not necessary to know the distribution Γ.

Buying, selling and investing decisions. Take the choice of k' as given. The entrepreneur's problem then reduces to choosing d, s_L, s_{NL} and i to maximize c. This program is linear so the entrepreneur will always choose corner solutions. The decision to keep or sell lemons is trivial: as long as $p > 0$ the entrepreneur will sell the lemons ($s_L = \lambda k$), since they are worthless to him if kept. The decisions to keep or sell nonlemons and to invest in new projects or in purchasing projects depend on A. The return (i.e. the number of $t+1$ projects obtained per coconut spent) from buying projects is $A^M \equiv \frac{\gamma(1-\lambda^M)}{p}$. I refer to this as the market rate of return.[10] Conversely, the number of $t+1$ nonlemon projects an entrepreneur must give up to obtain one coconut is $\frac{\gamma}{p} > A^M$. The return on investing is simply A. This implies that the optimal choices of d, s_{NL} and i are given by two cutoffs, shown in figure 1.2.

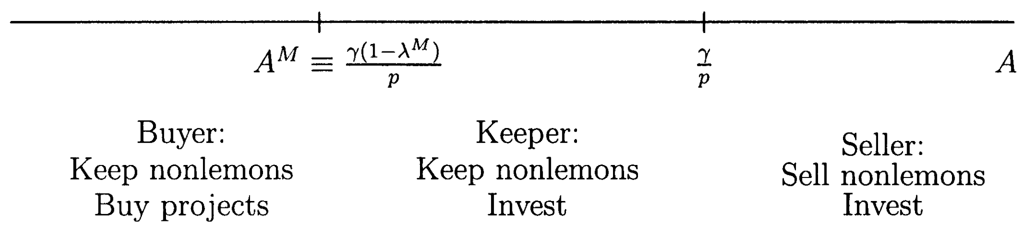


Figure 1.2: Buying, selling and investing decision as a function of A

Suppose first that $k' > \gamma(1-\lambda)k$ so the entrepreneur wants to save more than by just keeping his nonlemons. For $A < A^M$, entrepreneurs are Buyers: the return from buying is greater than the return from investing so $i \geq 0$ and $s_{NL} \geq 0$ bind and $d > 0$. For $A \in [A^M, \frac{\gamma}{p}]$ entrepreneurs are Keepers: investing offers a higher return than buying but not higher than the opportunity cost of selling nonlemons at the market price, so the entrepreneur neither buys projects nor sells nonlemons; $d \geq 0$ and $s_{NL} \geq 0$ bind and $i > 0$. For $A > \frac{\gamma}{p}$ entrepreneurs are Sellers: the return from investing is high enough for the entrepreneurs to sell nonlemons in order to finance investment; $d \geq 0$ and $s_{NL} \leq (1-\lambda)k$ bind and $i > 0$. If instead $k' < \gamma(1-\lambda)k$ (which by lemma 4 below is inconsistent with equilibrium), then Buyers and Keepers would choose $i = d = 0$ and $s_{NL} > 0$ while Sellers would still choose $d = 0$, $s_{NL} = (1-\lambda)k$ and $i > 0$. Combining these arbitrage conditions with the constraint from program (1.13) yields the following lemma:

Lemma 2. *Given k', the optimal d, s_L, s_{NL} and i are given by*

	Buyers: $A \in [0, A^M]$	*Keepers:* $A \in [A^M, \frac{\gamma}{p}]$	*Sellers:* $A \in (\frac{\gamma}{p}, \infty)$
$s_L =$	λk	λk	λk
$d =$	$\max\left\{\frac{k'-\gamma(1-\lambda)k}{\gamma(1-\lambda^M)}, 0\right\}$	0	0
$s_{NL} =$	$\max\left\{\frac{\gamma(1-\lambda)k-k'}{\gamma}, 0\right\}$	$\max\left\{\frac{\gamma(1-\lambda)k-k'}{\gamma}, 0\right\}$	$(1-\lambda)k$
$i =$	0	$\max\left\{\frac{k'-\gamma(1-\lambda)k}{A}, 0\right\}$	$\frac{k'}{A}$

(1.14)

[10]Noting, however, that it involves two different goods (projects and coconuts) as well as two different dates.

Consumption-savings decision under a relaxed budget set. An entrepreneur with investment opportunity is A must choose $\frac{c}{k}$ and $\frac{k'}{k}$ from his budget set, shown in figure 1.3.

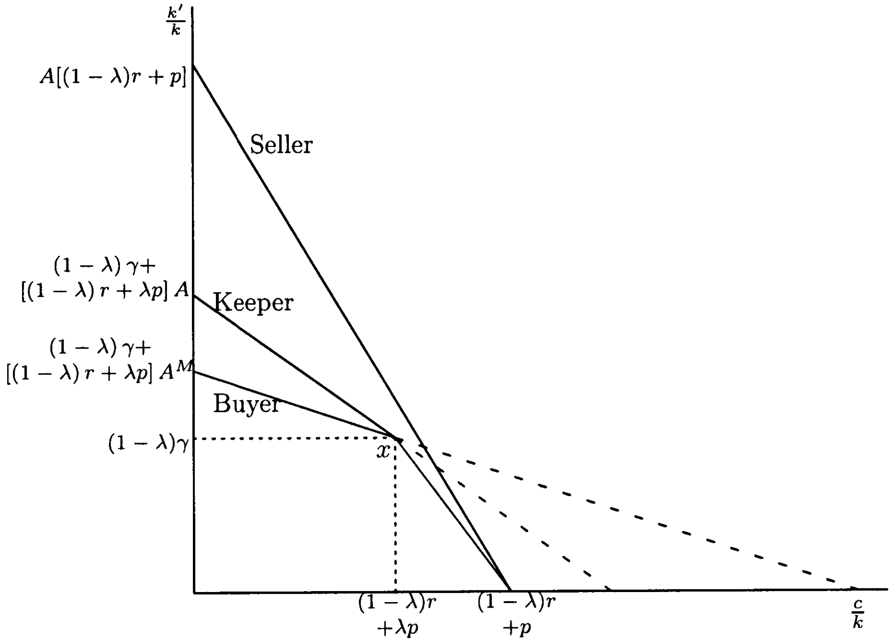


Figure 1.3: Budget sets

Point x represents an entrepreneur who chooses $s_L = \lambda k$ and $i = s_{NL} = d = 0$, an option available to all entrepreneurs. He simply consumes the dividends $(1-\lambda)rk$ and the proceeds from selling lemons λpk, and enters period $t+1$ with $(1-\lambda)\gamma k$ projects.

Consider first the decision of a Keeper. If he wishes to increase consumption beyond point x he must sell nonlemons, which means giving up $\frac{\gamma}{p}$ projects for each additional coconut of consumption. If instead he wishes to carry more projects into $t+1$, he invests with productivity A. Hence the budget constraint is kinked: to the right of x the slope is $-\frac{\gamma}{p}$ whereas to the left it is $-A$. Consider next a Buyer. His budget set is the same as for the Keeper except that the return he obtains from saving beyond point x is the market return A^M, which is higher than his individual return on investment A but lower than that of Keepers. Lastly, a Seller will sell all his projects and his budget constraint is linear with constant slope $-A$.

Define the entrepreneur's *virtual wealth* as

$$W(k,A,X) \equiv \left[\lambda p(X) + (1-\lambda)\left(r(X) + \max\left\{p(X), \frac{\gamma}{\max\{A, A^M(X)\}}\right\}\right)\right]k \quad (1.15)$$

Virtual wealth corresponds to to the projection of the left half of the budget constraint onto the horizontal axis. It consists of the coconuts the entrepreneur has (dividends plus proceeds of selling lemons) plus the nonlemon projects, valued at the maximum of either their sale price p or their replacement cost $\frac{\gamma}{\max\{A, A^M(X)\}}$. The linear budget set $\frac{k'}{k} \leq \max\{A, A^M(X)\}\left[\frac{W(k,A,X)}{k} - \frac{c}{k}\right]$ is weakly larger than the true kinked budget, so substituting it in program (1.13) leads to the following relaxed program:

$$\begin{aligned} V(k, A, X) &= \max_{c,k'} [u(c) + \beta \mathbb{E}[V(k', A', X')\,|X]] \\ & s.t. \\ k' &= \max\{A, A^M(X)\}[W(k, A, X) - c] \end{aligned} \tag{1.16}$$

Lemma 3. *Under the relaxed program (1.16), the entrepreneur's consumption is* $c(k, A, X) = (1-\beta) W(k, A, X)$

Due to logarithmic preferences, entrepreneurs will always choose to consume a fraction $1-\beta$ of their virtual wealth and save the remaining β, by some combination of keeping their old nonlemons, buying projects and physical investment. Note that the entrepreneur's decision, while rational and forward looking, does not depend on the transition density $\Pi(X'|X)$ or on the stochastic process for A. This feature will make it possible to solve for the equilibrium statically.

Notice that the function W is decreasing in A. Different agents have different relative valuations of projects and coconuts but asymmetric information prevents them from trading away those differences. In that sense, capital is illiquid. Furthermore, Lemma 3 implies that agents who value projects the least also consume less, so project valuation is negatively correlated with the marginal utility of consumption. Therefore agents would be willing to save in a risk-free asset with a lower expected return, a premium that would disappear if there was symmetric information. See Appendix 1.7.2 for a formal derivation. Kiyotaki and Moore (2008) obtain a similar result by assuming that *resaleability constraints* prevent entrepreneurs from reselling a fraction of their projects. Here instead the difference between the values placed on projects by entrepreneurs with different investment opportunities is derived endogenously as a result of asymmetric information.

Supply and demand under the relaxed program. Take p as given. By (1.14), the supply of projects will include all the lemons plus the nonlemons from Sellers. Hence

$$S(p) = \left[\lambda + (1-\lambda)\left(1 - F\left(\frac{\gamma}{p}\right)\right)\right] K \tag{1.17}$$

This implies a market proportion of lemons of

$$\lambda^M(p) = \frac{\lambda}{\lambda + (1-\lambda)\left(1 - F\left(\frac{\gamma}{p}\right)\right)} \tag{1.18}$$

and a market rate of return of:[11]

$$A^M(p) = \frac{\gamma}{p}\left(1-\lambda^M(p)\right) = \frac{\gamma}{p}\frac{(1-\lambda)\left(1-F\left(\frac{\gamma}{p}\right)\right)}{\lambda+(1-\lambda)\left(1-F\left(\frac{\gamma}{p}\right)\right)} \tag{1.19}$$

Demand for projects will come from Buyers. By Lemma 3, under the relaxed program they choose $k' = \beta A^M W\left(k, A^M, X\right)$. By Lemma 2, they each demand $\frac{k'-\gamma(1-\lambda)k}{\gamma(1-\lambda^M)}$ projects. Using (1.15) and adding over all Buyers, demand for projects will be:

$$D(p) = \left(\beta\left[\lambda+(1-\lambda)\frac{r}{p}\right] - \frac{(1-\beta)(1-\lambda)}{1-\lambda^M(p)}\right) F\left(A^M(p)\right) K \tag{1.20}$$

Market clearing implies

$$S(p^*) \geq D(p^*) \text{ with equality whenever } p^* > 0 \tag{1.21}$$

Equilibrium conditions under the true program.

Lemma 4. *$D > 0$ only if the solutions to programs (1.13) and (1.16) coincide for all entrepreneurs*

The solutions to programs (1.13) and (1.16) will not coincide whenever in the relaxed program, some entrepreneurs wish to choose points to the right of x. Lemma 4 states that if this is the case there will be no demand for projects and the price must be zero.

This implies the following result:

Proposition 2.

1. *In any recursive equilibrium, the function $p(X)$ satisfies (1.21) for all X*

2. *For any $p(X)$ that satisfies (1.21), there exists a recursive competitive equilibrium where the price is given by $p(X)$*

3. *There exists at least one function $p(X)$ that satisfies (1.21)*

Proposition 2 establishes that a recursive equilibrium exists and must satisfy (1.21) regardless of whether or not the solutions to programs (1.13) and (1.16) coincide. Therefore it is possible to find equilibrium prices statically simply by solving (1.21). Once p^* is determined, it is straightforward to solve, also statically, for the rest of the equilibrium objects. λ^M and A^M follow from (1.18) and (1.19). If $p^* > 0$ then virtual wealth and, by Lemma 3,

[11] Define $A(0) \equiv 0$.

consumption for each entrepreneur can be found using (1.15) and s_L, s_{NL}, d and i are given by (1.14). If instead the only solution to (1.21) is $p^* = 0$, I refer to the situation as one of *market shutdown*. It is still possible to solve the relaxed problem (1.16), which results in

$$k' = \beta (1 - \lambda) (Ar + \gamma) k$$

This satisfies $k' \geq \gamma (1 - \lambda) k$ iff $A \geq \bar{A} \equiv \frac{\gamma}{r} \frac{(1-\beta)}{\beta}$. Hence for entrepreneurs with $A \geq \bar{A}$, consumption and investment can be computed in the same way as when the market does not shut down whereas entrepreneurs with $A < \bar{A}$ chose $c = (1 - \lambda) rk$ and $k' = \gamma (1 - \lambda) k$.

Aggregate capital accumulation is found by replacing the equilibrium values of i into the law of motion of capital (1.2),[12] yielding

$$\begin{aligned} \frac{K'}{K} &= \gamma (1 - \lambda) + \int_{A^M}^{\frac{\gamma}{p}} [\beta A [\lambda p + (1 - \lambda) r] - (1 - \beta) (1 - \lambda) \gamma] \, dF(A) \\ &\quad + \int_{\frac{\gamma}{p}}^{\infty} \beta A [p + (1 - \lambda) r] \, dF(A) \end{aligned} \tag{1.22}$$

In general, the market return $A^M(p)$ can be either increasing or decreasing. An increase in the price has a direct effect of lowering returns by making projects more expensive and an indirect effect of improving returns by increasing the proportion of entrepreneurs who choose to sell their nonlemons. This implies that there could be more than one solution to (1.21). In this case, I will assume that the equilibrium price is given by the highest solution. More worryingly, there could exist a price $p' > p^*$ such that $A^M(p') > A^M(p^*)$ even when p^* is the highest solution to (1.21). This will be the case when selection effects are strong enough that the return from buying projects would be higher at a price higher than the highest market-clearing one. Both Buyers and Sellers would be better off if there was sufficient demand to sustain such a price. Stiglitz and Weiss (1981) argue that when this is the case the equilibrium concept used above is not reasonable and it would be more sensible to assume that Buyers set a price above p^* that maximizes their return and ration the excess supply. Appendix 1.7.1 discusses how the definition of equilibrium may be adapted to allow for rationing, a change that makes little difference for the results. In section 1.5, I consider signals that segment the market into a continuum of different submarkets. In that variant there exists a unique equilibrium in which Buyers can never benefit from raising prices in any submarket (see Lemma 8), so the issue of what is the right equilibrium concept becomes moot, a point first made by Riley (1987) in the context of the Stiglitz-Weiss model.[13] For

[12]By Walras' Law, it is equivalent to just sum $k'(k, A, X)$ over all entrepreneurs.

[13]In their terminology, there will be redlining (exclusion of arbitrarily similar yet distinct groups) but not pure rationing (partial exclusion of observationally identical projects).

some of the results below, it will simplify the analysis to simply assume that parameters are such that the issue does not arise:[14]

Assumption 1. *$A^M(p)$ is decreasing*

1.4.2 Equivalence with an economy with taxes

As shown in figure 1.2, asymmetric information introduces a wedge between the return obtained by Buyers, A^M, and the return given up by Sellers of nonlemons, $\frac{\gamma}{p}$. The magnitude of this wedge depends on $\lambda^M(X)$. It turns out that this wedge is exactly isomorphic to the wedge that would be introduced by imposing state-dependent taxes on the sales of projects.

Consider the economy with borrowing constraints and symmetric information of section 1.3.2, but now assume that the government imposes an ad-valorem tax of $\tau(X)p$ coconuts on sales of projects. The total revenue $T(X) = \tau p(X)S(p(X))$ collected from this tax is rebated lump-sum to all entrepreneurs. Entrepreneurs solve the following program:

$$V(k, A, X) = \max_{c,k',i,s_{NL},d} [u(c) + \beta\mathbb{E}[V(k', A', X')|X]] \tag{1.23}$$

$$s.t.$$

$$c + i + p(X)[d(1 + \tau(X)) - s_{NL}] \leq (1 - \lambda)r(X)k + T(X)$$

$$k' = \gamma[(1 - \lambda)k + d - s_{NL}] + Ai$$

$$i \geq 0,\ d \geq 0$$

$$s_{NL} \in [0, (1 - \lambda)k]$$

This problem can be solved by the same steps used to solve program (1.13). Solving for the equilibrium conditions leads to the following equivalence result.

Proposition 3. *Suppose $\tau(X) = \frac{\lambda^{M*}(X)}{1-\lambda^{M*}(X)}$, where $\lambda^{M*}(X)$ is the equilibrium value of the asymmetric information economy. Then prices and allocations of the symmetric-information-with-taxes and the asymmetric information economies are identical.*

By Proposition 3, the distortionary effect of having a proportion λ^M of lemons in the market is exactly equivalent to the one that would result from a tax at the rate $\tau = \frac{\lambda^M}{1-\lambda^M}$. Moreover, asymmetric information gives all entrepreneurs the possibility of earning λpk coconuts from selling lemons to others. This has an exact counterpart in the lump-sum redistribution of the government's revenue.

[14] A sufficient condition for Assumption 1 to hold is $h(x) \leq \frac{1}{x}\left[1 + \frac{1-\lambda}{\lambda}(1 - F(x))\right]$ for all x, where h is the hazard function of A. Results do not rely on Assumption 1 unless otherwise stated.

Chari, Kehoe, and McGrattan (2007) propose a way to decompose economic fluctuations into the movements of an efficiency wedge, a labour wedge, an intertemporal wedge and a government spending wedge. The implicit taxes that result from asymmetric information do not translate neatly into a single one of these wedges. They distort both the consumption-saving decision (resulting in an intertemporal wedge) and the allocation of investment across different entrepreneurs (resulting in an efficiency wedge). Furthermore, the model would have an intertemporal wedge even under symmetric information due to borrowing constraints and the fact that workers do not participate in asset markets.

It is reasonably simple to analyze the effects of exogenous changes in tax rates. This will be useful when looking at the economy with asymmetric information because Proposition 3 implies these are exactly isomorphic to the effects of endogenous changes in λ^M.

Lemma 5. *For any state X*

1. $\frac{dp}{d\tau} < 0$
2. $\frac{dA^M}{d\tau} < 0$
3. $\left.\frac{dK'}{d\tau}\right|_{\tau=0} < 0$

An increase in taxes increases the wedge between A^M and $\frac{\gamma}{p}$. Parts 1 and 2 of Lemma 5 establish that this increase in the wedge manifests itself through *both* lower returns for Buyers and lower prices for Sellers. Both of these effects tend to lower capital accumulation. In addition, taxes have the effect of redistributing resources form Buyers and Sellers to all entrepreneurs, including Keepers. As with any tax, the relative incidence on Buyers and Sellers depends on elasticities. For small enough τ, the elasticities of supply and demand are mechanically linked, as the density of marginal Buyers, $f\left(A^M\right)$, approaches that of marginal Sellers, $f\left(\frac{\gamma}{p}\right)$. Part 3 of Lemma 5 establishes that in this case the redistributive effect always goes against the higher-A agents, reinforcing the effect of lower capital accumulation.[15]

1.4.3 Comparative statics and aggregate shocks

The equilibrium conditions derived in section 1.4.1 are static. This feature is a consequence of assuming that entrepreneurs have log preferences, no labour income and a single asset to invest in. This simplifies the analysis of the effects of aggregate shocks. In particular, it

[15]For τ away from zero, it is possible to construct counterexamples where $f\left(\frac{\gamma}{p}\right)$ is much higher than $f\left(A^M\right)$, so supply is much more elastic than demand. In this case it is possible for Sellers to be net beneficiaries of redistribution, so taxes can conceivably increase capital accumulation. Working in the opposite direction is the fact that the direct marginal distortion increases with τ.

implies that shocks will have the same effects whether or not they are anticipated. Thus by answering the comparative statics question "how would the features of the model change if a parameter were different?" one also answers the impulse response question "how would the economy respond to a shock to one of the parameters?"

Consider first the effects of a productivity shock. It makes a difference in this model whether the shock affects primarily the coconut-producing capacity of the economy or its project-producing capacity. Suppose first that there is a proportional shock to coconut-productivity. This would affect the equilibrium conditions through its effect on the marginal product of capital $r = Y_K$. Its effects can therefore be understood through the comparative statics of the equilibrium with respect to r.

Proposition 4. *If in equilibrium $p^* > 0$ then*

1. *p^* is increasing in r.*
2. *Under Assumption 1, A^{M*} is decreasing in r.*
3. *λ^{M*} is decreasing in r.*
4. *Under Assumption 1, $\frac{K'}{K}$ is increasing in r*

Favourable shocks will mean that entrepreneurs hold a higher number of coconuts. Other things being equal, entrepreneurs would want to save a fraction β of the additional coconuts. Sellers and Keepers would do so through physical investment but Buyers would attempt to buy more projects, thus bidding up the price (part 1) and lowering returns (part 2). Note that it is not productivity per se that matters but rather the effect of the productivity shock on current dividends. A similar effect would result, for instance, if there was a temporary shock to the capital share of output leaving total output unchanged or simply a helicopter drop of coconuts from outside the economy. Part 3 of Proposition 4 has the important implication that the severity of the lemons problem, as measured by the equivalent tax wedge $\tau = \frac{\lambda^M}{1-\lambda^M}$ will respond to aggregate shocks. Higher prices persuade marginal Keepers to sell their nonlemons and therefore a favourable shock to the coconut-producing capacity of the economy will alleviate the lemons problem.

Turn now to an investment-productivity shock. This can be represented as a proportional change in the investment opportunity of every entrepreneur, from A to ϕA, so that the distribution of A becomes F', where $F'(A) = F\left(\frac{A}{\phi}\right)$.

Proposition 5.

1. *λ^{M*} is decreasing in ϕ.*

2. *Under Assumption 1,* $\frac{K'}{K}$ *is increasing in* ϕ

Proposition 5 implies that higher productivity in the project-producing sector also alleviates the lemons problem. In this case, the effect comes from the supply side rather than the demand side. Because physical investment has become more attractive, marginal Keepers decide to sell their nonlemons, improving the mix of projects.

Propositions 4 and 5 jointly show that positive shocks lessen financial market wedges and negative shocks worsen them. Liquidity, as measured by (the inverse of) the size of these wedges, is procyclical.

The endogenous response of liquidity has the important consequence of amplifying the response of the economy to exogenous shocks. To show this, I compare the responses of economies with symmetric and asymmetric information to the same exogenous shock. In order to make sure that the economies are otherwise identical, I assume that in the symmetric information economy there are (fixed) taxes on transactions at a rate such that, absent the shock, prices and allocations in both economies would be exactly the same. Denote equilibrium variables in both economies by the superscripts SI and AI respectively.

Proposition 6.

1. $\frac{dp^{AI}}{dr} > \frac{dp^{SI}}{dr}$
2. $\frac{dA^{M,SI}}{dr} < \frac{dA^{M,AI}}{dr}$
3. $\frac{dK^{\prime,AI}}{dr} > \frac{dK^{\prime,SI}}{dr}$ *for* λ *small enough*

Proposition 6 implies that, in response to a productivity shock which increases r, asymmetric information amplifies the rise in asset prices, moderates the drop in rates of return and amplifies the increase in the rate of capital accumulation compared to the symmetric information benchmark.

For negative shocks, the adverse selection effect can be sufficiently strong to lead to a complete shutdown of financial markets.

Proposition 7.

1. *If*

$$\max_p A^M(p) < \frac{\gamma}{r}\frac{(1-\beta)}{\beta} \tag{1.24}$$

then the market shuts down

2. *Sufficiently large negative shocks to coconut-productivity or project-productivity lead to market shutdowns*

When r is sufficiently low, then entrepreneurs have very few coconuts for each project they own. The return from buying that would be needed to entice Buyers to choose k' above the kink in figure 1.3 is so high that it cannot be obtained at any price, and the market shuts down. When project-productivity is low, the measure of entrepreneurs who are willing to sell nonlemons at any given price becomes low. The market becomes full of lemons, lowering returns to the point where Buyers choose to stay at the kink in figure 1.3 and the market shuts down. Notice that asymmetric information is essential for the result. In the symmetric information benchmark of section 1.3.2, as in the complete markets benchmark of section 1.3.1, markets would only shut down when no entrepreneurs wish to invest, so there are no gains from trade.

It is also possible to analyze shocks whose only effect is due to informational asymmetry. Consider a temporary increase in λ, compensated by an increase in K such that $(1-\lambda)K$ remains unchanged. This shock has no effect on the production possibility frontier of the economy and, with symmetric information, would have no effect on allocations. With asymmetric information, the fact that there are more lemons mixed in with the nonlemons makes a difference.

Proposition 8. *An simultaneous increase in λ and K that leaves $(1-\lambda)K$ unchanged increases λ^{M*}*

The increase in λ^{M*} that results from this type of shock is equivalent to an increase in taxes, so the results in Lemma 5 regarding the effects on asset prices, rates of return and capital accumulation can be applied directly.

One interpretation of this type shock may be the following. Suppose every period entrepreneurs receive an endowment of $\Delta_\lambda K$ useless lemons, so the total number of lemons is $(\lambda+\Delta_\lambda)K$ rather than λK. However, in ordinary times it is possible to tell apart the endowment-lemons from the nonlemons, so their existence is irrelevant. A shock to λ of the kind described above is equivalent to entrepreneurs losing the ability to detect endowment-lemons, a form of deterioration of information. Effects of this sort will play a role in section 1.5 where I make the quality of information endogenous.

1.4.4 Simulations

In this section I compute dynamic examples of the response of the economy to shocks. In order to highlight the role of asymmetric information, I compare the impulse responses to those of an economy with a fixed level of taxes on financial transactions such that steady state allocations are identical.

While I choose parameter values that are close to those used in quantitative models, the spirit of the exercise is to illustrate the mechanisms underlying the results stated above and give a rough idea of the potential magnitudes rather than to constitute a quantitatively precise simulation.

Parameter	Value
β	0.85
γ	.9
λ	0.15
$F(A)$	Gamma distribution with $E(A) = 1$ and $std(A) = 2$
Y	$Z[(1-\lambda)K]^{\alpha} L^{1-\alpha}$ with $\alpha = 0.4$
L	1
Z	1

Table 1.1: Parameter values used in simulations

The production function is a standard Cobb-Douglas. The length of the period is approximately two years.[16] Usually one would calibrate discount rates in order to match either observed interest rates or rates of return to capital but in this model this is complicated by the fact that these are different for different agents (see Appendix 1.7.2). Under the choice of $\beta = 0.85$, the gross annualized risk-free rate is 0.93 for Buyers and 1.14 on average. $\gamma = 0.9$ and $\lambda = 0.15$ imply an annual rate of depreciation of 13%. There is less guidance as to what are reasonable values for λ and F. The values were chosen primarily to ensure that the adverse selection problem is not severe enough to make markets shut down in steady state. A standard deviation of 2 for A implies that the investment opportunity of the marginal Seller, who is at the 91^{st} percentile of investment productivity, is 3.3 times better than that of the marginal Buyer, who is at the 76^{th} percentile. Under these parameters, Assumption 1 holds in the relevant range where equilibria take place.

In all cases I assume the economy begins at a steady state and is hit by a shock at $t = 3$. As a preliminary remark, I show that such a steady state exists.

Lemma 6. *Under Assumption 1, for any fixed Z there exists a unique steady state level of capital.*

The first exercise is to simulate a productivity shock in the consumption goods sector lasting a single period. Panel 2 of figure 1.4 shows the response of output to the TFP shock. It rises mechanically at the time of the shock and remains slightly above steady state because of capital accumulation. Panel 3 shows how capital accumulation responds to the

[16] A relatively long period length makes the investment opportunity of a given entrepreneur somewhat persistent despite the assumption that A is *iid*.

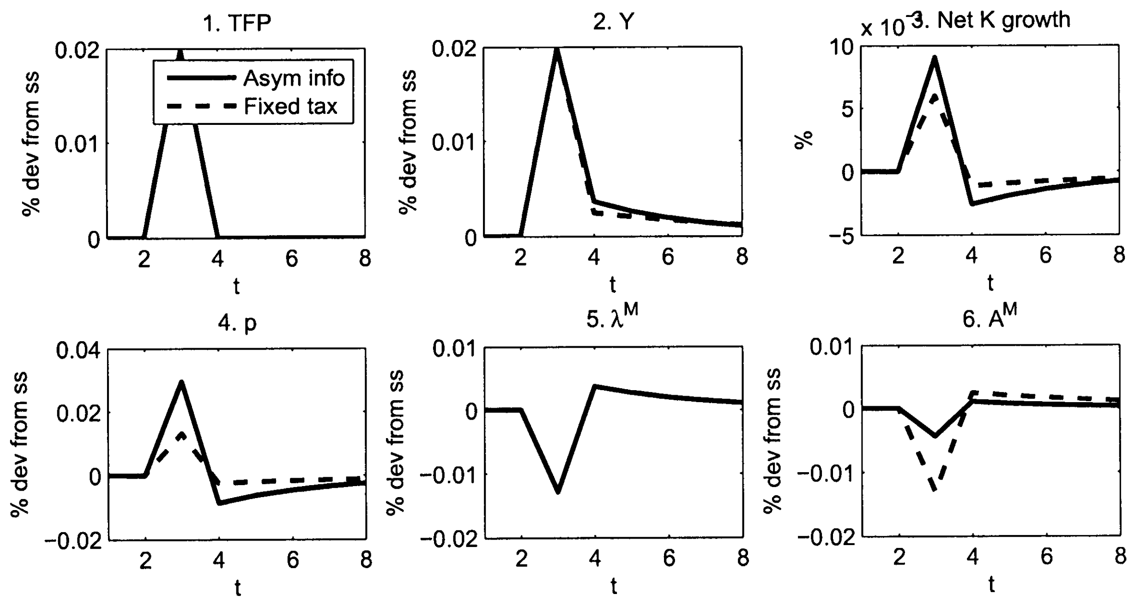


Figure 1.4: Transitory productivity shock in the consumption goods sector

shock, illustrating the amplifying effect of asymmetric information. Panel 4 illustrates the response of asset prices. Because the increase in the marginal product of capital increases the supply of savings, these would rise even with symmetric information; they rise even more because of the selection effect. The response of λ^M is shown on panel 5. Panel 6 shows the response of market rates of return. The increase in the supply of savings drives them down, but selection effects moderate the effect. After the shock is over at $t = 4$, the marginal product of capital is slightly below its steady state level due to diminishing marginal product, so the effects are reversed.

If the productivity shock followed an AR(1) process (with persistence of 0.9), the effects would be similar to the nonpersistent shock. The main difference is that output in the asymmetric information economy remains above that of the fixed-tax economy for longer due to the sum of several periods of more capital accumulation.

The next exercise is to simulate a productivity shock in the investment sector, i.e. a shift in $F(A)$ of the kind considered in Proposition 5, lasting only one period. The most interesting difference compared to the standard TFP shock lies in the response of A^M and p. The increase in investment-productivity means that more entrepreneurs wish to sell their assets to obtain financing. Even with symmetric information, this raises market returns, as shown on panel 6 of figure 1.6. The selection effect means that this is even stronger with asymmetric information. With symmetric information, the increase in the supply of assets necessarily lowers asset prices, as shown on panel 4. With asymmetric information the increase in the proportion of nonlemons can lead to the opposite effect, as is the case in

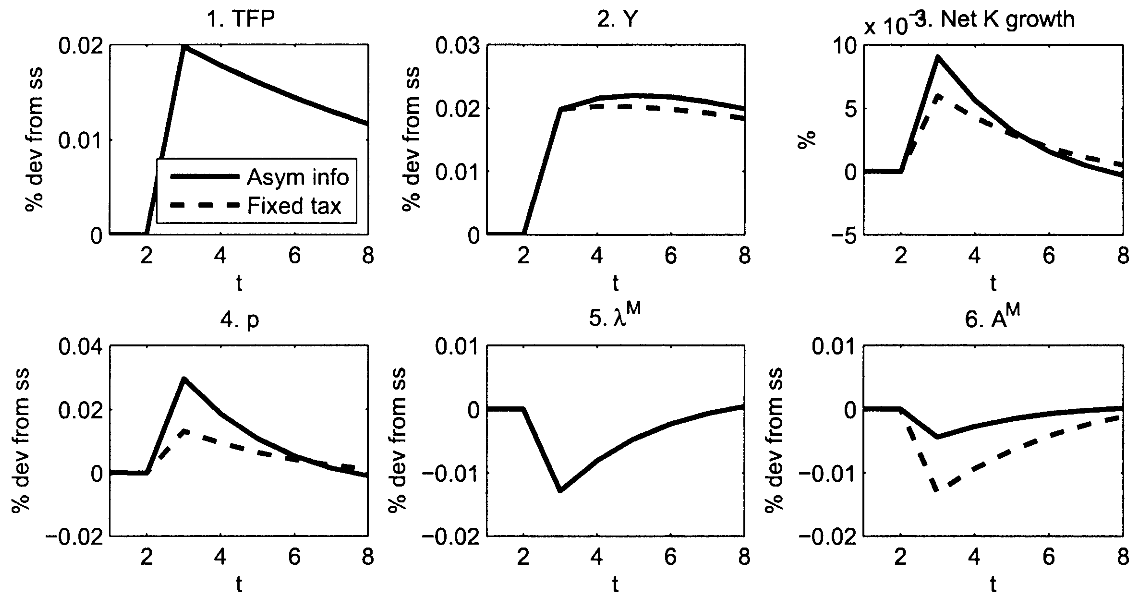


Figure 1.5: AR(1) productivity shock in the consumption goods sector

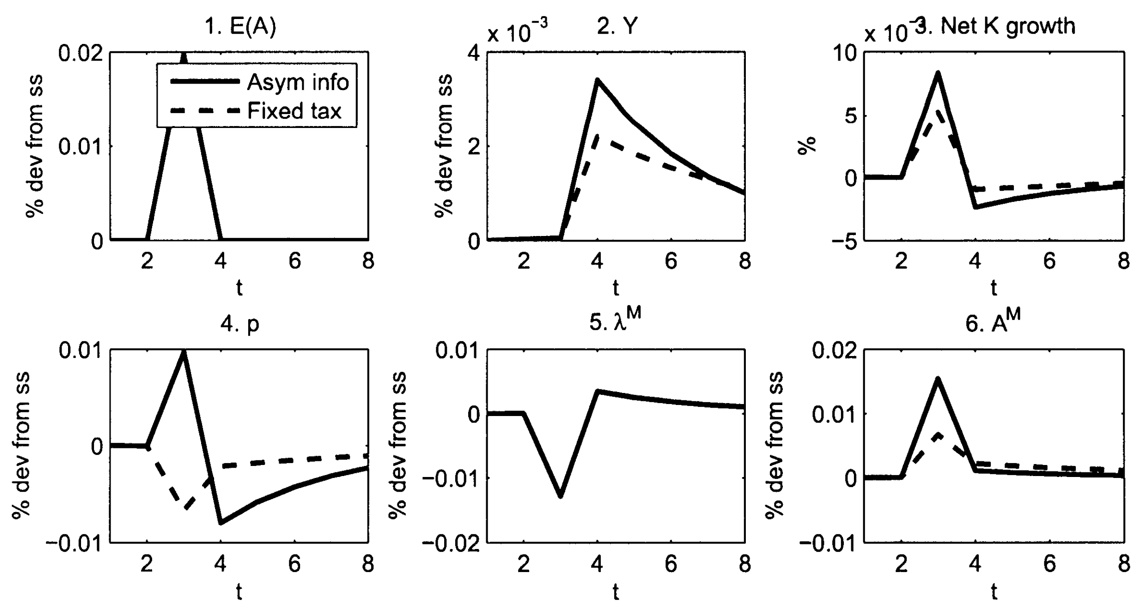


Figure 1.6: Transitory productivity shock in the investment sector

this example.

1.5 Informative signals and learning

In section 1.4 I made the extreme assumption that potential buyers of projects do not know anything about an individual project. In reality there are many sources of information about assets that potential buyers may consult, such as financial statements and analyst reports. All of these are imperfect but contain some useful information that helps buyers update their beliefs about whether a given project is a lemon. In this section I extend the model by introducing publicly observable imperfect signals about individual projects' quality and explore how equilibrium outcomes are affected by the existence of these sources of information.

The structure of information is as shown in figure 1.7. Each project receives a random

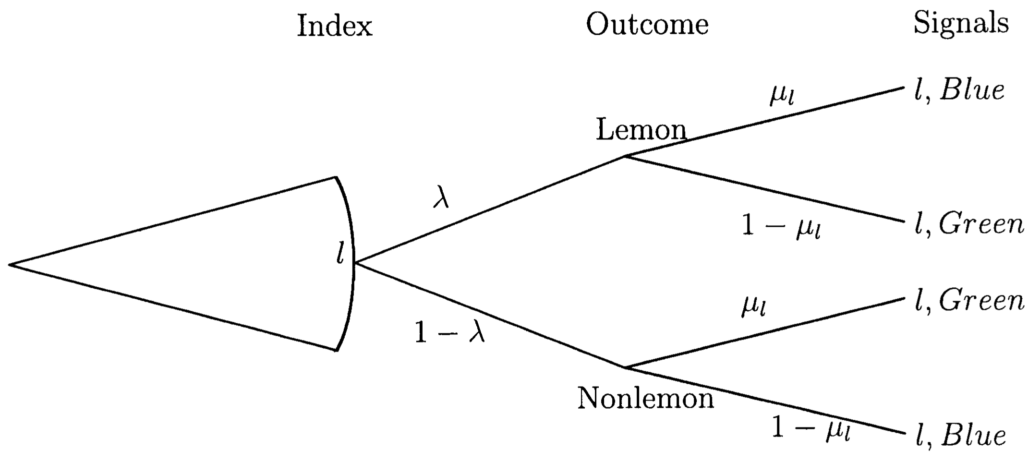


Figure 1.7: Information structure

index l uniformly distributed in $[0,1]$. After it has either become a lemon or not it emits a publicly observable message $s \in \{Blue, Green\}$. A function $\mu : [0,1] \rightarrow [0,1]$ governs the conditional probability of emitting each of the two messages for a given index l; I denote $\mu(l)$ by μ_l, where $\mu_l \equiv \Pr[s = Blue|l, Lemon] = \Pr[s = Green|l, Nonlemon]$. The index l, as well as the messages *Blue* or *Green* are publicly observable, so a signal consists of a pair $l, s \in [0,1] \times \{Blue, Green\}$. Conditional on the signal, the probability that a given project is a lemon is given respectively by

$$\lambda_{l,B} \equiv \Pr[Lemon|l, Blue] = \frac{\lambda\mu_l}{\lambda\mu_l + (1-\lambda)(1-\mu_l)}$$
$$\lambda_{l,G} \equiv \Pr[Lemon|l, Green] = \frac{\lambda(1-\mu_l)}{\lambda(1-\mu_l) + (1-\lambda)\mu_l} \quad (1.25)$$

The index l represents the different pieces of information that a firm can issue in any given period: a financial statement, news about a labour dispute, a profit forecast, consumer reports about its products, a rumour about production delays, etc. The message s represents the actual content of that piece of information, such as "profits increased 5% in the second quarter" or "the product ranks third in customer satisfaction", simplified so that each message can only take two values. To abstract from issues of strategic release of information, I assume that both the index l and the message s are beyond the entrepreneur's control.

Define

$$I_l(\mu_l) \equiv \left(\mu_l - \frac{1}{2}\right)^2$$

I_l (or any monotone transformation thereof) measures the quality of signals for projects with index l. When $I_l = 0$ then $\lambda_{l,s} = \lambda$ and signals with index l are completely uninformative; if this is true for all l the model reduces to the one without signals. If $I_l = \frac{1}{4}$ then signals are perfectly informative; if this is true for all l, the model reduces to the symmetric information benchmark of section 1.3.2. For intermediate values of I_l the signals are partially correlated with the project outcome.

It will be useful below to discuss the overall distribution of information in the economy. To do so, reorder the indices l (without loss of generality) so that μ_l is weakly increasing and define the distribution function $H(\mu)$:[17]

$$H(\mu) \equiv \sup\{l : \mu_l \leq \mu\}$$

1.5.1 General equilibrium

A definition of equilibrium, which is just a generalization of Definition 1 can be found in Appendix 1.7.3. Equilibrium conditions can be found statically in the same way as in section 1.4.1. Signals have the effect of segmenting the market into a continuum of separate submarkets, with prices $p_{l,s}$. Denote the price vector by $p = \{p_{l,s}\}_{l=[0,1],s=B,G}$. Entrepreneurs

[17]Note the slight abuse of notation: μ refers to both the function $\mu(l) : [0,1] \to [0,1]$ and to the values that function may take.

might decide to sell their nonlemons into certain l, s submarkets but not others[18] and may also decide which submarkets to buy from. Entrepreneurs will sell their nonlemons into pool l, s iff $A > \frac{\gamma}{p_{l,s}}$, so

$$\lambda_{l,s}^{M}(p_{l,s}) = \frac{\lambda_{l,s}}{\lambda_{l,s} + (1-\lambda_{l,s})\left(1 - F\left(\frac{\gamma}{p_{l,s}}\right)\right)} \tag{1.26}$$

and the return obtained by Buyers who purchase from market l, s is

$$A_{l,s}^{M}(p_{l,s}) = \frac{\gamma}{p_{l,s}}\left(1 - \lambda_{l,s}^{M}(p_{l,s})\right) \tag{1.27}$$

The marginal Buyer must be indifferent between buying in any of the submarkets where demand is positive or investing on his own. Denoting his investment opportunity by A^*, this implies that

$$A_{l,s}^{M}(p_{l,s}) \leq A^*, \text{ with equality if } p_{l,s} > 0, \text{ for every } l, s \tag{1.28}$$

By continuity, for every l, s there is at least one price that satisfies condition (1.28). If $A_{l,s}^{M}(p_{l,s}) < A^*$ for all $p_{l,s}$ then the l, s market shuts down and $p_{l,s} = 0$. Otherwise there exists at least one price $p_{l,s}$ where (1.28) holds as equality. Assume that for any l, s, $p_{l,s}$ will be given by highest price that satisfies $p_{l,s}$ and denote this price by $p_{l,s}(A^*)$.[19]

Lemma 7. *$p_{l,s}(A^*)$ is decreasing in A^**

If $\lambda_{l,s}^{M}$ did not respond to the price, Lemma 7 would be a trivial statement: if Buyers demand higher returns, this lowers asset prices. With $\lambda_{l,s}^{M}$ endogenous, this depends on the selection effect not dominating the direct effect. Lemma 7 holds because this is always the case for high enough p, including the highest solution to condition (1.28).

Since Buyers are indifferent between buying in any submarket, demand in each submarket is not uniquely determined. However, total spending across submarkets is, and is given by

$$TS(p, A^*) = K\left[\beta\left[\lambda\int_0^1 \left[\mu_l p_{l,B} + (1-\mu_l)\, p_{l,G}\right] dl + (1-\lambda)\, r\right] - \frac{(1-\beta)(1-\lambda)\gamma}{A^*}\right] F(A^*) \tag{1.29}$$

The supply in each submarket is determined by arbitrage between selling and keeping, leading

[18] Assume they are sufficiently well diversified that, at the level of the individual entrepreneur, their holdings of projects are uniformly distributed across l and the proportion of messages *Blue* and *Green* for each l is given by μ_l.

[19] Under Assumption 1, there is always a unique price that satisfies (1.28). Otherwise, the focus on the highest solution is justified by the fact that otherwise Buyers could improve their returns by raising prices. See Appendix 1.7.1 for a discussion of this case.

to total revenue from selling projects equal to

$$TR(p) = K \int_0^1 \left[\begin{array}{c} p_{l,B} \left[\lambda \mu_l + (1-\lambda)(1-\mu_l) \left(1 - F\left(\frac{\gamma}{p_{l,B}}\right)\right)\right] + \\ p_{l,G} \left[\lambda (1-\mu_l) + (1-\lambda) \mu_l \left(1 - F\left(\frac{\gamma}{p_{l,G}}\right)\right)\right] \end{array} \right] dl \tag{1.30}$$

Market clearing implies that the value of excess demand must be zero, i.e.

$$E(p, A^*) \equiv TS(p, A^*) - TR(p) = 0 \tag{1.31}$$

It is easily seen that (1.26)-(1.31) are necessary and sufficient conditions for equilibrium. However, there is nothing in the definition of equilibrium that implies that if there are many solutions to (1.28) the highest must be selected. The following equilibrium refinement imposes this selection rule.

Definition 2. *A robust equilibrium with signals is a recursive equilibrium with signals and a value of $A^*(X)$ such that, for any l, s and any $\tilde{p} \geq p_{l,s}(X)$, the following inequality holds: $A^M_{l,s}(\tilde{p}) \leq A^*(X)$, with equality if $\tilde{p} = p_{l,s}(X) > 0$*

Mild regularity conditions ensure existence and uniqueness of a robust equilibrium.

Lemma 8. *If $H(\mu)$ is continuous, then there exists a unique robust equilibrium with signals*

Lemma 8 states that there is a unique equilibrium in which it is never the case that Buyers would prefer higher prices, without requiring Assumption 1 or allowing for rationing in the definition of equilibrium. For any A^*, prices and allocations are uniquely determined, so the proof rests on showing that there exists a unique value of A^* such that $E(p(A^*), A^*) = 0$. Uniqueness is guaranteed because, using Lemma 7, $E(p(A^*), A^*)$ is monotonic. Existence requires the assumption that H be continuous. A small change in A^* can lead to a discrete shutdown of a particular submarket, but continuity implies that each submarket is small so the excess demand function is continuous and must intersect zero at some point. A similar argument can be found in Riley (1987).

1.5.2 Effects of information

For concreteness assume that $\mu_l \geq \frac{1}{2} \forall l$; the symmetry of the information structure implies that this is without loss of generality.[20] In this case, $\lambda_{l,G} < \lambda < \lambda_{l,B}$, so message *Green* is good news while message *Blue* is bad news. For some indices l, μ_l will be near $\frac{1}{2} \iff I_l$

[20]The possibility that μ_l might take values both below and above $\frac{1}{2}$ is important in section 1.5.4, where I study the effect of uncertainty about the value of μ_l.

near 0, so signals will be relatively uninformative, whereas for other indices μ_l will be near $1 \iff I_l$ near $\frac{1}{4}$, so signals will be very informative. Taking A^* as given for now, compare prices in these different submarkets.

Lemma 9. *For given A^*, $p_{l,G}$ is increasing in μ_l and $p_{l,B}$ is decreasing in μ_l*

For a given l, μ_l determines what proportion of the lemons end up in each of the *Green* or *Blue* submarkets. Better information implies that the lemons will be more highly concentrated in just one of the two submarkets, the *Blue* one in case $\mu_l > \frac{1}{2}$. To maintain the same return for Buyers in both submarkets, this means the price must rise in the relatively better *Green* submarket and drop in the relatively worse *Blue* submarket. This can be interpreted in terms of the implicit tax that the presence of lemons imposes, as in section 1.4.2. Concentrating the lemons in the *Blue* submarket is akin to increasing the tax on *Blue*-market transactions and lowering it in the *Green*-market transactions.

Lemma 10. *Given any A^*, $\exists \hat{\mu} \in (0,1)$ such that $p_{l,B} = 0$ and $p_{l,G} > 0$ iff $\mu_l \geq \hat{\mu}$*

If signals are sufficiently informative, lemons will be so exclusively concentrated in the bad submarket that it will necessarily shut down, while the good submarket will be almost lemon-free and have a positive price. What Lemma 10 does not specify is whether there exist values of μ_l such that either both submarkets shut down or both have a positive price. In fact, either of these cases is possible for values of μ_l close to $\frac{1}{2}$.

Define the *amount of indispensable information* $\varepsilon(X)$ for a given state X as the maximum number $\varepsilon \geq 0$ such that $p_{l,B}(X) = p_{l,G}(X) = 0$ whenever $I_l < \varepsilon$. If $\varepsilon(X) > 0$, then some amount of information is necessary for trade to take place; markets with uninformative signals will shut down. Instead if $\varepsilon = 0$, markets with uninformative signals will be open. Price patterns in cases where $\varepsilon > 0$ and $\varepsilon = 0$ are illustrated in figure 1.8.

The condition $\varepsilon(X) > 0$ will always hold when λ is sufficiently large. The economic interpretation of this is that poor quality assets are so common that Buyers will only be willing to buy assets that they have received good reports about.

A general increase in the quality of information can be represented by an increase in I_l for all l or equivalently by a shift in the distribution $H(\mu)$ away from $\mu = \frac{1}{2}$, i.e. a decrease in $H\left(\frac{1}{2} + \delta\right) - H\left(\frac{1}{2} - \delta\right)$ for every δ. This increases the overall degree of sorting of lemons and nonlemons in different submarkets. By Lemmas 9 and 10, for given A^* this will mean a higher fraction of markets with either little adverse selection and high prices or extreme adverse selection and market shutdown and a lower fraction of markets with intermediate degrees of adverse selection and moderate prices. In addition, there will be general-equilibrium effects from changes in A^*.

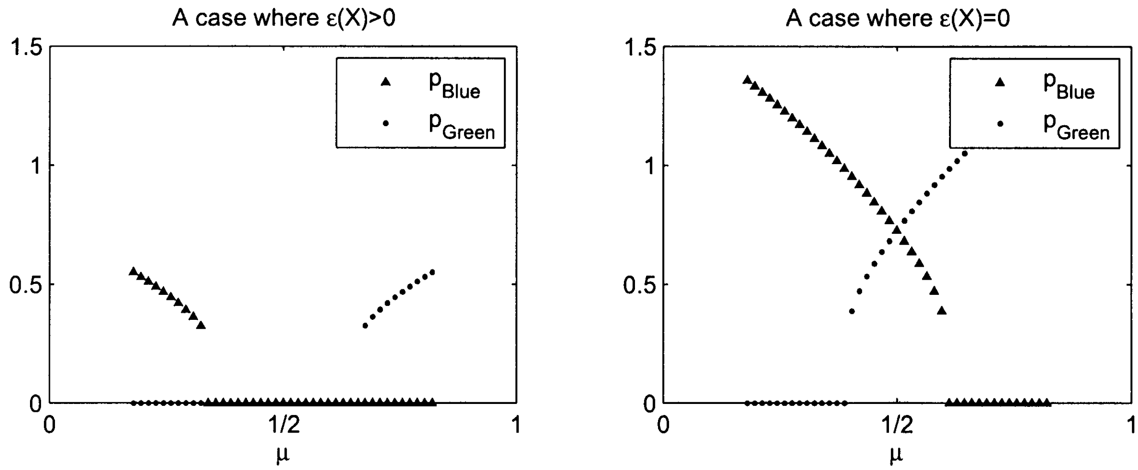


Figure 1.8: Price patterns when $\varepsilon > 0$ and when $\varepsilon = 0$

Let $\xi(X) \equiv \{\mu \in [0,1] : I(\mu) \geq \varepsilon(X)\}$ be the set of values of μ_l such that the level of information conveyed is sufficient for markets not to shut down.

Lemma 11. *A shift in $H(\mu)$ away from $\mu = \frac{1}{2}$*

1. *May increase or decrease $A^*(X)$*

2. *Necessarily increases $A^*(X)$ if there is no interval $(a,b) \subset \xi(X)$ such that $H(b) - H(a)$ decreases.*

In order to interpret Lemma 11, consider a case where $\varepsilon(X) > 0$, so the set $\xi(X)$ does not include the entire unit interval. Submarkets where the pool is relatively mixed because signals are uninformative shut down, as in the left panel of figure 1.8, so the implicit taxes in those markets are infinite. For those cases, increasing the probability that μ_l takes values in $\xi(X)$ (as implied by the conditions of part 2 of the Lemma) necessarily gives Buyers more relatively-less-adversely-selected submarkets to purchase from, which increases market rates of return. One way to interpret this is by noting that increasing the probability that μ_l takes values in $\xi(X)$ means that nonlemons tend to end up in lower-implicit-tax submarkets. As in part 2 of Lemma 5, lowering implicit taxes improves the rate of return for Buyers.

Improving the information has several effects on the process of capital accumulation. If $\varepsilon(X) > 0$, then by Lemma 9, more information increases asset prices for a given A^*. This increases wealth and therefore capital accumulation. Furthermore, price increases benefit only those entrepreneurs who sell projects, who have better-than-average investment opportunities. This raises the average productivity of investment. However, general equilibrium effects complicate the picture. By Lemma 11, improving information increases market returns. On the one hand, this persuades marginal Keepers to become Buyers and since they have worse-than-average investment opportunities, this further increases the average productivity of investment. On the other hand, by Lemma 7 the increase in A^* counteracts the increase in asset prices. The following proposition identifies conditions under which *all* entrepreneurs unambiguously increase their capital accumulation in response to an improvement of information.

Proposition 9. *Suppose* $\Pr\left[\mu_l \in \xi\left(X\right)\right] = 0$*. Then a shift in* H *such that* $\Pr\left[\mu_l \in \xi\left(X\right)\right] > 0$ *increases* $k'(k, A, X)$ *for every entrepreneur.*

An immediate corollary of Proposition 9 is that aggregate capital accumulation increases as well. Admittedly, the conditions under which the proposition holds are extreme, as they only refer to situations where improving information moves the economy from complete market shutdown to some positive level of financial activity. Still, they provide a useful benchmark from which to explore the generality of the result by numerical simulation. These explorations suggest that the result that aggregate capital accumulation increases with improvements in information holds fairly generally. However, it is possible to construct counterexamples where it does not, in the same way that it is possible to construct counterexamples to part 3 of Lemma 5 where $\frac{dK'}{d\tau} > 0$.

1.5.3 Uncertainty about the information structure

In the model, when Buyers are thinking about whether to buy a project, they observe its signal, form a Bayesian posterior about whether it is a lemon and then choose whether to buy at the equilibrium price. In reality, inferring the the true value of a project on the basis of signals is a difficult task. Messages *Blue* and *Green* are intended to represent complicated composites of the information published by a firm in a given period. Suppose a signal is "market share increased from 17% to 22% in the past year but profit margins declined". This signal could have been issued by a healthy firm (a nonlemon), if it operated in a market where customers have high brand loyalty (which makes market share valuable) and profit margins declined due to cyclical factors. Alternatively, this signal could have been issued by a struggling firm (a lemon) if market share increased simply as a result of pricing its products too low, and management's inability to control costs will continue to hurt profits. If Buyers are inexperienced they will find it difficult to assess which of these two explanations is more likely and will therefore find the signal relatively uninformative. Instead, if Buyers are more experienced, they will have observed firms in this industry increase market share at the expense of profits several times in the past and will know how frequently this turned out successfully. Therefore they will find the signal informative and form a more accurate posterior. They may have to worry, however, about whether their experience continues to be relevant or whether changes in the environment have rendered it obsolete.

Formally, assume that at any point in time agents do not know the function μ, which they need in order to form posteriors $\lambda_{l,s}$ using equation (1.25). Their beliefs about μ are given by a distribution B over all possible functions $\mu : [0,1] \to [0,1]$. Beliefs about a particular μ_l are given by a marginal distribution B_l with density b_l. Beliefs are *independent* if μ_l is independent of $\mu_{l'}$ for all $l \neq l'$. The mean of $B_l(\mu_l)$ is denoted $\hat{\mu}_l$.

Lemma 12. *The posterior $\lambda_{l,s}$ depends on beliefs B only through $\hat{\mu}_l$.*

The binary structure of both signals and project quality means that the mean of beliefs about μ_l is a sufficient statistic for the problem that Buyers care about, which is inferring project quality from a given signal. This implies that uncertainty about the true value of μ_l, for a given mean, makes no difference to the informativeness of signals. Better knowledge of the information structure matters only to the extent that it shifts $\hat{\mu}_l$. As before, values of $\hat{\mu}_l$ further away from $\frac{1}{2}$ mean signals are more informative, so informativeness can be indexed by $I_l \equiv \left(\hat{\mu}_l - \frac{1}{2}\right)^2$.

Lemma 13. *Let χ be any random variable, possibly correlated with μ_l. In expectation, observing χ increases I_l.*

Lemma 13 follows from the law of total variance. Uncertainty about whether a project is a lemon given a signal can be decomposed into uncertainty given a value of μ_l and uncertainty about μ_l. Observing a variable that is informative about μ_l will reduce this second component and on average make signals more informative. One limitation of this result is that it only refers to mean informativeness and the mean need not be the only moment of economic interest. Nevertheless, it implies that on average signals will convey more information if the structure by which they are generated is better understood.

A special extreme case of Lemma 13 occurs when $\hat{\mu}_l = \frac{1}{2}$, as would be the case for instance if $B_l(\mu_l)$ is symmetric around $\frac{1}{2}$. Under these beliefs signals are completely uninformative, while for someone who knew the true value of μ_l they would convey information. For example, if beliefs are that μ_l might take the values 0 or 1 with equal probability, then $I_l = 0$ but knowing μ_l would make signals perfectly informative.

The nature of the equilibrium will depend on where exactly beliefs B come from. I will assume that, starting from some B_0, beliefs are the result of Bayesian updating given a set of variables that agents are able to observe every period. As is standard in rational expectations equilibria, agents are able to observe prices and, if these are informative about the function μ, they can simultaneously update their beliefs B and adjust their demand accordingly. They are not, however, able to observe the quantity of projects traded in any given submarket. Furthermore, they do not learn μ from observing the signals emitted by the lemons and nonlemons in their own portfolio of projects.[21] At the end of each period, all agents are also able to observe a random vector χ, whose probability distribution may depend on μ and other equilibrium objects, and use this observation to update B. For now leave the exact nature of χ unspecified, a concrete description is in section 1.5.4 below. Finally, assume the true μ is not fixed but follows a known Markov process, and agents take this into account to form their beliefs B_{t+1} for the following period. Note that since all signals are public, all agents will have the same beliefs.

Let the state variable be $X = \{Z, \Gamma, B\}$, where B refers to the beliefs that agents hold when entering the period, before prices are realized. The entrepreneur solves

$$V(k, A, X) = \max_{c, k'(\mu), i, s_{L,l,s}(\mu), s_{NL,l,s}(\mu), d_{l,s}} \left[u(c) + \beta \mathbb{E}\left[V\left(k'(\mu), A', X'\right) | X, p(X, \mu)\right]\right] \tag{1.32}$$

$$s.t.$$

$$c + i + \int_0^1 \sum_s p_{l,s}(X, \mu)\left[d_{l,s} - \left(s_{L,l,s}(\mu) + s_{NL,l,s}(\mu)\right)\right] dl \leq (1 - \lambda)\, r(X)\, k$$

$$k'(\mu) = \gamma \left[(1 - \lambda) k + \int_0^1 \sum_s \left[\left(1 - \lambda_{l,s}^M(X, \mu)\right) d_{l,s} - s_{NL,l,s}\right] dl\right] + Ai$$

$$i \geq 0,\ d_{l,s} \geq 0$$

$$s_{L,l,B} \in [0, \lambda \mu_l k],\ s_{L,l,G} \in [0, \lambda(1 - \mu_l) k]$$

$$s_{NL,l,B} \in [0, (1 - \lambda)(1 - \mu_l) k],\ s_{NL,l,G} \in [0, (1 - \lambda) \mu_l k]$$

Notice that the buying, selling and investing decisions are not necessarily sufficient to determine k'. In case $d_{l,s}$ is positive for some submarket l, s, then $\lambda_{l,s}^M$ determines how many nonlemons the entrepreneur obtains from that purchase depends. The realized $\lambda_{l,s}^M$ depends on the realized $\mu_{l,s}$, which is unknown to the entrepreneur. Conditional expectations about the future state, including future beliefs (which will in turn depend on the realized χ) and k' are formed knowing the current state X and any information the current prices provide about current μ.

Definition 3. *A recursive rational expectations equilibrium with signals consists of prices* $\{p(X, \mu), r(X), w(X)\}$*; market proportions of lemons* $\lambda^M(X, \mu)$*; laws of motion* $\Gamma'(X, \mu)$ *and beliefs* $B'(X, \mu, \chi)$ *and associated transition density* $\Pi(X'|X)$*; a value function* $V(k, A, X)$

[21] This may seem inconsistent with the fact that they are fully diversified. However, full diversification can be achieved by holding a countably infinite number of projects. Learning the true μ_l for a countable number of indices l would provide information about a zero-measure subset of submarkets.

and decision rules $\{c^w(X), c(k,A,X), k'(k,A,X;\mu), i(k,A,X), s_{L,l,s}(k,A,X;\mu), s_{NL,l,s}(k,A,X;\mu), d_s(k,A,X;\mu)\}$ *such that (i) factor prices equal marginal products:* $w(X) = Y_L(X)$, $r(X) = Y_K(X)$; *(ii) workers consume their wage* $c^w(X) = w(X)$; *(iii)* $\{c(k,A,X), k'(k,A,X;\mu), i(k,A,X), s_{L,l,s}(k,A,X;\mu), s_{NL,l,s}(k,A,X;\mu), d_{l,s}(k,A,X;\mu)\}$ *and* $V(k,A,X)$ *solve program (1.32) taking* $p(X;\mu)$, $r(X)$, $\lambda^M(X;\mu)$ *and* $\Pi(X'|X)$ *as given; (iv) each submarket* l,s *clears:* $S_{l,s}(X;\mu) \geq D_{l,s}(X;\mu)$, *with equality whenever* $p_{l,s}(X;\mu) > 0$; *(v) in each market the proportion of lemons is consistent with individual selling decisions:* $\lambda^M_{l,s}(X;\mu) = \frac{S_{L,l,s}(X;\mu)}{S_{l,s}(X;\mu)}$; *(vi) the law of motion of* Γ *is consistent with individual decisions:* $\Gamma'(k,A)(X) = \int_{k'(\tilde{k},\tilde{A},X)\leq k} d\Gamma(\tilde{k},\tilde{A})\ F(A)$ *and (vii) beliefs evolve according to Bayes' rule*

Notice one subtlety about the definition of equilibrium. Consistent with the assumption that Buyers do not know μ, program (1.32) does not allow the choice of $d_{l,s}$ to depend on the realization of μ. However, in general program (1.32) may have many solutions if Buyers are indifferent between buying from different submarkets (making demand a correspondence rather than a function). If demand is indeed a correspondence, the definition of equilibrium allows $D(X;\mu)$ to take any value in that correspondence, possibly one that depends on μ. This corresponds to assuming that, when Buyers are indifferent, demand adjusts to meet supply.

For a given state X, define $\hat{\mu}^p_l \equiv \mathbb{E}[\mu_l|X,p]$. $\hat{\mu}^p_l$ represents the mean of beliefs about μ_l once the agent has observed equilibrium prices. The expected return that Buyers believe they will obtain if they buy projects in submarket l,s is still given by equation (1.27), except that μ is replaced by $\hat{\mu}^p$ in the definition of $A^M_{l,s}(p_{l,s})$, i.e.

$$\begin{aligned} A^M_{l,s}(p_{l,s};\hat{\mu}^p) &\equiv \frac{\gamma}{p_{l,s}}\left(1-\lambda^M_{l,s}(p_{l,s}\hat{\mu}^p_l)\right) \\ &= \frac{\gamma}{p_{l,s}}\frac{(1-\lambda_{l,s}(\hat{\mu}^p_l))\left(1-F\left(\frac{\gamma}{p_{l,s}}\right)\right)}{\lambda_{l,s}(\hat{\mu}^p_l)+(1-\lambda_{l,s}(\hat{\mu}^p_l))\left(1-F\left(\frac{\gamma}{p_{l,s}}\right)\right)} \end{aligned}$$

If the actual proportion of lemons among projects in the l,s submarket is higher than Buyers thought (for instance if $s = Blue$ and $\mu > \hat{\mu}^p$), then returns will be lower; conversely if the true proportion of lemons is lower, returns will be higher. However, if beliefs are independent, agents are able to diversify away this risk (and it will be optimal for them to do so) and will only care about the expected return when deciding whether to buy from submarket l,s. Using this fact it is possible to prove that there will exist an equilibrium where prices do not reveal anything about μ.

Lemma 14. *There exists a rational expectations equilibrium such that, in any state* X *where beliefs are independent, prices do not depend on* μ. *In this equilibrium,* $\hat{\mu}^p = \hat{\mu}$ *and (1.28)* *and (1.31) hold, replacing* $\hat{\mu}$ *for* μ *in the definition of* $A^M_{l,s}(p_{l,s})$, $E(p, A^*)$ *and* $W(k, A, X)$.

Lemma 14 implies that for any state in which beliefs are independent, it is possible to characterize prices and allocations for that state on the basis of beliefs only, without knowing the true μ. This implies that the analysis from section 1.5.2 regarding the effects of information on equilibrium outcomes still holds, with information given by the function $\hat{\mu}$ rather than μ. The true μ will affect the quantity of projects sold in each submarket, but the shocks to these cancel out and do not affect aggregate variables. It remains to show that the learning process will indeed lead to beliefs that are independent across submarkets.

1.5.4 Learning process

Assume that the variable χ that agents observe after every period consists of sample of size N_l of signal-outcome pairs for each index l. Each observation consists of the signal the project issued plus whether it turned out to be a lemon or not. This is a formalization of the idea that market participants learn from experience. The more times they have gone through the process of analyzing information about a firm and monitoring its subsequent performance the better they will become at inferring a firm's prospects from its published information.

N_l is random and follows a Poisson distribution with mean

$$\omega_l = [f_l \omega_S + (1 - f_l)\omega_K] \tag{1.33}$$

where

$$\begin{aligned} f_l &= \mathbb{I}(p_{l,B} > 0)\left[\lambda\mu_l + (1-\lambda)(1-\mu_l)\left(1 - F\left(\frac{\gamma}{p_{l,B}}\right)\right)\right] \\ &\quad + \mathbb{I}(p_{l,G} > 0)\left[\lambda(1-\mu_l) + (1-\lambda)\mu_l\left(1 - F\left(\frac{\gamma}{p_{l,G}}\right)\right)\right] \end{aligned}$$

is the fraction of projects of index l that are sold and ω_S and ω_K are parameters, with $\omega_S > \omega_K$. Equation (1.33) says that, for each project, there is a Poisson probability that the market finds out what happened to it. This probability is higher for projects that were sold than for projects that were kept by their owner.

The rationale for the assumption that $\omega_S > \omega_K$ is that firms that raise funds from the market usually provide investors with much more detailed information about their financial condition than those that do not, both at the time of raising funds and thereafter. Part of this is due to legal reasons, such as reporting requirements for publicly held companies, and part may be because firms are purposefully attempting to alleviate the lemons problem. The information that investors observe after investing gives them feedback about how accurate their assessment of the firm was at the time they decided whether to invest in it. Furthermore, it is not sufficient that information exist, someone must take the trouble to analyze it order to learn from it. The main reason someone would do that is to help them decide whether to trade. When the volume of trade decreases, the amount of attention paid to analyzing information is likely to decrease as well. Anecdotal evidence certainly suggests that this is the case. To take just one example, the investment bank Paribas laid off its entire Malaysian research team in 1998 in response to reduced business during the Asian crisis.[22]

In the model, observations of χ are used by agents to update beliefs B using Bayes' rule. It is useful to analyze the updating in two steps. First, since the number of l-indexed projects actually sold (and therefore ω_l) depends on the true value of μ_l, the number of signals itself is a source of information.[23] Secondly, given N_l, each observation can be treated as a Bernoulli trial, where observing $Blue, Lemon$ or $Green, Nonlemon$ is a success, which happens with probability μ_l, and observing $Blue, Nonlemon$ or $Green, Lemon$ is a failure, which happens with probability $1 - \mu_l$.

By Lemma 13, each updating of $B_l(\mu_l)$ increases the expected informativeness of signals for index l, as agents learn from experience. If μ were constant over time, in the long run the market would observe enough realizations of l-indexed signal-outcome pairs to learn the true value of μ with arbitrarily high precision.

Instead, if the true value of μ_l follows a nondegenerate Markov process, then agents face a filtering problem. After each period, they update their beliefs about μ_t on the basis of signal-outcome observations and use those to form beliefs about $\mu_{l,t+1}$, taking into account the stochastic process followed by μ_l. The solution of this filtering problem for general stochastic processes can be quite complex, but it is easy to compute in some special cases. Suppose that each $\mu_{l,t}$ follows an independent two-state Markov process, taking values $\bar{\mu} > \frac{1}{2}$ and $1 - \bar{\mu}$, with switching probability $\sigma < \frac{1}{2}$. At any given period, beliefs about $\mu_{l,t}$ are summarized by a single number $b_{l,t} \equiv \Pr[\mu_{l,t} = \bar{\mu}]$, and are independent. Applying Bayes' rule, after observing n_l successes out of N_l observations, $b_{l,t+1}$ is given by:

$$b_{l,t+1} = \frac{(1-\sigma)\,\bar{\mu}^{n_l}(1-\bar{\mu})^{N_l-n_l}(\omega_{l\bar{\mu}})^{N_l}e^{-\omega_{l\bar{\mu}}}b_{l,t} + \sigma(1-\bar{\mu})^{n_l}\bar{\mu}^{N_l-n_l}(\omega_{l1-\bar{\mu}})^{N_l}e^{-\omega_{l1-\bar{\mu}}}(1-b_{l,t})}{\bar{\mu}^{n_l}(1-\bar{\mu})^{N_l-n_l}(\omega_{l\bar{\mu}})^{N_l}e^{-\omega_{l\bar{\mu}}}b_{l,t} + (1-\bar{\mu})^{n_l}\bar{\mu}^{N_l-n_l}(\omega_{l1-\bar{\mu}})^{N_l}e^{-\omega_{l1-\bar{\mu}}}(1-b_{l,t})} \tag{1.34}$$

where $\omega_{l\bar{\mu}}$ and $\omega_{l1-\bar{\mu}}$ denote the values of ω_l when μ_l takes the values $\bar{\mu}$ and $1-\bar{\mu}$ respectively.

[22] *Wall Street Journal*, October 26, 1998.

[23] Quantitatively, this source of information is negligible compared to the information derived from the actual content of the signals

$\hat{\mu}_{l,t}$ is simply given by

$$\hat{\mu}_{l,t} = b_{l,t}\bar{\mu} + (1 - b_{l,t})(1 - \bar{\mu})$$

In order to interpret equation (1.34), consider the extreme case in which $\omega_l = 0$ (which implies $N_l = 0$), i.e. entrepreneurs do not observe anything regarding index l. In this case,

$$b_{l,t+1} = (1 - \sigma)\, b_{l,t} + \sigma\,(1 - b_{l,t})$$

so b_l (and therefore $\hat{\mu}_l$) moves towards $\frac{1}{2}$, meaning that signals at $t+1$ are less informative than they were at t. The reason for this is that, because there is always a possibility that the signal structure might change, not learning anything about index l for one period means that the agents' understanding of the information structure has become less precise. Experience is a form of intangible capital, and can depreciate. Conversely, suppose that the realized value of N_l is very large. The law of large numbers implies that the number of Bernoulli successes observed will be close to the true $\mu_{l,t}$ with high probability. In the limit, agents will know $\mu_{l,t}$ exactly and $b_{l,t+1}$ approaches $1 - \sigma$ or σ. For intermediate cases, equation (1.34) implies that b_l will move towards $\frac{1}{2}$ whenever (i) $\left|b_{l,t} - \frac{1}{2}\right|$ is large (mean reversion); (ii) few signals are observed (experience becomes outdated) or (iii) $\frac{n_l}{N_l}$ is close to $\frac{1}{2}$ (different observations conflict with each other).

By Lemma 14, the function $\hat{\mu}$ or, equivalently, the distribution of mean beliefs $H(\hat{\mu})$, is sufficient to characterize allocations in any given state. Furthermore, the following lemma establishes that it is possible to characterize the distribution of next-period mean beliefs H' without knowing the true realized μ.

Lemma 15. *H' is a deterministic function of X*

By Lemma 15, the realized value of μ in any given state is irrelevant not only for the determination of prices and allocations in that state, but also for the learning process. This makes it possible to characterize the entire dynamic path of the economy by keeping track of beliefs, aggregate capital and productivity Z, without any reference to realized μ at all.

Computationally, the only complication is the need to carry the infinite-dimensional state variable H and compute its trasition density. However, H itself can be well approximated by a finite grid and its transition density computed by simulation. The fact that prices and quantities can be found statically means there is no need to compute the entrepreneur's value function.

1.5.5 Persistence

It is straightforward to verify that, taking H as given, the comparative statics of the economy with signals regarding the response to shocks are the same as those in the economy without signals. In addition, learning introduces a dynamic feedback mechanism between activity in financial markets and the real economy. Suppose the economy suffers a negative productivity shock. This lowers r, which lowers demand, increases A^* and lowers asset prices. At these lower asset prices, marginal Sellers in each submarket become Keepers, lowering the number of transactions. Since $\omega_K < \omega_S$, equation (1.33) implies that the sample sizes from which entrepreneurs will learn about μ will be lower. Equation (1.34) then implies that this will lead to a distribution of beliefs H' that is more concentrated towards $\frac{1}{2}$, increasing the overall level of informational asymmetry as signals become less informative. This will affect asset prices, the amount of financial market activity, the amount of learning and capital accumulation in future periods. Thus temporary shocks can have long-lasting real effects. In fact, under certain conditions a temporary shock can lead to an arbitrarily long recession. To prove this, I first establish the following preliminary result.

Lemma 16. *$\varepsilon(X)$ is decreasing in r*

Since r is decreasing in K, Lemma 16 implies that, other things being equal, more information is necessary to sustain financial-market activity in economies with higher levels of capital.

Consider the steady states of two otherwise identical economies, one with no signals and another with signals and endogenous learning. Without signals, the steady state simply consists of a level of capital K_0 such that $K' = K_0$. With signals, the steady state is a level of capital K_{ss} and a distribution of beliefs H_{ss} such that $K' = K_{ss}$ and $H' = H_{ss}$. Denote the steady state levels of output in both economies by Y_0 and Y_{ss} and the amounts of indispensable information by ε_0 and ε_{ss} respectively. If $\varepsilon_0 > 0$, this means that the steady state without signals is such that the market shuts down.

Proposition 10. *Fix any integer $T > 0$ and real number $\delta > 0$. Suppose $\varepsilon_0 > 0$ and suppose that, starting from steady state, the economy with signals suffers a negative productivity shock lasting n periods. If*

1. *The productivity shock is sufficiently large*

2. $$n \in \left(\frac{\log\left(\bar{\mu} - \frac{1}{2}\right) - \log\left(\sqrt{\varepsilon_0}\right)}{-\log(1-2\sigma)} - 1, \frac{\log K_{ss} - \log K_0}{-\log\left[\gamma(1-\lambda)\right]} \right)$$

3. ω_K is sufficiently small

then there is a $T' \geq T$ such that $|Y_{t+T'} - Y_0| < \delta$

Values of ω_K close to zero mean that it is very unlikely that entrepreneurs will observe the outcomes of projects that were not sold. If this is the case, negative productivity shocks that are sufficiently large to lead to market shutdowns will imply an almost complete interruption of the learning process, and entrepreneurs' understanding of the information structure will deteriorate. If positive amounts of information are indispensable for trade and the shock lasts long enough, then when the shock is over financial market activity will not recover because the information needed to sustain it will have been destroyed. The bounds on n in the statement of the proposition ensure that the shock lasts long enough for knowledge to depreciate but not long enough so that the capital stock falls below the informationless steady state level. Reconstructing the stock of knowledge will require learning mostly from non-sold projects, and small sample sizes imply that this process will be slow. Hence the levels of output can remain close to those of the informationless steady state for a long time. As long as the steady state of the economy with signals was originally above that of the economy without signals, then Proposition 10 implies that temporary shocks can lead to arbitrarily long recessions. By Proposition 9, this will be true whenever the economy with signals has enough information so that there is a positive amount of financial transactions.

1.5.6 Simulations

In this section I compute examples of how the economy responds to various shocks, taking into account the endogenous learning process. The examples are intended as explorations of the effects that are possible in the model and rough indications of potential magnitudes rather than as quantitatively precise estimates. To highlight the role of learning, in each case I compare the impulse responses to those of an economy with no learning where H is fixed at its steady state value. The parameter values I use differ from those in the simulations in section 1.4.4 because I wish to focus on economies where information is indispensable for trade, so that markets shut down when there is no information. I do this by choosing $\lambda = 0.5$, which makes lemons abundant and the asymmetric information problem severe. The length of the period is about a year. $\beta = 0.92$ leads to an implicit gross risk-free rate of 0.63 for Buyers and 1.27 on average. $\gamma = 1.78$ and $\lambda = 0.5$ imply an annual rate of depreciation of 11%. σ parameterizes the Markov process followed by μ_l. Define the half-life of that Markov process as the number of periods of no learning that it would take for $\mathbb{E}[\mu_l]$ to mean-revert half way back to $\frac{1}{2}$. A simple calculation shows that it is given by $-\frac{\log 2}{\log(1-2\sigma)}$. $\sigma = 0.2$ implies a half-life of 1.36 periods. $\bar{\mu} = 0.9$ implies that the true correlation between

Parameter	Value
β	0.92
γ	1.78
λ	0.5
σ	0.2
$\bar{\mu}$	0.9
$F(A)$	Gamma distribution with $E(A) = 1$ and $std(A) = 2$
Y	$Z[(1-\lambda)K]^{\alpha} L^{1-\alpha}$ with $\alpha = 0.3$
L	1
Z	1

signals an asset quality is very high, so learning it well has the potential to greatly reduce informational asymmetry. As to the intensity-of-learning parameters ω_S and ω_K, different examples use different values.

The first simulation is an illustration of Proposition 10. I assume that the probability of observing the outcomes of sold projects is very large ($\omega_S = 400$). Whenever a given l, s market is open, agents will have many observations from which to learn whether $\mu_{l,t} = \bar{\mu}$ or $\mu_{l,t} = 1 - \bar{\mu}$. It turns out that in steady state $I_l[(1-\sigma)\bar{\mu} + \sigma(1-\bar{\mu})] > \varepsilon(X)$, i.e. learning $\mu_{l,t}$ very precisely provides sufficient information to sustain trade at $t+1$. Therefore an l such that the market is open at t will be open at $t+1$ with very high probability, making market openness close to an absorbing state. It is not quite absorbing because, due to the assumption that N_l is Poisson, there is always a small probability there will be few observations and $\hat{\mu}_l$ will move towards $\frac{1}{2}$. I also assume that the probability of observing the outcome of nonsold projects is very low ($\omega_K = 0.007$). Whenever markets $l, Blue$ and $l, Green$ are shut for a given l, it is very likely that $N_l = 0$, so there will be no learning and l-markets will remain shut the following period. Hence market shutdown will also be nearly absorbing.

Figures 1.10 and 1.11 illustrate the response of this economy to a negative 10% productivity shock taking place at $t = 2$ and lasting only one period, starting from steady state.

Begin with the evolution of H, shown in figure 1.11. It is initially highly concentrated at either $\hat{\mu}_l = (1-\sigma)\bar{\mu} + \sigma(1-\bar{\mu}) = 0.74$ (and symmetrically $\hat{\mu}_l = 0.26$), where markets are open or $\hat{\mu}_l = \frac{1}{2}$, where markets are shut. The shock is sufficiently large to shut down all markets for one period; in this period ω_l is close to zero for all markets, so $\hat{\mu}_l$ shifts towards $\frac{1}{2}$, according to equation (1.34). The distribution H becomes more concentrated around $\frac{1}{2}$, corresponding to less informative signals. This loss of information turns out to be sufficient to prevent most markets from reopening at $t = 3$ when the productivity shock is over. Hence the distribution H continues to concentrate towards $\frac{1}{2}$. Eventually the percentage of markets with $\hat{\mu}_l$ away from $\frac{1}{2}$ begins to recover as some observations emerge even from shut markets.

Panel 2 of figure 1.10 shows the response of output. The response to the initial shock at $t = 2$ is mechanical and is reverted at $t = 3$. Then the impact of increased informational asymmetry on capital accumulation is felt and output drops steadily for several periods. Thus the model is able to generate the long recessions following financial crises that have been documented by Cecchetti, Kohler, and Upper (2009), Claessens, Kose, and Terrones (2009) and Cerra and Saxena (2008). In this (admittedly extreme) example, output remains close to 5% below its steady state value for over twenty periods.

Increased informational asymmetry affects capital accumulation by lowering both investment, shown n panel 4, and the average marginal rate of transformation of consumption goods into capital (average A), shown on panel 5. Average A drops because informational asymmetry interferes with the flow of coconuts towards higher-A entrepreneurs. The return to buying (shown on panel 6) drops, persuading marginal Buyers to become Keepers and undertake some investment. In contrast, the marginal Sellers in each submarket, who have relatively higher A decide to become Keepers due to lower asset prices and therefore reduce their investment. Finally, those very-high A entrepreneurs who decide to remain Sellers reduce their investment because lower asset prices reduce their wealth. This may provide an explanation for the pattern identified by Justiniano, Primiceri, and Tambalotti (2008b) who find, in an estimated quantitative model, that productivity in the investment sector is correlated with disturbances to the functioning of financial markets.

One summary measure of whether investment is being carried out by the most efficient entrepreneurs is to compute the standard deviation of physical investment across entrepreneurs. This should be high if investment is concentrated in the best entrepreneurs and low if mediocre entrepreneurs also undertake investment. Consistent with the drop in average A, this measure, shown on panel 7, drops after the shock and recovers gradually.

The model is silent about whether entrepreneurs with higher A will transform a given amount of coconuts into *more* machines or *better* machines. In reality it is likely that both effects are present to some extent. Panel 8 shows the result of the following exercise. Assume that all the effect of different values of A is due to better machines. An econometrician does not observe how good the machines are and measures capital formation by just adding investment, using the steady state average rate of transformation (which can be normalized to 1). Using this mismeasured capital stock, the econometrician then proceeds to compute Solow residuals. Since average A has decreased, the econometrician's procedure overestimates capital formation and leads to lower estimated Solow residuals. This may help explain the long periods of low (measured) productivity growth that follow some financial crises, as documented for instance by Hayashi and Prescott (2002) for Japan in the 1990s.

In this example, low measured Solow residuals account for about half the decrease in output and lower investment accounts for the rest.

The final panel of the figure tracks the drop and then recovery of financial activity as knowledge of the information structure is first destroyed and then reconstructed.

Beyond the immediate impact, the effects of the productivity shock are due to the deterioration of the economy's stock of financial knowledge. The next exercise is to consider a shock to that affects that knowledge directly. Suppose that σ increases from 0.2 to 0.5 for one period. $\sigma = 0.5$ means that $\Pr\left[\mu_{l,t+1} = \bar{\mu} | \mu_{l,t} = \bar{\mu}\right] = \Pr\left[\mu_{l,t+1} = \bar{\mu} | \mu_{l,t} = 1 - \bar{\mu}\right] = \frac{1}{2}$. There is a 50% chance that signals will change meaning between periods, which makes any knowledge of the time-t information structure irrelevant as of time-$t+1$. Effectively, the shock destroys the stock of financial knowledge. Figures 1.12 and 1.13 show that, aside from the initial period, the effects on the quality of information and therefore on other variables in the economy are very similar to those of a large productivity shock.

Due to the extreme values of the parameters ω_S and ω_K, the response of the economy changes in a highly nonlinear way with the size of the shock. The next exercise (figures 1.14 and 1.15) looks at a productivity shock of only 5% rather than 10% which, for these parameter values, is not enough to shut down the market completely. The number of projects sold decreases in response to the shock, as seen on panel 9 of figure 1.14, but is not close to the point where the market shuts down. Sample sizes for learning about μ decrease roughly in the same proportion as the drop in the number of sold projects, but since ω_S is very high they are still large enough that there is virtually no effect on the learning process and there is no information-induced recession.

Learning effects can also lead to high persistence after positive shocks. Consider an economy with $\omega_K = 0$, so there is never any learning from markets that are shut, making market shutdown an absorbing state. The rest of the parameters are as before, except that $\lambda = 0.35$ and $\gamma = 1.37$, so that informational asymmetry is slightly less severe but the rate of depreciation is the same.[24] For any level of capital above the informationless steady state, Lemma 16 implies $\varepsilon(X) > 0$, so there always exists a region near $\frac{1}{2}$ such that if $\hat{\mu}_l$ falls in that region, the l-markets shut down. Since $\omega_S < \infty$, $\hat{\mu}_l$ could always fall into that region even if $l-$markets are open, so market openness is not absorbing. This implies that the only steady state of this economy will be one with no trade and no information. However, a positive productivity shock that led markets to reopen for one period would lead to a large amount of learning, which could sustain financial market activity for a long time. Figures 1.16 and 1.17 illustrate the response of the economy to such a shock. Thanks to the restarting of the

[24]With these parameters, markets still shut down in the informationless steady state but the size of the positive shock that would be required to reopen them is smaller than with $\lambda = 0.5$.

learning process, information improves a lot at first and, because ω_S is high, it depreciates very slowly. This leads to a sustained increase in output. If the capital stock is computed without adjusting for the higher average A, around a third of the increase in output would be attributed to higher TFP.

The final experiment consists of modeling a stabilization of the information structure, i.e. a decrease in σ. This helps the learning process by slowing the rate at which knowledge of the information becomes outdated. I simulate a permanent decrease in σ from 0.2 to 0.1, using less extreme parameters for the Poisson sample sizes: $\omega_S = 3$ and $\omega_K = 1$. The results are shown in figures 1.18 and 1.19.

As a result of the stabilization, the distribution of H gradually spreads out, improving the quality of information. This increases the average productivity of investment, leading to a new steady state with higher output. Around half the increase in output would be attributed to higher TFP. This experiment is suggestive of some of the channels by which a more stable economic environment, which does not need to be re-learned every period can lead to higher levels of output.

1.6 Final remarks

This paper has explored the macroeconomic implications of asymmetric information about asset quality when assets are necessary collateral for financial transactions. Informational asymmetry acts like a tax on transactions, which has the potential to greatly distort the flow of investment. Furthermore, the distortions are sensitive to macroeconomic shocks and amplify their effects.

Public information about asset quality may alleviate informational asymmetry, provided agents have the experience necessary to interpret this information. By modeling the gaining of experience as the result of financial market activity, the model captures a new notion of market liquidity that emphasizes an economy's accumulated financial knowledge. The dynamics of gaining and losing experience can create a powerful propagation mechanism that leads from temporary shocks to long-lasting consequences for market liquidity, capital-accumulation, productivity and output, in ways that are consistent with stylized facts about financial crises.

At the center of the learning mechanism lies an externality: by choosing to sell their projects, entrepreneurs contribute to the generation of knowledge. The externality is especially strong when financial markets are close to shutting down. Is there something the government should do to correct this? The model abstracts from any costs of preparing information for Sellers or of analyzing it for Buyers. If these costs were literally zero, then it would be simple to compel agents to produce and analyze information regardless of market conditions, severing the link between learning and financial activity and eliminating the externality. If instead knowledge generation is costly and is only undertaken as a side-product of financial transactions, there may be a case for the government to try to prevent a complete market shutdown in order to preserve the stock of financial knowledge.

The idea that learning-by-doing about how to interpret information may affect informational asymmetries could have wider applicability beyond the types of setting explored in this paper. Exploring whether these mechanisms may account for differing levels of liquidity across different markets is a promising question for further research.

1.7 Appendix

1.7.1 Increasing $A^M(p)$ and rationing

Define

$$\begin{aligned} p^m(p) &\equiv \arg\max_{\tilde{p}\geq p} A^M(\tilde{p}) \\ p^M &\equiv p^m(0) \\ P^m &\equiv \{p \in \mathbb{R}_+ : p \in p^m(p)\} \end{aligned}$$

$p^m(p)$ is the price (or prices) above p that maximize the return for Buyers. If $A^M(p)$ has multiple local maxima, there may be values of p such that $p^m(p)$ contains more than one element.

Assumption 2. *$A^M(p)$ may have many local maxima, but no two are equal*

Assumption 2 implies that $p^m(p)$ contains at most two elements and if it contains two, one of them must be p. The results in this appendix would still hold without it, but making this assumption simplifies the proofs without ruling out any cases of economic interest. p^M is the global maximizer of $A^M(p)$, which exists because $A^M(p)$ is bounded and continuous and must be unique by Assumption 2. P^m is the set of prices such that Buyers cannot be made better off with higher prices. Suppose Assumption 1 does not hold, i.e. $p^* \notin P^m$, where p^* is defined by (1.21). What should we expect from an equilibrium? Stiglitz and Weiss (1981) argue that the Buyers will offer to pay a price $p^m(p^*)$ and ration the excess supply (and possibly if $A^M(p)$ has multiple maxima, buy the projects rationed out of the market at some lower price $p^l(p^*) < p^*$). This is illustrated in Figure 1.9.

Here $P^m = [p^m(p_a), p^l(p_c)] \cup [p^m(p_c), \infty)$. If the highest Walrasian (nonrationing) equilibrium price lies at a point like $p_b \in P^m$ or $p_d \in P^m$, then these are reasonable equilibrium

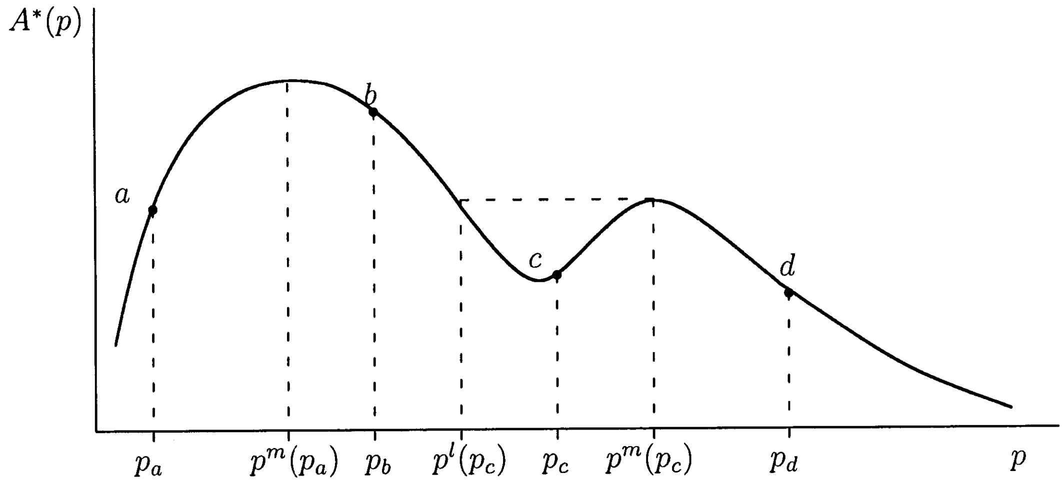


Figure 1.9: $A^M(p)$ and equilibrium prices with rationing

prices. If the highest Walrasian equilibrium price lies at a point like p_a, then Buyers prefer to offer $p^m(p_a)$, which improves the proportion of nonlemons enough to improve their returns. At that price, there is excess supply, so a fraction of Sellers are rationed out of the market. No matter how cheaply they offer to sell their projects, no one will be willing to buy them. If the highest Walrasian equilibrium price lies at a point like p_c, then Buyers prefer to raise prices up to $p^m(p_c)$ and ration the excess supply. Unlike case a, if those rationed out of the market offer to sell their projects at a price below $p^l(p_c)$, then this provides a return to Sellers which is better than that obtained at price $p^m(p^c)$. In equilibrium, Buyers anticipate the possibility of a second round market, which implies that the return from buying in each round must be the same. Therefore the second-round price must be $p^l(p_c)$, such that $A^M(p^m(p_c)) = A^M(p^l(p_c))$. The number of projects actually bought in the first round must be exactly such that, given the projects that remain unsold, the second-round market clears.

Formally, this notion of equilibrium is captured as follows.[25] Let $\rho_n(X)$ be the fraction of Sellers who manage to sell in each of the two rounds, at a price $p_n(X)$.The entrepreneur

[25] Arnold (2005) applies this equilibrium concept to the Stiglitz and Weiss (1981) model

solves

$$V(k,A,X) = \max_{c,k',l,s_{L,n},s_{NL,n},d_n} \left[u(c) + \beta \mathbb{E}\left[V(k',A',X')\,|X\right]\right] \tag{1.35}$$

$$s.t.$$

$$c + l + \sum_{n=1,2} p_n(X)\left[d_n - \rho_n(X)(s_{L,n} + s_{NL,n})\right] \le (1-\lambda) r(X) k$$

$$k' = \gamma\left[(1-\lambda)k + \sum_{n=1,2}\left[\left(1-\lambda_n^M(X)\right)d_n - r_n(X) s_{NL,n}\right]\right] + Ai$$

$$l \ge 0,\ d_n \ge 0$$

$$s_{L,1} \in [0, \lambda k],\ s_{NL,1} \in [0, (1-\lambda)k]$$

$$s_{L,2} \in [0, \lambda k - \rho_1(X) s_{L,1}],\ s_{NL,2} \in [0, (1-\lambda)k - \rho_1(X) s_{NL,1}]$$

In this formulation, $s_{L,n}$ and $s_{NL,n}$ represent the lemons and nonlemons respectively that the entrepreneur attempts to sell in round n; he only manages to sell $\rho_n(X) s_{L,n}$ and $\rho_n(X) s_{NL,n}$ respectively.

Supply and demand are defined in the obvious way

$$\begin{aligned} S_{L,n}(X) &\equiv \int s_{L,n}(k,A,X)\, d\Gamma(k,A) \\ S_{NL,n}(X) &\equiv \int s_{NL,n}(k,A,X)\, d\Gamma(k,A) \\ S_n(X) &\equiv S_{L,n}(X) + S_{NL,n}(X) \\ D_n(X) &\equiv \int d_n(k,A,X)\, d\Gamma(k,A) \end{aligned}$$

Definition 4. *A recursive competitive equilibrium with rationing consists of prices* $\{p_n(X), r(X), w(X)\}$*; rationing coefficients* $\rho_n(X)$*; market proportions of lemons* $\lambda^M(X)$*; a law of motion* $\Gamma'(X)$ *and associated transition density* $\Pi(X'|X)$*; a value function* $V(k,A,X)$ *and decision rules* $\{c^w(X), c(k,A,X), k'(k,A,X), l(k,A,X), s_{L,n}(k,A,X), s_{NL,n}(k,A,X), d_n(k,A,X)\}$ *such that (l) factor prices equal marginal products:* $w(X) = Y_L(X)$*,* $r(X) = Y_K(X)$*; (ii) workers consume their wage* $c^w(X) = w(X)$*; (iii)* $\{c(k,A,X), k'(k,A,X), l(k,A,X), s_{L,n}(k,A,X), s_{NL,n}(k,A,X), d_n(k,A,X)\}$ *and* $V(k,A,X)$ *solve program (1.35) taking* $p_n(X)$*,* $\rho_n(X)$*,* $r(X)$*,* $\lambda_n^M(X)$ *and* $\Pi(X'|X)$ *as given; (iv) either (a) the market clears at a price that Buyers do not wish to increase, i.e.* $S_1(X) = D_1(X)$*,* $S_2(X) = D_2(X) = 0$*,* $\rho_1(X) = \rho_2(X) = 1$*,* $p_1(X) = p_2(X) \in P^m$ *(d) there is rationing at* p^M*, i.e.* $p_1(X) = p_2(X) = p^M$*,* $\rho_1(X) = \frac{D_1(X)}{S_1(X)} \le 1$*,* $\rho_2(X) = \frac{D_2(X)}{S_2(X)} = 0$ *or (c) there is rationing in the first round and market clearing in the second, i.e.* $\rho_1(X) = \frac{D_1(X)}{S_1(X)} \le 1$*,* $\rho_2(X) = \frac{D_2(X)}{S_2(X)} = 1$*,*

$p_1(X) \in P^m$, $p_2(X) \in P^m$; *(v) the market proportions of lemons are consistent with individual selling decisions:* $\lambda_n^M(X) = \frac{S_{L,n}(X)}{S_n(X)}$ *and (vi) the law of motion of* Γ *is consistent with individual decisions:* $\Gamma'(k, A)(X) = \int_{k'(\tilde{k},\tilde{A},X)\leq k} d\Gamma(\tilde{k}, \tilde{A})F(A)$

Lemma 17. *The equilibrium exists and is unique*

Proof. Take any state X and Let A^* be the investment opportunity of the marginal Buyer. Total spending on projects is

$$TS(p_1, p_2, \rho_1, \rho_2, A^*) = K\left[\beta\left[\lambda\left[\rho_1 p_1 + \rho_2(1-\rho_1)p_2\right] + (1-\lambda)r\right] - (1-\beta)(1-\lambda)\frac{\gamma}{A^*}\right]F(A^*)$$

and total revenue from sales is

$$TR(p_1, p_2, \rho_1, \rho_2) = K\left[\begin{array}{c} \rho_1 p_1\left[\lambda + (1-\lambda)\left(1 - F\left(\frac{\gamma}{p_1}\right)\right)\right] + \\ \rho_2 p_2\left[\lambda + (1-\lambda)\left(1 - F\left(\frac{\gamma}{p_2}\right)\right) - \rho_1\left[\lambda + (1-\lambda)\left(1 - F\left(\frac{\gamma}{p_1}\right)\right)\right]\right] \end{array}\right]$$

Equilibrium condition (iv) implies

$$E(p_1, p_2, \rho_1, \rho_2, A^*) \equiv TS(p_1, p_2, \rho_1, \rho_2, A^*) - TR(p_1, p_2, \rho_1, \rho_2) = 0$$

The function $E(p_1, p_2, \rho_1, \rho_2, A^*)$ is increasing in A^* and decreasing in p_1, p_2, ρ_1 and ρ_2.

Let

$$p^h(A^*) \equiv \begin{cases} \text{the highest solution to } A^M(p) = A^* & \text{if a solution exists} \\ p^M & \text{otherwise} \end{cases}$$

$$\rho^h(A^*) \equiv \begin{cases} 1 & \text{if a solution exists} \\ 0 & \text{otherwise} \end{cases}$$

Both $p^h(A^*)$ and $\rho^h(A^*)$ are decreasing, which implies that $E^h(A^*) \equiv E(p^h(A^*), p^h(A^*), \rho^h(A^*), \rho^h(A^*), A^*)$ is increasing in A^*. By definition, in equilibrium either $E^h(A^*) = 0$ or $E^h(A^*) = 0$ crosses zero discontinuously at A^* Since $E^h(A^*)$ is increasing, this implies uniqueness.

To establish existence, distinguish three cases:

1. $E^h(A^*) = 0$ for some A^*. Then the following values constitute an equilibrium: $p_1^* = p_2^* = p^h(A^*)$, $\rho_1^* = 1$, $\rho_2^* = 0$.

2. $E^h(A^*)$ crosses zero discontinuously at $A^* = A^*(p^M)$. Then $E\left(p^M, p^M, 1, 0, A^*\right) < 0 < E\left(p^M, p^M, 0, 0, A^*\right)$ so there exists a value of $\rho_1^* \in (0,1)$ such that $E\left(p^M, p^M, \rho_1, 0, A^*\right) = 0$. Then the following values constitute an equilibrium: $p_1^* = p_2^* = p^M$, ρ_1^*, $\rho_2^* = 0$

3. $E^h(A^*)$ crosses zero discontinuously at some other value of A^*. This implies that $p^h(A^*)$ is discontinuous at A^*, which, by Assumption 2, implies that $A^M(p) = A^*$ must have exactly two solutions in P^m, the higher one of which of which is local maximum. Denote them $p^h(A^*)$ and $p^l(A^*)$. We have that $E(p^h(A^*), p^h(A^*), 1, 1, A^*) < 0 < E\left(p^l(A^*), p^l(A^*), 1, 1, A^*\right)$, which implies there is a value of $\rho_1^* \in (0,1)$ such that $E\left(p^h(A^*), p^l(A^*), \rho_1, 1, A^*\right) = 0$. Then the following values constitute an equilibrium: $p_1^* = p^h(A^*)$, $p_2^* = p^l(A^*)$, ρ_1^*, $\rho_2^* = 1$.

□

Lemma 18. *Consider the equilibrium with signals (Definition 5), given by conditions (1.31) and (1.28) and suppose $\mu_l = \frac{1}{2} + \epsilon(l - \frac{1}{2})$. In the limit as $\epsilon \to 0$, the equilibrium with signals converges to the rationing equilibrium.*

Proof. Suppose in a given state X the rationing equilibrium is given by $\{A^*, p_1, p_2, \rho_1, \rho_2\}$, with $A^M(p_1) = A^M(p_2) = A^*$. Recall that the function $A^M_{l,s}(p; \mu_l)$ is continuous in μ_l and equal to $A^M(p)$ when $\mu_l = \frac{1}{2}$. Consider any $\delta_A > 0$, $\delta_p > 0$ and $\rho_1, \rho_2 \in [0,1]$. By continuity, there exists $\epsilon(\delta_A, \delta_p)$ small enough that, for any $\epsilon < \epsilon(\delta_A, \delta_p)$, there exists $A_1^{*\prime}(\epsilon)$ satisfying $\left|A^{*\prime}(\epsilon) - A^*\right| < \delta_A$ such that the fraction of submarkets l, s where the equation $A^M_{l,s}(p; \mu_l(\epsilon)) = A^{*\prime}$ has a solution $p_{l,s}(A^{*\prime})$ with $\left|p_{l,s}(A^{*\prime}) - p_1\right| < \delta_p$ is exactly ρ_1 and the fraction of the remaining submarkets where the equation $A^M_{l,s}(p; \mu_l(\epsilon)) = A^{*\prime}$ has a solution $p_{l,s}(A^{*\prime})$ with $\left|p_{l,s}(A^{*\prime}) - p_2\right| < \delta_p$ is exactly ρ_2. The result then follows from noting that if for every ϵ a fraction ρ_1 of submarkets have prices $p^*_{l,s}(\epsilon)$ satisfying $\lim_{\epsilon\to 0} p^*_{l,s}(\epsilon) = p_1$, a fraction ρ_2 of the remaining ones have prices $p^*_{l,s}(\epsilon)$ satisfying $\lim_{\epsilon\to 0} p^*_{l,s}(\epsilon) = p_2$ and the rest have $p^*_{l,s}(\epsilon) = 0$, and $\lim_{\epsilon\to 0} A^{*\prime}(\epsilon) = A^*$, then

$$\lim_{\epsilon\to 0} E\left(p^*_{l,s}(\epsilon), A^{*\prime}(\epsilon); \mu_l(\epsilon)\right) \equiv E(p_1, p_2, \rho_1, \rho_2, A^*)$$

□

1.7.2 Liquidity premia

Entrepreneurs in the model do not face a portfolio problem. If they wish to carry wealth from one period to the next, the only way to do it is to buy or create projects. For each coconut they save, they obtain $\max\left\{A, A^M\right\}$ projects at $t+1$, which they consider equivalent to obtaining a (risky) amount of $\max\left\{A, A^M\right\} W_k(A', X')$ coconuts at $t+1$. Still, it is possible to define the implicit risk-free rate R^f for entrepreneur j by assuming he has access to an alternative safe technology that converts t-dated coconuts into $t+1$-dated coconuts (and hence faces a portfolio problem) and asking what the return on that technology would need to be for him not to change his equilibrium decisions.

Formally, consider an entrepreneur who has access to a technology that delivers R coconuts tomorrow in exchange for a coconut today. Letting m be the coconuts he receives from this safe investment, he solves

$$V(k,m,A,X) = \max_{c,k',m',i,s_L,s_{NL},d} \left[u(c) + \beta \mathbb{E}\left[V(k',m',A',X')\,|X\right]\right]$$

$$s.t.$$

$$c + i + p(X)\left[d - s_L - s_{NL}\right] + \frac{m'}{R} \leq (1-\lambda)\, r(X)\, k + m$$

$$k' = \gamma\left[(1-\lambda)k + \left(1-\lambda^M(X)\right)d - s_{NL}\right] + Ai$$

$$i \geq 0,\ d \geq 0,\ m' \geq 0$$

$$s_L \in [0,\lambda k],\ s_{NL} \in [0,(1-\lambda)k]$$

Assume for concreteness that the equilibrium is such that $p > 0$ and the solutions of programs (1.13) and (1.16) coincide[26] and define $W(k,m,A,X) = W(k,A,X) + m$. The entrepreneur's (relaxed) problem reduces to

$$V(W,A) = \max_{c,\pi,W'} \left[u(c) + \beta \mathbb{E}\left[V(W',A',X')\,|X\right]\right]$$

$$s.t.$$

$$W' = \left[\pi \max\left\{A, A^M\right\} W_k(A',X') + (1-\pi)R\right](W-c)$$

$$\pi \in [0,1]$$

where π is the fraction of his savings $W - c$ that he invests in projects. Define R^f as the maximum value of R that is consistent with $\pi = 1$ being an optimal choice.

Proposition 11.

1. $R^f < \max\left\{A, A^M\right\} \mathbb{E}\left[W_k(A',X')\right]$

2. *If there was symmetric information and X' was deterministic, then* $R^f = \max\left\{A, A^M\right\} \mathbb{E}\left[W_k(A',X')\right]$

Proof.

[26]Proposition 11 below does not depend on this assumption; it is straightforward to adapt the proof to the case where markets shut down.

1. The first order condition for π is

$$\pi = \begin{cases} 1 & \text{if } \mathbb{E}\left[V_W\left(W', A', X'\right)\left(\max\left\{A, A^M\right\} W_k\left(A', X'\right) - R\right)|X\right] > 0 \\ \text{anything} & \text{if } \mathbb{E}\left[V_W\left(W', A', X'\right)\left(\max\left\{A, A^M\right\} W_k\left(A', X'\right) - R\right)|X\right] = 0 \\ 0 & \text{if } \mathbb{E}\left[V_W\left(W', A', X'\right)\left(\max\left\{A, A^M\right\} W_k\left(A', X'\right) - R\right)|X\right] > 0 \end{cases}$$

so R^f must satisfy:

$$\begin{aligned} R^f &= \frac{\mathbb{E}\left[V_W\left(W', A', X'\right)\max\left\{A, A^M\right\} W_k\left(A', X'\right)|X\right]}{\mathbb{E}\left[V_W\left(W', A', X'\right)|X\right]} && (1.36) \\ &= \mathbb{E}\left[\max\left\{A, A^M\right\} W_k\left(A', X'\right)|X\right] + \frac{cov\left[V_W\left(W', A', X'\right), \max\left\{A, A^M\right\} W_k\left(A', X'\right)|X\right]}{\mathbb{E}\left[V_W\left(W', A', X'\right)|X\right]} \end{aligned}$$

when evaluated at $\pi = 1$. Using $c = (1-\beta) W$ and $u_c = V_W$ and evaluating at $\pi = 1$:

$$\begin{aligned} V_W\left(W', A', X'\right) &= \frac{1}{(1-\beta) W'} = \frac{1}{(1-\beta)\left[\pi \max\left\{A, A^M\right\} W_k\left(A', X'\right) + (1-\pi) R^f\right](W-c)} \\ &= \frac{1}{(1-\beta)\max\left\{A, A^M\right\} W_k\left(A', X'\right)(W-c)} && (1.37) \end{aligned}$$

Equation (1.37) implies that the covariance term in (1.36) is weakly negative, strictly so if $W_k\left(A', X'\right)$ is not a constant. Finally, equation (1.15) implies that $W_k\left(A', X'\right)$ is indeed not constant as long as $p(X) \neq \frac{\gamma}{A^M(X)} \iff \lambda^M(X) \neq 0$.

2. Under symmetric information the price of nonlemons is $p_{NL} = \frac{\gamma}{A^M}$ and the price of lemons is zero, so $W_k = \left[(1-\lambda)\left(r(X) + \frac{\gamma}{A^M(X)}\right)\right]$, which does not depend on the realization of A'. If in addition X is deterministic, then W_k is constant and therefore the covariance term in equation (1.36) is zero, which gives the result.

□

1.7.3 Equilibrium with signals

With a continuum of submarkets and signals, the entrepreneur's program becomes

$$V(k,A,X) = \max_{c,k',i,s_{L,l,s},s_{NL,l,s},d_{l,s}} \left[u(c) + \beta\mathbb{E}\left[V(k',A',X')\,|X\right]\right] \tag{1.38}$$

$$s.t.$$

$$c + i + \int_0^1 \sum_s p_{l,s}(X)\left[d_{l,s} - \left(s_{L,l,s} + s_{NL,l,s}\right)\right] dl \leq (1-\lambda)\, r(X)\, k$$

$$k' = \gamma\left[(1-\lambda)\,k + \int_0^1 \sum_s \left[\left(1-\lambda^M_{l,s}(X)\right) d_{l,s} - s_{NL,l,s}\right] dl\right] + Ai$$

$$i \geq 0,\ d_{l,s} \geq 0$$

$$s_{L,l,B} \in [0, \lambda\mu_l k],\ s_{L,l,G} \in [0, \lambda(1-\mu_l)\,k]$$

$$s_{NL,l,B} \in [0, (1-\lambda)(1-\mu_l)\,k],\ s_{NL,l,G} \in [0, (1-\lambda)\,\mu_l k]$$

Denote the vector of market proportions of lemons by $\lambda^M = \left\{\lambda^M_{l,s}\right\}_{l=[0,1],s=B,G}$ and define supply and demand for a submarket l, s as:

$$\begin{aligned}
S_{L,l,s}(X) &\equiv \int s_{L,l,s}(k,A,X)\, d\Gamma(k,A) \\
S_{NL,l,s}(X) &\equiv \int s_{NL,l,s}(k,A,X)\, d\Gamma(k,A) \\
S_{l,s}(X) &\equiv S_{L,l,s}(X) + S_{NL,l,s}(X) \\
D_{l,s}(X) &\equiv \int d_{l,s}(k,A,X)\, d\Gamma(k,A)
\end{aligned}$$

Definition 5. *A recursive equilibrium with signals consists of prices* $\{p(X),\ r(X),\ w(X)\}$*; market proportions of lemons* $\lambda^M(X)$*; a law of motion* $\Gamma'(X)$ *and associated transition density* $\Pi(X'|X)$*; a value function* $V(k,A,X)$ *and decision rules* $\{c^w(X),\ c(k,A,X),\ k'(k,A,X),\ i(k,A,X),\ s_{L,l,s}(k,A,X),\ s_{NL,l,s}(k,A,X),\ d_s(k,A,X)\}$ *such that (i) factor prices equal marginal products:* $w(X) = Y_L(X)$*,* $r(X) = Y_K(X)$*; (ii) workers consume their wage* $c^w(X) = w(X)$*; (iii)* $\{c(k,A,X),\ k'(k,A,X),\ i(k,A,X),\ s_{L,l,s}(k,A,X),\ s_{NL,l,s}(k,A,X),\ d_{l,s}(k,A,X)\}$ *and* $V(k,A,X)$ *solve program (1.38) taking* $p(X)$*,* $r(X)$*,* $\lambda^M(X)$ *and* $\Pi(X'|X)$ *as given; (iv) each submarket* l, s *clears:* $S_{l,s}(X) \geq D_{l,s}(X)$*, with equality whenever* $p_{l,s}(X) > 0$*; (v) in each market the proportion of lemons is consistent with individual selling decisions:* $\lambda^M_{l,s}(X) = \frac{S_{L,l,s}(X)}{S_{l,s}(X)}$ *and (vi) the law of motion of* Γ *is consistent with individual decisions:* $\Gamma'(k,A)(X) = \int_{k'(\tilde{k},\tilde{A},X)\leq k} d\Gamma(\tilde{k},\tilde{A})F(A)$.

Given A^* and p, aggregate capital accumulation can be found by simply adding

$$k'(k, A, X) = \beta \max\{A, A^*\} W(k, A, X) \tag{1.39}$$

across all entrepreneurs, where

$$W(k, A, X) = k \left[\int \begin{pmatrix} \lambda \left[\mu p_B(\mu) + (1-\mu) p_G(\mu)\right] \\ +(1-\lambda) r \\ +(1-\lambda)\mu \max\left\{\frac{\gamma}{\max\{A, A^*\}}, p_G(\mu)\right\} \\ +(1-\lambda)(1-\mu) \max\left\{\frac{\gamma}{\max\{A, A^*\}}, p_B(\mu)\right\} \end{pmatrix} dH(\mu) \right] \tag{1.40}$$

and $p_s(\mu)$ denotes the price $p_{l,s}$ in submarkets with index l such that $\mu_l = \mu$. Using the linearity of policy functions:

$$\frac{K'}{K} = \int \beta \max\{A, A^*\} W(1, A, X) dF(A) \tag{1.41}$$

1.7.4 Proofs

Proof of Lemma 1. $r(X)$ does not depend on the distribution of k because Y does not. For any given p and λ^M, linearity of the policy functions and the fact that A^j is independent of k^j imply that S_L S_{NL} and D do not depend on the distribution of k and therefore neither do the market clearing values of $p(X)$ and $\lambda^M(X)$. Linearity then implies that neither do aggregate quantities. □

Proof of Lemma 3. The first order and envelope conditions are

$$\begin{aligned} u_c &= \beta \max\left\{A, A^M(X)\right\} \mathbb{E}\left[V_{k'}(k', A', X') \,|X\right] \\ V_k(k, A, X) &= W_k(k, A, X)\, u_c \end{aligned}$$

and the Euler equation is:

$$u_c = \beta \max\left\{A, A^M(X)\right\} \mathbb{E}\left[W_{k'}(k', A', X') \,|X\right] u_{c'}$$

With logarithmic preferences, the Euler equation becomes

$$\frac{1}{c} = \beta \max\left\{A, A^M(X)\right\} \mathbb{E}\left[\frac{W_{k'}(k', A', X')}{c'} |X\right]$$

Conjecture that $c = aW(k, A, X)$, which implies

$$W(k', A', X') = W_{k'}(k', A', X') \max\left\{A, A^M(X)\right\}(1-a) W(k, A, X)$$

and replace in the Euler equation:

$$\frac{1}{aW(k,A,X)} = \beta \max\left\{A, A^M(X)\right\} \mathbb{E}\left[\frac{W_{k'}(k', A', X')}{aW_{k'}(k', A', X') \max\left\{A, A^M(X)\right\}(1-a) W(k, A, X)} | X\right]$$

which reduces to $a = 1 - \beta$. □

Proof of Lemma 4.

Assume there is an entrepreneur for whom the solutions to both programs differ. For Sellers both programs are identical so it must be that at least one Buyer or Keeper chooses $k' < (1-\lambda)\gamma k$. Then by revealed preference all Buyers choose $k' < (1-\lambda)\gamma k$. Replacing in 1.14 yields a negative demand. □

Proof of Proposition 2.

1. This follows immediately from Lemma 4. Whenever the solutions to the two programs do not coincide, $p^* = 0$ satisfies (1.21), which therefore holds in either case.

2. In the text.

3. Take any X. For sufficiently large p, $S(p) > D(p)$. If there exists a price such that $D(p) \geq S(p)$, then the result follows by continuity. If $D(p) < S(p)$ for all p, then $p^* = 0$ is a solution.

□

Proof of Proposition 3. Take any state X and let r^*, p^*, λ^{M*} and A^{M*} represent equilibrium values under asymmetric information in that state. Multiplying supply and demand by $(1 - \lambda^{M*})$ to express them in quantities of nonlemons rather than total projects, market clearing condition (1.21) can be reexpressed as

$$\left[\frac{\beta}{\gamma} A^{M*}\left[\lambda p^* + (1-\lambda) r^*\right] - (1-\beta)(1-\lambda)\right] F\left(A^{M*}\right) K \leq (1-\lambda)\left[1 - F\left(\frac{\gamma}{p^*}\right)\right]$$

Turn now to the economy with symmetric information and taxes. Virtual wealth is

$$W(k, A, X) \equiv \left[T + (1-\lambda)\left(r(X) + \max\left\{p(X), \frac{\gamma}{\max\left\{A, A^M(X)\right\}}\right\}\right)\right] k$$

At price p^* the supply of projects is $S = (1-\lambda)\left(1 - F\left(\frac{\gamma}{p^*}\right)\right)$ and tax revenue is

$$\begin{aligned} T &= \tau p^* (1-\lambda)\left(1 - F\left(\frac{\gamma}{p^*}\right)\right) \\ &= \frac{\lambda^{M*}}{1-\lambda^{M*}} (1-\lambda) p^* \left(1 - F\left(\frac{\gamma}{p^*}\right)\right) \\ &= \lambda p^* \end{aligned}$$

The return to buying projects is $A^M = \frac{\gamma}{p^*(1+\tau)} = \frac{\gamma\left(1-\lambda^M\right)}{p^*} = A^{M*}$ and, because K is the same, $r = r^*$. Therefore the virtual wealth is

$$W = \begin{cases} \left[\lambda p^* + (1-\lambda)\left(r^* + \frac{\gamma}{A^{M*}}\right)\right] k & \text{if } A \leq A^{M*} \\ \left[\lambda p^* + (1-\lambda)\left(r^* + \frac{\gamma}{A}\right)\right] k & \text{if } A \in \left(A^{M*}, \frac{\gamma}{p}\right] \\ \left[\lambda p^* + (1-\lambda)\left(r^* + p^*\right)\right] k & \text{if } A > \frac{\gamma}{p} \end{cases}$$

which is the same as with asymmetric information. This implies that demand for nonlemons is the same as with asymmetric information and the market clearing condition must hold, confirming that p^* is an equilibrium price. Since this is true for every state X, programs (1.13) and (1.23) are identical and allocations also coincide. □

Proof of Lemma 5.

1. Market clearing implies

$$\frac{dp}{d\tau} = \frac{\frac{\partial S}{\partial \tau} - \frac{\partial D}{\partial \tau}}{\frac{\partial D}{\partial p} - \frac{\partial S}{\partial p}}$$

   where

$$\begin{aligned} D(p,\tau) &= \left[\frac{\beta}{\gamma} A^M(p)\left[\tau(1-\lambda) p\left(1 - F\left(\frac{\gamma}{p}\right)\right) + (1-\lambda) r\right] - (1-\beta)(1-\lambda)\right] F\left(A^M(p)\right) \\ S(p,\tau) &= (1-\lambda)\left[1 - F\left(\frac{\gamma}{p}\right)\right] \\ A^M(p,\tau) &= \frac{\gamma}{p(1+\tau)} \end{aligned}$$

   Taking derivatives and substituting:

$$\frac{dp}{d\tau} = -\frac{\frac{\frac{\beta}{\gamma} F\left(A^M\right) A^M}{1+\tau}\left[r - p\left(1 - F\left(\frac{\gamma}{p}\right)\right)\right] + \left[\frac{\beta}{\gamma} A^M\left[\tau p\left(1 - F\left(\frac{\gamma}{p}\right)\right) + r\right] - (1-\beta)\right] f\left(A^M\right) \frac{A^M}{1+\tau}}{\frac{\beta}{\gamma} r F\left(A^M\right) \frac{A^M}{p} + \left[\frac{\beta}{\gamma} A^M\left[\tau p\left(1 - F\left(\frac{\gamma}{p}\right)\right) + r\right] - (1-\beta)\right] f\left(A^M\right) \frac{A^M}{p} + f\left(\frac{\gamma}{p}\right) \frac{\gamma}{p^2}\left[1 - \frac{\beta F\left(A^M\right)\tau}{1+\tau}\right]} < 0$$

2. Market clearing implies

$$\frac{dA^M}{d\tau} = \frac{\frac{\partial S}{\partial \tau} - \frac{\partial D}{\partial \tau}}{\frac{\partial D}{\partial A^M} - \frac{\partial S}{\partial A^M}}$$

where

$$\begin{aligned} D\left(A^M,\tau\right) &= \left[\frac{\beta}{\gamma} A^M \left[\tau(1-\lambda)\, p\left(A^M,\tau\right)\left(1 - F\left(\frac{\gamma}{p\left(A^M,\tau\right)}\right)\right) + (1-\lambda)\, r\right] - (1-\beta)(1-\lambda)\right] F\left(A^M\right) \\ S\left(A^M,\tau\right) &= (1-\lambda)\left[1 - F\left(A^M(1+\tau)\right)\right] \\ p\left(A^M,\tau\right) &= \frac{\gamma}{A^M(1+\tau)} \end{aligned}$$

Taking derivatives and substituting:

$$\frac{dA^M}{d\tau} = -\frac{f\left(A^M(1+\tau)\right) A^M \left[1 - \frac{\tau}{1+\tau}\beta F\left(A^M\right)\right] + \frac{\beta F\left(A^M\right)}{(1+\tau)^2}\left(1 - F\left(A^M(1+\tau)\right)\right)}{\frac{\beta}{\gamma} F\left(A^M\right) r + \left[\frac{\beta}{\gamma} A^M \left[\tau p \left(1 - F\left(\frac{\gamma}{p}\right)\right) + r\right] - (1-\beta)\right] f\left(A^M\right) + f\left(A^M(1+\tau)\right)\left[1 + \tau - \tau\beta F\left(A^M\right)\right]} < 0$$

3. Integrating k' over all entrepreneurs, K' is given by

$$\begin{aligned} K' = &\int_0^{A^M} \left[\beta A^M \left(T + (1-\lambda)\, r\right) + \beta(1-\lambda)\gamma\right] dF(A) \\ &+ \int_{A^M}^{\frac{\gamma}{p}} \left[\beta A\left(T + (1-\lambda)\, r\right) + \beta(1-\lambda)\gamma\right] dF(A) \\ &+ \int_{\frac{\gamma}{p}}^{\infty} \left[\beta A\left(T + (1-\lambda)\, r\right) + \beta A(1-\lambda)\, p\right] dF(A) \end{aligned}$$

where

$$T = \tau(1-\lambda)\, p\left(1 - F\left(\frac{\gamma}{p}\right)\right)$$

Taking derivatives:

$$\frac{dK'}{d\tau} = (1-\lambda)\beta \left[\begin{array}{c} p\left(1 - F\left(\frac{\gamma}{p}\right)\right)\left[A^M F\left(A^M\right) + \int_{A^M}^{\infty} A dF(A)\right] + \left(\tau p \left(1 - F\left(\frac{\gamma}{p}\right)\right) + r\right)\frac{dA^M}{d\tau} \\ + \left[\int_{\frac{\gamma}{p}}^{\infty} A dF(A) + \tau\left[1 - F\left(\frac{\gamma}{p}\right) + p f\left(\frac{\gamma}{p}\right)\frac{\gamma}{p^2}\right]\left[A^M F\left(A^M\right) + \int_{A^M}^{\infty} A dF(A)\right]\right]\frac{dp}{d\tau} \end{array} \right]$$

Replacing with the expressions from parts 1 and 2 and rearranging:

$$\begin{aligned} \frac{x}{(1-\lambda)\beta}\frac{dK'}{d\tau} = &\left[\begin{array}{c} \left[\frac{\beta}{\gamma} A^M \left[\tau p\left(1 - F\left(\frac{\gamma}{p}\right)\right) + r\right] - (1-\beta)\right] \frac{p}{1+\tau}\left(\left(1 - F\left(\frac{\gamma}{p}\right)\right)\left[A^M F\left(A^M\right) + \int_{A^M}^{\infty} A dF(A)\right] - \int_{\frac{\gamma}{p}}^{\infty} A dF(A)\right) \\ -\frac{p}{1+\tau}\left[\frac{\beta}{\gamma} A^M \left[\tau p\left(1 - F\left(\frac{\gamma}{p}\right)\right) + r\right] - (1-\beta)\right]\tau p \frac{\gamma}{p^2}\left[A^M F\left(A^M\right) + \int_{A^M}^{\infty} A dF(A)\right] f\left(\frac{\gamma}{p}\right) \end{array} \right] f\left(A^M\right) \\ &+ \left[\begin{array}{c} \left(\left[A^M F\left(A^M\right) + \int_{A^M}^{\infty} A dF(A)\right] - A^M\right)\left[p\left(1 - F\left(\frac{\gamma}{p}\right)\right)\left[\tau - \frac{\tau^2}{1+\tau}\beta F\left(A^M\right)\right] - \frac{r\tau}{1+\tau}\beta F\left(A^M\right)\right] \\ + \left[A^M F\left(A^M\right) + \int_{A^M}^{\infty} A dF(A)\right] p\left(1 - F\left(\frac{\gamma}{p}\right)\right) - A^M r \end{array} \right] f\left(\frac{\gamma}{p}\right) \\ &+ \left[\begin{array}{c} \frac{\frac{\beta}{\gamma} F\left(A^M\right)}{(1+\tau)} pr \left[-A^M\left(1 - F\left(A^M\right)\right)\left(1 - F\left(\frac{\gamma}{p}\right)\right) + \left(1 - F\left(\frac{\gamma}{p}\right)\right)\int_{A^M}^{\infty} A dF(A) - \left(\left[1 - \frac{p}{r}\left(1 - F\left(\frac{\gamma}{p}\right)\right)\right]\right)\int_{\frac{\gamma}{p}}^{\infty} A dF(A)\right] \\ + \frac{\tau}{1+\tau}\left[p\left(1 - F\left(\frac{\gamma}{p}\right)\right)\right]^2 \frac{\beta}{\gamma} F\left(A^M\right)\left(\left[A^M F\left(A^M\right) + \int_{A^M}^{\infty} A dF(A)\right] - A^M\right) \end{array} \right] \end{aligned}$$

where

$$x \equiv \frac{\beta}{\gamma}F\left(A^{M}\right)r+\left[\frac{\beta}{\gamma}A^{M}\left[\tau p\left(1-F\left(\frac{\gamma}{p}\right)\right)+r\right]-(1-\beta)\right]f\left(A^{M}\right)+f\left(\frac{\gamma}{p}\right)(1+\tau)\left[1-\beta F\left(A^{M}\right)\frac{\tau}{1+\tau}\right]>0$$

Using the market clearing condition and the fact that as $\tau \to 0$, $F\left(A^{M}\right) \to F\left(\frac{\gamma}{p}\right)$ and $f\left(A^{M}\right) \to f\left(\frac{\gamma}{p}\right)$, this expression reduces to

$$\begin{aligned}\frac{x}{(1-\lambda)\beta}\frac{dK'}{d\tau} &= -\left[\left(\frac{r}{p}-\beta\left[\frac{r}{p}+1\right](1-F(A^{M}))F\left(A^{M}\right)\right)pA^{M}\right]f\left(A^{M}\right) \\ &\quad -\left[\frac{\beta}{\gamma}F\left(A^{M}\right)pr\left[A^{M}\left(1-F\left(A^{M}\right)\right)^{2}+F\left(A^{M}\right)\left(\frac{p}{r}+1\right)(1-\beta)\int_{A^{M}}^{\infty}AdF(A)\right]\right]<0\end{aligned}$$

□

Proof of Proposition 4. 1. Fixing p, higher r increases demand but has no effect on supply. If $\frac{\partial[D(p)-S(p)]}{\partial p}<0$ the equilibrium price must rise to restore market clearing. While this inequality need not hold for every p, it holds at the p that constitutes the highest solution to (1.21).

2. The result follows from part 1 and Assumption 1.

3. The result follows from part 1 and (1.18).

4. By part 1, the terms inside the integrals of equation 1.22 are increasing in r. By part 3, A^{M} is decreasing in r. Since both terms inside the integrals are positive but the second is greater than the first, the results follows.

□

Proof of Proposition 5. Denote the original equilibrium by $\left\{p^{*},\lambda^{M*},A^{M*}\right\}$ and decompose the effect of an increase in ϕ into two steps: (i) the effect of increasing ϕ while decreasing r to leave ϕr constant and (ii) the effect of restoring r to its original value. For step (i), equation (1.21) implies that $\left\{p,\lambda^{M},A^{M}\right\}=\left\{\frac{p^{*}}{\phi},\lambda^{M*},\phi A^{M*}\right\}$ is an equilibrium for any ϕ. Furthermore, equation (1.15) implies that each entrepreneur's proportional increase in $\max\left\{A^{M},A\right\}$ is exactly offset by a proportional decrease in virtual wealth and $\frac{K'}{K}$ does not change with ϕ. Step (ii) consists of increasing r, so the results follow from Proposition 4. □

Proof of Proposition 6. The effect of r on each of the endogenous variables in the asymmetric information economy can be decomposed into the effect it has in the fixed-wedge symmetric information economy plus the effect of the change in the implicit τ. By part 3 of Proposition 4, the implicit τ is decreasing in r. The inequalities then follow from Lemma 5. □

Proof of Proposition 7.

1. Rearranging (1.20):

$$D(p) = \frac{1}{p}\left[\beta\left[\lambda p + (1-\lambda)r\right] - \frac{\gamma(1-\beta)(1-\lambda)}{A^M(p)}\right] F\left(A^M(p)\right)$$

   Condition (1.24) ensures that

$$D(p) < \beta\lambda F\left(A^M(p)\right)$$

   for any p. Since the supply of lemons from Buyers is $\lambda F\left(A^M(p)\right) > D(p)$, there is no price that equalizes supply and demand, which implies $p^* = 0$.

2. First note that $A^M(p)$ is bounded because (i) it is continuous in p, (ii) $\lim_{p\to\infty} A^M(p) = 0$ and (iii) using l'Hôpital's Rule

$$\lim_{p\to 0} A^M(p) = \lim_{p\to 0} \frac{f\left(\frac{\gamma}{p}\right)}{p^2} \frac{\gamma^2(1-\lambda)}{\lambda}$$

   which must be equal to zero for A to have a finite mean.

   Since $A^M(p)$ is bounded, condition (1.24) is met for sufficiently low r, which proves the result for coconut-productivity shocks. Also because $A^M(p)$ is bounded, then

$$A^M(p,\phi) = \frac{\gamma}{p}\frac{(1-\lambda)\left(1-F\left(\frac{\gamma}{p\phi}\right)\right)}{\lambda + (1-\lambda)\left(1-F\left(\frac{\gamma}{p\phi}\right)\right)}$$

   converges uniformly to zero as $\phi \longrightarrow 0$, so a sufficiently large project-productivity shock also ensures that condition (1.24) is met.

□

Proof of Proposition 8. For given prices, equation (1.18) implies that λ^{M*} is increasing in λ. In addition, $D(p) - S(p)$ is decreasing in λ, so p must fall to restore market clearing. By (1.18), this reinforces the increase in λ^{M*}. □

Proof of Lemma 6. In equilibrium, $r = Y_K$, which is decreasing in K. By part 4 of Proposition 4, $\frac{K'}{K}$ is decreasing in K. For K large enough, r will be arbitrarily close to zero and, by equation (1.21), $p = 0$, which implies $\frac{K'}{K} < 1$. For K small enough, r will be arbitrarily large and equation (1.21) implies p will also be arbitrarily large, so $\frac{K'}{K} > 1$. It remains to show that $\frac{K'}{K}$ is continuous in r. To see this, define

$$E(p,r) \equiv p[D(p;r) - S(p;r)]$$

$E(p,r)$ represents the value of excess demand, which must be zero in equilibrium. $E(p,r)$ is continuous in p and r and, under Assumption 1, decreasing in p, so $p^*(r)$ defined by $E(p^*,r) = 0$ is continuous in r. Since $\frac{K'}{K}(p,r)$ is continuous in p and r, then $\frac{K'}{K}(p^*(r),r)$ is continuous in r and therefore there is a unique value of r such that $\frac{K'}{K}(p(r),r) = 1$ □

Proof of Lemma 7. The result follows from the fact that $A^M_{l,s}(p_{l,s})$ is continuous in $p_{l,s}$, $\lim_{p_{l,s}\to\infty} A^M_{l,s}(p_{l,s}) = 0$ and the definition of $p_{l,s}(A^*)$. □

Proof of Lemma 9. Dropping the l subscript for clarity By (1.28),

$$\frac{\partial p_G}{\partial \mu} = -\frac{\frac{\partial A^M_G}{\partial \mu}}{\frac{\partial A^M_G}{\partial p_G}}, \frac{\partial p_B}{\partial \mu} = -\frac{\frac{\partial A^M_B}{\partial \mu}}{\frac{\partial A^M_B}{\partial pA}}$$

The denominators in both expressions are negative by lemma 7 and, replacing (1.25) and (1.26) in (1.27) and differentiating, the numerators are

$$\begin{aligned}
\frac{\partial A^M_B}{\partial \mu} &= -\frac{\gamma}{p_B}\frac{\lambda(1-\lambda)\left(1-F\left(\frac{\gamma}{p_B}\right)\right)}{\left[\lambda\mu + (1-\lambda)(1-\mu)\left(1-F\left(\frac{\gamma}{p_B}\right)\right)\right]^2} < 0 \\
\frac{\partial A^M_G}{\partial \mu} &= \frac{\gamma}{p_G}\frac{\lambda(1-\lambda)\left(1-F\left(\frac{\gamma}{p_G}\right)\right)}{\left[\lambda(1-\mu) + (1-\lambda)\mu\left(1-F\left(\frac{\gamma}{p_G}\right)\right)\right]^2} > 0
\end{aligned}$$

so the result follows. □

Proof of Lemma 10. Using equations (1.25), (1.26) and (1.27), the market return in submarkets *Blue* and *Green* are respectively

$$\begin{aligned}
A^M_{l,B}(p_{l,B}) &= \frac{\gamma}{p_{l,B}}\frac{(1-\lambda)(1-\mu_l)\left(1-F\left(\frac{\gamma}{p_{l,B}}\right)\right)}{\lambda\mu_l + (1-\lambda)(1-\mu_l)\left(1-F\left(\frac{\gamma}{p_{l,B}}\right)\right)} \\
A^M_{l,G}(p_{l,G}) &= \frac{\gamma}{p_{l,G}}\frac{(1-\lambda)\mu_l\left(1-F\left(\frac{\gamma}{p_{l,G}}\right)\right)}{\lambda(1-\mu_l) + (1-\lambda)\mu_l\left(1-F\left(\frac{\gamma}{p_{l,G}}\right)\right)}
\end{aligned}$$

By the same argument used in the proof of Proposition 7, both $A^M_{l,B}$ and $A^M_{l,G}$ are bounded. $A^M_{l,B}$ is decreasing and continuous in μ_l with $\lim_{\mu_l \to 1} A^M_{l,B} = 0$, so for sufficiently highμ_l, $\max_{p_{l,B}} A^M_{l,B}(p_{l,B}) < A^*$. $A^M_{l,G}$ is increasing in μ_l with $\lim_{\mu_l \to 1} A^M_{l,G} = \frac{\gamma}{p_{l,G}}$, so for sufficiently highμ_l, $\max_{pG} A^M_{l,G}(p_{l,G}) > A^*$. □

Proof of Lemma 8. A robust equilibrium requires that $p^*_{l,s} = p_{l,s}(A^*)$ for all l, s. Letting $E(A^*) \equiv E(p(A^*), A^*)$, (1.31) can be rewritten as $E(A^*) = 0$. For sufficiently low A^*, $p_{l,s}(A^*)$ is arbitrarily large for all l, s, so $E(A^*)$ is necessarily negative. For sufficiently high A^*, $p_{l,s}(A^*) = 0$, so $E(A^*)$ is necessarily positive. If $E(A^*)$ is continuous, this implies that a solution exists and if it is monotonically increasing, the solution must be unique. Continuity follows because $E(p, A^*)$ is continuous in all its arguments and, since $A^M_{l,s}(p; \mu_l)$ is continuous in μ_l and $H(\mu)$ is continuous, then $p_{l,s}(A^*)$ can only be discontinuous on a zero-measure set. Monotonicity follows because $E(p, A^*)$ is increasing in A^* and decreasing in $p_{l,s}$ and, by lemma 7, $p_{l,s}(A^*)$ is decreasing. □

Proof of Lemma 11.

1. Suppose w.l.o.g. that the change in H consists of $\Delta h = H(\mu_1 + \delta) - H(\mu_1 - \delta) = -[H(\mu_0 + \delta) - H(\mu_0 - \delta)]$ for some small δ, with the probability that μ lies in any other interval unchanged, with $\frac{1}{2} \le \mu_0 < \mu_1$. Fixing A^*, the change in the value of excess demand is

$$\begin{aligned} \Delta E \;\approx\; & \left[\begin{array}{c} p_B(\mu_0)\left[\lambda\mu_0(1-\beta F(A^*)) + (1-\lambda)(1-\mu_0)\left(1 - F\left(\frac{\gamma}{p_B(\mu_0)}\right)\right)\right] + \\ p_G(\mu_0)\left[\lambda(1-\mu_0)(1-\beta F(A^*)) + (1-\lambda)\mu_0\left(1 - F\left(\frac{\gamma}{p_G(\mu_0)}\right)\right)\right] \end{array} \right] \Delta h \\ - & \left[\begin{array}{c} p_B(\mu_1)\left[\lambda\mu_1(1-\beta F(A^*)) + (1-\lambda)(1-\mu_1)\left(1 - F\left(\frac{\gamma}{p_B(\mu_1)}\right)\right)\right] + \\ p_G(\mu_1)\left[\lambda(1-\mu_1)(1-\beta F(A^*)) + (1-\lambda)\mu_1\left(1 - F\left(\frac{\gamma}{p_G(\mu_1)}\right)\right)\right] \end{array} \right] \Delta h \end{aligned}$$

   where $p_s(\mu)$ denotes $p_{l,s}$ for l such that $\mu_l = \mu$. By Lemma (9), $p_B(\mu_1) < p_B(\mu_0) \le p_G(\mu_0) < p_G(\mu_1)$, so the sign of ΔE is ambiguous in general. Since E is increasing in A^*, ΔA^* must have the opposite sign as ΔE to restore $E = 0$, so the effect on A^* is also ambiguous.

2. In this case, by assumption the shift can only be from a μ such that both *Blue* and *Green* markets are close to one where the *Green* one is open, so $p_B(\mu_0) = p_G(\mu_0) = p_B(\mu_1) = 0$ and $p_G(\mu_1) > 0$. This implies that $\Delta E < 0$, so A^* must rise.

□

Proof of Proposition 9. Using (1.39) and (1.40),

$$k'(k,A,X) = k\left[\int\left(\begin{array}{c}\lambda\left[\mu p_B(\mu)+(1-\mu)\,p_G(\mu)\right]\max\{A,A^*\}\\ +(1-\lambda)r\max\{A,A^*\}\\ +(1-\lambda)\mu\max\left\{\gamma,p_G(\mu)\max\{A,A^*\}\right\}\\ +(1-\lambda)(1-\mu)\max\left\{\gamma,p_B(\mu)\max\{A,A^*\}\right\}\end{array}\right)dH(\mu)k\right]$$

Every term in the integral is increasing in A^* and $p_s(\mu)$. The result then follows because, by Lemma 11, the shift in probability increases A^* and also shifts probability from shut markets to markets where prices are positive. □

Proof of Lemma 12. Assume w.l.o.g. that $s = G$ and drop the l subscript for clarity.

$$\begin{aligned}\lambda_G &\equiv \Pr\left[Lemon|G\right]\\ &= \Pr\left[G|Lemon\right]\frac{\lambda}{\Pr\left[G\right]}\\ &= \int \Pr\left[G|Lemon,\mu\right]dB(\mu)\frac{\lambda}{\int\Pr\left[G|\mu\right]dB(\mu)}\\ &= \int(1-\mu)dB(\mu)\frac{\lambda}{\int\left[\lambda(1-\mu)+(1-\lambda)\mu\right]dB(\mu)}\\ &= \frac{\lambda(1-\hat{\mu})}{\lambda(1-\hat{\mu})+(1-\lambda)\,\hat{\mu}}\end{aligned}$$

An equivalent reasoning applies when $s = B$. □

Proof of Lemma 13. Denote the unconditional distribution of χ by F_χ. The expected informativeness of signals conditional on having observed χ is

$$\mathbb{E}I\left(\tilde{\chi}\right) = \int\left(\hat{\mu}_l\left(\tilde{\chi}\right)-\frac{1}{2}\right)^2 dF_\chi\left(\tilde{\chi}\right)$$

where $\hat{\mu}_l\left(\tilde{\chi}\right) \equiv \mathbb{E}\left(\mu_l|\chi=\tilde{\chi}\right)$ is the informativeness of signals given the updated beliefs about μ_l conditional on χ having the realized value $\tilde{\chi}$. Using the law of iterated expectations

$$\begin{aligned}\mathbb{E}I\left(\tilde{\chi}\right)-I &= \int\left(\hat{\mu}_l\left(\tilde{\chi}\right)-\frac{1}{2}\right)^2 dF_\chi\left(\tilde{\chi}\right)-\left(\hat{\mu}_l-\frac{1}{2}\right)^2\\ &= \int\hat{\mu}_l\left(\tilde{\chi}\right)^2 dF_{\tilde{\chi}}\left(\tilde{\chi}\right)-\hat{\mu}_l^2\\ &= Var\left[\hat{\mu}_l\left(\chi\right)\right]\geq 0\end{aligned}$$

and therefore any information that χ provides about the true value of μ_l increases the expected informativeness of the signals. □

Proof of Lemma 14. Conjecture that $p(X,\mu)$ does not depend on μ. This immediately implies that $\hat{\mu}^p = \hat{\mu}$. Since $\lambda_{l,s}^M$ is independent for each value of l and agents are risk averse, they will be effectively risk neutral when buying projects. This implies that the expected return in every submarket where demand is positive must be the same, which implies that (1.28) must hold and $p(A^*)$ indeed does not depend on μ. It remains to show that the value of A^* that ensures market-clearing does not depend on realized μ. The market clearing condition is

$$E(p(A^*), A^*, \mu) = K\left[\int_0^1 \mu_l \varphi_l(A^*)dl + \kappa(A^*)\right] = 0$$

where

$$\begin{aligned}\varphi_l(A^*) \equiv & \beta\lambda\left(p_{l,B}(A^*) - p_{l,G}(A^*)\right) F\left(A^*\right) \\ & + p_{l,B}(A^*)\left[\lambda - (1-\lambda)\left(1 - F\left(\frac{\gamma}{p_{l,B}(A^*)}\right)\right)\right] \\ & - p_{l,G}(A^*)\left[\lambda - (1-\lambda)\left(1 - F\left(\frac{\gamma}{p_{l,G}(A^*)}\right)\right)\right]\end{aligned}$$

and

$$\begin{aligned}\kappa(A^*) = & \left[\beta\left[\lambda\int_0^1 p_{l,G}(A^*)dl + (1-\lambda)r\right] - (1-\beta)(1-\lambda)\frac{\gamma}{A^*}\right] F\left(A^*\right) \\ & + \int_0^1 \left[p_{l,B}(A^*)(1-\lambda)\left(1 - F\left(\frac{\gamma}{p_{l,B}(A^*)}\right)\right) + \lambda p_{l,G}(A^*)\right] dl\end{aligned}$$

Since the values of μ_l are independent draws from $B_l(\mu_l)$ and φ_l is bounded, then by the law of large numbers $E(p(A^*), A^*, \mu)$ is a constant and therefore so is the value of A^* that ensures market clearing. The capital accumulation equation (1.41) holds because entrepreneurs are diversified both with respect to the projects they hold and those they buy. □

Proof of Lemma 15.

$$\begin{aligned}H'(\hat{\mu}) & \equiv \int \mathbb{I}\left[\hat{\mu}_{l,t+1} \le \hat{\mu}\right] dl \\ & = \int \mathbb{I}\left[b_{l,t+1} \le \frac{\hat{\mu} - (1-\bar{\mu})}{2\bar{\mu} - 1}\right] dl\end{aligned}$$

$b_{l,t+1}$ is a function of the random variables μ_l, N_l and n_l. The distributions of μ_l, N_l and n_l are a function of the state (up to a reordering of the indices) and the realizations are independent across l. The result then follows from the law of large numbers. □

Proof of Proposition 16. By equation (1.29), total spending is increasing in r, which implies A^* is decreasing in r. This implies that the set values of μ such that $A^M_{l,s}(p_{l,s};\mu) = A^*$ has a solution increases. □

Proof of Proposition 10. Since $\sigma > 0$, no signal in the economy is perfectly informative and there is some residual informational asymmetry in all submarkets. This implies that, by the same reasoning used in the proof of Lemma 7, a sufficiently large negative productivity shock will lead all submarkets to shut down for n periods. Suppose $\omega_K = 0$, so there is no learning while markets shut down. Equation (1.34) implies that for any submarket l

$$b_{l,t+n} = \left[b_{l,t} - \frac{1}{2}\right](1-2\sigma)^n + \frac{1}{2}$$

Since $b_{l,t} \leq 1-\sigma$ for all l, this implies

$$b_{l,t+n} \leq \frac{1}{2}(1-2\sigma)^{n+1} + \frac{1}{2} \tag{1.42}$$

for all l. Using that $\hat{\mu}_{l,t} = b_{l,t}[2\bar{\mu}-1] + (1-\bar{\mu})$ and $I_{l,t} = \left(\hat{\mu}_{l,t} - \frac{1}{2}\right)^2$, equation (1.42) implies

$$I_{l,t+n} \leq \left(\left(\frac{1}{2}(1-2\sigma)^{n+1}\right)[2\bar{\mu}-1]\right)^2$$

for all l. Condition $n > \frac{\log\left(\bar{\mu}-\frac{1}{2}\right)-\log(\sqrt{\varepsilon_0})}{-\log(1-2\sigma)} - 1$ then ensures that

$$I_{l,t+n} < \varepsilon_0 \tag{1.43}$$

for all l.

Furthermore, condition $n < \frac{\log K_{ss} - \log K_0}{-\log[\gamma(1-\lambda)]}$ ensures that $K_{t+n} > K_0$ and therefore, by Lemma 16, this implies that $\varepsilon(X_{t+n}) > \varepsilon_0$. Using equation (1.43):

$$I_{l,t+n} < \varepsilon_0 < \varepsilon(X_{t+n})$$

for all l. Therefore, all submarkets continue to be shut after the shock is over, and will continue to be shut as long as $K \geq K_0$. Therefore the economy will converge to X_0. Convergence implies that there is a $\tilde{T}$ such that for $m > \tilde{T}$, $|Y_{t+m} - Y_0| < \delta$, so setting $T' = \max\left\{\tilde{T}, T\right\}$, the result would hold if $\omega_K = 0$.

$\Pr[\mu_l \in \xi(X)]$ is continuous in ω_K so for ω_K sufficiently close to zero the condition

$I_{l,t+m} < \varepsilon(X_{t+m})$ holds for an arbitrarily high proportion of submarkets for all $m \leq T'$. Since K' is continuous in H, the result follows. $\square$

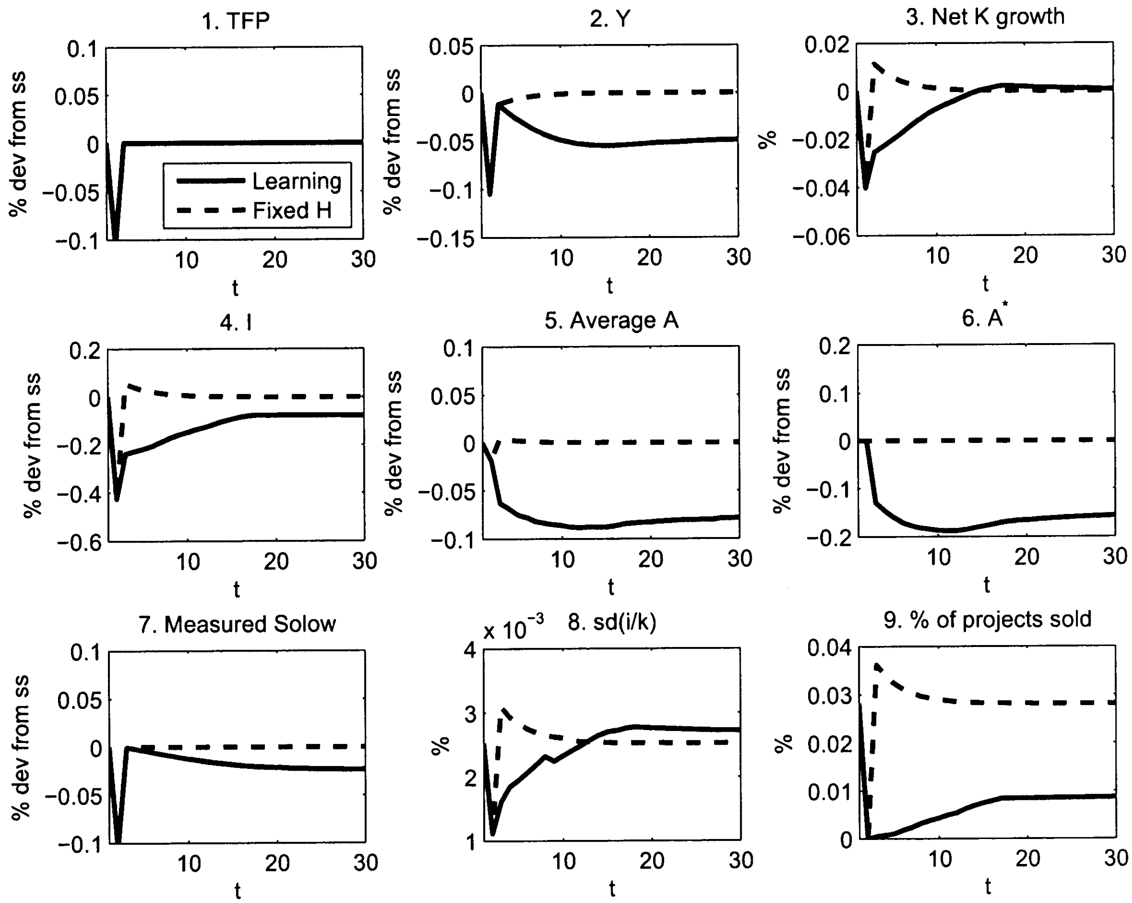


Figure 1.10: Transitory TFP shock that leads to market shutdown

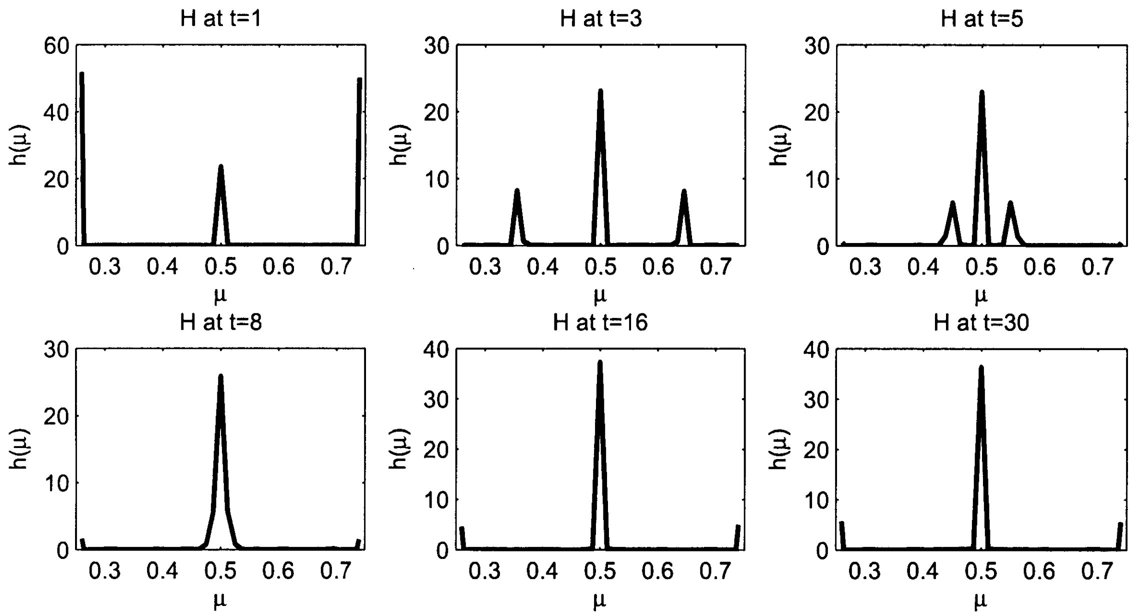


Figure 1.11: Evolution of H after a transitory TFP shock that leads to market shutdown

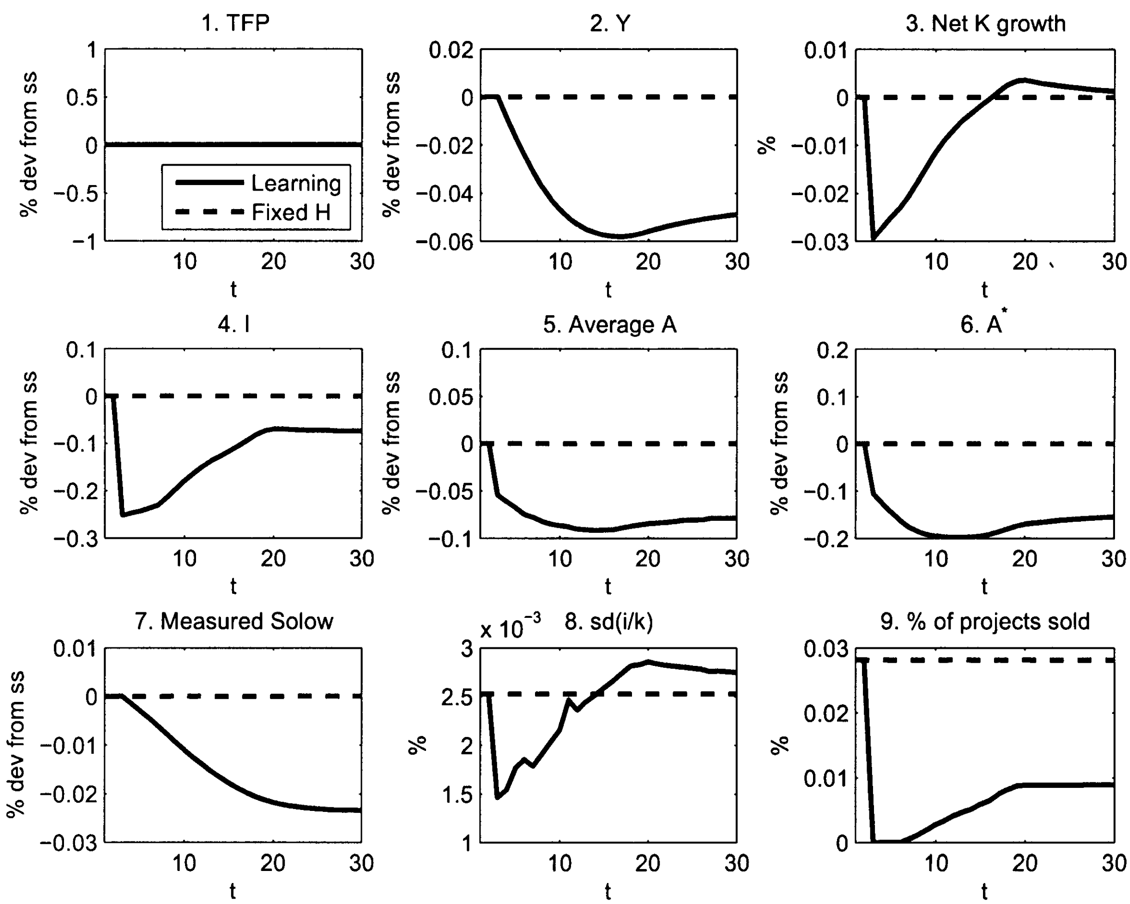


Figure 1.12: A shock that destroys the stock of financial knowledge

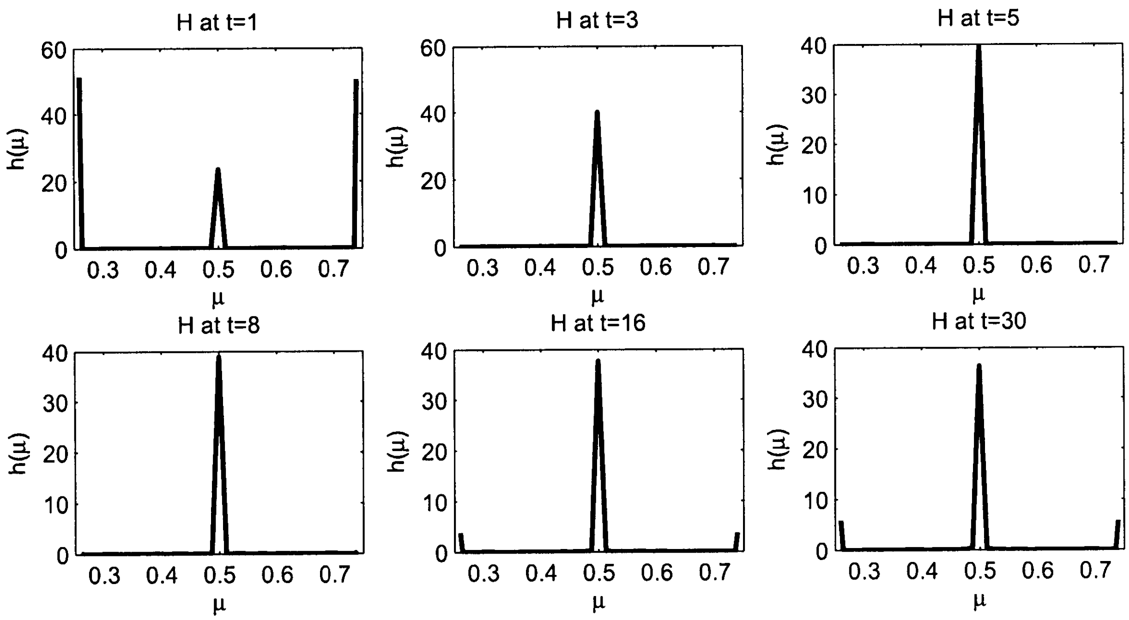


Figure 1.13: Evolution of H after a shock that destroys the stock of financial knowledge

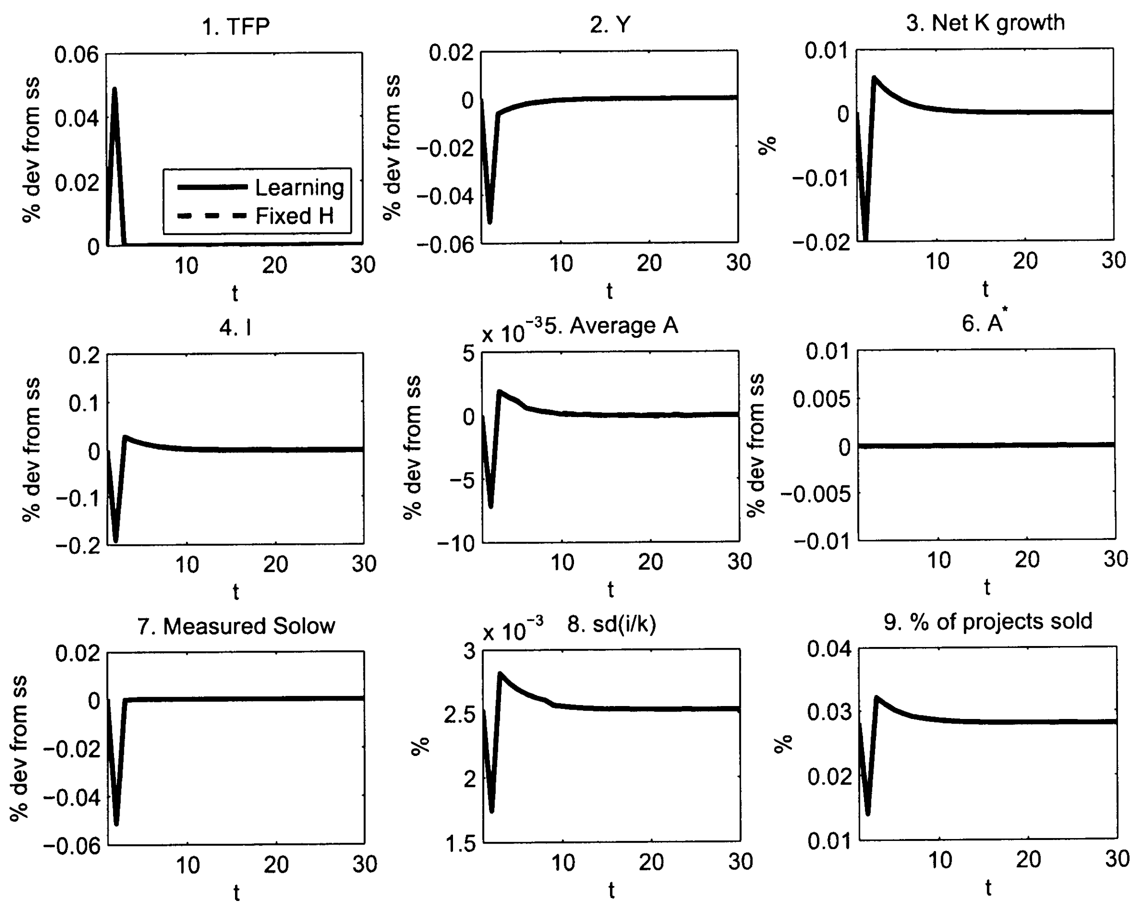


Figure 1.14: Transitory TFP shock that does not lead to market shutdown

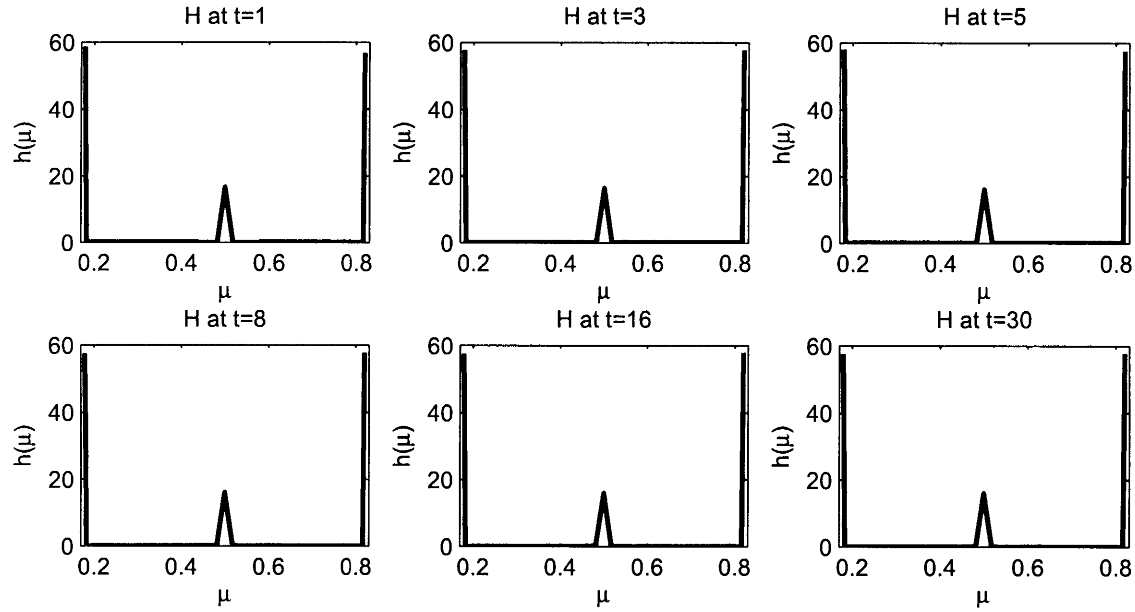


Figure 1.15: Evolution of H after a transitory TFP shock that does not lead to market shutdown

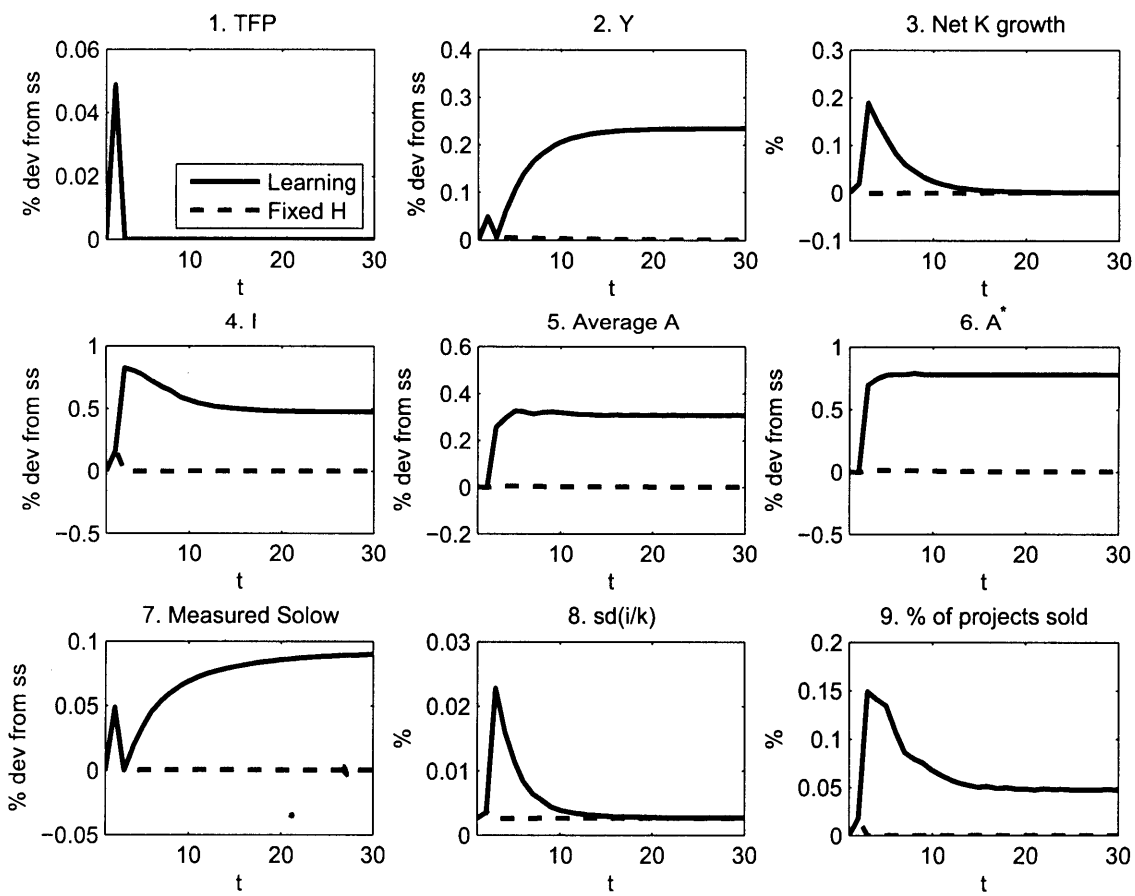


Figure 1.16: Positive TFP shock that leads markets to reopen

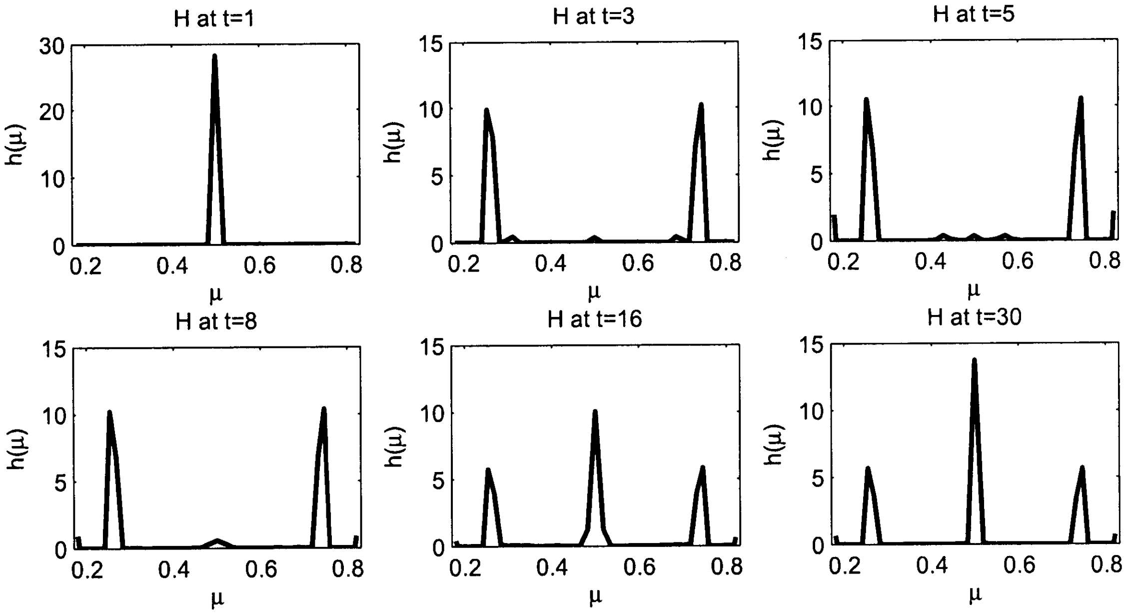


Figure 1.17: Evolution of H after a positive TFP shock that leads markets to reopen

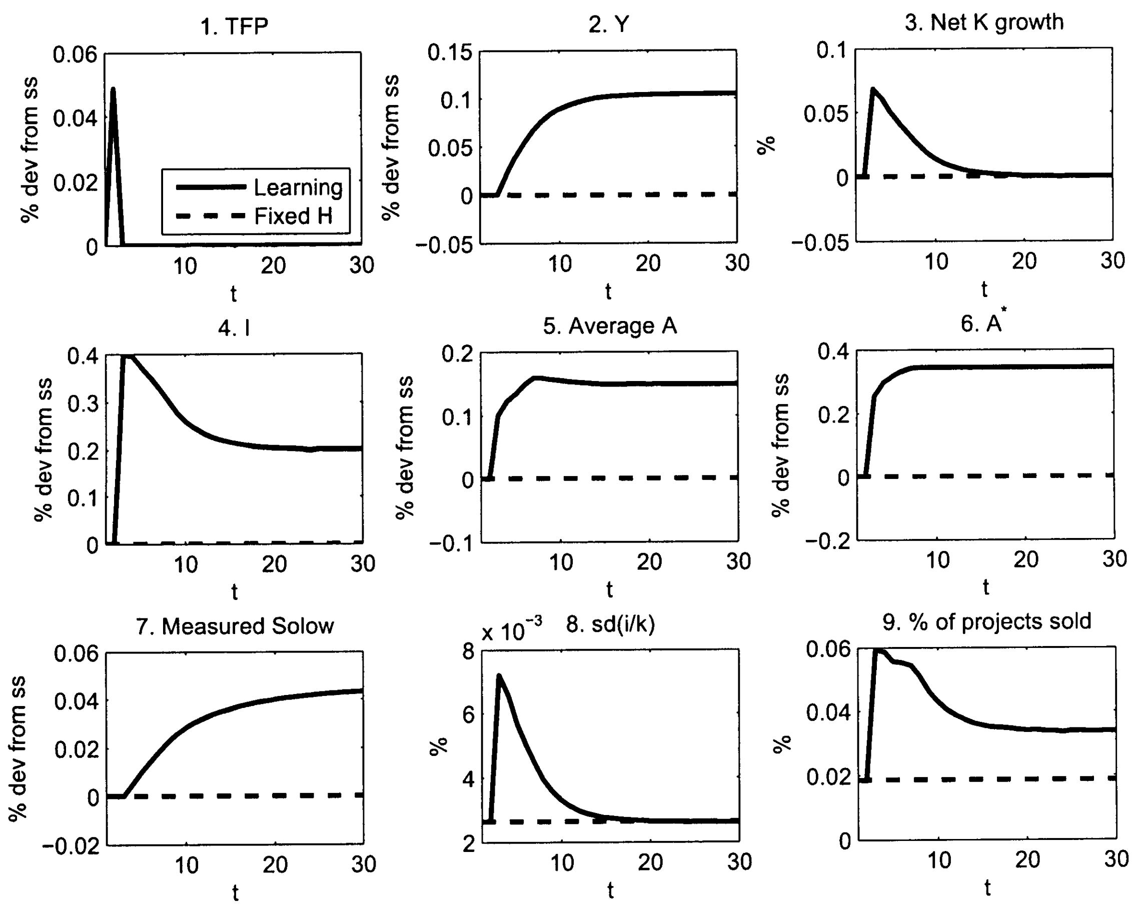


Figure 1.18: A stabilization of the information structure

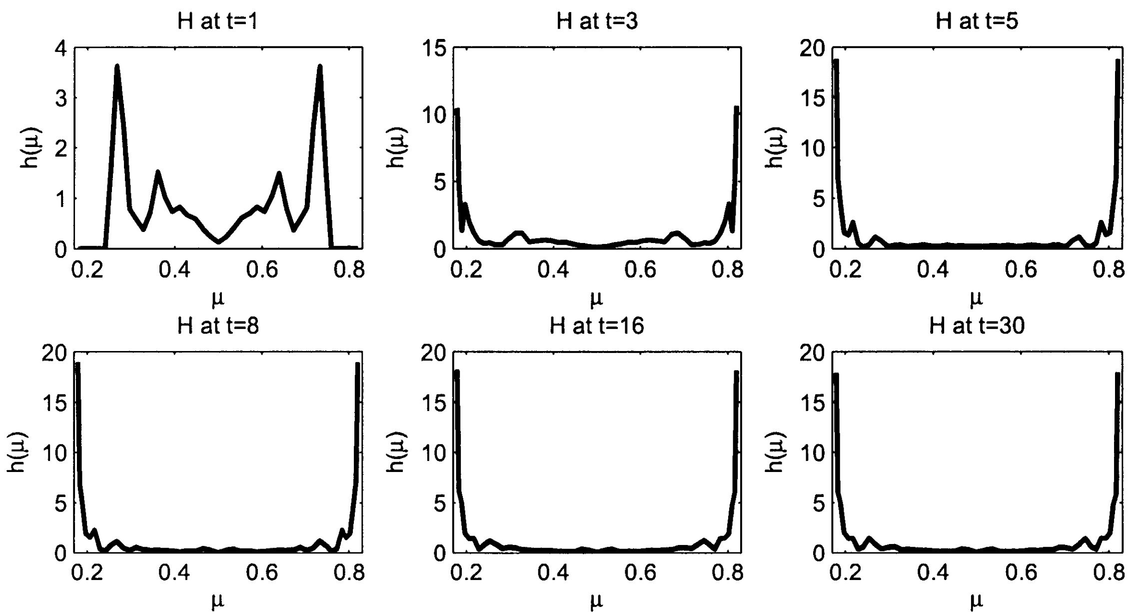


Figure 1.19: Evolution of H after a stabilization of the information structure

Chapter 2

Optimal Financial Fragility

2.1 Introduction

When a firm is financed with short term debt from multiple lenders but has slow-maturing investment projects, it exposes itself to runs. If a large fraction of the lenders simultaneously refuse to refinance their claims, the firm will be forced to seek emergency financing, liquidate its projects prematurely or request bankruptcy protection, all of which can be costly. Yet many firms are financed exactly in this way.

Broadly speaking, there are two main families of explanations of why firms would choose this kind of fragile financial structure. One of them, pioneered by Bryant (1980) and Diamond and Dybvig (1983), is that certain firms, "banks", do this in order to provide insurance against investors' idiosyncratic liquidity shocks. Deposit contracts allow investors who face a liquidity shock to obtain funds on demand while allowing greater risk pooling than if they had to invest for themselves and rely on asset sales for liquidity. However, a panic-based run may also occur: the belief that everyone will withdraw their deposit may prove self-fulfilling. The framework has the feature, common to most models of multiple equilibria, that it provides a theory of what *may* occur and not of what *will* occur. Accordingly, a central question in this literature, addressed among others by Diamond and Dybvig (1983), Wallace (1988), Green and Lin (2003) and Peck and Shell (2003), has been whether there exist contractual or policy schemes such that runs cease to be possible equilibrium outcomes. A complementary approach, followed by Goldstein and Pauzner (2005), has been to apply the equilibrium selection techniques developed by Carlsson and van Damme (1993) and Morris and Shin (1998) to transform the multiple equilibria model into one with definite predictions. This makes it possible to predict in what states of the world runs will take place and hence how likely they are ex-ante. The model can then be used to assess the consequences of different financial arrangements, even in a setting where panic-based runs sometimes happen in in equilibrium.

The second main theory of financial fragility is that some firms deliberately choose it in order to discipline managers, who might otherwise abscond (Calomiris and Kahn, 1991), divert assets from their best uses to obtain private benefits (Diamond, 2004) or attempt to bargain down the value of liabilities (Diamond and Rajan, 2000, 2001, 2005; von Thadden, Berglöf, and Roland, 2003). The goal in making firms vulnerable to runs is to ensure that managerial misbehaviour triggers a run that reduces or destroys managers' private benefits, so that ex-ante there is no incentive to misbehave. While in the literature that followed Diamond and Dybvig (1983) runs are an unfortunate side effect of arrangements designed to provide liquidity, in this view they are a useful (although possibly off-equilibrium) instrument in overcoming an agency problem. For this instrument to be effective, however, it must be the case that runs only occur when they are supposed to. If there are other Nash equilibria besides the one where runs happen if and only if there is evidence of misbehaviour, then a theory of equilibrium selection is required to assess the consequences of financial fragility.

Yet the fragility-for-discipline literature has not examined the question of multiple equilibria to the same extent that the fragility-due-to-liquidity-provision literature has. The demand deposit contracts studied by Calomiris and Kahn (1991) and Diamond and Rajan (2001) lead to optimal allocations provided investors play the equilibrium where they demand payment only upon observing bad signals about productivity but they are easily seen to permit an equilibrium where everybody always demands payment, leading to inefficient liquidation.[1]

In this paper I argue that the economic mechanism studied in this literature can, for the most part, survive these objections but the details of how the problem is modeled matter. I first construct a generalized version of the model in Diamond (2004). An entrepreneur needs to raise funds for a risky project. If the project goes ahead, the entrepreneur may engage in a diversionary activity that yields private benefits but reduces expected returns; the entrepreneur must credibly commit not to do so. At an intermediate date there is publicly observable but nonverifiable news about the project returns and it is possible to liquidate the project, which reduces output and destroys the entrepreneur's private benefit. The optimal incentive compatible allocation involves assigning the entrepreneur part of the project returns and, possibly, liquidating the project if early news is bad. Since this is ex-post

[1]The focus on the equilibrium without panic-based runs in good states of the world can be justified in several ways. One is a forward-induction argument: if investors expected to coordinate on the panic equilibrium they would not invest in the first place. However, they might still invest if they expect to coordinate on the panic equilibrium based on a sunspot with sufficiently low probability. Another justification is that in good states of the world it may be possible to raise new funds to withstand a run, removing the incentive to run. I return to this argument in section 2.4.1.

inefficient, a single outside investor who is unable to commit to liquidation would renegotiate with the entrepreneur and thus be unable to provide the necessary ex-ante incentives. Hence some other means of achieving the optimal allocation must be found.

I examine the special case studied by Diamond (which has binary output levels and where early news is a sufficient statistic for the entrepreneur's action) and his proposed solution that the firm should issue dispersed short term debt. I show that this implementation has two separate difficulties. Firstly, the game played by the investors under his proposed contract has multiple equilibria. It is possible to apply equilibrium selection criteria but these would always select the same equilibrium, whereas the optimal allocation requires selection contingent on early news. Alternatively, the right contingent equilibrium selection can be obtained if one assumes that it is costless to obtain emergency refinancing when the firm is solvent, which may be reasonable in some applications but not in others. The second difficulty is that even if one assumes that the correct equilibrium is selected, in general the resulting allocation is suboptimal and leads to more frequent liquidation than necessary.

Implementation is successful, however, in a slightly different special case, where the distribution of output is continuous rather than binary and early signals are perfectly precise. Intuitively, a richer distribution of output makes it possible to select different equilibria depending on early news while using a single equilibrium selection criterion. In particular, focusing on the unique equilibrium that is robust to small amounts of idiosyncratic noise (as in the global games literature) it is possible to obtain the kind of contingent equilibrium selection that is required to implement the optimal allocation. Furthermore, the fact that early news reveals realized output perfectly means that it is possible to assign the project returns optimally without relying on unverifiable variables, so the optimal allocation can be achieved using simple contracts. An exception is when the required investment is small but agency problems are very severe, since this requires liquidating the project but assigning the proceeds to the entrepreneur and the simple contracts considered here are unable to do so.

Overall, the model predicts that dispersed short term debt should be used for projects that are not too profitable and have no better instruments for disciplining managers, which is broadly in line with the empirical evidence.

The paper complements other explanations that have been offered of why firms choose to issue short term debt. Hart and Moore (1998) also view short term debt, and in particular the right it gives investors to foreclose on assets, as a means to discipline self-interested managers. However, they allow costless renegotiation so that ex-post inefficient outcomes are ruled out. Diamond (1991, 1993) shows that firms may prefer to issue short term debt if they are asymmetrically informed about how their credit rating is likely to evolve. Broner, Lorenzoni, and Schmukler (2007) show that short term debt may be preferred if investors have short horizons and are risk averse. None of these theories, however, distinguishes between widely held and concentrated debt or has a role for coordination failure. More broadly, the paper relates to a large literature, dating back at least to Jensen and Meckling (1976) and Grossman and Hart (1982) that uses agency theory to study firms' optimal financial structure.

The issue of why firms choose to have multiple creditors has also been addressed by several authors. Bolton and Scharfstein (1996) suggest that having multiple creditors leads to inefficient renegotiation after default, which is helpful in deterring strategic default but costly when default is involuntary. Berglöf and von Thadden (1994) show that it can be useful for different investors to hold short and long term debt because this gives the short term creditors the possibility to impose the losses from liquidation on (junior) long term creditors, strengthening their bargaining position and deterring strategic default. In their paper, however, renegotiation prevents inefficient liquidation so their mechanism would not solve the kind of agency problem that I consider. Detragiache, Garella, and Guiso (2000) present a model where lenders acquire information about borrowers but may suffer liquidity shocks that prevent them from refinancing their loans; firms prefer to rely on several of them to avoid adverse selection when attempting to refinance from uninformed investors.

The possibility of coordination failure when a firm borrows short term from multiple creditors is addressed most directly by Morris and Shin (2004). Their focus, however, is on the proper way to price this kind of debt instrument rather than on with firms rely on this financial structure in the first place.

Section 2.2 describes the economic environment. Section 2.3 characterizes the optimal incentive compatible allocations in this environment. Section 2.4 examines whether it is possible to implement optimal allocations using simple contracts that do not condition on nonverifiable variables and do not require commitment. Section 2.5 discusses interpretations, limitations and extensions of the results, as well as empirical evidence.

2.2 The environment

An entrepreneur has a project that requires investing K at $t = 0$. If the project is undertaken the entrepreneur must choose an action $a \in \{0, 1\}$. $a = 1$ represents a diversionary action that yields a private benefit of v to the entrepreneur but reduces expected output. The entrepreneur's choice of a is private information. At $t = 1$ a random variable $z \in [\underline{z}, \bar{z}]$ is realized. z represents some imperfect indicator of the firm's probable performance such as a customer survey or a preliminary sales report. It is publicly observable but not verifiable. At $t = 2$ the project yields output q. Let $f_{zq}(z, q|a)$ denote the joint *pdf* or *pmf* of z and q

conditional on a.

Assumption 3. *The conditional cdf $F(q|z,a)$ is weakly decreasing in z.*

Assumption 4. *The marginal distributions $f_z(z|a)$ and $f_q(q|a)$ have a strictly monotone likelihood ratio, i.e. $\frac{f_z(z|0)}{f_z(z|1)}$ and $\frac{f_q(q|0)}{f_q(q|1)}$ are strictly increasing functions of z and q respectively.*

Assumption 5. *The conditional distributions $f_{z|q}(z|a,q)$ and $f_{q|z}(q|a,z)$ have a weakly monotone likelihood ratio, i.e. $\frac{f_{z|q}(z|0,q)}{f_{z|q}(z|1,q)}$ and $\frac{f_{q|z}(q|0,z)}{f_{q|z}(q|1,z)}$ are weakly increasing functions of z (for any q) and q (for any z) respectively.*

The signal z provides information both about what output is likely to be and about what the action the entrepreneur took was. Assumption 3 says that a higher signal implies (weakly) higher output in a FOSD sense, conditional on a given action by the entrepreneur. Assumption 4 implies that (unconditionally) better signals z or higher output q are always evidence of the entrepreneur having chosen $a = 0$.[2] Assumption 5 implies that the same is (weakly) true conditional on the realized value of q and z respectively.

After observing z, the project can be liquidated early, in which case it yields λq instead of q, where $\lambda \in (0,1)$. Assume that the act of liquidation accelerates the realization of q up to $t = 1$ and is irreversible. Liquidation may be partial, i.e liquidating a fraction α yields $\alpha\lambda q$ at $t = 1$ and $(1-\alpha)q$ at $t = 2$.[3] $1-\lambda$ may represent the loss from physically interrupting the project or the various possible costs involved in rapidly selling it to a third party. The entrepreneur's private benefit from $a = 1$ is destroyed proportionally with early liquidation, i.e. it is equal to $a(1-\alpha)v$. This is intended to capture a specific kind of agency problem. $a = 1$ does not represent lack of effort or outright stealing, since there is no reason to believe that liquidating a firm's assets would eliminate the private benefits of those actions. Instead, $a = 1$ is intended to represent a diversionary action, such as empire-building of self-dealing, that the entrepreneur profits from only to the extent that the firm continues to operate until $t = 2$.

The entrepreneur has no assets and therefore needs to obtain K from outside investors in exchange for some promise of future payments. There is a competitive market of potential outside investors. Both the entrepreneur and the investors are risk neutral and do not discount the future.

[2]Together with assumption 3, this implies that higher z brings a FOSD increase in q unconditionally as well as conditionally on a.

[3]It is not possible, however, to carry out a partial liquidation, observe the proceeds of this to learn about realized q and, depending on the results of this, decide whether or not to carry out further liquidation.

2.3 Optimal allocations

In this section I derive a benchmark constrained efficient allocation, subject to incentive and participation constraints and a monotonicity constraint discussed below. It can be thought of as the allocation that would be achieved by the parties if they could contract on nonverifiable variables and commit to enforcing ex-post inefficient outcomes. An allocation consists of a liquidation rule $\alpha(z)$ and a rule that determines the amount $B(z,q)$ that outside investors receive as a function of z and q, leaving $q[1-\alpha(z)(1-\lambda)]-B(z,q)$ for the entrepreneur.

Assumption 6. $\mathbb{E}(q|a=1)<K$

Assumption 6 implies that even if the entrepreneur pledges the entire cash flow to the outside investors, they still would not invest if they expect the entrepreneur to divert. This means that any allocation that gives investors nonnegative profits must be such that $a=0$ is incentive compatible. In addition, assume that both the entrepreneur and outside investors have limited liability so $B(z,q)$ is constrained to lie in the interval $[0,q[1-\alpha(z)(1-\lambda)]]$.

The optimal allocation must provide sufficient incentives for the entrepreneur to choose $a=0$. This is done by rewarding the entrepreneur (by assigning him output) for outcomes that are likely under $a=0$ and punishing him (by not assigning him output and liquidating the firm to destroy his private benefits) for outcomes that are likely under $a=1$, to the greatest extent consistent with limited liability.

Without further assumptions, this will produce a nonmonotonic payment schedule $B(z,q)$ which assigns no output to outside investors when q is sufficiently high. Innes (1990) suggests two possible reasons why nonmonotonic schedules are seldom observed. One is that outside investors might somehow be able to (at least partially) sabotage the firm; if $B(z,q)$ were decreasing in q they would choose to do so for some realizations of q. An allocation that is immune to this possibility should satisfy that $B(z,q)$ be nondecreasing in q. The second possible reason is that the entrepreneur may be able to temporarily simulate higher q than was really produced (perhaps by secret borrowing);[4] an allocation that is immune to this possibility should satisfy that $\frac{\partial}{\partial q}[q[1-\alpha(z)(1-\lambda)]-B(z,q)]\leq 1$. When $\alpha(z)=0$ the two requirements are equivalent; otherwise they may differ, though not in ways that are significant for the rest of the analysis. In what follows, I adopt the first version of the constraint.

[4]He cannot, however, simulate *lower* q than was really produced.

The optimal allocation is thus given by the solution to the following program:

$$\max_{\alpha,B} \int \left(\int ([1-\alpha(z)(1-\lambda)]\, q - B(z,q))\, f_{q|z}(q|z,0)\, dq \right) f_z(z|0)\, dz \tag{2.1}$$

$s.t$

$$\int \left(\int ([1-\alpha(z)(1-\lambda)]\, q - B(z,q))\, f_{q|z}(q|z,0)\, dq \right) f_z(z|0)\, dz \geq$$

$$\int \left(\int ([1-\alpha(z)(1-\lambda)]\, q - B(z,q))\, f_{q|z}(q|z,1)\, dq + v(1-\alpha(z)) \right) f_z(z|1)\, dz \tag{2.2}$$

$$\int \left(\int B(z,q)\, f_{q|z}(q|z,0)\, dq \right) f_z(z|0)\, dz \geq K \tag{2.3}$$

$$q[1-\alpha(z)(1-\lambda)] - B(z,q) \geq 0 \tag{2.4}$$

$$B(z,q) \geq 0 \tag{2.5}$$

$$B(z,q) \text{ nondecreasing in } q \tag{2.6}$$

where (2.2) is the incentive compatibility condition for the entrepreneur to choose $a = 0$ and (2.3) is the outside investors' zero-profit condition.

Proposition 12. *The optimal allocation is given by $B(z,q)$ and $\alpha(z)$ such that*

$$B(z,q) = \min\{q, q^B(z)\}\,[1-\alpha(z)(1-\lambda)] \tag{2.7}$$

$$\alpha(z) = \begin{cases} 1 & \text{if } z < z^* \\ 0 & \text{if } z \geq z^* \end{cases} \tag{2.8}$$

for some number z^* and some weakly decreasing function $q^B(z)$.

This is echoes the result in Innes (1990), who finds that (without advance signals z and the possibility of liquidation) the optimal contract is standard debt. Standard debt gives the entrepreneur a payoff of zero for low output (an outcome likely under $a = 1$) and the maximal possible payoffs consistent with monotonicity for high output (an outcome likely under $a = 0$).

The same logic applies in this setting, with two differences. First, the realized signal z provides additional information about the choice of a. Since higher z is indicative of $a = 0$, this means that $q^B(z)$, the cutoff value of q where the entrepreneur starts being rewarded rather than punished, can be lower for higher z. Second, the possibility of liquidation gives an additional instrument for providing incentives. Liquidation will be used only when z is sufficiently low, due to two reinforcing effects. Firstly, liquidating eliminates the private benefit from $a = 1$ and thus always relaxes the incentive compatibility constraint, but does so more strongly when it follows low signals z that are particularly likely under $a = 1$. Secondly, liquidating reduces output by a fraction λ. Since output is likely to be low following a low signal, the expected cost of liquidation is lower for low z. Hence, the firm will be liquidated only for z below some cutoff z^*. There exist parameters such that $z^* = \underline{z}$. In these cases it is possible to satisfy the incentive compatibility constraint without liquidation, and the outcome is first best.

2.3.1 Comparative statics

Note that since the entrepreneur obtains all the surplus, the optimal allocation problem can simply be restated as the problem of choosing z^* and $q^B(z)$ to minimize the inefficiency from liquidation, i.e:

$$\min_{z^*, q^B(z)} z^*$$

$$s.t.$$

$$c\left(z^*, q^B(z)\right) \equiv \int\int \max\left\{q - q^B(z), 0\right\}\left[f_{zq}(z, q|0) - f_{zq}(z, q|1)\right]\left[1 - \mathbb{I}(z < z^*)(1-\lambda)\right] dqdz \tag{2.9}$$

$$- v\left[1 - F_z(z^*|1)\right] \geq 0$$

$$r\left(z^*, q^B(z)\right) \equiv \int\int \min\left\{q, q^B(z)\right\} f_{zq}(z, q|0)\left[1 - \mathbb{I}(z < z^*)(1-\lambda)\right] dqdz - K \geq 0 \tag{2.10}$$

This makes the analysis of how the optimal allocation is affected by parameters relatively straightforward.

Proposition 13.

1. *z^* and $q^B(z)$ are weakly increasing in K and v.*

2. *z^* and $q^B(z)$ fall in response to a FOSD increase in $f_q(q|0)$ (with $f_{z|q}(z|q, 0)$ unchanged) and rise in response to a FOSD increase in $f_q(q|1)$ (with $f_{z|q}(z|q, 1)$ unchanged).*

3. *The response of z^* and $q^B(z)$ to changes in λ is ambiguous.*

The need for higher investment K means that more of the output must be assigned to outside investors, so incentives for the entrepreneur must rely more on the threat of liquidation. More severe incentive problems (higher v) mean that incentives must be strengthened, which also requires more liquidation. Higher expected output means that the entrepreneur will have stronger incentives from receiving ouput, so liquidation may be used less. Higher expected output conditional on $a = 1$ makes incentive problems more severe, so liquitation must be used more. Higher λ increases output upon liquidation, which relaxes the zero-profit constraint. It also affects incentives due to the regions of z, q where the optimal allocation mandates liquidation ($z < z^*$) but assigns part of the liquidation proceeds to the entrepreneur ($q > q^B(z)$). If these regions are likelier under $a = 0$ that under $a = 1$, then higher λ helps incentives, while the opposite is the case if these regions are likelier under $a = 1$. In this latter case, higher λ relaxes the zero profit constraint but tightens the incentive compatibility constraint so the overall effect might be that more liquidation (higher z^*) is required when assets are more liquid.

2.4 Implementation without commitment

The optimal allocation described in section 2.3 is ex-post inefficient and relies in part on nonverifiable information to assign output. If $z < z^*$, then the parties will have an incentive to renegotiate at $t = 1$ in order not to liquidate and gain $(1 - \lambda)\,\mathbb{E}\,(q|z, 0)$ between them. If, as is reasonable in many circumstances, they cannot commit to abstain from renegotiating, the entrepreneur will know that his private benefit will never be destroyed and will choose $a = 1$. Anticipating this, outside investors will be unwilling to invest. Is there a way to implement the optimal allocation despite the inability to commit, while also not relying on nonverifiable information?

2.4.1 Diamond's proposed implementation

Diamond (2004) studies a special case of the model where q can only take the values H and 0. This considerably simplifies the analysis because the payment rule only needs to specify how the output will be split in case the realized value is H; because $B(z, 0)$ necessarily equals 0, the issue of the shape of the $B(z, q)$ function (including in particular whether monotonicity is a reasonable requirement and the distinction between debt and equity) does not arise.[5] Furthermore, Diamond assumes that the conditional pmf of q is

$$\begin{aligned} f(H|z, a) &= z \\ f(0|z, a) &= 1 - z \end{aligned}$$

[5]It does arise, however, in the slightly more general case where q can take two different positive values. Diamond mentions this case but does not fully analyze it.

so that all the effect of a on output is subsumed by its effect on the signal z (in other words, z is a sufficient statistic for a) and the entrepreneur retains no private information at $t = 1$. This may capture, for instance, an application where z is the quality of the product, which depends in part on the entrepreneur's action, and q depends on both quality and market conditions that the entrepreneur has no influence over.

Diamond proposes that the firm issue debt with face value $\frac{b}{N}$ to each of N investors, where $b \in (\lambda H, H]$, payable on demand on a first-come-first serve basis either at $t = 1$ or at $t = 2$ (each investor chooses at $t = 1$ whether to demand payment or to wait until $t = 2$). For each investor who demands payment at $t = 1$, the firm must liquidate enough output to satisfy his full claim (recall that the act of liquidation accelerates the realization of q), at least until output is exhausted. This means that if an investor decides to demand payment he imposes the full cost of liquidation on the remaining investors, who are either next in line or have decided to wait until $t = 2$, and therefore he has no incentive to renegotiate.[6]

This financial arrangement induces a game played by investors at $t = 1$, each of whom must choose whether to "Run" or "Wait". Assume they decide simultaneously and those who run arrive in line in random order. The payoffs of this game, contingent on z, are shown in table 2.1 (for simplicity I illustrate the case of $N = 2$ and $\lambda \in \left(\frac{1}{2}, \frac{b}{H}\right)$ but the argument applies more generally).

Table 2.1: Payoffs in Diamond (2004)

	Run	Wait
Run	$\frac{z\lambda H}{2}$, $\frac{z\lambda H}{2}$	$\frac{zb}{2}$, $z\left(H - \frac{b}{2\lambda}\right)$
Wait	$z\left(H - \frac{b}{2\lambda}\right)$, $\frac{zb}{2}$	$\frac{zb}{2}$, $\frac{zb}{2}$

The reason for these payoffs is as follows. If no one runs, there will be no liquidation at $t = 1$ and if $q = H$ each investor will collect $\frac{b}{2}$ at $t = 2$, so that expected payoff is $\frac{zb}{2}$. If only one player runs it will be necessary to liquidate a fraction $\alpha = \frac{b}{2\lambda H}$ of output to fully satisfy his claim of $\frac{b}{2}$. This will leave $(1 - \alpha)H - \frac{b}{2} = H - \frac{b}{2\lambda}$ for the player who does not run. Finally, if both players run it will be necessary to liquidate all the output and one would still not satisfy both claims fully, so each investor gets half the expected output of $z\lambda H$.

Equilibrium selection. It is immediate from the table that, for any z, the game has two Nash equilibria: (Run,Run) and (Wait,Wait), the first of which leads to full liquidation and the second to no liquidation. Allowing for arbitrary equilibrium selection contingent on z it is possible for this kind of financial structure to implement the kind of contingent liquidation

[6]In general if $b < H$ the entrepreneur might offer the investor an increased $t = 2$ claim in exchange for the investor abstaining from demanding liquidation. See the discussion in section 2.5 on whether this offer would be credible.

rule required by (2.8). Unfortunately, the effectiveness of the mechanism hinges entirely on right equilibrium being selected for each z.

Furthermore, the (Wait, Wait) equilibrium that is supposed to be played when $z > z^*$ requires that players choose weakly dominated strategies. This difficulty can be avoided by a slight modification of the contract whereby the firm offers a positive interest rate $B - b$, with $B \leq H$, for those not demanding payment, so that payoffs after (Wait, Wait) are $\frac{zB}{2}, \frac{zB}{2}$ and (Wait, Wait) is a strict Nash equilibrium. However, z-contingent equilibrium selection is still required. Since z simply scales up the payoffs of every strategy profile, any equilibrium selection criterion that is employed would select the same equilibrium for every z.

Refinancing. Diamond argues that if the firm is allowed to raise new funds at $t = 1$, a slightly different financial structure may implement (2.8) even if investors always play (Run, Run). The firm should issue debt for an aggregate value of z^*H due at $t = 1$. At $t = 1$ the entrepreneur retains no private information so, the argument goes, he should be able to raise new financing up to the full value of the project, which is zH. This will be sufficient to pay all the maturing debt, thus avoiding liquidation, if and only if $z > z^*$, as required. Whether or not the assumption that it is possible to raise new financing quickly and costlessly is reasonable depends on the context. If the project is very transparent to outsiders or if current investors have deep pockets and can provide new financing then it is plausible. If current investors are small and z is not observable by outsiders, as may be the case for instance for small firms financed with trade credit, then it is a less attractive assumption. In general, it is possible to think of λ as not only representing the cost of physical liquidation but the cost of obtaining emergency financing. In that interpretation, values of z above z^* will not prevent runs.

Equilibrium and optimal allocations. Another problem with the allocation induced by this financial structure, both if it relies on equilibrium selection and if it relies on refinancing, is that neither of these arrangements actually reproduces the optimal allocation, except in a limiting case. Since by assumption q provides no additional information about a, both liquidation and output assignment depend only on z, and the optimal assignment rule (2.7) reduces to

$$B(z, H) = \begin{cases} & \text{if } z \leq z^B & \text{if } z > z^B \\ \text{if } z \geq z^*: & H & 0 \\ \text{if } z < z^*: & \lambda H & 0 \end{cases} \tag{2.11}$$

for some z^B.[7] (2.11) says that it will be optimal to assign all the output (or the output that remains after liquidation, as the case may be) to the investors for sufficiently low z and to

[7]Formally, $q^B(z)$ takes either the value 0 when $z > z^B$ or the value ∞ when $z < z^B$

the entrepreneur for sufficiently high z.[8] This is done in order to use the output assignment rule to provide incentives to the greatest extent possible.

Assuming equilibrium is selected correctly without the need for refinancing, the output assignment rule that arises from the equilibrium in Diamond's game is

$$B(z,H) = \begin{cases} \text{if } z \geq z^*: & B \\ \text{if } z < z^*: & \lambda H \end{cases} \tag{2.12}$$

This is because when $z < z^*$ then the (Run, Run) equilibrium will be selected and since $b > \lambda H$, then investor's claims will exhaust all the output, leaving nothing for the entrepreneur. When $z > z^*$ the (Wait, Wait) equilibrium will be selected and investors collect their claim B. (2.11) and (2.12) can coincide only in the case where $z^B = \bar{z}$ (setting $B = H$) i.e. when all the output is always assigned to the investors and only the threat of liquidation is used to provide incentives to the entrepreneur. If $B = H$, incentive compatibility also requires that $F_z(z^*|1) = 1$ because the entrepreneur never gets any output, so the only way to ensure that he does not choose $a = 1$ is if that action is always detected.

Instead, if the equilibrium is selected correctly by issuing debt for z^*H and refinancing it when $z > z^*$, the resulting output assignment rule is

$$B(z,H) = \begin{cases} \text{if } z \geq z^*: & \frac{z^*}{z}H \\ \text{if } z < z^*: & \lambda H \end{cases} \tag{2.13}$$

When $z < z^*$, the value of the assets is less than the value of liabilities so the firm will not be able to obtain fresh financing. The contract becomes just like the no-refinancing case and it is assumed that the agents will coordinate on the (Run,Run) equilibrium. Assuming $z^* > \lambda$, agents' claims will exhaust output. When $z > z^*$ the firm can issue fresh debt with face value $\frac{z^*}{z}H$, which fetches a price of z^*H and thus is sufficient to pay maturing debt without resorting to liquidating output. (2.11) and (2.13) can coincide only in the case where $z^B = z^* = \bar{z}$, i.e. when in addition to all output being assigned to investors, liquidation is only avoided for the highest possible value of the signal z.

I conclude that outside very special limiting examples, the implementation proposed by Diamond will produce suboptimal allocations, even allowing for arbitrary equilibrium selection or refinancing. This does not mean that the contracts are not useful. They may produce allocations that satisfy incentive compatibility which, by assumption 6, is indispensable for the project to take place. But in general they produce more liquidation than would be

[8]There is no presumption as to whether z^* or z^B is greater. This will depend on the relative tightness of the incentive compatibility and zero profit constraints.

necessary if the investors could commit to an ex-post inefficient liquidation rule and to an output assignment rule contingent on the nonverifiable signal z.

2.4.2 An example of successful implementation

Consider the special case of the model where z is a perfect signal of q or, equivalently, where at $t = 1$ agents already observe what the realization of q is going to be, and $f_q(q|a)$ is a continuous *pdf* with full support on $\mathbb{R}^+$. In this case both the liquidation decision and the output assignment rules are functions of q only. Conditions (2.7) and (2.8) for an optimal allocation still apply, but q^B is a constant rather that a function of z and the liquidation threshold is simply q^*. Either q^* or q^B could be greater or they could be equal. Figure 2.1 illustrates the three possibilities.

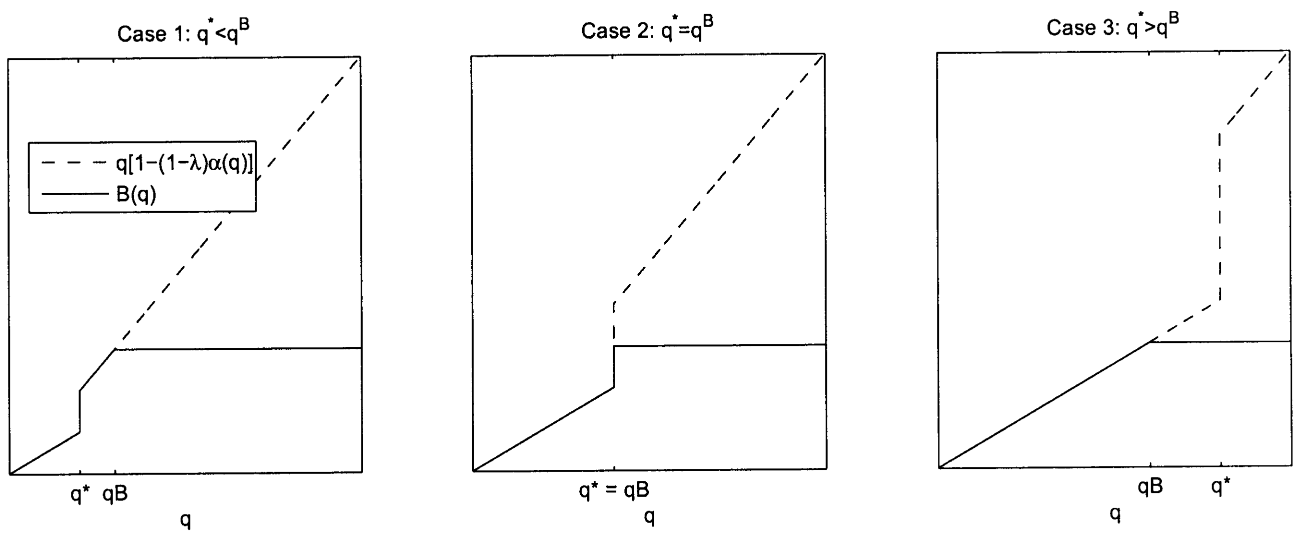


Figure 2.1: Optimal allocation when q is observed at $t = 1$

In case 1, with $q^* < q^B$, the set of states where there is liquidation is a subset of the set of states where the entrepreneur gets nothing. In case 2, with $q^* = q^B$, the set of non-liquidation and positive payoff for entrepreneur outputs exactly coincide. In case 3, with $q^* > q^B$, there are states where there is early liquidation but the entrepreneur keeps part of the proceeds. In all cases, if the investors were able to commit, the optimal allocation could be implemented by a simple debt contract plus a liquidation rule, i.e.

$$B(q) = \begin{cases} \text{if } q \geq q^*: & \min\{B, q\} \\ \text{if } q < q^*: & \min\{B, \lambda q\} \end{cases} \tag{2.14}$$

$$\alpha(q) = \mathbb{I}\{q < q^*\} \tag{2.15}$$

In case 1, the face value B of debt must be $B = q^B$, in case 2, $B \in (\lambda q^B, q^B)$ and in case 3, $B = \lambda q^B$. Case 1 (high face value of debt, low q^*) will arise when the required investment K is high but incentive problems (measured by v) are not so severe whereas case 3 will arise when the required investment is not high but strong deterrence is required to prevent the entrepreneur from choosing $a = 1$.

Consider attempting to implement the optimal allocation with the following contract, closely related to the one proposed by Diamond. The entrepreneur borrows K from a continuum of mass 1 of outside investors in exchange for a promised payment of either b at $t = 1$ or $B > b$ at $t = 2$ (each investor chooses at $t = 1$ which option he prefers). At $t = 1$, each investor can demand that the entrepreneur liquidate enough of the project (as long as any remains) to pay him b. These claims are met on a first-come-first-serve basis until either there is no more output left or all creditors who demand payment are satisfied.[9] At $t = 2$, any output left is first used to pay up to B to creditors who waited and the entrepreneur keeps the rest.

The firm is *solvent* whenever $q > B$, so that it could pay all its debts if investors wait and *liquid* whenever $\lambda q > b$, so that it can pay all its debts if investors run. It is *super-liquid* if $\lambda q > B$, so that it can pay all its long term debt even if it has to liquidate its assets at $t = 1$. If $b > \lambda B$, the financial structure of the firm is *fragile*, since there are realizations of q that make the firm solvent but illiquid, thus vulnerable to runs. The game induced by this contract is slightly different depending on whether or not the financial structure is fragile (although the equilibrium is continuous at $b = \lambda B$), so I analyze each of these cases in turn.

Fragile structure: $b \in (\lambda B, B)$

Let A denote the fraction of investors who run. Let u_R denote the payoff an investor obtains from running, u_W the payoff from waiting and u_E the payoff for the entrepreneur. Payoffs are shown in table 2.2.

The reason is as follows. *Region 1.* The maximum cash that the firm can obtain at $t = 1$ is λq, and it has to pay Ab. If $A \geq \frac{\lambda q}{b}$, this is not enough, and the firm pays the first $f = \frac{\lambda q}{b}$ investors. Since they arrive in random order, each investor who runs has a probability $\frac{f}{A} = \frac{\lambda q}{Ab}$ of collecting b, so he receives an expected payoff of $\frac{\lambda q}{A}$. The entrepreneur and investors who wait get nothing. *Region 2.* Since $A < \frac{\lambda q}{b}$, the firm can satisfy the claims of all the investors

[9]Since investors are risk-neutral, the equilibrium would be unchanged if payments were pro-rated among all the investors that demand payment. What is key is that investors who do not demand payment get nothing if the output runs out. This is probably a more reasonable assumption if the firm commits to a sequential-service policy than if it is allowed to take time to count how many investors demand payment in order to pro-rate payments. Note that this requires that it be possible to liquidate output at cost $1 - \lambda$ at the same speed as investors run.

Table 2.2: Payoffs when $b \in (\lambda B, B)$

Condition	Region 1 $A \geq \frac{\lambda q}{b}$	Region 2 $\frac{\lambda q}{b} \geq A \geq \frac{q-B}{\frac{b}{\lambda}-B}$	Region 3 $\frac{q-B}{\frac{b}{\lambda}-B} \geq A$
u_R	$\frac{\lambda q}{A}$	b	b
u_W	0	$\frac{q-\frac{Ab}{\lambda}}{1-A}$	B
u_E	0	0	$q - \frac{Ab}{\lambda} - (1-A)B$

who run. In order to pay Ab it must liquidate a fraction $\alpha = \frac{Ab}{\lambda q}$ of output, so that at $t = 2$, there is $(1-\alpha)q = q - \frac{Ab}{\lambda}$ left, but the firm owes $(1-A)B$ to the remaining investors. If $A \geq \frac{q-B}{\frac{b}{\lambda}-B}$, then there is not enough output left to satisfy all these claims and by distributing either on a first-come-first-serve or pro-rata basis, each investor gets and expected payoff of $\frac{q-\frac{Ab}{\lambda}}{1-A}$, while the entrepreneur gets nothing. *Region 3.* If $A < \frac{q-B}{\frac{b}{\lambda}-B}$ the output that remains at $t = 2$ is sufficient to pay $(1-A)B$ to the remaining investors, each of which gets B, while the entrepreneur gets the rest.

Like most models of runs, this game does not feature global strategic complementarities: $u_R(A) - u_W(A)$ is not monotonically increasing in A, but instead reaches a maximum at $A = \frac{\lambda q}{b}$, the point where the run exactly exhausts output. However, like Goldstein and Pauzner (2005)'s model of bank runs, it exhibits one-sided strategic complementarity: $u_R(A) - u_W(A)$ is increasing in A whenever it is negative.

It is immediate from table 2.2 that if $q > \frac{b}{\lambda}$, then waiting is a dominant action for investors, since regions 1 and 2 do not exist and $B > b$. This is a difference with Goldstein and Pauzner (2005), who need to assume that the technology is qualitatively different for sufficiently high fundamentals in order to make sure there is an upper dominance region. Here, since q is unbounded rather than binary, the upper dominance region arises naturally. Conversely, when $q < b$, running is dominant since region 3 does not exist and the condition $b > \frac{q-\frac{Ab}{\lambda}}{1-A}$ always holds. When $q \in \left(b, \frac{b}{\lambda}\right)$, however, investors' best response depends on A. If $A \geq \frac{q-b}{b\left(\frac{1}{\lambda}-1\right)}$, then running is a best response, whereas if $A < \frac{q-b}{b\left(\frac{1}{\lambda}-1\right)}$, then waiting is a best response. This naturally leads to multiple equilibria, as in the example studied by Diamond (2004), Diamond and Dybvig (1983) and many other models of runs.[10] The allocation induced by contract $\{B, b\}$ will depend on how this multiplicity is resolved. However, unlike the example in section 2.4.1, q does not simply scale up the payoffs of every strategy profile proportionately. This creates the potential for a single equilibrium selection criterion to select different equilibria depending on q. Following Morris and Shin (1998), I will focus on the unique equilibrium that is robust to small amounts of idiosyncratic noise in the observation of q by investors.

[10]Since the premium for refinancing $B - b$ is strictly positive, both are strict Nash equilibria.

Assume that instead of observing q, each investor i observes an idiosyncratic signal $x_i = q + \sigma\varepsilon_i$, where ε_i has a standard Normal distribution and is *iid* across investors (I will focus on the limit as $\sigma \to 0$).[11] A strategy for each investor now consists of a function $s_i : \mathbb{R} \to \{R, W\}$ that indicates whether they will Run or Wait depending on the signal they observe. I look for a symmetric threshold equilibrium, where $s_i(x_i) = R$ iff $x_i < q^*$ for some q^* that is the same for all i. By the law of large numbers, the proportion of investors that demand payment is

$$A(q) = \Pr[x_i < q^*|q] = \Phi\left[\frac{q^* - q}{\sigma}\right] \tag{2.16}$$

where Φ is the *cdf* of a standard Normal. Using Bayes' rule, the posterior about q of an agent that observes x_i is

$$f(q|0, x_i) = \frac{\phi\left(\frac{x_i - q}{\sigma}\right) f(q|0)}{\int \phi\left(\frac{x_i - \hat{q}}{\sigma}\right) f(\hat{q}|0)\, d\hat{q}}$$

where ϕ is the *pdf* of a standard Normal and therefore, using (2.16), the posterior about A is

$$f_A(A|x_i) = \sigma \frac{\phi\left(\frac{x_i - q^* + \sigma\Phi^{-1}(A)}{\sigma}\right) f\left(q^* - \sigma\Phi^{-1}(A)\,|0\right)}{\int \phi\left(\frac{x_i - \hat{q}}{\sigma}\right) f(\hat{q}|0)\, d\hat{q}} \frac{1}{\phi\left(\Phi^{-1}(A)\right)} \tag{2.17}$$

Using (2.17), the posterior for the marginal agent, who observes $x_i = q^*$, is

$$f_A(A|x_i) = \frac{\sigma f\left(q^* - \sigma\Phi^{-1}(A)\,|0\right)}{\int \phi\left(\frac{x_i - \hat{q}}{\sigma}\right) f(\hat{q}|0)\, d\hat{q}}$$

As $\sigma \to 0$, both numerator and denominator converge to $\sigma f(x^*|0)$, so it must be that the agent who observes q^* believes that A follows a uniform distribution. Morris and Shin (2003) refer to these as Laplacian beliefs and offer a discussion of their role in coordination games.

Let $u_R(q)$ and $u_W(q)$ denote the expected payoffs from running and waiting respectively obtained by an investor who believes $A \sim U[0, 1]$ and knows q with certainty. Using table

[11] Normality is assumed for illustration purposes but is not essential to the argument. Frankel, Morris, and Pauzner (2003) show that for games with two actions and symmetric players equilibrium selection in the limit is independent of the distribution of noise.

2.2,

$$u_R(q) = \int_0^{\frac{\lambda q}{b}} b dA + \int_{\frac{\lambda q}{b}}^1 \frac{\lambda q}{A} dA = \lambda q \left[1 - \log\left(\frac{\lambda q}{b}\right)\right] \tag{2.18}$$

$$\begin{aligned} u_W(q) &= \begin{cases} \int_0^{\frac{q-B}{\frac{b}{\lambda}-B}} B dA + \int_{\frac{q-B}{\frac{b}{\lambda}-B}}^{\frac{\lambda q}{b}} \frac{q-\frac{Ab}{\lambda}}{1-A} dA = B - \left(q - \frac{b}{\lambda}\right)\log\left(1 - \frac{B\lambda}{b}\right) & \text{if } q \geq B \\ \int_0^{\frac{\lambda q}{b}} \frac{q-\frac{Ab}{\lambda}}{1-A} dA = q - \left(q - \frac{b}{\lambda}\right)\log\left(1 - \frac{q\lambda}{b}\right) & \text{if } q < B \end{cases} \\ &= \min\{B,q\} - \left(q - \frac{b}{\lambda}\right)\log\left(1 - \frac{\min\{B,q\}\lambda}{b}\right) \end{aligned} \tag{2.19}$$

Since the agent that observes q^* must be exactly indifferent between running and not running, must believe $A \sim U[0,1]$ and knows q with arbitrarily high precision, equating (2.18) and (2.19) we obtain the following equation for q^*:

$$\lambda q^* \left[1 - \log\left(\frac{\lambda q^*}{b}\right)\right] = \min\{B,q^*\} - \left(q^* - \frac{b}{\lambda}\right)\log\left(1 - \frac{\min\{B,q^*\}\lambda}{b}\right) \tag{2.20}$$

Non-fragile structure: $b < \lambda B$

In this case the payoffs are as given in table 2.3.

Table 2.3: Payoffs when $b < \lambda B$

	Region 1	Region 2	Region 3
Condition	$A \geq \frac{\lambda q}{b}$	$\min\left\{\frac{\lambda q}{b}, \frac{B-q}{B-\frac{b}{\lambda}}\right\} \geq A$	$\frac{\lambda q}{b} \geq A \geq \frac{B-q}{B-\frac{b}{\lambda}}$
u_R	$\frac{\lambda q}{A}$	b	b
u_W	0	$\frac{q-\frac{Ab}{\lambda}}{1-A}$	B
u_E	0	0	$q - \frac{Ab}{\lambda} - (1-A)B$

The reason is as follows. *Region 1.* This is as before. If $A \geq \frac{\lambda q}{b}$, the run exhausts the firm's resources. *Region 2.* At $t = 2$ the firm has $q - \frac{Ab}{\lambda}$ left and must pay $(1-A)B$. If $A < \frac{B-q}{B-\frac{b}{\lambda}}$, these resources are insufficient to satisfy all remaining investors, so each obtains $\frac{q-\frac{Ab}{\lambda}}{1-A}$. Notice that since $\frac{b}{\lambda} < B$, the firm's $t = 2$ solvency is actually helped by investors who demand payment at $t = 1$. *Region 3.* In this region A is not large enough to cause the firm to exhaust output at $t = 1$, but is sufficiently large to enable the firm to repay all $t = 2$ claims.

From table 2.3, when $q > \frac{b}{\lambda}$, waiting is dominant since region 1 does not exist and the condition $\frac{q-\frac{Ab}{\lambda}}{1-A} > b$, relevant for region 2, always holds. When $q < b$, running is dominant since region 3 does not exist (because $\frac{\lambda q}{b} < \frac{B-q}{B-\frac{b}{\lambda}}$) and the condition $b > \frac{q-\frac{Ab}{\lambda}}{1-A}$ always holds.

When $q \in \left(b, \frac{b}{\lambda}\right)$, investors' best response depends on A. Region 3 again does not exist, so focusing on region 2, investors will prefer to run as long as $b > \frac{q - \frac{Ab}{\lambda}}{1-A}$, which is equivalent to $A > \frac{q-b}{b\left(1-\frac{1}{\lambda}\right)}$. Hence there are multiple equilibria. Computing $u_R(q)$ and $u_W(q)$ as before, the equation defining q^* is

$$\lambda q^* \left[1 - \log\left(\frac{\lambda q^*}{b}\right)\right] = q^* - \left(q^* - \frac{b}{\lambda}\right) \log\left(1 - \frac{q^* \lambda}{b}\right)$$

Since $q^* < \frac{b}{\lambda} < B$, this is simply a special case of (2.20), which describes the solution in this case as well.

Properties of the equilibrium

Lemma 19.

i. (2.20) has a unique positive solution.

ii. The solution q^* is a continuous function of b, B and λ

Equilibrium selection is based on the fact that at q^*, $\frac{\partial u_W(q)}{\partial q} > \frac{\partial u_R(q)}{\partial q}$, i.e. at the margin, additional output benefits those who wait more than those who run and for that reason, higher output will lead agents to coordinate on the no-run equilibrium. Note however that, as illustrated in figure 2.2, this inequality does not for every q (if it did hold for every q, the proof of Lemma 19 would be trivial). Indeed, $lim_{q \to 0} \frac{\partial u_W(q)}{\partial q} = 0$ and $lim_{q \to 0} \frac{\partial u_W(q)}{\partial q} = \infty$. For low values of q, given Laplacian beliefs about A, it is very likely that the firm will be liquidated entirely at $t = 1$. Hence at the margin additional output benefits an agent who decides to run more than an agent who decides not to run.

Lemma 19 shows that a symmetric threshold equilibrium exists, and that it is unique. Standard arguments show that it is unique not just in the class of symmetric threshold equilibria and that indeed it is the only rationalizable outcome. Using (2.16) and the fact that $\sigma \to 0$, we also know that whenever $q > q^*$, $A(q) \approx 0$ and whenever $q < q^*$, $A(q) \approx 1$. Since $q^* < \frac{b}{\lambda}$, this means that whenever q is below q^*, it will be necessary to liquidate all the output and it will still be insufficient to satisfy all the $t = 1$ claimants.

Lemma 20. *q^* is*

i. increasing in b and

ii. weakly decreasing in B

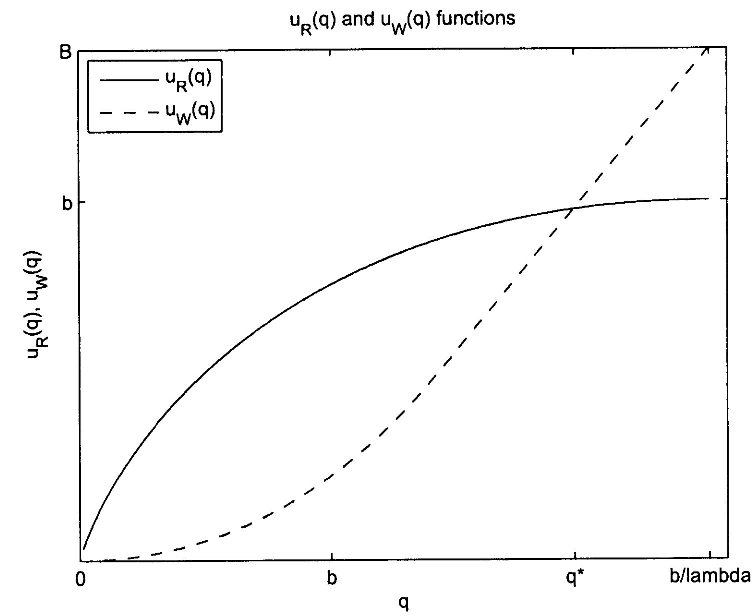


Figure 2.2: Payoffs from running and waiting under Laplacian beliefs

Lemma 20 says that the greater the promised payment at $t = 1$ or the smaller the promised payment at $t = 2$, the higher the output required to prevent a run on the firm.

Lemma 21.

i. $\lim_{b \to B} q^* = \frac{B}{\lambda} = \frac{b}{\lambda}$

ii. $lim_{b \to 0} q^* = 0$

Lemma 21 says that if the promised payment for an investor who runs becomes very similar to the promised payment for one who does not run, any illiquid firm will face a run. On the other hand, as the promised payment for those who run becomes small, runs never take place. Lemma 21 also implies that q^* could be greater or less than B. If $q^* > B$ then there are realizations of q such that the firm is solvent but faces a run; if $q^* < B$ then solvent firms never face runs (even if they are fragile).

The output assignment and liquidation rules that arise from the equilibrium of this game are:

$$B(q) = \begin{cases} \text{if } q \geq q^*: & \min\{B, q\} \\ \text{if } q < q^*: & \lambda q \end{cases} \tag{2.21}$$

$$\alpha(q) = \mathbb{I}\{q < q^*\} \tag{2.22}$$

Notice that b only enters (2.21) through the dependence of q^* on b, but not directly. This is because, as noted above, $q^* < \frac{b}{\lambda}$, so that whenever q is low enough to induce a run, this run exhausts all the output. b only affects how many investors get to receive a payment before output is exhausted but not how much they receive in aggregate. Furthermore, by lemmas 19-21 q^* is a continuous, increasing function of b, ranging from 0 to $\frac{B}{\lambda}$. This means that, given a value of B, the entrepreneur can engineer any desired value of $q^* \in \left(0, \frac{B}{\lambda}\right)$ by an appropriate choice of b.

Comparison of equilibrium conditions (2.21), (2.22) with the optimality conditions (2.14), (2.15) immediately implies the following proposition:

Proposition 14. *The optimal allocation can be reproduced by a contract $\{B, b\}$ if it falls under cases 1 or 2, but not if it falls under case 3.*

Proposition 14 provides a characterization of the conditions under which financial fragility is a solution to the problem of inability to commit. When the required investment is large and the agency problems are not too severe, a contract that makes the firm financially fragile will be able to reproduce the optimal allocation. However, contracts of this kind are unable to lead to early liquidation while allowing the entrepreneur to keep part of the proceeds, as under the optimal contract in case 3. In other words, a contract $\{B, b\}$ cannot induce the liquidation of super-liquid firms. This would be part of optimal allocation when the required investment is not too large, so that (2.3) is easily satisfied, but the entrepreneur's incentive to choose $a = 1$ is strong, so liquidation up to a high level of q is required to satisfy (2.2).

When the optimal allocation falls under case 1, then the equilibrium under the contract $\{B, b\}$ that reproduces it never has runs against solvent but illiquid firms, only a against a subset of insolvent firms (because $q^* < B$). When it falls under case 2, all insolvent firms and some solvent ones suffer runs.

2.5 Discussion

Sections 2.4.1 and 2.4.2 show that seemingly innocuous variations in the basic model lead to either the possibility or the impossibility of implementing the optimal allocation with simple fragile contracts. Why do these variations matter? Two things are required in order to reproduce the optimal allocation: the correct equilibrium selection contingent on early news and an output assignment rule that provides maximal incentives. Variations in the setting affect the possibility of satisfying each of these requirements.

To obtain correct equilibrium selection, using the Morris-Shin criterion, one needs the relative payoffs of running and waiting to vary with early news. This cannot be achieved in the simple binary output case since news affects all payoffs proportionately. Outside this extreme case, however, the relative payoffs of running and waiting will depend on z and therefore the equilibrium that will be played may depend on z, as required. Unfortunately, a richer distribution of output comes at a price: it makes it necessary to specify nontrivial output assignment rules and to consider what restrictions on the shape of these rules are reasonable. Note that if z were contractible, contingent equilibrium selection would be easy to achieve: the contract would simply state that $b = 0$ if $z > z^*$ and $B = 0$ if $z < z^*$, making the decision to run only in the correct states of the world a dominant strategy. It is only the noncontractability of z that makes the problem difficult.

In order to provide maximal incentives with simple contracts one needs to be able to contract upon all the variables that would optimally be used to provide incentives. In general, this includes both q and z since both provide information about a. Simple fragile contracts that do not condition on z do not in general provide maximal incentives. The example considered in section 2.4.2 is special in that optimal allocations can be described in terms of q only and therefore are reproducible with simple fragile contracts. A slightly less special case where a similar result could be achieved would be if z is an imperfect signal of q (as in the general case) but q is a sufficient statistic for a so that the optimal assignment rule does not depend on z.[12] For instance, z could simply be a noisy forecast of q.

An open question remains as to what financial arrangement should be adopted when the optimal allocation defined in section 2.3 cannot be reproduced with simple contracts. It is possible that some degree of financial fragility remains desirable in that it allows the correct equilibrium selection, providing the necessary incentives for the entrepreneur to choose $a = 0$ even if the efficiency cost is greater than in the benchmark optimal allocation.

The model uses the three-period structure that is often used in models of liquidity, but this is for simplicity and is not essential to the argument. Similar issues arise, for instance, in the infinite horizon models of DeMarzo and Fishman (2007) and DeMarzo and Sannikov (2006). In these models ex-post inefficient liquidation is part of an optimal incentive scheme but a single investor who cannot commit would renegotiate rather than liquidate. Mutually incompatible claims satisfied on a first come first serve basis could be used to engineer the right threshold for runs in those models as well.

A subtle issue is whether the financial structure designed to prevent renegotiating away inefficient outcomes does indeed achieve this. When faced with a run, the entrepreneur could

[12]Strictly speaking, $B(z, q) = \min\{q, q^B(z)\}\,[1 - \alpha(z)(1 - \lambda)]$ does depend on z in this case because although $\min\{q, q^B(z)\}$ does not, α does. In order to achieve results similar to those in section 2.4.2 one needs to consider, for instance, contracts that promise a payment b to those who run and a conditional payment $B(\alpha)$, which is a function of the degree of liquidation but not directly of the news, to those who wait.

offer an extra payment to all investors if they abstain from running. If $q^* < B$ then this offer is never credible because in the region where there are runs the firm is insolvent and cannot even satisfy its original claims, let alone increased ones. If $q^* > B$, an increased payment is feasible. Introducing a renegotiation stage changes the nature of the game and introduces several questions: is the renegotiated offer announced to everyone or to each investor as they arrive to the front of the queue? Do agents at the back of the queue observe what has happened at the front? What do they infer from the fact that an offer has been made? One way to give concrete answers to these questions while ruling out renegotiation in equilibrium is the following. Suppose investors (who, recall, observe q with noise and are thus asymmetrically informed) form the belief that in the off-equilibrium event that the entrepreneur makes any offer to renegotiate, it must mean that q is extremely low and therefore running is dominant.[13] This belief sustains a perfect Bayesian equilibrium where the entrepreneur indeed never attempts to renegotiate.

Related to this last issue is the question of bankruptcy. A firm that faces a run will typically seek some form of bankruptcy protection rather than, as the model assumes, comply sequentially with investors' demands for liquidation. Within the logic of the model, bankruptcy protection is harmful since it interferes with ex-ante desirable liquidation.[14] One reinterpretation of the model is that λ simply represents the administrative cost of bankruptcy proceedings. When investors run to demand payment, the firm files for bankruptcy, destroying $(1-\lambda)q$ in the process. This, however, is not fully satisfactory. Bankruptcy typically implies settling debts of equal priority pro-rata disregarding the order in which creditors attempted to collect them. If investors expect a run to lead to bankruptcy and pro-rata payments this removes the incentive to run, defeating the purpose of the fragile structure. In practice, however, it is possible that the firm may pay the first investors who demand payment in full before requesting bankruptcy protection, perhaps because it does not immediately realize the extent of the problem. In Chapter 3 I explore a model of speculative attacks where the agent that is speculated against (in this case the firm) is not sure about the proportion of speculators who attack for nonstrategic reasons (for instance, for liquidity needs) and find conditions under which it is optimal for the firm to pay the first few speculators who run before declaring bankruptcy in order to learn about the severity of the attack. If this is the case then there is indeed an incentive to run despite the possibility of bankruptcy.

[13] What must necessarily be true is that they infer $q < q^*$ since otherwise the entrepreneur should not expect a run and should not make an offer. However, for running to be dominant we need to assume that they infer the stronger condition $q < b$, which makes running dominant no matter how high the renegotiation offer

[14] Diamond (2004) discusses a role for bailouts when λ is random: a rule that prevents liquidation only when it turns out to be very costly can be valuable.

Contracts of the form $\{B, b\}$ can be interpreted and indeed instrumented in different ways. Literally, they are putable debt with principal-plus-interest b up to $t = 1$ and interest $B - b$ accruing between $t = 1$ and $t = 2$. Equivalently, they could be debt b due at $t = 1$ plus an option, in favour of the lender, to refinance for $B - b$. They could also be implemented by short-term debt b alone in the following way: the firm announces (without committing to a contract) that it will offer lenders the option to refinance at $t = 1$ for an interest rate of $B - b$. As discussed above, if lenders interpret any offer other than $B - b$ as a sign that q is very low, then there is a perfect Bayesian equilibrium where the entrepreneur always makes the "standard" refinancing offer $B - b$. Therefore the model predicts that firms that need to discipline management will use some, possibly very simple, form of dispersed short term debt.

While exact empirical counterparts of the main parameters of the model are usually not available, the predictions of the model seem broadly in line with empirical evidence. One premise of the model is that having multiple creditors can be a means of achieving ex-post inefficient outcomes. In a study of creditor pools of distressed German firms, Brunner and Krahnen (2008) find that the turnaround probability is lower when there are many creditors. In a study of UK firms, Franks and Sussman (2005) find that when liabilities are concentrated there are few instances of creditor runs or coordination failure. Furthermore, the model predicts that firms should rely on fragility when agency problems are severe (high v) or the project is not too profitable (high K or low $E(q|0)$). In a sample of Italian firms, Detragiache, Garella, and Guiso (2000) find that those that are less profitable are more likely to borrow from multiple banks. Direct measures of v are hard to find but it seems plausible that it could be correlated with various measures of the difficulty of holding managers accountable. Demirgüç-Kunt and Maksimovic (1999) and Giannetti (2003) find international cross-section evidence that poor legal institutions for protecting creditor rights are associated with a greater proportion of short term debt. Ongena and Smith (2000) and Esty and Megginson (2003) find that it is also associated with borrowing from multiple sources.

Overall, the model provides a way to study multiple equilibria in a framework where they may arise due to the purposeful design of a fragile financial structure. This kind of structure may be useful in simultaneously overcoming the entrepreneur's moral hazard and investors' inability to commit to liquidate the firm.

2.6 Appendix

Proof or Proposition 12. Restate the monotonicity constraint (2.6) as

$$\frac{\partial B(z,q)}{\partial q} = \gamma(z,q) \quad (2.23)$$
$$\gamma(z,q) \geq 0 \quad (2.24)$$

Program (2.1) becomes an optimal control problem, with B as a state variable and γ and α as controls.

The Hamiltonian for the transformed problem is

$$\begin{aligned} H(\alpha,\gamma,B,q) = {} & (1+\rho)\left[\left[1-\alpha(z)(1-\lambda)\right]q - B(z,q)\right] f_{q|z}(q|z,0)\, f_z(z|0) \\ & -\rho\left(\left[1-\alpha(z)(1-\lambda)\right]q - B(z,q)\, f_{q|z}(q|z,1) + v(1-\alpha(z))\right) f_z(z|1) \\ & +\mu\left[B(z,q)-K\right] f_{q|z}(q|z,0)\, f_z(z|0) + \eta(z,q)\left[q\left[1-\alpha(z)(1-\lambda)\right] - B(z,q)\right] \\ & +\varsigma(z,q)\, B(z,q) + \chi(z,q)\,\gamma(z,q) + \psi(z,q)\,\gamma(z,q) \end{aligned}$$

where ρ, μ, $\eta(z,q)$, $\varsigma(z,q)$ and $\chi(z,q)$ are the multipliers on (2.2), (2.3), (2.4), (2.5) and (2.24) respectively and $\psi(z,q)$ is the co-state variable. The first order conditions for α, B and γ respectively are:

$$\alpha = \mathbb{I}\left\{\left(\begin{array}{c} -(1+\rho)(1-\lambda)\left(\int q f_{q|z}(q|z,0)\,dq\right) f_z(z|0) \\ +\rho\left[\left((1-\lambda)\left(\int q f_{q|z}(q|z,1)\,dq\right)+v\right) f_z(z|1)\right] \\ -\int \eta(z,q)\, q(1-\lambda)\,dq \end{array}\right) > 0\right\} \quad (2.25)$$

$$-\frac{d\psi(z,q)}{dq} = f_{zq}(z,q|0)\, d(z,q) - \eta(z,q) + \varsigma(z,q) \quad (2.26)$$

$$0 = \chi(z,q) + \psi(z,q) \quad (2.27)$$

where

$$d(z,q) \equiv \left[-(1+\rho)+\mu\right] + \rho\frac{f_{zq}(z,q|1)}{f_{zq}(z,q|0)} \quad (2.28)$$

Suppose $-(1+\rho)+\mu > 0$. In that case $d(z,q)$ is always positive so it is always profitable to increase B up to the limited liability constraint. But this would mean that the entrepreneur receives nothing. Hence in any solution where the project produces a positive surplus it must be that $-(1+\rho)+\mu < 0$. Assumptions 4 and 5 imply that $d(z,q)$ is weakly decreasing in both q and z. For any given z, define $q^B(z) \in \mathbb{R}^+ \cup \infty$ as the lowest q such that $d(z,q) < 0$ (where $q^B(z) = \infty$ means that $d(z,q) > 0, \forall q$). Since $d(z,q)$ is weakly decreasing in both q and z, it follows that $q^B(z)$ is weakly decreasing. If, for a given z,

$q^B(z) > 0$, then condition (2.26) implies that constraint (2.4) will bind for $q < q^B(z)$ and constraint (2.24) will bind for $q > q^B(z)$. Instead, if $q^B(z) = 0$, then constraint (2.5) for every q. This implies (2.7).

For $q > q^B(z)$ constraint (2.4) does not bind, so $\eta(z,q) = 0$, whereas for $q < q^B(z)$ neither (2.24) nor (2.5) bind, so $\varsigma(z,q) = \frac{d\psi(z,q)}{dq} = 0$ and $\eta(z,q) = f_{zq}(z,q|0)\,d(z,q)$. Overall, this means that

$$\eta(z,q) = \max\{f_{zq}(z,q|0)\,d(z,q), 0\} \tag{2.29}$$

Replacing (2.29) in (2.25) and rearranging:

$$\alpha = \mathbb{I}\left\{(1-\lambda)\int qg(z,q)\,f_{q|z}(q|z,0)\,dq + \rho v\frac{f_z(z|1)}{f_z(z|0)} > 0\right\} \tag{2.30}$$

where

$$g(z,q) \equiv \min\left\{-(1+\rho) + \rho\frac{f_{zq}(z,q|1)}{f_{zq}(z,q|0)}, -\mu\right\}$$

Assumption 4 implies that the second term in (2.30) is decreasing in z. Assumption 3 implies that an increase in z brings about a FOSD increase in q and since $g(z,q)$ is negative and weakly decreasing in both z and q. this means that the first term is decreasing in z as well. Together, this implies that there will be a threshold level z^* such that (2.8) holds. □

Proof of Proposition 13. Note that

$$\frac{\partial r}{\partial z^*} = (\lambda-1)\int \min\{q, q^B(z)\}\,f_{zq}(z,q|0)\,dz \leq 0 \tag{2.31}$$

$$\frac{\partial r}{\partial q^B(z)} = \left[1 - F\left(q^B|z,0\right)\right]\left[1 - \mathbb{I}(z<z^*)(1-\lambda)\right] f_z(z|0) > 0 \tag{2.32}$$

Since $r\left(z^*, q^B(z)\right)$ is decreasing in z^*, (2.31) implies that either $z^* = -\infty$ or $\frac{\partial c}{\partial z^*} > 0$, since otherwise z^* could be lowered without violating any constraints.

Furthermore, suppose that in response to some parameter change not involving f, $q^B(z)$ increases for some z. Using (2.28), this implies that $\frac{-(1+\rho)+\mu}{\rho}$ has increased, which in turn implies that $q^B(z)$ rises for every z.

1. $\frac{\partial c}{\partial K} = 0$ and $\frac{\partial r}{\partial K} = -1$ and $\frac{\partial c}{\partial v} = -\left[1 - F_z(z^*|1)\right] < 0$ and $\frac{\partial r}{\partial v} = 0$. Increasing K or v reduces the constraint set, which implies that z^* must increase. By equation (2.31), a higher z^* tightens constraint (2.10), so, by equation (2.32), higher $q^B(z)$ is required to satisfy it, so $q^B(z)$ must be higher for all z.

2. Rewrite $f_{zq}(z,q|a)$ as $f_q(q|a)f_{z|q}(z|q,a)$. Using this, a FOSD increase in $f_q(q|0)$ (with

$f_{z|q}(z|q,0)$ unchanged) relaxes both constraints, so z^* decreases and $q^B(z)$ decreases for all z. A FOSD increase in $f_q(q|1)$ (with $f_{z|q}(z|q,1)$ unchanged) tightens constraint (2.9), leaving constraint (2.10) unchanged, so z^* increases and $q^B(z)$ increases for all z.

3. An increase in λ relaxes constraint (2.10) but has ambiguous effects on (2.9):

$$\begin{aligned}\frac{\partial r}{\partial \lambda} &= \int_{\underline{z}}^{z^*}\int \min\left\{q, q^B(z)\right\} f_{zq}(z,q|0)\,dqdz > 0 \\ \frac{\partial c}{\partial \lambda} &= \int_{\underline{z}}^{z^*}\int \max\left\{q - q^B(z), 0\right\}\left[f_{zq}(z,q|0) - f_{zq}(z,q|1)\right] dqdz\end{aligned}$$

Hence the overall effect is ambiguous.

□

Proof of Lemma 19. i. From (2.18),

$$\frac{\partial^2 u_R(q)}{\partial q^2} = -\lambda q$$

so $u_R(q)$ is strictly concave. From (2.19),

$$\frac{\partial^2 u_W(q)}{\partial q^2} = \begin{cases} 0 & \text{if } q > B \\ \frac{\lambda}{b-\lambda q} & \text{if } q \le B \end{cases}$$

so $u_W(q)$ is weakly convex, which implies that $u_R(q)$ and $u_W(q)$ intersect at most twice. Furthermore, whenever $q < b$ then $u_R(q) > u_W(q)$ and whenever $0 > q > \frac{b}{\lambda}$ then $u_R(q) < u_W(q)$, which means that they must intersect an odd number of times for $q > 0$. This implies that they intersect only once for $q > 0$ and that at the point of intersection q^*, we have $\frac{\partial u_W(q^*)}{\partial q} > \frac{\partial u_R(q^*)}{\partial q}$.

ii. This follows from the fact that $u_R(q)$ and $u_W(q)$ are both continuous in b, B and λ and $u_R(q) = u_W(q)$ has a unique solution.

□

Proof of Lemma 20. i. Differentiating (2.20) implicitly,

$$\frac{\partial q^*}{\partial b} = \frac{\frac{\partial u_R}{\partial b} - \frac{\partial u_W}{\partial b}}{\frac{\partial u_W}{\partial q} - \frac{\partial u_R}{\partial q}}$$

Lemma 19 shows that the denominator is positive. The numerator is also positive since $\frac{\partial u_R}{\partial b} = \frac{\lambda q}{b} > 0$ (which means that the more investors are promised if they run the more attractive it is to run) and

$$\begin{aligned}
\frac{\partial u_W}{\partial b} &= \frac{b - \lambda q}{b - \lambda \min\{B, q\}} \frac{\min\{B, q\}}{b} + \frac{1}{\lambda} \log\left(1 - \frac{\lambda \min\{B, q\}}{b}\right) \\
&\leq \frac{\min\{B, q\}}{b} + \frac{1}{\lambda} \log\left(1 - \frac{\lambda \min\{B, q\}}{b}\right) \\
&\leq \frac{\min\{B, q\}}{b} + \lim_{\lambda \to 0} \frac{1}{\lambda} \log\left(1 - \frac{\lambda \min\{B, q\}}{b}\right) \\
&\quad \text{(since the last expression is decreasing in } \lambda\text{)} \\
&= 0 \text{ (using l'Hopital's rule)}
\end{aligned}$$

(which means that the more is paid to those who run, the worse off those who do not run are).

ii. Differentiating (2.20) implicitly,

$$\frac{\partial q^*}{\partial B} = \frac{\frac{\partial u_R}{\partial B} - \frac{\partial u_W}{\partial B}}{\frac{\partial u_W}{\partial q} - \frac{\partial u_R}{\partial q}}$$

The numerator in this case is nonpositive since $\frac{\partial u_R}{\partial B} = 0$ (the payoff of those who run does not depend on how much is promised to those who do not run) and

$$\frac{\partial u_W}{\partial B} = \begin{cases} \frac{\lambda(q-B)}{b-\lambda B} & \text{if } q > B \\ 0 & \text{if } q \leq B \end{cases} \geq 0$$

□

Proof of Lemma 21. By direct computation using (2.20). □

Chapter 3

Speculative Attacks against a Strategic Agent with Incomplete Information

3.1 Introduction

In August and September of 1992, the Bank of England sold billions of dollars of foreign reserves in an attempt to maintain the pound's exchange rate within the bands of the European ERM. On September 16 it finally gave up and abandoned the ERM. The cost of having attempted to defend the parity was later estimated at approximately £ 3.3 billion.

This pattern, of first attempting to defend the existing regime and giving up after some time, is a common feature of speculative attacks and explaining it presents a theoretical challenge. So-called first-generation models based on Krugman (1979), such as Flood and Garber (1984) and Broner (2007) account for it in a very simple way: by assuming the government follows and attempts to defend an unsustainable policy for exogenous reasons and abandons it only when forced to do so. However, these models leave unanswered the question of why a government would behave this way.

As formalized by Obstfeld (1996) and others, speculative attacks often have a self-fulfilling aspect: if enough agents believe the government will abandon a regime, they will act in ways that make it optimal for the government to indeed abandon it. The unsatisfying conclusion of models of self-fulfilling equilibria is that, at least within some range of parameters, the outcome is arbitrary or depends on ad-hoc unmodeled factors. Following Morris and Shin (1998), many authors have argued that modifying the common-knowledge assumptions of games that have self-fulfilling equilibria may help to resolve this indeterminacy and provide more definite predictions.

The basic building block of models in this literature is a game played by many small agents ("speculators") who are incompletely informed about the relevant parameters of the economy, and one large agent ("the bank") who has complete information. Typically, the focus of the analysis is on the structure of actions and information of the game played by the speculators. In contrast, the bank's information and objectives are usually described in very simple terms so that its strategy can be summarized, or even replaced, by a simple rule such as "defend the existing regime unless a mass of speculators larger than A^* attacks it".

This paradigm (and for that matter the multiple-equilibria paradigm too) fails to account for why the bank, acting rationally, would ever engage in an unsuccessful partial defence of the regime, as the Bank of England did in 1992. In these models, the bank knows the "fundamentals" of the economy and can therefore perfectly predict (or in some versions observe) what the size of the speculative attack is going to be. As long as defending the regime is costly, it would never be the case that it attempts to defend it but surrenders after some time, since this failure would have been foreseen.

However, in some contexts the possibility of a temporary, unsuccessful defence of the status quo makes an important difference. For example, if we wish to apply these methods to the study of bank runs, as Goldstein and Pauzner (2005) do, the *only* reason why depositors would run is if they believe that the bank will pay some of them before falling or deciding to suspend convertibility.

How can the theory account for the phenomenon of unsuccessful defences? One possibility, implicity assumed by Morris and Shin (1998), is that defending the regime is not costly at the margin; conditional on regime change, the bank does not have a preference for how far it held out. In many contexts this is not a reasonable assumption: the losses to the central bank's balance sheet are greater the more reserves it has spent trying to defend a fixed exchange rate; the liquidation costs a banks incurs in are smaller the sooner it suspends convertibility; the retaliation against a dictator is likely to be harsher the longer it held on to power.

If unsuccessful defences are costly, a theory that accounts for them must somehow allow the bank to have uncertainty about the size of the attack it is going to face.[1] With uncertainty and suitable timing assumptions, the bank's decision may be viewed as an optimal stopping problem: as the attack escalates, it must decide whether to surrender or to continue to defend the regime in the hope that the attack will be over soon, using its appropriately updated

[1]This is also noted by Goldstein, Ozdenoren, and Yuan (2008). In their model the central bank has uncertainty about the value of maintaining a fixed exchange rate and may learn about this by observing the speculative attack. Their model does not, however, allow the central bank to surrender to the attack midway.

beliefs about how large the attack is likely to be.

One way to introduce uncertainty is to abandon Nash equilibrium as a solution concept. In any Nash equilibrium, the bank knows the strategies of the speculators, and is thus able to predict the size of the speculative attack with no uncertainty.[2] However, under appropriate conditions (although not in the Morris-Shin limit), both attacking a regime and not attacking it are rationalizable actions. If the requirement that the bank know the speculators' strategies is dropped, it is possible to simply endow it with beliefs about the joint distribution of (rationalizable) actions the speculators might take, and under these beliefs a policy of partial defence may indeed be optimal. The trouble with explaining the phenomenon along these lines is that this explanation relies on arbitrary assumptions about the bank's beliefs and simply picks one of the many rationalizable action profiles.

This paper introduces uncertainty into the bank's decision problem in a different way. As in Diamond and Dybvig (1983), there is aggregate uncertainty about the distribution of (heterogeneous) preferences in the population. This distribution is governed by a single random parameter θ, and neither the bank nor the speculators know its realization. Although the bank knows the equilibrium strategies, the equilibrium size of the attack, conditional on the bank's information, is a random variable, so it faces a nontrivial optimal stopping problem.

For this problem to have an interior solution, in which the bank surrenders after some time despite having incurred sunk costs of defence, it must be that as the attack progresses the bank becomes sufficiently more pessimistic about the magnitude of the attack that it will face. In particular, it must think that an attack that is not over by the stopping point is unlikely to be over soon after that, which is consistent with some ex-ante beliefs $f(A)$ about the size of the attack A but not with others. However, I show that given any probability distribution $f(A)$, it is possible to reverse-engineer a prior about θ such that $f(A)$ is indeed the endogenous probability distribution in an equilibrium of the game. Hence a basic finding is that the model is able to deliver the kinds of uncertainty that could justify partial defences.

This alone, however, provides no clear guidance as to what factors make unsuccessful defences likely to arise. To answer this question, I specialize the model to a simple example with linear payoffs and normal uncertainty. In this case, I show that partial defences will occur if heterogeneity in preferences is sufficiently small relative to aggregate uncertainty about average preferences. The model can be shown to have multiple equilibria in certain cases. The bank's uncertainty and the possibility of partial unsuccessful defences, however, are not due to this but to uncertainty about outcomes *within* a given equilibrium.

[2]This is strictly true in pure strategies, though possibly not in correlated mixed strategies if the bank does not observe the correlating device.

The model introduces dynamics into the model in an extremely limited way. The speculators must still decide whether or not to attack at the beginning of the game and are not allowed to learn from one another's actions. This abstracts from what is certainly a very important dimension in real-life speculative attacks. Furthermore, although I will informally describe the bank's actions as "waiting" there is no real temporal dimension to the problem: the attack is assumed to build up continuously and the bank must choose a point in this continuum to stop defending the attack.

Section 3.2 introduces the model and defines equilibrium conditions. Section 3.3 explores, in the general case, how uncertainty about preferences translates into uncertainty about outcomes. Section 3.4 discusses the special case of normal uncertainty and linear payoffs. Section 3.5 briefly concludes. The Appendix contains the proofs omitted in the text.

3.2 The Model

The backbone of the model is a variant of the simple binary action game of Morris and Shin (2003). There is a measure-one continuum of speculators, indexed by $i \in [0,1]$ and a bank. At the beginning of the game, each speculator chooses one of two actions: either attack the current policy regime, $a_i = 1$, or not attack, $a_i = 0$. Agents who decide to attack form a queue and inform the bank of their decision one by one. The bank cannot observe the length of the queue. After each one of them announces his decision the bank has two options: either abandon the regime or to continue to support it. If the bank abandons the regime the game ends in Defeat for the bank. If it continues to support it, two things may happen. If the attacker was the last in the queue, the game ends in Survival for the bank; otherwise, the game continues with the next attacker.

3.2.1 Speculators' Payoffs

The payoff from not attacking is normalized to zero whereas attacking has an idiosyncratic cost c_i and brings a benefit of 1 if the regime is defeated.[3] Agent i's payoff is therefore given by

$$u_i = a_i \left[\mathbb{I}\{\text{Defeat}\} - c_i\right]$$

[3] A more complete model of bank runs would require modeling the dependence of speculators' payoff on the size of the attack and not just the survival of the bank. For instance, some speculators may join the queue but never get paid because the bank surrenders before they reach the front of the queue. This introduces strategic substitutability as well as strategic complementarity in speculators' actions since, beyond the point where the run topples the bank, the incentive to attack diminishes with the size of the attack. Goldstein and Pauzner (2005) show how to adapt global games techniques to account for this.

Costs of attacking are distributed in the population according to the distribution $G(c_i|\theta)$ with density $g(c_i|\theta)$, where θ is a parameter. $G(c_i|\theta)$ is assumed to be decreasing in θ, so c_i is increasing in θ in a FOSD sense. c_i is not constrained to lie in $[0,1]$ so one of the actions may be dominant for some of the speculators. In the context of bank runs, where attacking means withdrawing a deposit, $G(0|\theta)$ can represent the fraction of speculators that receive a liquidity shock and need to attack, as in Diamond and Dybvig (1983), and $1-G(1|\theta)$ may represent the fraction of speculators who have long-term deposits, or are out of town or are otherwise unable to participate in the run. In the context of currency attacks, $G(0|\theta)$ may represent the demand for foreign currency to pay for imports and $1-G(1|\theta)$ may represent the post-devaluation domestic money demand, as in Krugman (1979).

3.2.2 Bank's payoff

Let $A = \int_0^1 a_i d_i$ denote the total mass of speculators who attack. The bank's payoff is given by a pair of functions: $S(A)$, $D(\tau)$. $S(A)$ is the payoff the bank obtains if the game ends in Survival after A attackers. $D(\tau)$ is the payoff the bank obtains if the game ends in Defeat after τ attackers, i.e. if the bank abandons the regime after supporting it against τ attackers. Assume S and D are weakly decreasing, so defending the regime is costly at the margin, and $S(0) > D(0)$, so if no one attacks the bank prefers to survive.

The four panels in Figure 3.1 illustrate different possible cases of payoffs the bank might have. Panel (i) shows the payoffs assumed by Morris and Shin (1998), which are the special case where $S(A) = v - A$ and $D(\tau) = 0$. As mentioned in the introduction, for these payoffs the bank's problem is trivial since it would always defend the regime up to the point where $A = v$. Panel (ii) shows an interpretation where the bank has finite liquidity reserves. As proposed by Bagehot (1873), the bank does not mind using reserves as long as it succeeds in maintaining the regime, but it would rather not waste them on an unsuccessful defence. Panel (iii) shows a case where there is a fixed benefit of maintaining the regime and the cost of defending it is linear in the size of the attack, both for successful and failed defences. Panel (iv) shows a similar example but with increasing marginal cost of using reserves.

3.2.3 Information

At the beginning of the game, nature draws the random variable θ from some prior density $p(\theta)$. No one observes the realized θ, but speculators observe their own realized c_i. A speculator's individual cost is informative about the distribution of costs in the population

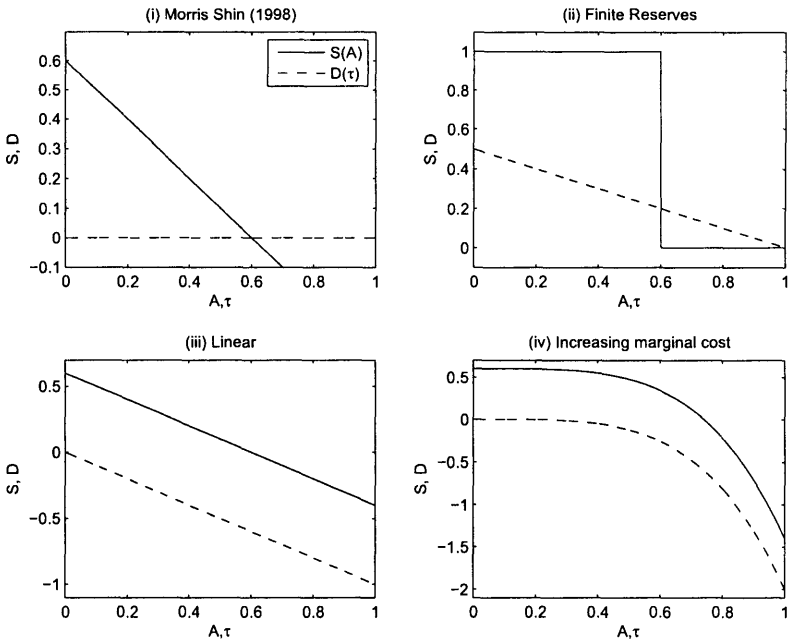


Figure 3.1: Examples of payoffs for the bank

summarized by θ; applying Bayes' rule, a speculator's posterior is:

$$p\left(\theta|c_i\right) = \frac{g\left(c_i|\theta\right)p\left(\theta\right)}{\int_\theta g\left(c_i|\theta\right)p\left(\theta\right)d\theta} \tag{3.1}$$

Definition 6. *$g\left(c_i|\theta\right)$ satisfies the monotone inference property if, for any $p\left(\theta\right)$, the posterior distribution $P\left(\theta|c_i\right)$ is decreasing in c_i*

If $g\left(c_i|\theta\right)$ satisfies this property then a speculator who observes a higher cost c_i for himself will infer that the parameter θ is likely to be higher, which implies that the costs of the other speculators are also likely to be higher.

3.2.4 Equilibrium

I focus on monotone equilibria, defined as perfect Bayesian equilibria such that a speculator attacks if and only if c_i is less than some threshold c^*, which is identical for all speculators. In such an equilibrium, the aggregate size of the attack is $A\left(\theta\right) = \Pr\left[c_i \leq c^*|\theta\right] = G(c^*|\theta)$. Since the bank does not know θ, it does not know the realized value of A. Instead, it has beliefs given by the density

$$f\left(A\right) = p\left(G^{-1}\left(c^*;A\right)\right)\left|\frac{\partial G^{-1}\left(c^*;A\right)}{\partial A}\right| \tag{3.2}$$

where G^{-1} is the inverse of G with respect to its second argument, i.e. $A \equiv G(c|G^{-1}(c;A))$. While the attack is taking place, the bank gradually learns about the realized value of A but in a limited way: it only learns from the fact that the attack is not over yet. Hence its optimal stopping problem can be formulated simply as the following one-dimensional optimization problem:[4]

$$\max_{\tau \in [0,1]} V(\tau) = \int_0^\tau S(A) f(A) dA + [1 - F(\tau)] D(\tau) \tag{3.3}$$

The first term of (3.3) is the value the bank obtains from the possibility of surviving if the size of the attack turns out to be less than τ. The second term is the value it will obtain if it surrenders after τ attackers. Program (3.3) may or may not have an interior optimum. In case it does, the first order necessary condition is

$$\begin{aligned} V'(\tau) &= [S(\tau) - D(\tau)] f(\tau) + [1 - F(\tau)] D'(\tau) = 0 \\ \Rightarrow h(\tau) [S(\tau) - D(\tau)] &= -D'(\tau) \end{aligned} \tag{3.4}$$

where $h(A) \equiv \frac{f(A)}{1-F(A)}$ is the hazard function of the size of the attack. The marginal benefit of waiting, given by the difference between the value of survival and defeat, $S(\tau) - D(\tau)$ times the instantaneous probability that the attack will be over, $h(\tau)$, must equal the marginal cost of waiting, which is $-D'(\tau)$. Since the function $V(\tau)$ is not necessarily concave, condition (3.4) could also denote a local minimum. The second order condition for a local maximum is:

$$V''(\tau) = [S'(\tau) - 2D'(\tau)] f(\tau) + [S(\tau) - D(\tau)] f'(\tau) + [1 - F(\tau)] D''(\tau) \leq 0 \tag{3.5}$$

Even if (3.4) and (3.5) hold, they may identify a local but not global maximum, so in general it is only possible to say that the bank's best response is given by $\tau^* \in \arg\max_{\tau \in [0,1]} V(\tau)$.

The distinction between interior and corner solutions to program (3.3) is important. If $\tau^* = 0$ then the bank does not attempt to resist at all and falls as soon as the attack begins. If $\tau^* = 1$ then the bank withstands the attack no matter how large it is (although this does not rule out that it may ex-post regret having done so if the realized value of $S(A(\theta))$ is less than $D(0)$). Unsuccessful partial defences only occur when $\tau^* \in (0,1)$ and the realized $A(\theta)$ happens to be greater than τ^*.

[4]Denoting by $W(a)$ the value function conditional on having resisted an attack of size a, then the problem can be represented by the differential equation $W'(a) = -h(a)[S(a) - W(a)]$, with value matching condition $W(\tau) = D(\tau)$ and smooth pasting condition $W'(\tau) = D'(\tau)$. These are equivalent to the first order conditions in the text.

The regime will survive iff $A(\theta) = G(c^*|\theta) \leq \tau^*$. Higher values of θ are associated with FOSD higher costs of attacking and therefore, for given c^*, smaller attacks. If $\lim_{\theta\to\infty} G(c^*|\theta) > \tau^*$, then the smallest possible attack is sufficiently large for the bank to fail; conversely if $\lim_{\theta\to-\infty} G(c^*|\theta) \leq \tau^*$, then the largest possible attack is too small to make the bank fail. Otherwise there exists a unique θ^* such that the bank fails whenever $\theta < \theta^*$. This critical value is defined by

$$G(c^*|\theta^*) = \tau^* \tag{3.6}$$

For an individual speculator, attacking is a best response if, given his information, the probability of the bank failing is greater than the cost of attacking, i.e. if

$$\Pr[\theta \leq \theta^*|c_i] = P(\theta^*|c_i) > c_i \tag{3.7}$$

If $g(c_i|\theta)$ satisfies the monotone inference property then the LHS of (3.7) is decreasing in c_i, so given θ^* there exists a unique $c^* \in [0,1]$ such that the speculators attack iff $c_i \leq c^*$. This shows that the best response to threshold strategies are threshold strategies. The speculator who has cost $c_i = c^*$ must be indifferent between attacking and not attacking, which implies

$$P(\theta^*|c^*) = c^* \tag{3.8}$$

Definition 7. *A monotone equilibrium consists of a threshold* $\theta^* \in \mathbb{R}\cup\{-\infty, +\infty\}$*, strategies* $\tau^* \in [0,1]$ *and* $c^* \in [0,1]$ *and beliefs* $f(A)$ *such that*

1. *Either (i) (3.6) holds, (ii)* $G(c^*|\theta) > \tau^*, \forall\theta$ *and* $\theta^* = +\infty$ *or (iii)* $G(c^*|\theta) < \tau^*, \forall\theta$ *and* $\theta^* = -\infty$.

2. τ^* *solves program (3.3) using* $f(A)$*, so the bank is best-responding given its beliefs.*

3. *(3.8) holds, so the speculators are best-responding.*

4. $f(A)$ *satisfies (3.2), so the bank's beliefs are consistent with the speculators' strategies.*

3.3 Beliefs about the size of the attack

Suppose the probability distribution of the sizes of attacks A were known to be given by some density function $f(A)$. In principle, $f(A)$ could be estimated empirically from the sizes of actual (successful or unsuccessful) speculative attacks. Could the model account for $f(A)$ as arising from the equilibrium of the game described above? The answer is that it is always possible to reverse-engineer some prior $p(\theta)$ such that the game has an equilibrium where the unconditional density of A is $f(A)$.

Proposition 15. *Let $f(A)$ be an arbitrary continuous pdf on $[0,1]$ and let $g(c_i|\theta)$ be a continuous pdf on $\mathbb{R}$ such that*

1. *$g(c_i|\theta)$ satisfies the monotone inference property*

2. *$lim_{\theta\to\infty}G(c_i|\theta) = 0, \forall c_i$ and $lim_{\theta\to-\infty}G(c_i|\theta) = 1, \forall c_i$*

Then there exists a prior $p(\theta)$ such that under primitives $g(c_i|\theta)$, $p(\theta)$ there is an equilibrium where the unconditional distribution of A is $f(A)$

The way to construct such an equilibrium is as follows. Beliefs $f(A)$ immediately imply a best response τ^* for the bank. Given any strategy c^* for the speculators, it is mechanically possible to find a prior about the value of θ, $p_{c^*}(\theta)$, such that the posterior belief about the size of the attack is indeed $f(A)$. Under prior $p_{c^*}(\theta)$, the speculators' best response to a cutoff strategy c^* will be some other cutoff strategy $T(c^*, p_{c^*})$. An equilibrium consists of a fixed point such that $T(c^*, p_{c^*}) = c^*$ and continuity implies that such a fixed point exists.

The importance of Proposition 15 resides in that, depending on the functional forms of $S(A)$ and $D(\tau)$, it could be that program (3.3) has an interior solution only for some functions $f(A)$ and not for others. This means that some functions $f(A)$ will be consistent with the possibility of observing failed partial defences while others will not. Proposition 15 says that it is always possible to generate examples that produce $f(A)$ that are consistent with an equilibrium with interior τ^*.

Table 3.1 shows example of what priors could justify different $f(A)$ functions. In all cases $f(A)$ is some Beta distribution, the bank's payoffs are $S(A) = 0.6 - A$, $D(\tau) = -\tau$ and $g(c_i|\theta) = \phi(c_i - \theta)$, where ϕ is the density for a standard normal, which satisfies the conditions of Proposition 15, as shown in Lemma 22 below.

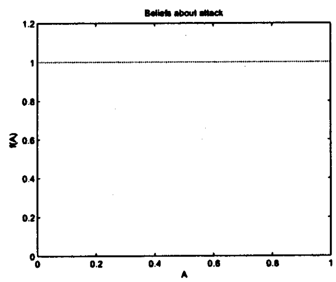
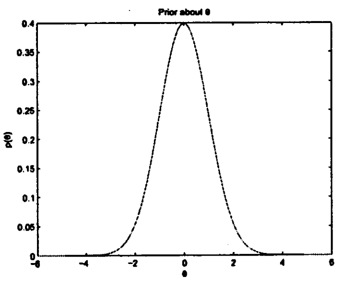
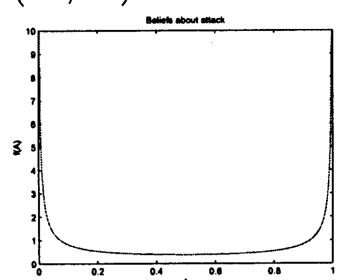
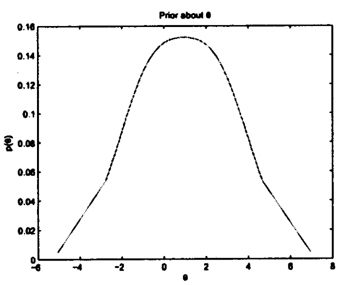
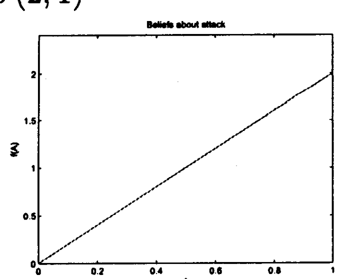
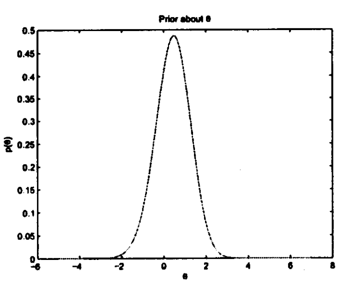
Lemma 22. $g(c_i|\theta) = \sqrt{\alpha}\phi(\sqrt{\alpha}(c_i - \theta))$ *satisfies the monotone inference property*

Proposition 15 also implies that making precise predictions requires imposing more structure on the problem, since under the general specification virtually anything could happen. The following section explores a simple special case of the model.

3.4 Normal uncertainty and linear payoffs

Consider the following special case of the model. The bank's payoffs are $S(A) = v - A$ and $D(\tau) = -\tau$, so there is a fixed value of survival $v \in (0,1)$ and a constant marginal

Table 3.1: Examples of priors that would lead to beliefs about A

		Equilibrium		
$f(A)$	$p(\theta)$	c^*	τ^*	θ^*
$\beta(1,1)$ (uniform) 		0	1	$-\infty$
$\beta(.25,.25)$ 		0.95	0.056	2.54
$\beta(2,1)$ 		1	0	∞

cost of defending the regime, as in panel (iii) of Figure 3.1. The costs of attacking are normally distributed in the population, with mean θ and variance $\frac{1}{\alpha_c}$ and the prior on θ is also normal, with mean μ and variance $\frac{1}{\alpha_\mu}$, i.e. $g(c_i|\theta) = \sqrt{\alpha_c}\phi\left(\sqrt{\alpha_c}\left(c_i - \theta\right)\right)$ and $p\left(\theta\right) = \sqrt{\alpha_\mu}\phi\left(\sqrt{\alpha_\mu}\left(\theta - \mu\right)\right)$. $\frac{1}{\alpha_c}$ and $\frac{1}{\alpha_\mu}$ are measures of heterogeneity and aggregate uncertainty respectively.

In order to highlight the role of the bank's uncertainty and speculators' incomplete information, I first analyze variants of the game with common knowledge and where the bank is informed.

3.4.1 Common knowledge benchmark

Assume first that θ is common knowledge for both the bank and the speculators. The bank's decision becomes simpler because, in any Nash equilibrium, it knows what size of attack it will face. If the bank decides that it will not outlast the attack then it finds it optimal to choose $\tau^* = 0$. As in Morris and Shin (1998), the bank will abandon the regime iff $A\left(\theta\right) > v$.

A mass $\Phi\left(-\sqrt{\alpha_c}\theta\right)$ of speculators have $c_i < 0$ so attacking is dominant for them. Conversely, a mass $1 - \Phi\left(\sqrt{\alpha_c}\left(1-\theta\right)\right)$ have $c_i > 1$ so not attacking is dominant for them. If θ is such that $v - \Phi\left(-\sqrt{\alpha_c}\theta\right) > 0 > v - \Phi\left(\sqrt{\alpha_c}\left(1-\theta\right)\right)$, i.e. if $\theta \in \left[-\frac{\Phi^{-1}(v)}{\sqrt{\alpha_c}}, 1 - \frac{\Phi^{-1}(v)}{\sqrt{\alpha_c}}\right]$ then the game will have multiple equilibria. If the bank expects all the speculators who have $c_i \in [0,1]$ to attack, then $\tau = 0$ is a best response, which in turn justifies their decision to attack. Conversely, if the bank expects all the speculators who have $c_i \in [0,1]$ not to attack, then any $\tau^* > \Phi\left(-\sqrt{\alpha_c}\theta\right)$ is a best response, which justifies the speculators' decision.

3.4.2 Informed bank benchmark

Now assume instead that the bank knows the realized value of θ but the speculators do not. As with common knowledge, in any Nash equilibrium the bank will always choose either $\tau^* = 0$ or $\tau^* = 1$ because it will know the size of the attack. I will look for a monotone equilibrium such that the bank chooses $\tau^* = 1$ iff $\theta \geq \theta^*$.

From equation (3.1), a speculator's Bayesian posterior about θ is a normal distribution, with mean $\frac{\alpha_\mu\mu+\alpha_c c^*}{\alpha_\mu+\alpha_c}$ and variance $\frac{1}{\alpha_\mu+\alpha_c}$. Given a cutoff θ^* for the bank's strategy, speculator's best response cutoff c^* is given by:

$$\Phi\left(\sqrt{\alpha_\mu + \alpha_c}\left(\theta^* - \frac{\alpha_\mu\mu + \alpha_c c^*}{\alpha_\mu + \alpha_c}\right)\right) = c^* \tag{3.9}$$

which is just a special case of the indifference condition (3.8).

The size of the attack will be given by

$$A(\theta) = \Phi\left(\sqrt{\alpha_c}\,(c^* - \theta)\right)$$

which is decreasing in θ. This implies that, as long as $v \in (0,1)$, there will be a unique cutoff θ^* such that the bank prefers $\tau = 1$ to $\tau = 0$ iff $\theta \geq \theta^*$. The cutoff is given by the indifference condition

$$\Phi\left(\sqrt{\alpha_c}\,(c^* - \theta^*)\right) = v \tag{3.10}$$

Solving (3.9) and (3.10), the speculators' cutoff must satisfy

$$\Phi\left(\sqrt{\alpha_c}\left(\frac{\alpha_\mu (c^* - \mu)}{\alpha_\mu + \alpha_c} - \frac{\Phi^{-1}(c^*)}{\sqrt{\alpha_\mu + \alpha_c}}\right)\right) = v \tag{3.11}$$

The derivative of the left hand side of (3.11) is

$$\phi(\cdot)\sqrt{\alpha_c}\left(\frac{\alpha_\mu}{\alpha_\mu + \alpha_c} - \frac{1}{\phi\left(\Phi^{-1}(c^*)\right)\sqrt{\alpha_\mu + \alpha_c}}\right) \leq \phi(\cdot)\sqrt{\alpha_c}\left(\frac{\alpha_\mu}{\alpha_\mu + \alpha_c} - \frac{\sqrt{2\pi}}{\sqrt{\alpha_\mu + \alpha_c}}\right)$$

Therefore if

$$\frac{\alpha_\mu}{\sqrt{\alpha_\mu + \alpha_c}} \leq \sqrt{2\pi} \tag{3.12}$$

the left hand side is decreasing and (3.11) has a unique solution. Furthermore, if condition (3.12) does not hold, there exists values of μ and v such that (3.11) has multiple solutions. Condition (3.12) says that in order to guarantee uniqueness in the game where the bank has complete information, heterogeneity $\frac{1}{\alpha_c}$ must be small relative to aggregate uncertainty $\frac{1}{\alpha_\mu}$. When heterogeneity is small, the idiosyncratic cost c_i is a very good signal about θ, which rules out multiple self-fulfilling equilibria.

The following proposition summarizes the above benchmark results.

Proposition 16.

1. *Under common knowledge, there are multiple equilibria if* $\theta \in \left[-\frac{\Phi^{-1}(v)}{\sqrt{\alpha_c}}, 1 - \frac{\Phi^{-1}(v)}{\sqrt{\alpha_c}}\right]$.

2. *If the bank knows the realized* θ *but the speculators do not, there is a unique equilibrium for every* μ, v *iff* $\frac{\alpha_\mu}{\sqrt{\alpha_\mu+\alpha_c}} \leq \sqrt{2\pi}$.

3. *In both cases* $\tau^* \in \{0,1\}$, *so there are never unsuccessful defences.*

3.4.3 Uninformed bank

Now consider the game as described in section 3.2, where the bank does not observe θ. Given a threshold θ^* for the bank's survival, speculators' best response cutoff is still given by (3.9). The threshold θ^* is related to the bank's strategy by equation (3.6), which reduces to

$$\theta^* = c^* - \frac{\Phi^{-1}(\tau^*)}{\sqrt{\alpha_c}} \tag{3.13}$$

if $\tau \in (0,1)$, $\theta^* = +\infty$ if $\tau^* = 0$ and $\theta^* = -\infty$ if $\tau^* = 1$.

With linear $S(\cdot)$ and $D(\cdot)$ payoff functions, the FOC and SOC for an interior solution to the bank's optimal stopping problem (3.3) simplify to:

$$vh(\tau) = 1 \tag{3.14}$$

$$h'(\tau) \leq 0 \tag{3.15}$$

The SOC (3.15) says that for the bank to find it optimal to abandon its defence at some interior point τ, $f(A)$ must be such that it is decreasingly likely that the attack will be over soon.

Conditions (3.14) and (3.15) only identify a local optimum. For a global optimum, the bank must compare the value it obtains from an interior τ to the value of offering no resistance $V(0) = 0$ and the value of resisting any possible attack $V(1) = v - E(A)$ (as well as comparing it to other local optima if there are any).

Finally, by equation (3.2), the bank's beliefs about the attack it will face are given by:

$$f(A) = \frac{\sqrt{\frac{\alpha_\mu}{\alpha_c}}\phi\left(\sqrt{\frac{\alpha_\mu}{\alpha_c}}\Phi^{-1}(A) - \sqrt{\alpha_\mu}(c^* - \mu)\right)}{\phi\left(\Phi^{-1}(A)\right)} \tag{3.16}$$

In what follows I make use of the following properties of $f(A)$

Lemma 23.

1. $E(A) = \Phi\left(\frac{c^*-\mu}{\sqrt{\frac{1}{\alpha_\mu}+\frac{1}{\alpha_c}}}\right)$

2. *If* $\alpha_\mu < \alpha_c$, *(or* $\alpha_\mu = \alpha_c$ *and* $c^* < \mu$*), then* $\lim_{A\to 0} f(A) = \infty$

3. *If* $\alpha_\mu > \alpha_c$ *and* $c^* > \mu$, *then* $f(A)$ *has increasing hazard*

Part 1 of Lemma 3.16 simply computes the expected size of the attack from the point of view of the bank. If the cutoff c^* for not attacking is high compared to μ, the prior mean of c_i, then attacks will tend to be larger. The expected attack will be more sensitive to this difference when the bank has less overall (aggregate plus idiosyncratic) uncertainty about any speculator's c_i. Part 2 of Lemma 3.16 states that, if aggregate uncertainty is large relative to heterogeneity, then the probability that attacks will be very small is high. The reason is that, for a given c^*, small heterogeneity makes the size of the attack more sensitive to the realization of θ, while large aggregate uncertainty makes realizations of θ themselves more variable, which makes realized values of A near the extremes (and in particular near $A = 0$) more likely. Part 3 of Lemma 3.16 says that if instead aggregate uncertainty is small relative to heterogeneity and in addition the cutoff c^* is higher than the prior mean of θ, then $f(A)$ has increasing hazard. The reason is that relatively small aggregate uncertainty shifts $f(A)$ towards the center rather than the extremes, while $c^* > \mu$ shifts it to the right, which suffices for the hazard function to be increasing.

I distinguish between three different kinds of equilibria: "no resistance" equilibria, with $\tau^* = 0$; "full resistance" equilibria, with $\tau^* = 1$ and "waiting" equilibria, with $\tau \in (0, 1)$ and characterize necessary and sufficient conditions under which each may exist.

3.4.4 No resistance equilibria

In a no resistance equilibrium, the bank settles for obtaining $V(0) = 0$ by giving up immediately. Since the regime always falls, i.e. $\theta^* = \infty$, then any speculator for whom not attacking is not dominant will attack, i.e. $c^* = 1$.

A simple necessary condition for no resistance to be optimal is that $V'(0) = vf(0) - 1 \leq 0$.[5] As long as the density at 0 is greater than $\frac{1}{v}$, i.e. as long as there is sufficient chance that the attack will be very small, at least a little resistance will be preferable to no resistance at all. Hence a necessary condition for a no resistance equilibrium is that the conditions of Lemma 23.2 not hold, i.e. $\alpha_\mu > \alpha_c$, (or $\alpha_\mu = \alpha_c$ and $\mu \leq 1$).

Under the conditions of Lemma 23.3, there can never be an interior solution to the bank's problem since this would contradict (3.15). The bank will not value the option to wait and will simply choose between $\tau = 0$ and $\tau = 1$. By Lemma 23.1, the former is preferred if $\Phi\left((1-\mu)/\sqrt{\frac{1}{\alpha_\mu} + \frac{1}{\alpha_c}}\right) > v$. Hence the conditions $\alpha_\mu > \alpha_c$, $1 > \mu$ and $\Phi\left((1-\mu)/\sqrt{\frac{1}{\alpha_\mu} + \frac{1}{\alpha_c}}\right) > v$ are sufficient to ensure that there is a no resistance equilibrium.

[5] Strictly speaking, the density at 0 may not be well defined. Let $f(0)$ mean $\lim_{A \to 0} f(A)$.

3.4.5 Full resistance equilibria

In a full resistance equilibrium, the bank obtains a value $V(1) = v - E(A)$. The regime never falls, so $\theta^* = -\infty$ and $c^* = 0$.

Since the bank could always obtain zero by choosing $\tau = 0$, a necessary condition for this kind of equilibrium to exist is $v \geq E(A) = \Phi\left(-\mu/\sqrt{\frac{1}{\alpha_\mu} + \frac{1}{\alpha_c}}\right)$. Furthermore, under the conditions for Lemma 23.3, there can never be an interior solution for the bank's problem, so conditions $\alpha_\mu > \alpha_c$, $\mu < 0$ and $v \geq \Phi\left(-\mu/\sqrt{\frac{1}{\alpha_\mu} + \frac{1}{\alpha_c}}\right)$ are sufficient to ensure that there is a full resistance equilibrium.

3.4.6 Waiting equilibria

In a waiting equilibrium, the bank chooses some intermediate $\tau^* \in (0,1)$. By (3.15), this requires that the hazard function be decreasing at some point. By Lemma 23.3, this means that either $\alpha_\mu \leq \alpha_c$ or $\mu \geq 0$ must hold. Furthermore, if $V'(0) > 0$ and $V(0) > V(1)$ then any optimum must necessarily be interior. Therefore, if the conditions for Lemma 23.2 hold and $v < \Phi\left(-\mu/\sqrt{\frac{1}{\alpha_\mu} + \frac{1}{\alpha_c}}\right)$ we know that any equilibrium must be a waiting equilibrium.

3.4.7 Discussion

The conditions for existence of the various types of equilibria are summarized in the following proposition.

Proposition 17. *The following are necessary and sufficient conditions for no resistance, full resistance and waiting equilibria respectively*

	Necessary conditions	*Sufficient conditions*
No resistance	$\Phi\left(\frac{1-\mu}{\sqrt{\frac{1}{\alpha_\mu}+\frac{1}{\alpha_c}}}\right) \geq v$ $\alpha_\mu > \alpha_c$, *(or* $\alpha_\mu = \alpha_c$ *and* $\mu \leq 1$*)*	$\Phi\left(\frac{1-\mu}{\sqrt{\frac{1}{\alpha_\mu}+\frac{1}{\alpha_c}}}\right) \geq v$ $\alpha_\mu > \alpha_c$ $\mu < 1$
Full resistance	$\Phi\left(\frac{-\mu}{\sqrt{\frac{1}{\alpha_\mu}+\frac{1}{\alpha_c}}}\right) \leq v$	$\Phi\left(\frac{-\mu}{\sqrt{\frac{1}{\alpha_\mu}+\frac{1}{\alpha_c}}}\right) \leq v$ $\alpha_\mu > \alpha_c$ $\mu < 0$
Waiting	$\alpha_\mu \leq \alpha_c$ *(or* $\mu \geq 0$*)*	$\alpha_\mu < \alpha_c$ $\Phi\left(\frac{-\mu}{\sqrt{\frac{1}{\alpha_\mu}+\frac{1}{\alpha_c}}}\right) > v$

Notice that Proposition 17 does not rule out the possibility of multiple equilibria. Indeed, if

$$v \in \left[\Phi\left(\frac{-\mu}{\sqrt{\frac{1}{\alpha_\mu} + \frac{1}{\alpha_c}}} \right), \Phi\left(\frac{1-\mu}{\sqrt{\frac{1}{\alpha_\mu} + \frac{1}{\alpha_c}}} \right) \right] \tag{3.17}$$

then the necessary conditions for both full resistance equilibria and no resistance equilibria hold. This interval always exists, although it becomes smaller as either α_μ or α_c become large. If, in addition, $\mu < 0$, the sufficient conditions for both full resistance and no resistance equilibria hold as well and we are certain to have multiple equilibria.

The logic behind self-fulfilling equilibria is not exactly the same as in the case where the bank knows θ. In the game where the bank is informed, multiplicity arises when information is sufficiently common across speculators to allow for coordination on different equilibria. This requires *small* aggregate uncertainty, so that speculators' common prior is highly informative and *large* heterogeneity so that speculator's idiosyncratic cost is not very informative about the aggregate state. In the game where the bank is uninformed, multiplicity arises if the expected mass of speculators who do not have a dominant strategy is sufficiently large to justify each of the bank's possible decision rules. This requires that the bank's desire to survive v take intermediate values and that the unconditional distribution of c_i be sufficiently concentrated, which requires *small* heterogeneity and *small* aggregate uncertainty.

The timing assumptions matter for the types of equilibria that may arise. Under the usual assumption that the bank moves after observing A, its decision is always trivial. If it could, the bank would want to commit to full resistance in order to steer the equilibrium towards its desired outcome, where the attack is small and the bank survives. However, its commitment is not credible: if the attack turns out to be large, the bank will choose to fail, making the attack self-fulfilling. Instead, when the bank has uncertainty and makes decisions continuously, a form of commitment may arise. In equilibria where $\tau^* = 1$, during the course of play the bank will always (correctly) believe that an ongoing attack is likely to be over soon. This justifies its persistence in defending the regime even when ex-post it would prefer not to have done so.

Interestingly, the same forces that lead to uniqueness in the informed-bank case lead to equilibria with waiting when the bank is uninformed. When there is little heterogeneity and large aggregate uncertainty, private sources of information (i.e. the idiosyncratic cost c_i), which the bank does not have access to, are relatively more informative. This force eliminates multiplicity in the informed-bank case, as in Morris and Shin (2000), but heightens the informational disadvantage of the bank in the uninformed-bank case. For the uninformed bank, an interior τ is justified when the attack is likely to be very small or very large: it is willing to wait up to τ because of the chance that the attack might be very small but if after

τ attackers the attack is not over then it realizes that the attack will not be over soon and it abandons its defence of the regime. As discussed above, probability distributions of this kind, which have most of the mass in the extremes, arise when there is little heterogeneity relative to the amount of aggregate uncertainty.

One prediction of the model is that one should expect to see failed partial defences against speculative attacks in the same kinds of environments when one also observes successful defences against small attacks. For instance, in a fixed-exchange regime where money demand is highly variable, the central bank will often experience what the model describes as small speculative attacks, simply from shifts in $G(0|\theta)$. It will therefore be more willing to engage in a partial defence than a central bank in a country where money demand is very stable and a speculative attack is not easily mistaken for day-to-day variation in money demand.

Proposition 17 does not fully characterize the possible equilibria that will arise for each combination of parameters, but it is possible to compute the equilibria numerically in order to find sharper boundaries for the regions in the parameter space where each equilibrium occurs. Figure 3.2 shows the regions of α_μ, α_c where each of the types of equilibrium occurs, fixing $\mu = 0.3$ and setting $v = 0.4$ or $v = 0.6$ in each case.

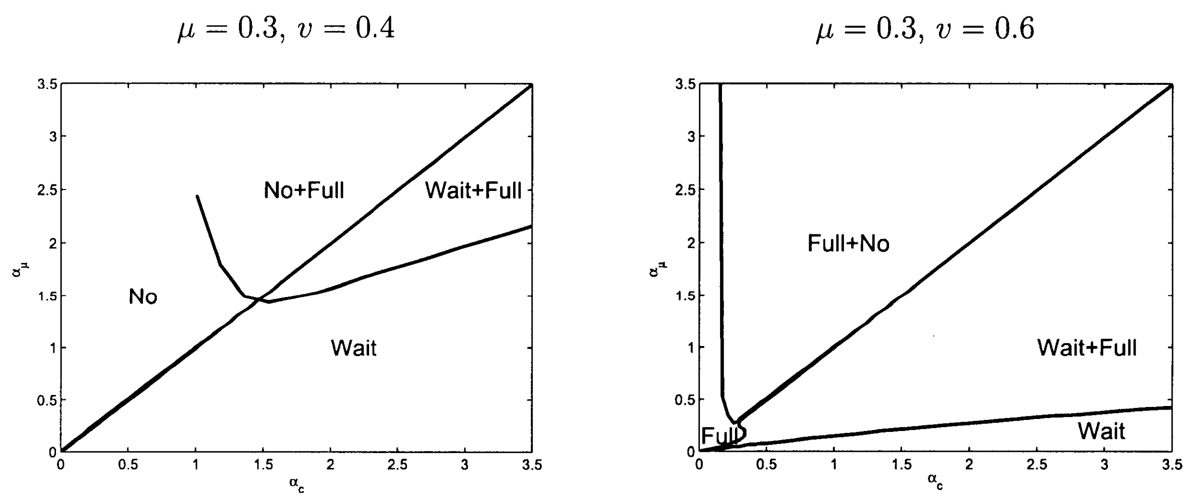


Figure 3.2: Regions of no resistance, full resistance and waiting equilibria

In the case where $v = 0.4$, Proposition 17 implies that no resistance equilibria will exist iff $\alpha_\mu > \alpha_c$, i.e. in the northwest half of the graph. When this condition does not hold, $V'(0) > 0$ so some resistance is preferable, and since v is not too high there exists a waiting equilibrium. Moreover, when the unconditional distribution of c_i is concentrated (the northeast of the graph), there also exists a full-resistance equilibrium. In it, speculators' knowledge that $\tau^* = 1$ shifts enough mass towards a no-attack strategy that full resistance is desirable. Hence there is a region of multiplicity with waiting and full resistance equilibria and a region of multiplicity with no resistance and full resistance equilibria.

In the case where $v = 0.6$, when α_c is sufficiently greater than α_μ (the southeast of the graph), then the distribution $f(A)$ is shifted towards the extremes and $E(A)$ approaches $\frac{1}{2}$. This means that, although $\tau = 1$ is preferable to $\tau = 0$ (since $v > \frac{1}{2}$), the bank can always do even better by choosing τ positive but small, since the likelihood of a very small attack is high and it is not very costly to wait to see if it indeed takes place. Hence, only waiting equilibria exist sufficiently southeast in the graph. However, as in the case where $v = 0.4$, there is a region where waiting and full resistance equilibria both exist. When $\alpha_\mu > \alpha_c$, waiting equilibria do not exist since $f(A)$ has more mass in the center, which makes waiting less desirable. If both α_c and α_μ are sufficiently high, no resistance equilibria may exist, as the bank's pessimism becomes self-justifying. Otherwise, only full resistance equilibria exist.

Within the waiting-equilibrium region in the right panel of Figure 3.2, Figure 3.3 shows how the equilibrium stopping point τ^* is affected by various parameters.

Panel (i) shows that the high ratios of heterogeneity to aggregate uncertainty lead to lower resistance in waiting equilibria (as well as ruling out waiting equilibria entirely if they are sufficiently high). This is because higher heterogeneity leads to a $f(A)$ with less mass at the extremes, which means that $h(\tau)$ will be decreasing only for very low values of τ. Panel (ii) shows that higher overall uncertainty for the bank (measured by $\frac{1}{\alpha_\mu} + \frac{1}{\alpha_\mu}$) leads to more waiting. This is because in these examples it happens that $c^* > \mu$, so by (3.16), this makes small attacks more likely, which justifies waiting more. Panel (iii) shows that the more the bank values survival the longer it is willing to wait. This is due to two reinforcing effects: firstly, given a function $f(A)$, (3.14) implies that higher v requires a lower hazard for the bank not to wish to continue defending; since the hazard must be decreasing at an optimum, higher v implies waiting more; secondly, a higher τ^* leads to a lower equilibrium c^* for the speculators, which makes a small attack more likely and justifies waiting more. Finally, panel (iv) shows that the higher the bank's prior belief about speculators' costs of attacking, the more it will be willing to wait, simply because this will lead it to believe that attacks are likely to be small.

3.5 Final Remarks

If banks and governments undertake costly defence measures when faced with speculative attacks and after some time decide to abandon them, then (assuming they are acting rationally) it must be that in the meantime they learned something about the environment or the actions of the speculators that they did not know at first. This paper provides one possible explanation of where their original uncertainty may stem from: uncertainty about

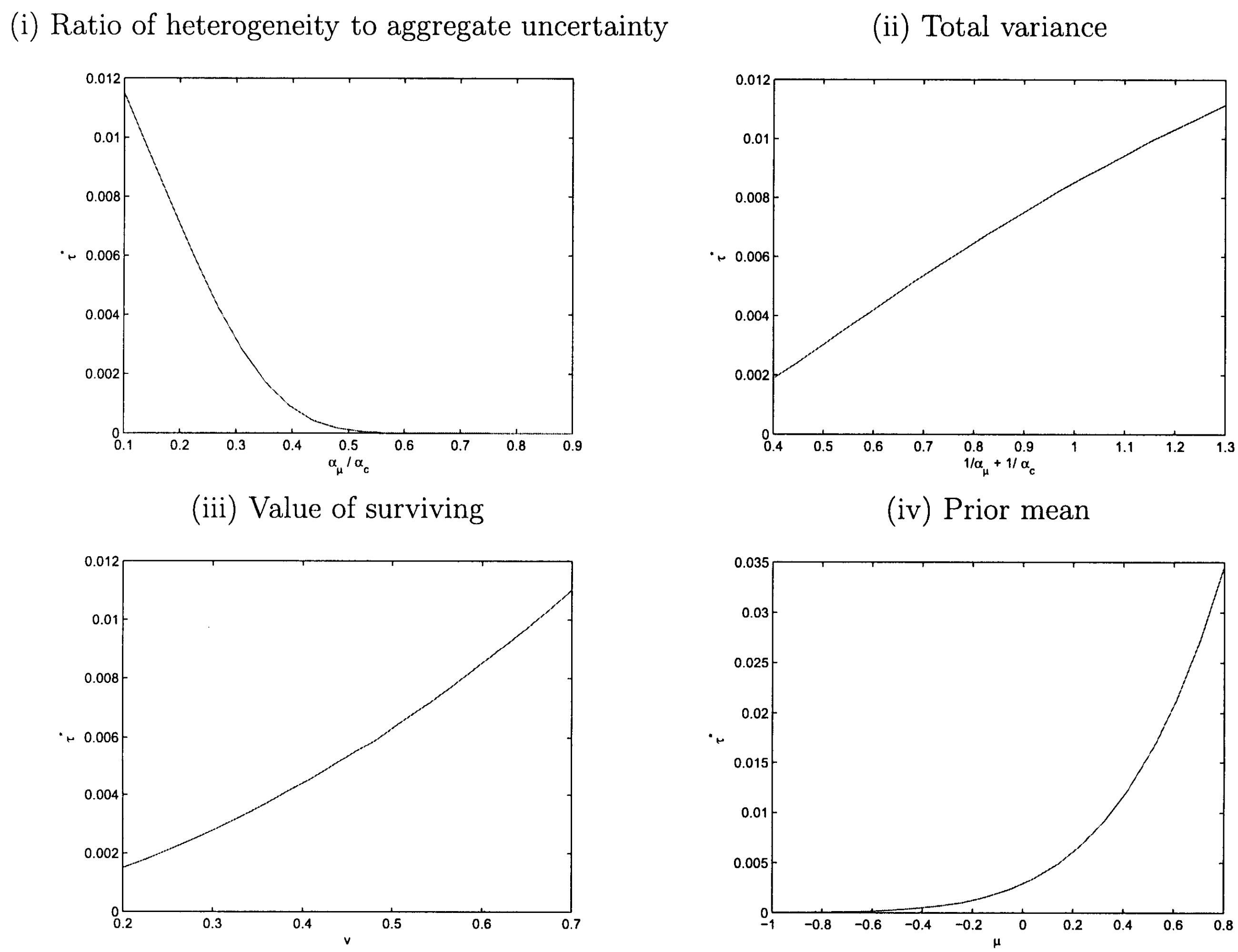


Figure 3.3: Equilibrium τ^* for different parameters

(some aspect of) the distribution of speculators' payoffs from attacking or not attacking the regime.

Under fairly general conditions, virtually any beliefs about the attack could be consistent with the equilibrium of a simple coordination game. In a special case with normal uncertainty and linear payoffs, beliefs that would justify some, but not complete, defence of the status quo arise when aggregate uncertainty is great compared to the degree of heterogeneity in the population, so that very small or very large attacks are likely.

Of course, this is not the only possible source of uncertainty that banks or governments may face in these episodes. They could be unsure, as in Goldstein, Ozdenoren, and Yuan (2008), about the costs of regime change (v in this model) or about what information the public has. Part of the analysis of the present model is likely to extend to these settings, such as the nature of the optimal stopping problem and the key role of the (endogenous) hazard function. Other aspects, such as the role of heterogeneity, are more special to the exact way uncertainty is introduced in the model.

One important feature of real speculative attacks that the model does not include is the possibility that speculators may learn as the attack progresses. An extension of the framework along those lines is left for future work.

3.6 Appendix

Proof of Proposition 15. Given $f(A)$, let τ^* solve program (3.3) and define the operator $T(c^*)$ by the following series of steps:

1. Given c^*, let θ^* satisfy (3.6) if a solution exists, $\theta = \infty$ if $G(c^*|\theta) > \tau^*, \forall\theta$ and $\theta = -\infty$ if $G(c^*|\theta) < \tau^*, \forall\theta$

2. Using (3.2), let

$$p(\theta) = \left. \frac{f(A)}{\left|\frac{\partial G^{-1}(c^*;A)}{\partial A}\right|} \right|_{A=G(c^*|\theta)} \tag{3.18}$$

   Property 2 in the statement of the proposition ensures that $G^{-1}(c^*; A)$ exists so $p(\theta)$ is well defined.

3. Let $T(c^*)$ be the solution to (3.8) where θ^* is the value obtained in step 1 and the function $p(\theta)$ derived in step 2 is used in (3.1) to compute $P(\theta^*|c)$. Since $g(c_i|\theta)$ satisfies the monotone inference property, this equation always has a unique solution.

Since $f(A)$ and $g(c_i|\theta)$ are continuous, then the operator $T(c^*) : \mathbb{R} \longrightarrow [0,1]$ is a continuous function, so it must have a fixed point in $[0,1]$. If c^* is such a fixed point, then under prior $p(\theta)$ given by (3.18), $\{\tau^*, c^*, \theta^*, f(A)\}$ is an equilibrium of the game.

□

Proof of Lemma 22. Using (3.1)

$$
\begin{aligned}
p\left(\hat{\theta}|c\right) &= \frac{\sqrt{\alpha}\phi\left(\sqrt{\alpha}\left(c-\theta\right)\right)p\left(\theta\right)}{\int_{\theta}\sqrt{\alpha}\phi\left(\sqrt{\alpha}\left(c-\theta\right)\right)p\left(\theta\right)d\theta} \\
P\left(\hat{\theta}|c\right) &= \frac{\int_{\theta\leq\hat{\theta}}\phi\left(\sqrt{\alpha}\left(c-\theta\right)\right)p\left(\theta\right)d\theta}{\int_{\theta}\phi\left(\sqrt{\alpha}\left(c-\theta\right)\right)p\left(\theta\right)d\theta}
\end{aligned}
$$

Taking derivatives and rearranging:

$$
\begin{aligned}
\frac{\partial P\left(\hat{\theta}|c\right)}{\partial c} &= \frac{\alpha}{\left[\int_{\theta}\phi\left(\sqrt{\alpha}\left(c-\theta\right)\right)p\left(\theta\right)d\theta\right]^2}\left[\begin{array}{c}\left(\int_{\theta\leq\hat{\theta}}\theta\phi\left(\sqrt{\alpha}\left(c-\theta\right)\right)p\left(\theta\right)d\theta\right)\left(\int_{\theta\geq\hat{\theta}}\phi\left(\sqrt{\alpha}\left(c-\theta\right)\right)p\left(\theta\right)d\theta\right)- \\ \left(\int_{\theta\geq\hat{\theta}}\theta\phi\left(\sqrt{\alpha}\left(c-\theta\right)\right)p\left(\theta\right)d\theta\right)\left(\int_{\theta\leq\hat{\theta}}\phi\left(\sqrt{\alpha}\left(c-\theta\right)\right)p\left(\theta\right)d\theta\right)\end{array}\right] \\
&\leq \frac{\alpha}{\left[\int_{\theta}\phi\left(\sqrt{\alpha}\left(c-\theta\right)\right)p\left(\theta\right)d\theta\right]^2}\left[\begin{array}{c}\hat{\theta}\left(\int_{\theta\leq\hat{\theta}}\phi\left(\sqrt{\alpha}\left(c-\theta\right)\right)p\left(\theta\right)d\theta\right)\left(\int_{\theta\geq\hat{\theta}}\phi\left(\sqrt{\alpha}\left(c-\theta\right)\right)p\left(\theta\right)d\theta\right)- \\ \hat{\theta}\left(\int_{\theta\geq\hat{\theta}}\phi\left(\sqrt{\alpha}\left(c-\theta\right)\right)p\left(\theta\right)d\theta\right)\left(\int_{\theta\leq\hat{\theta}}\phi\left(\sqrt{\alpha}\left(c-\theta\right)\right)p\left(\theta\right)d\theta\right)\end{array}\right] \\
&= 0
\end{aligned}
$$

□

Proof of Lemma 23.

1. By direct computation

2.

$$
\begin{aligned}
f(A) &= \frac{\sqrt{\frac{\alpha_\mu}{\alpha_c}}\phi\left(\sqrt{\frac{\alpha_\mu}{\alpha_c}}\Phi^{-1}(A)-\sqrt{\alpha_\mu}\left(c^*-\mu\right)\right)}{\phi\left(\Phi^{-1}(A)\right)} \\
&= \sqrt{\frac{\alpha_\mu}{\alpha_c}}\exp\left[\frac{1}{2}\left(1-\frac{\alpha_\mu}{\alpha_c}\right)\left[\Phi^{-1}(A)\right]^2+2\frac{\alpha_\mu}{\sqrt{\alpha_c}}\left(c^*-\mu\right)\Phi^{-1}(A)-\alpha_\mu\left(c^*-\mu\right)^2\right]
\end{aligned}
$$

   Taking the limit as $A\longrightarrow 0$ gives the result.

3. Let $\gamma=\sqrt{\frac{\alpha_\mu}{\alpha_c}}$ and $C=-\sqrt{\alpha_\mu}\left(c^*-\mu\right)$

$$
h(A)=\frac{\frac{\phi\left(\gamma\Phi^{-1}(A)+C\right)}{\phi\left(\Phi^{-1}(A)\right)}}{\int\limits_A^1\frac{\phi\left(\gamma\Phi^{-1}(a)+C\right)}{\phi\left(\Phi^{-1}(a)\right)}da}
$$

Define

$$
\begin{aligned}
u(A) &\equiv \phi\left(\gamma\Phi^{-1}(A)+C\right) \\
v(A) &\equiv \phi\left(\Phi^{-1}(A)\right) \\
w(A) &\equiv \int_A^1 \frac{\phi\left(\gamma\Phi^{-1}(a)+C\right)}{\phi\left(\Phi^{-1}(a)\right)}da
\end{aligned}
$$

Taking derivatives,

$$
\begin{aligned}
h'(A) &= \frac{u'(A)\,v(A)\,w(A)-u(A)\,v'(A)\,w(A)-u(A)\,v(A)\,w'(A)}{[w(A)]^2} \\
&= \frac{u(A)}{[w(A)]^2}\left\{\left[\left(1-\gamma^2\right)\Phi^{-1}(A)-\gamma C\right]\int_A^1 \frac{\phi\left(\gamma\Phi^{-1}(a)+C\right)}{\phi\left(\Phi^{-1}(a)\right)}da+\phi\left(\gamma\Phi^{-1}(A)+C\right)\right\} \\
&= \frac{u(A)}{[w(A)]^2}\left\{\frac{\left(1-\gamma^2\right)\Phi^{-1}(A)-\gamma C}{\gamma}\int_{\gamma\Phi^{-1}(A)+C}^1 \phi(x)\,dx+\phi\left(\gamma\Phi^{-1}(A)+C\right)\right\} \\
&= \frac{u(A)}{[w(A)]^2}\left[1-\Phi\left(\gamma\Phi^{-1}(A)+C\right)\right]\left\{\frac{\left(1-\gamma^2\right)\Phi^{-1}(A)-\gamma C}{\gamma}+H\left(\gamma\Phi^{-1}(A)+C\right)\right\}
\end{aligned}
$$

where I have used the change of variable $x=\gamma\Phi^{-1}(a)+C$ and H is the hazard function of the standard normal distribution. The assumptions of the Lemma can be restated as $\gamma>1$, $C<0$. For $A<\frac{1}{2}$, they imply that all the terms in brackets are positive, so the hazard must be increasing. For $A\geq\frac{1}{2}$, use the fact that $H(y)>y$ so

$$
\frac{\left(1-\gamma^2\right)\Phi^{-1}(A)-\gamma C}{\gamma}+H\left(\gamma\Phi^{-1}(A)+C\right)\geq-\gamma\Phi^{-1}(A)-C+H\left(\gamma\Phi^{-1}(A)+C\right)>0
$$

□

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