OBSERVATIONS OF SHORT PERIOD

INTERNAL WAVES IN MASSACHUSETTS BAY

by

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ABSTRACT

Short period (7-minutes) temperature fluctuations were
measured in 82 m of water from instrumented buoys moored 9 km
west of a prominent sill in Massachusetts Bay. The high fre-
quency oscillations begin with a large abrupt rise in temperature
and persist for about three hours. The average time interval
between the onset of successive groups of high frequency fluc-
tuations was 12.4 hours. Prior to their onset the temperature
is steady. The high frequency fluctuations are coherent and
of uniform phase in the vertical direction. The distribution
of the rms vertical displacements corresponds to the eigen-
function of the first mode of internal waves, which was com-
puted from measurements of density and velocity. Long crested,
short wavelength, dark narrow surface bands were related to
the high frequency temperature fluctuations. Groups of high
frequency large amplitude fluctuations occurred in velocity
measurements made 200 m from the temperature instrumented buoys.
The high frequency fluctuations occurring at regular intervals
of time seem to be internal waves of mode one propagating in
the same direction with respect to the moving medium.

Thesis Supervisor: Henry M. Stommel
Title: Professor of Oceanography

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Introduction

A form of organized fluctuations occurring in a stably stratified fluid and having periods varying from minutes to the half-pendulum day is internal gravity wave motion. These waves participate in the general variability of the ocean (Fofonoff, 1967), although the extent of participation is unknown. They may provide a sink for the ordinary tide (Munk and Macdonald, 1960, p.219; Cox and Sandstrom, 1962), and they represent a possible mechanism for mixing the ocean waters below the surface layer (Munk, 1966; Woods, 1968).

A complete statistical description of internal waves is both difficult and expensive, for it requires lengthy time series of a conservative property at many depths and at many horizontal locations. Haurwitz (1953) has shown that many identifications of internal waves may have been incorrect because the duration of the measurements was short and the fluctuations could have represented random or disorganized motion. Measurements exhibiting significant coherency (Sabinin and Shulepov, 1965) have been established less often than those containing a lack of coherency (Haurwitz et al, 1959; White, 1967).

During the 1966 summer, buoys instrumented for recording temperature at nine different depths were moored in 82 m of water 9 km west of a (magnetic) north-south trending submarine sill known as Stellwagen Bank (figure 1). This location (called

LOCATION OF STATION T

—·—·—· Vertical Section
Across Station T

(after CGS Chart 1106)

Figure 1.

2.
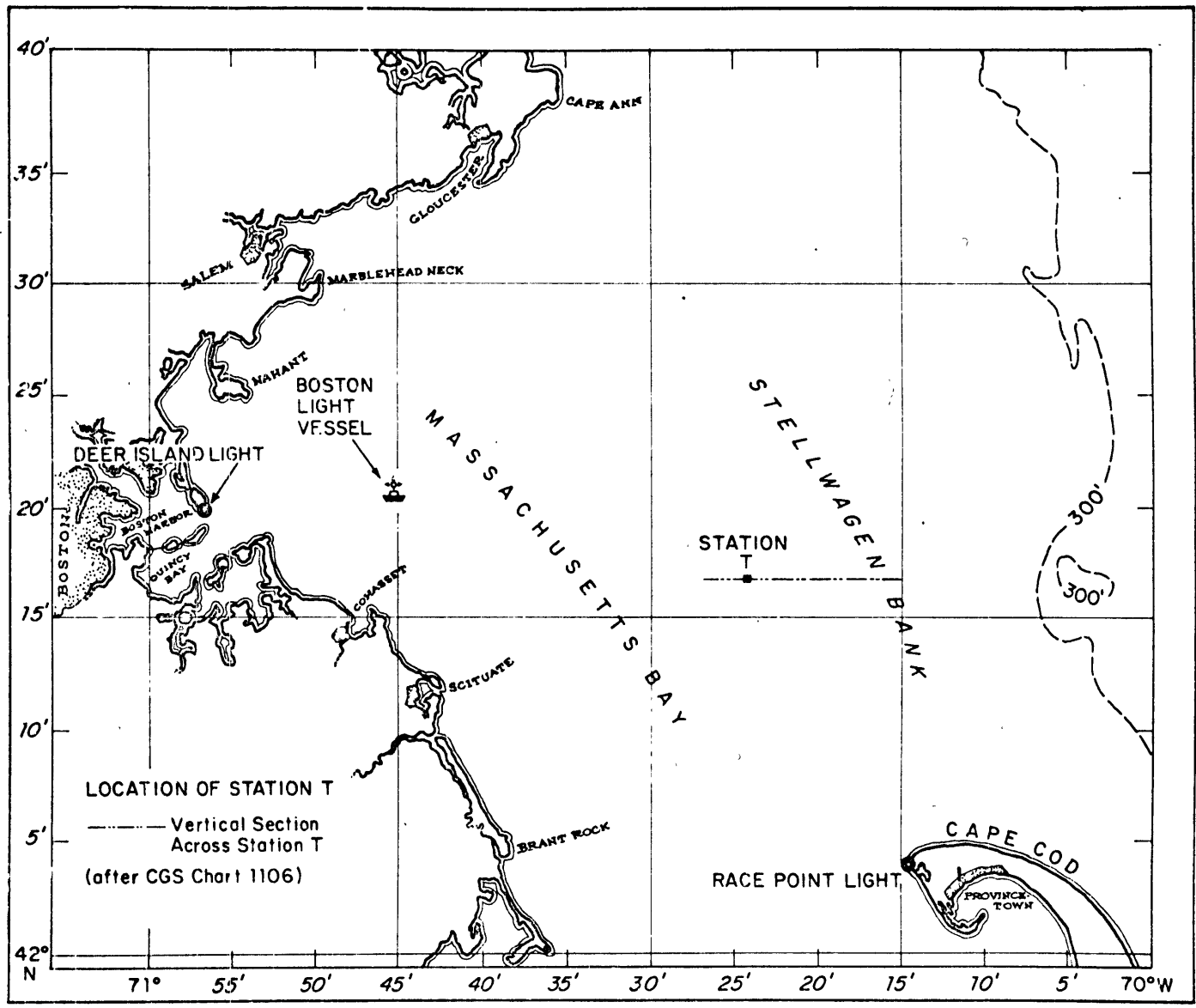
Station T) was situated in the center of a basin which extended half-way to the sill (figures 2 and 3). Stellwagen Bank has a uniform depth (27 m) along its crest, reaches to the bottom of the seasonal thermocline (Bigelow, 1927) and separates Massachusetts Bay from the Gulf of Maine. Massachusetts Bay, isothermal in winter, contains a well developed thermocline in summer. The location of Station T was chosen on the supposition that tidal flow over a submarine ridge represented a likely method of producing internal waves (Pickard, 1954, 1961; Long, 1954; Defant, 1961, p.559). Atmospheric disturbances are virtually nonexistent during the summer months.

Groups of high frequency temperature fluctuations occurring at regular intervals of time were recorded at Station T on both occasions that field measurements were conducted during the 1966 summer. Figure 4 is a Calcomp plot containing sequential (sampling interval = 2 minutes) temperature measurements at 11.1 m below the surface between July 28 and August 2, 1966. The high frequency fluctuations begin with a large abrupt rise in temperature and persist for about three hours. The time of maximum flood in Boston Harbor occurs about 70 minutes earlier than the initiation time of the high frequency fluctuations. Figure 5 contains the vertical distribution of temperature in a single 18-hour period. The onset of the feature occurred simultaneously at all depths. During the 1966 summer two flights were made over the buoys and on both occasions a pattern of multiple surface
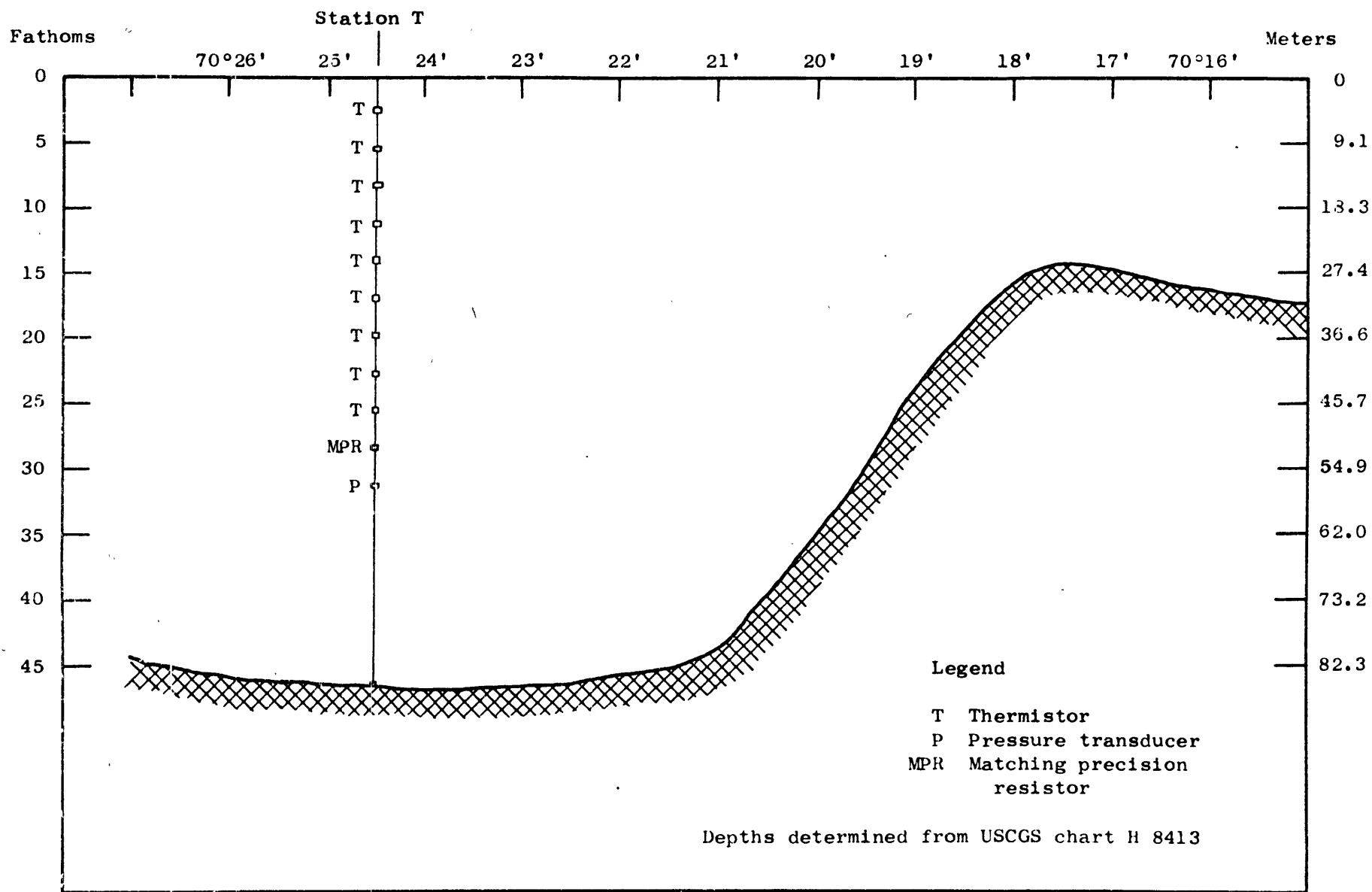
Figure 2.   Vertical section across Station T

70°30' 70°25' 70°20' 70°15'

40

50

42° 20'   42° 20'

40

50   30

40

40   20

STATION
T

40   30   20 15   15   20

15

40

42° 15'   42° 15'

Depths in fathoms

Data from USCGS chart H 8413

N

70°30'   70°25'   70°20'   70°15'   W

5.
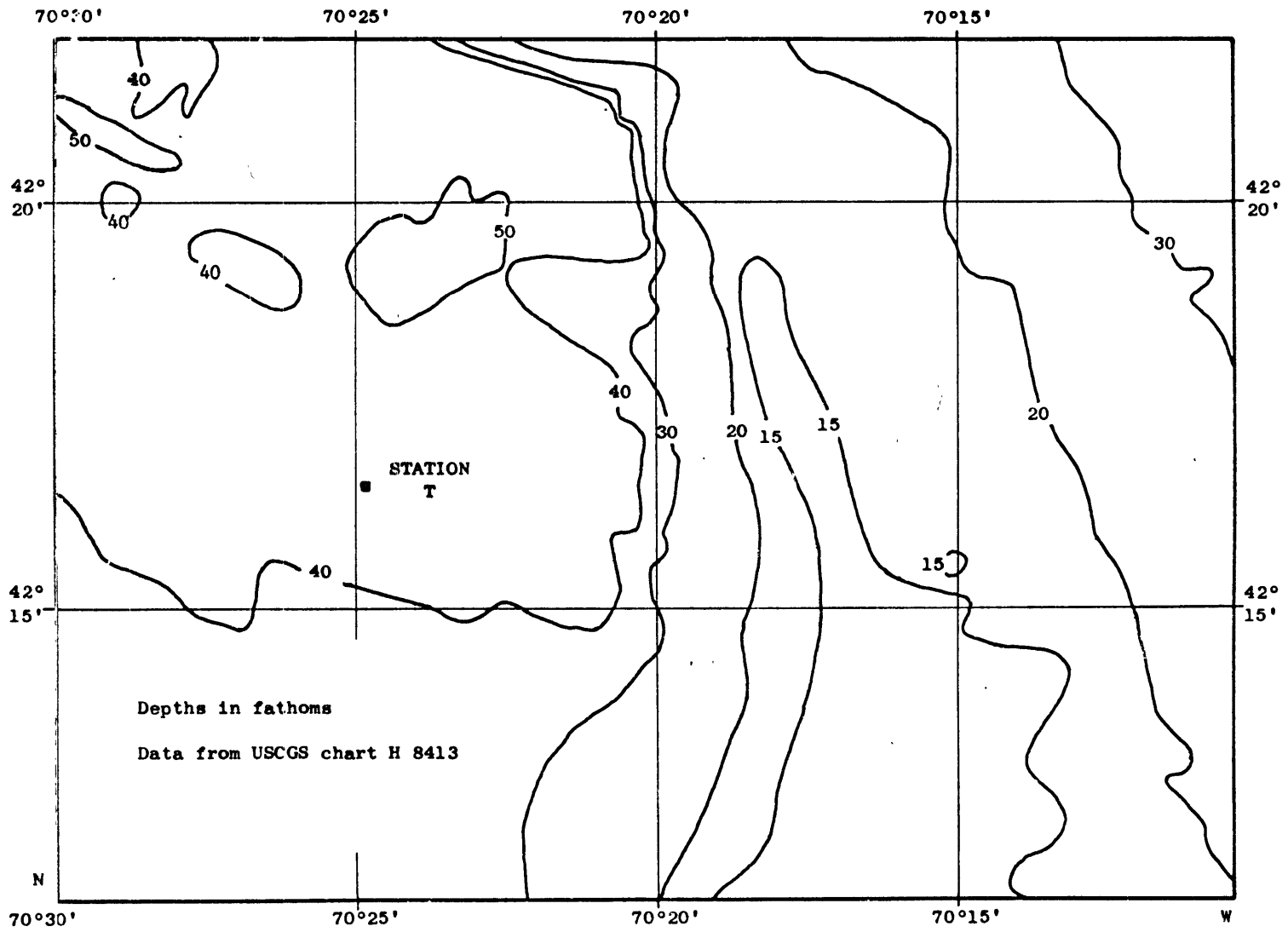
Figure 3.   Contour chart of station T and surrounding area

bands was viewed (figure 6). The high frequency fluctuations and the surface bands were investigated during the 1967 summer by means of (1) temperature and velocity measurements from moored buoys, (2) a towed thermistor, (3) aerial surveys, and (4) auxiliary data such as salinity and BT's; this paper describes the results.
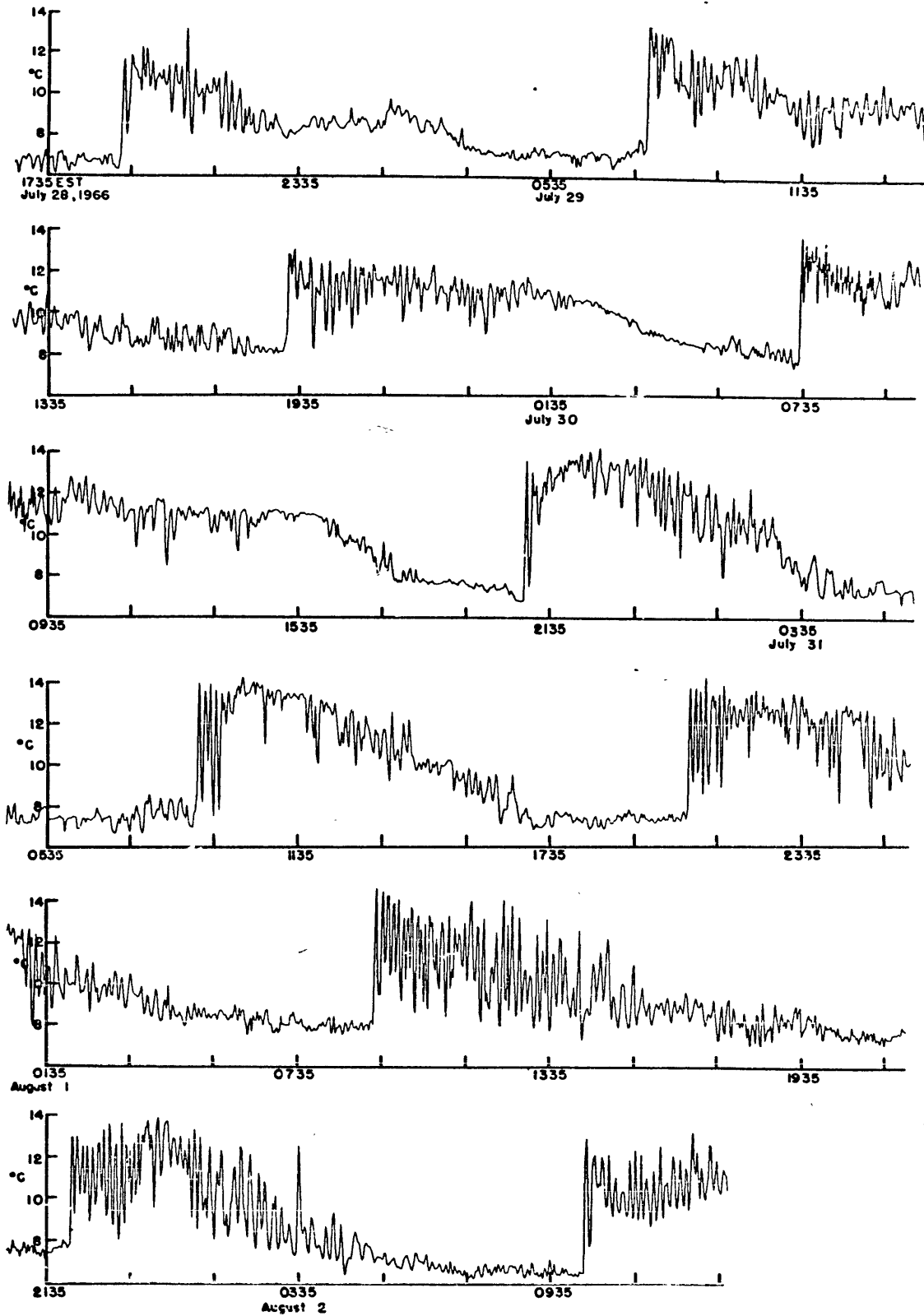
Figure 4 . A Calcomp plot of 3600 temperature measurements at 11.1m
at Station T (buoy A) between July 28 and August 2, 1966.
The sampling interval was 2-minutes.
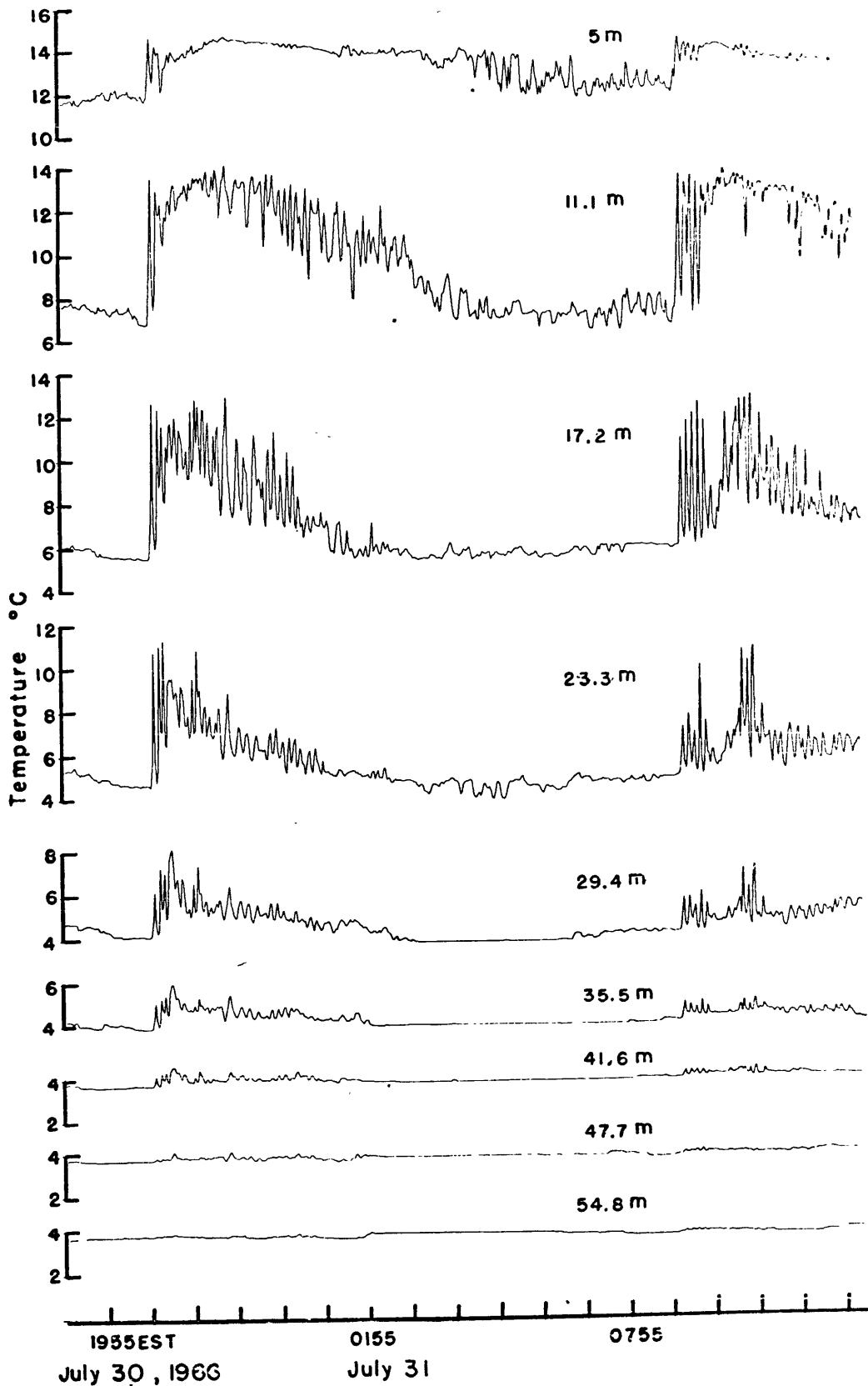
Figure 5 . A Calcomp plot of synchronous temperature measurements at 9 depths. The sampling interval was 2-minutes. (Buoy A)
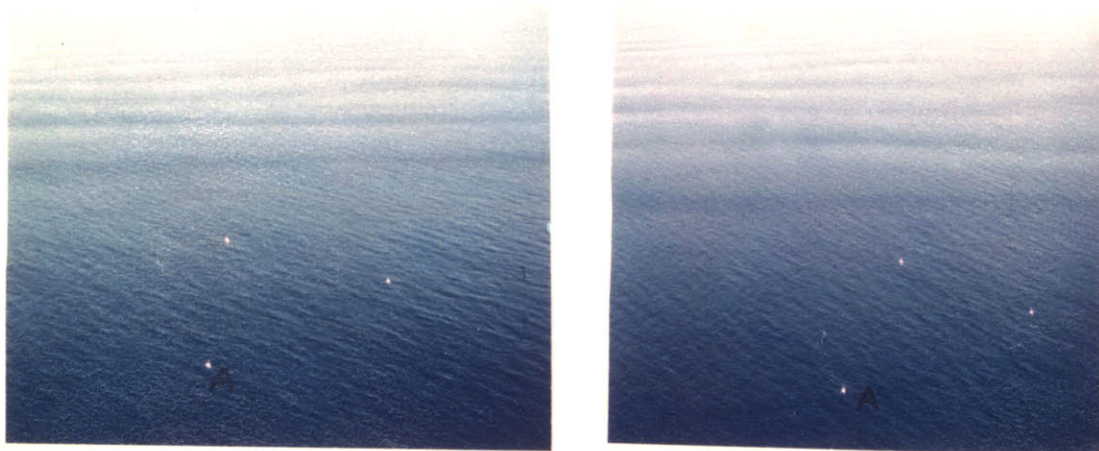
August 3, 1966.   Approximately 1100 EST.   Altitude 700 feet.
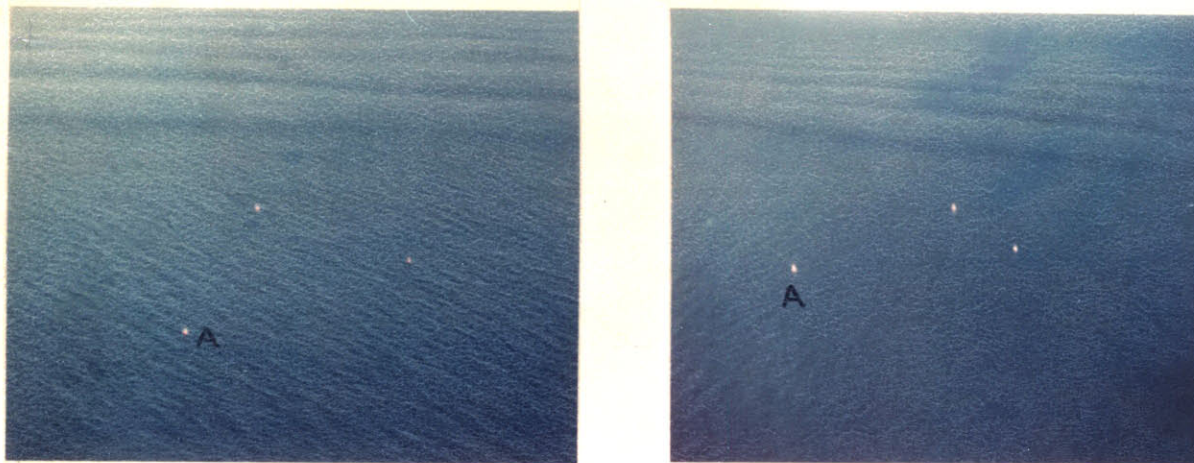
August 18, 1966.   Approximately 1030 EST.   Altitude 800 feet.

Oblique air photographs of surface bands approaching Station T.

Figure 6   Buoy A is identified in all photographs.

Data Acquisition

Three Richardson buoys (Richardson et al, 1963) were in-
strumented for measuring temperature at Station T. The buoy
consisted of a 10-foot tripod tower mounted onto an 8-foot out-
side diameter toroidal doughnut (figure 7). Designed to operate
in regions of great depth, it had an ultimate buoyancy of 6000
pounds, but in shallow water it was subject to tipping. The
stability was increased by attaching nine 50-pound weights to
the bottom of the toroid. The buoy bridle and ground tackle were
constructed from 1/2-inch proof coil chain (breaking strength =
13,200 pounds) and the mooring line consisted of 5/16-inch air-
craft cable (breaking strength = 9800 pounds). An 800-pound Stim-
son anchor (Richardson et al, 1963) was used, and in the silty
bottom it developed a holding power much in excess of its weight.

Nine thermistors, one pressure transducer and a reference
resistor were mated to a multiconductor cable, with the uppermost
thermistor 5 m below the water surface. The thermistor cable was
loosely attached to the aircraft cable with swivel snap hooks
and escaped the dynamic tensions encountered by the mooring line
(Paquette and Henderson, 1965). A 3/8-inch chain bridle prevented
the development of tensions between the temperature recording
digitizer and the thermistor cable.

Two models of the Geodyne temperature recording digitizer
(Perry and Smith, 1965 a,b) were used. One type stored the data
on a 150-foot roll of 16 mm Kodak film and had a storage capacity

Radar
reflector

PVC pipe

Navigation
light

Digitizer box

HIGH VOLTAGE
DANGER
MIT   54-1417

Thermistor cable

3/8"
chain

Tension line
for thermis-
tor cable

Three 50 lb
sash weights

Tape

1/2" chain

Fire hose
chafing gear

Pulling-up
pennant
(aircraft cable)

Swivel snap hook
Thermistor cable

5/16" air-
craft cable
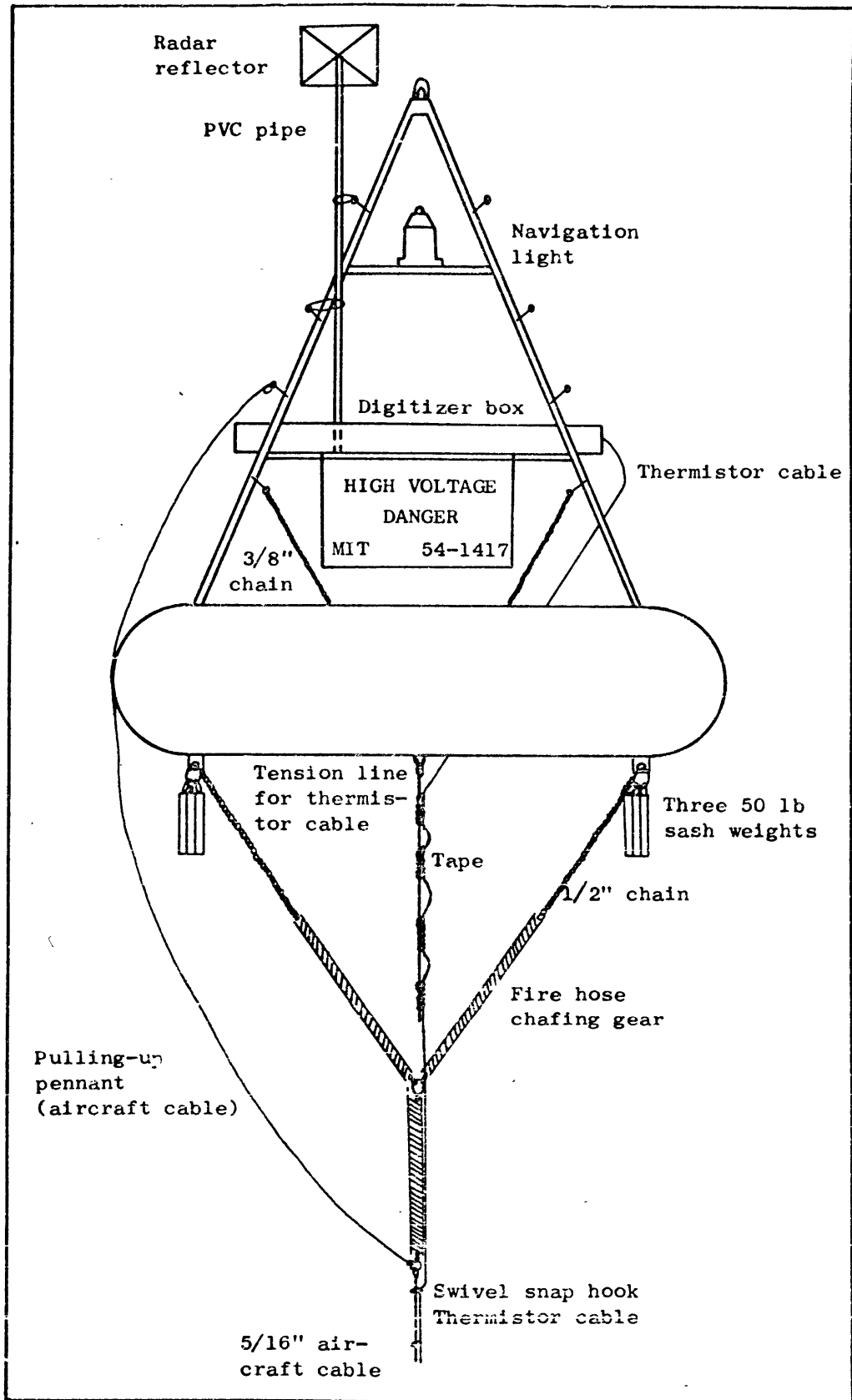
Figure 7.   A typical temperature recording buoy used at Station T

of 7500 samples; the other used a 394-foot, 4-inch, 2-track

closed loop magnetic tape which could contain 10,000 samples.

During an experiment each instrument recorded between 100,000

and 140,000 observations because 14 sensors were measured at

each sampling interval. A reference resistor, time, and tem-

perature were measured inside the instrument. The data, recorded

as electrical resistances in binary format, was processed by

Geodyne Corporation and stored on 7-track magnetic tapes com-

patible to IBM equipment. Subsurface temperature values greater

than 20° C or less than 2° C were considered erroneous measure-

ments and were replaced by linear interpolation using the pre-

ceding value and the next value. About 1/2 % of a time series

contained these errors. The pressure transducer and sensor

recording time failed to operate properly throughout the mea-

surement program.

In 1967 velocity was measured at three separate locations

using three Geodyne film-recording current meters (Richardson et

al, 1963) suspended from a Richardson buoy. Velocity and tem-

perature were measured at the same time at Station T. The cur-

rent meters were also placed on Stellwagen Bank and 11 km east

of the Bank. Velocity measurements consist of 50-second averages

of speed and direction each minute.

Mooring motion produces horizontal and vertical displacements

of the sensors which appear in fixed depth observations as noise

(Webster, 1964). The vertical displacements of the thermistors

that occur when a buoy moves from a position over the anchor to

an equilibrium position were estimated by the method described by Fritzlaff and Laniewski (1964) and Pode (1951). A mean velocity distribution at Station T was determined from the conservation of mass applied to fixed depth observations at Station T and on Stellwagen Bank. The value for the combined wind and water drag on the buoy was calculated assuming that 1/4 of the buoy was submerged with 0° tilt. The vertical displacements computed for the upper six thermistors are (in descending order): .05, .43, .85, .97, 1.09, and 1.21 m. The high frequency fluctuations are not influenced by heaving of the buoy due to the tide. Many periods of observations occurred during calm, glassy sea conditions. Horizontal excursions of the thermistors can be neglected because the water is inhomogeneous only in the vertical.

In 1966 only one instrumented buoy system operated successfully and in 1967 only one of the systems failed to operate. The precision and accuracy of temperature measurements were .02°C and .05°C, respectively. The sampling characteristics of the 1967 measurements were: Δt = .5 minutes, Δz = 5 m, Δx = 100 m, record length = 2-1/2 days; in 1966, Δt = 2 minutes, Δz = 6.1 m, record length = 5 days. The amount of aliasing was reduced by the thermistors, which had time constants of about one minute. The two buoys which recorded data in 1967 were separated by 100 m along an east-west line which was perpendicular to the crest of Stellwagen Bank. The

horizontal spacing was estimated from the surface bands which
had been aerially viewed in 1966. The measurement of the travel
time (order $5\Delta t$) for events moving from buoy T2 to buoy T1 was
unsuccessful. Experimental method introduced an uncertainty
of $2\Delta t$, the time of individual events could be identified only
within Δt, and the identification of similar events was diffi-
cult because the measuring systems differed.

Analysis of the Data

The groups of high frequency temperature fluctuations viewed in figures 4 and 5 were observed throughout the data. Daily values of mean temperature and standard deviations were quasi-stationary. No temperature inversions were observed in any portion of the measurements. The temperature and the increase in temperature occurring at the initiation of the high frequency fluctuations were uniform (Table 1). The time interval between the initiation of 29 groups of high frequency fluctuations was 12.4 hours and the standard deviation was 30 minutes.

Figure 8 contains a Calcomp plot of the speed measurements recorded at 10.6 m below the surface at Station T between July 13 and July 17, 1967. Groups of large amplitude high frequency fluctuations occurring at 12.4-hour intervals were also measured at 25.8 m and 42.6 m below the surface; they were absent from speed measurements recorded on Stellwagen Bank and at 11 km east of the Bank. The time origin of the velocity data had been destroyed by film exposure, and at each depth the onset of a group of large amplitude high frequency speed fluctuations was made to coincide. The measured directions shown in figure 9 occur at the time of individual maxima of the large amplitude high frequency fluctuations in the speed measurements. Current direction is referred to true north and is specified as the direction toward which the current is flowing. The time averaged

Table 1

STATISTICS OF THE INITIATION OF THE HIGH FREQUENCY TEMPERATURE FLUCTUATIONS

	July 6-9, 1966 z = 11.1 m		July 28-Aug 3, 1966		July 13-17, 1967		July 25-28, 1967 z = 10 m	
	T	ΔT	T	ΔT Temperature ($^{\circ}$C)	T	ΔT	T	ΔT
Mean	7.6	7.7	7.3	6.1	7.8	6.1	7.8	7.2
SD	1.3	1.0	0.5	0.7	0.3	0.5	0.3	0.6
Maximum	10.2	8.6	8.0	7.0	8.2	6.8	8.4	8.0
Minimum	6.2	5.4	6.3	4.7	7.4	5.5	7.5	6.5
Number (N)	6		10		5		5	
	z = 17.2 m						z = 15 m	
Mean	5.0	5.1	6.2	6.1	6.6	6.8	6.4	4.8
SD	0.5	2.5	0.8	0.9	0.2	0.5	0.4	1.8
Maximum	5.6	9.1	6.5	8.0	7.0	7.6	6.8	7.5
Minimum	4.2	2.0	5.6	5.0	6.4	6.2	5.7	2.6
Number (N)	6		10		5		5	

T   Temperature directly prior to the onset of the high frequency fluctuations
ΔT Increase in temperature at the onset of the high frequency fluctuations
z   Depth below the surface
SD   Standard deviation

17.
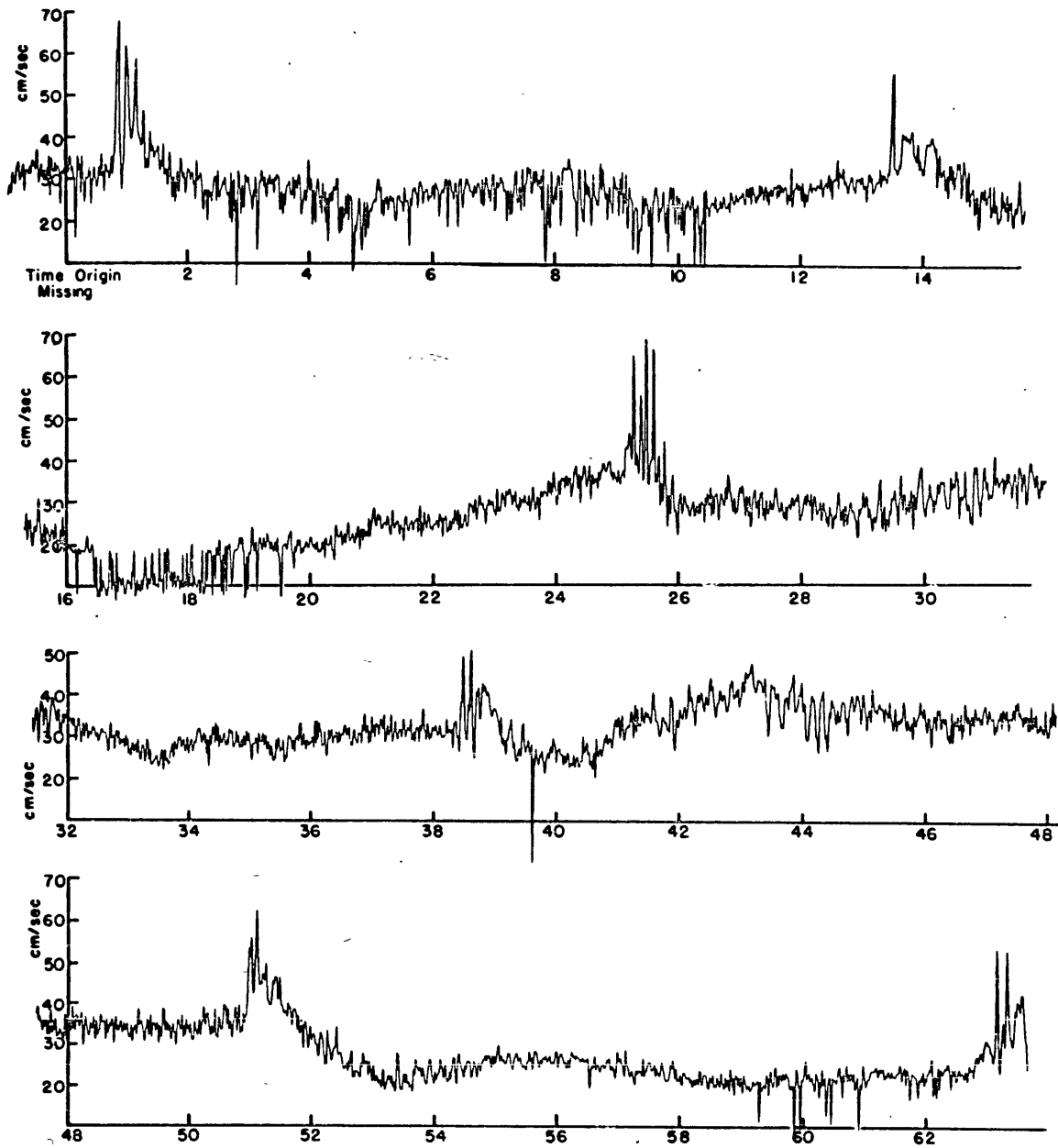
Figure 8 . A Calcomp plot of 3900 speed measurements at 10.6m at
Station T between July 13-17, 1967. The time origin is
missing and the interval between tick marks on the ab-
scissa is 2-hours.
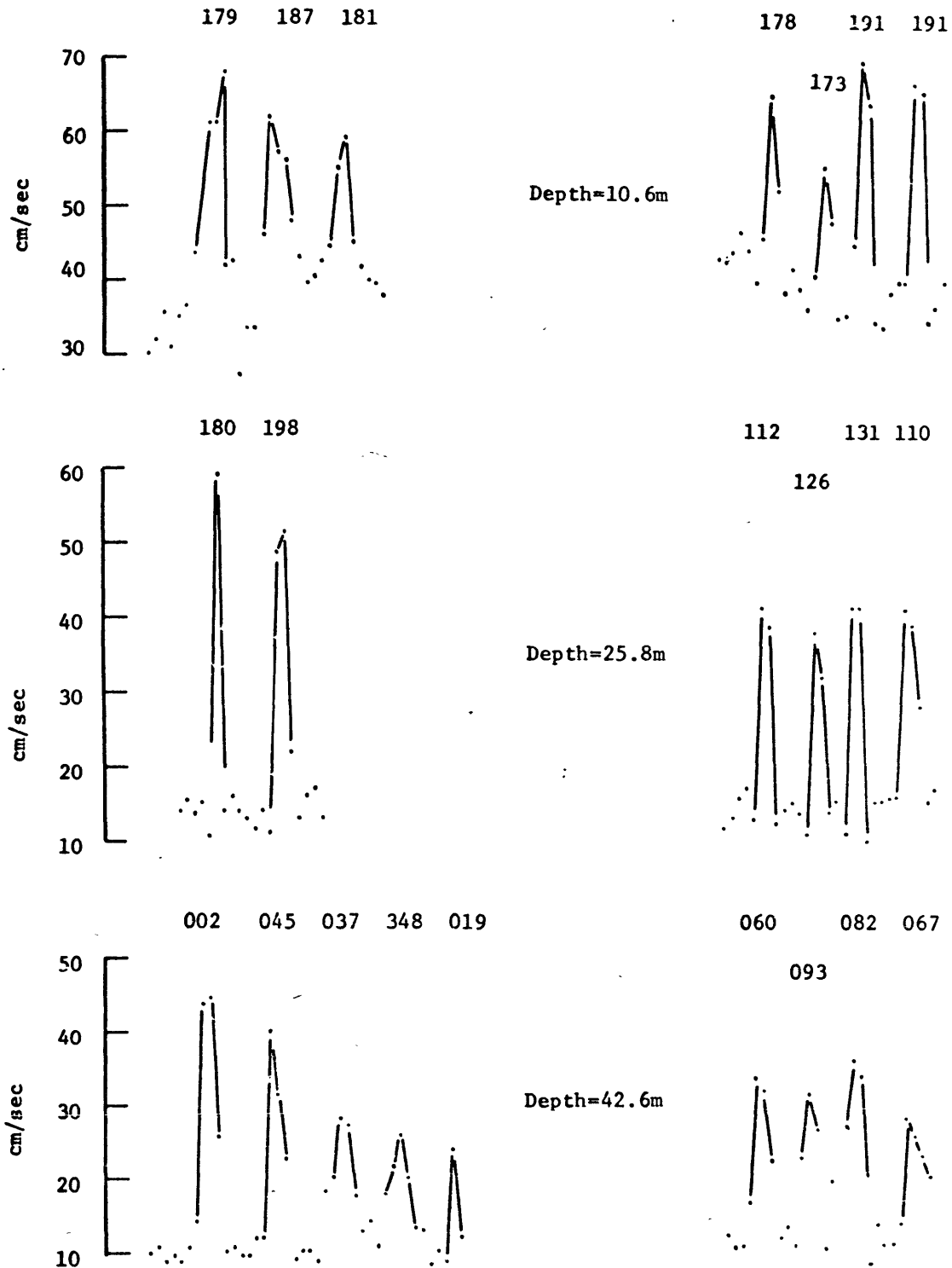
Figure 9 . A plot of speed (ordinate) and time (abscissa) for
two groups of large amplitude high frequency speed fluctuations
at three depths at Station Y. The time interval between each
point is 1-minute and the direction (degrees; referred to true
north) measured at the time of the peak value is given above
each peak. These groups occur at hour 1 and hour 25 in figure 8.

velocities measured during the 2-hour interval directly prior to the onset of the first group (left portion of figure 9) were (33 cm./sec; 130°) at 10.6 m, (15 cm/sec; 223°) at 25.8 m, and (10 cm/sec; 202°) at 42.6 m. At each of the three depths the range (maximum − minimum) in the average direction was 55°. In the absence of the mean current, the computed velocities occurring at the time of the individual maxima of the first group in figure 9 were (46 cm/sec; 213°) at 10.6 m, (39 cm/sec; 155°) at 25.8 m, and (50 cm/sec; 020°) at 42.6 m. Although a reversal in the computed directions occurs between the measurements at 42.6 m and those at the other two depths, the dominant north-south component is perplexing because the regularly occurring groups of large amplitude high frequency fluctuations in the speed and temperature measurements are considered to arise from the same process. Observations of long-crested narrow surface bands suggested that the high frequency temperature fluctuations represented two-dimensional (vertical and east-west directions) motion which moved westward. Measurements from a towed thermistor also indicated that the high frequency temperature fluctuations moved westward.

The mean and standard deviation of the speed measurements are given in Table 2. The omnidirectionality of the Savonius rotor produces an average speed greater than the true average (Hunkins, 1967; Hansen, 1964). A study of the speed and direction histograms indicates that Stellwagen Bank does not restrict

the tidal flow to the thermocline region. During flood tide the water below the thermocline on the east side of the sill participates in the flow over the Bank producing a uniform flow from surface to bottom on the sill. The westward flow then diffuses in the vertical direction and at Station T a vertical shear has developed above the sill depth. The flow at 10 m below the surface on both sides of the sill is more complicated because the current contains a rotary component. A rotary component is also present in the measurements at 25.8 m and 42.6 m at Station T. It is surprising that the large values of tidal speed measured on Stellwagen Bank have been unnoticed (Bigelow, 1927; Haight, 1942; U.S. Coast and Geodetic Tidal Current Tables, 1967).

Table 2

AVERAGE AND STANDARD DEVIATION OF THE SPEED MEASUREMENTS

Station T July 13-17, 1967			Stellwagen Bank July 24-28, 1967			East (11 km) of Stellwagen Bank August 22-25, 1967		
Depth m	Aver cm/sec	SD	Depth m	Aver cm/sec	SD	Depth m	Aver cm/sec	SD
10.6	28.9	7.8	7.6	41.3	11.3	10.6	28.2	11.7
25.8	15.0	4.0	15.2	35.0	13.0	25.8	20.9	8.3
42.6	13.4	4.4	22.8	35.4	16.5	42.6	17.1	8.3

Figure 10 . Temperature measurements from a thermistor towed at 16m
on September 19, 1967.

describes the vertical distribution of mean temperature, stan-

dard deviation, mean density and mean Brunt-Väisälä frequency

for the period July 25 to July 28. The time-averaged Brunt-

Väisälä frequency is defined by

$$N^2(z) = -\frac{g}{\rho_0}\frac{d\rho_0}{dz} \tag{1}$$

where ρ_0 is the mean density. The sequential temperature

measurements recorded between July 25 and July 27 were used to

evaluate the mean temperature and standard deviations at 5-meter

intervals from 5 m to 45 m below the surface. At depths

greater than 45 m the temperature distribution was measured from

a single bathythermograph lowering on July 28. A salinity

cast consisting of eight Van Dorn water sampling bottles (Van

Dorn, 1956) spaced at 10-meter intervals was conducted on

July 28. Density was computed from Knudsen's Formula (Lafond,

1951, p.91) using the mean values of temperature and the single

measurement of salinity. A thermal gradient exists between

5m and 25 m below the surface, and below the sill depth the

temperature is nearly isothermal. It is not surprising that

the amplitude of the high frequency temperature fluctuations

and the standard deviations are greatest in the region of

maximum temperature gradient. The distribution of salinity

(not shown) contained a halocline above the sill depth.

In figure 11 the period of the high frequency fluctuations

appears to increase with time for the group initiated at 1336,
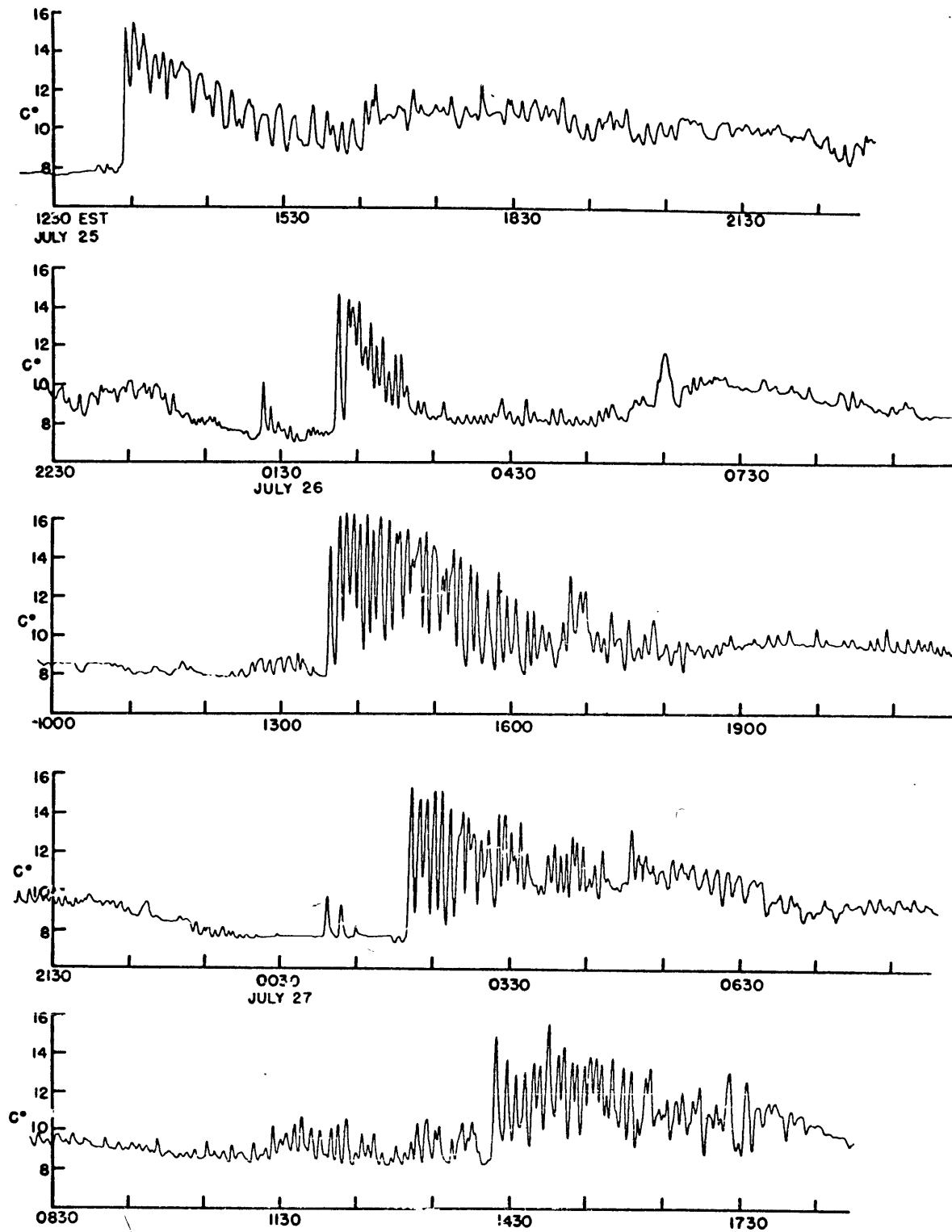
Figure II . A Calcomp plot of 6400 temperature measurements at 10m
at Station T (buoy T2) between July 25 and July 27, 1967.
Sampling interval was 30-seconds.
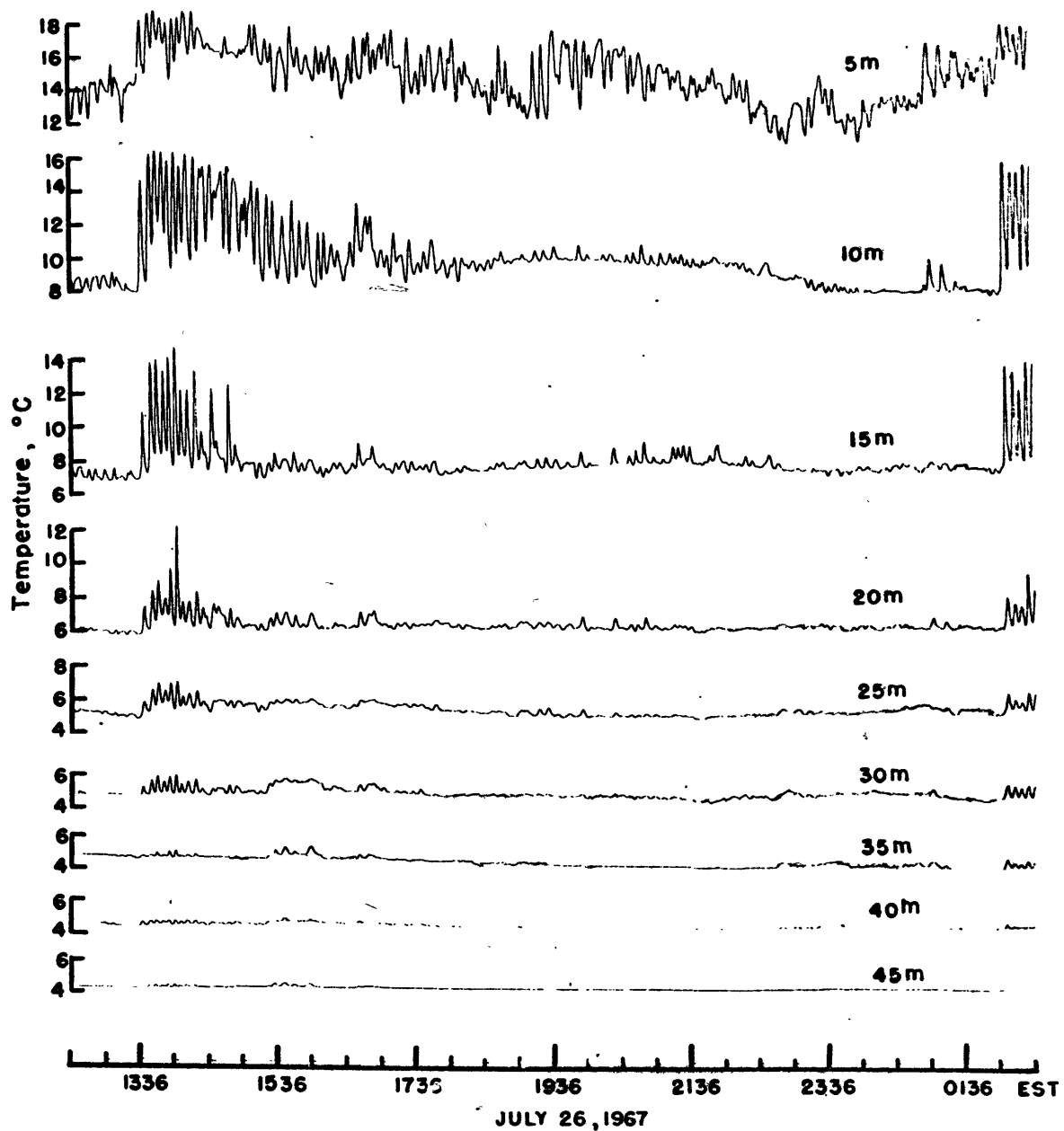
Figure 12 . A Calcomp plot of synchronous temperature measurements at
9 depths. Sampling interval was 30-seconds. (buoy T2)
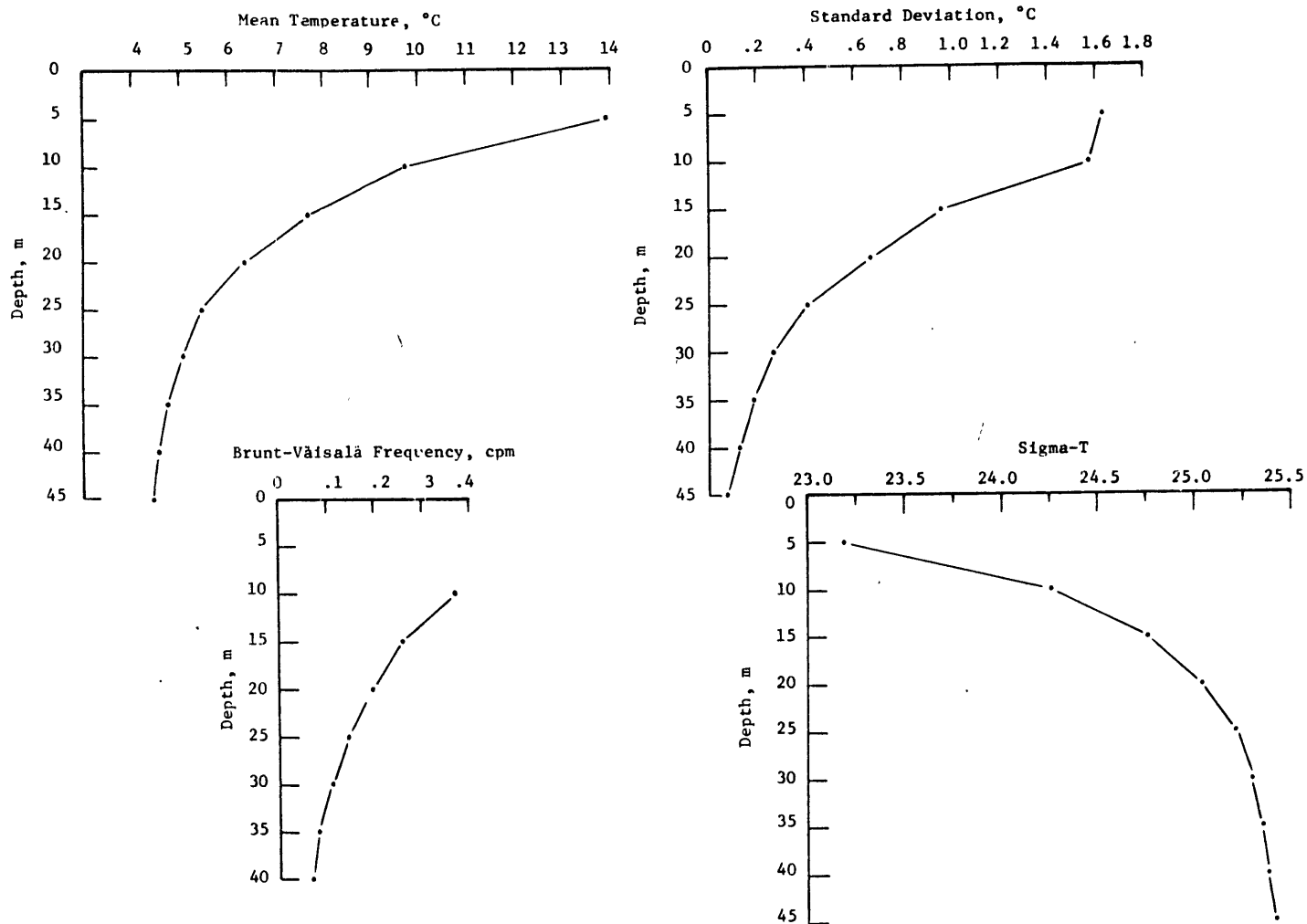
Figure 13. Average vertical distribution of temperature, density, and Brunt-Väisälä frequency at Station T between July 25 and July 27, 1967.

July 26; the opposite effect may be true for the group beginning at 1414, July 27.

Using the Woods Hole Oceanographic Institution heliocourier, a pattern of multiple surface bands was examined from an altitude of 700 feet between 1335 and 1500 EST on July 26, 1967. Figure 14 contains aerial photographs of the surface bands in the vicinity of buoys T1 and T2 and temperature measurements obtained at 10 m below the surface from the eastern buoy (buoy T2). Numbers on the photographs identify particular bands, i.e., a "1" designates the lead band. The arrival time of the lead band at the buoys coincided (within ±2 minutes) with the onset time of the high frequency temperature fluctuations. The westward speed of individual bands was determined by measuring their travel time between the buoys. The accuracy of a travel time measurement was ±30 seconds. In figure 14 narrow band "2" moved from the eastern buoy to twice the distance separating the buoys in five minutes beginning at 1345 EST. Band "3" travelled 100 m in two minutes beginning at 1350 EST. The average speed determined from a total of four travel time measurements was (88 ± 20) cm/sec. The distance separating the narrow bands was (175 ± 25) m. The crests of the bands paralleled Stellwagen Bank and exhibited the same curvature. Their length was 9.75 km and their northern edge occurred about 2.5 km north of Station T. Each narrow band consisted of about 30 m of choppy water. When the moored buoys were in the bands

Figure 14. Simultaneous measurement at Station T of surface

bands and high frequency temperature fluctuations on July 26,

1967. T^* represents 1336 EST, the onset time of the high

frequency temperature fluctuations. The dark narrow bands

represent the surface bands.

West →

1345 EST
2
1

West →
1350 EST

1350 EST
3
1
2

West →

1352 EST
1
2
3

Figure 14.

Thermistor 2 (10 meters)
Experiment 67 – 3 – T2

20 minutes

1153EST   1253   T* 1353   1453   1553

JULY 26, 1967

TEMPERATURE °C

18
16
14
12
10
8
6

29.
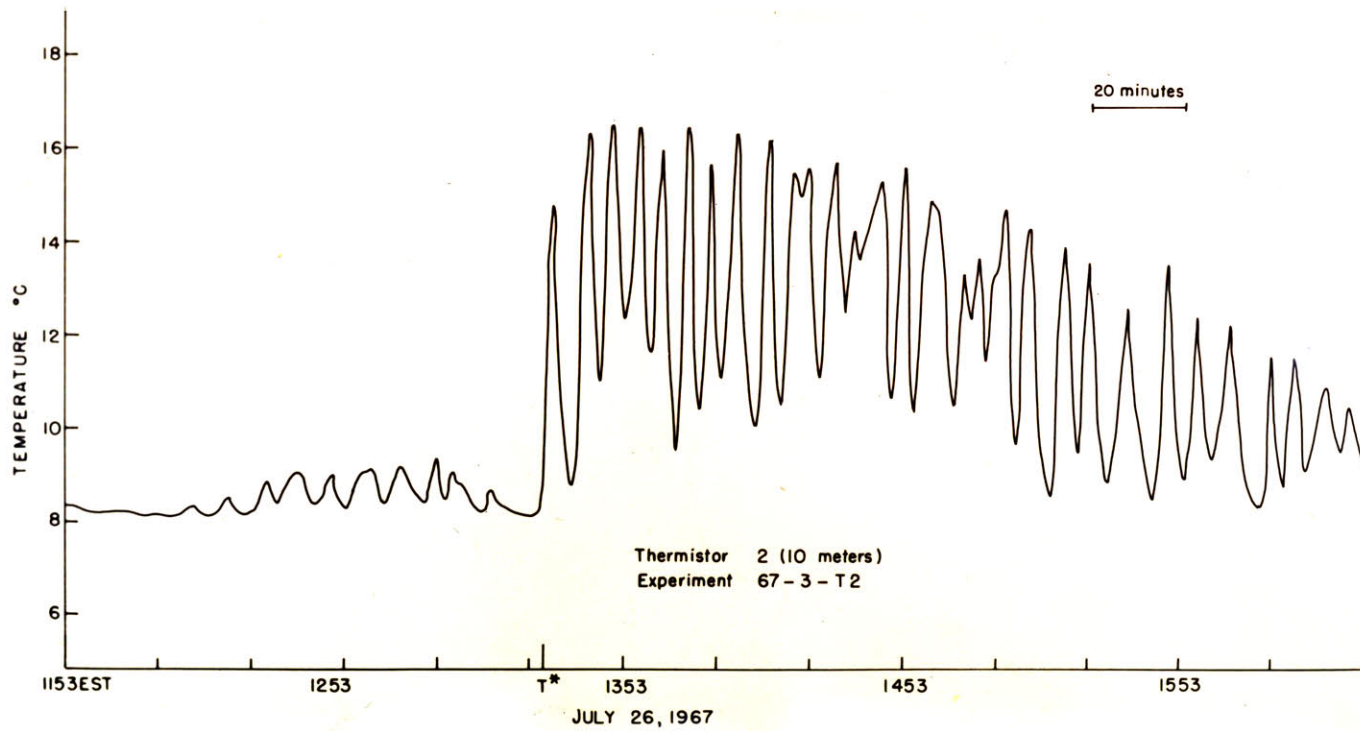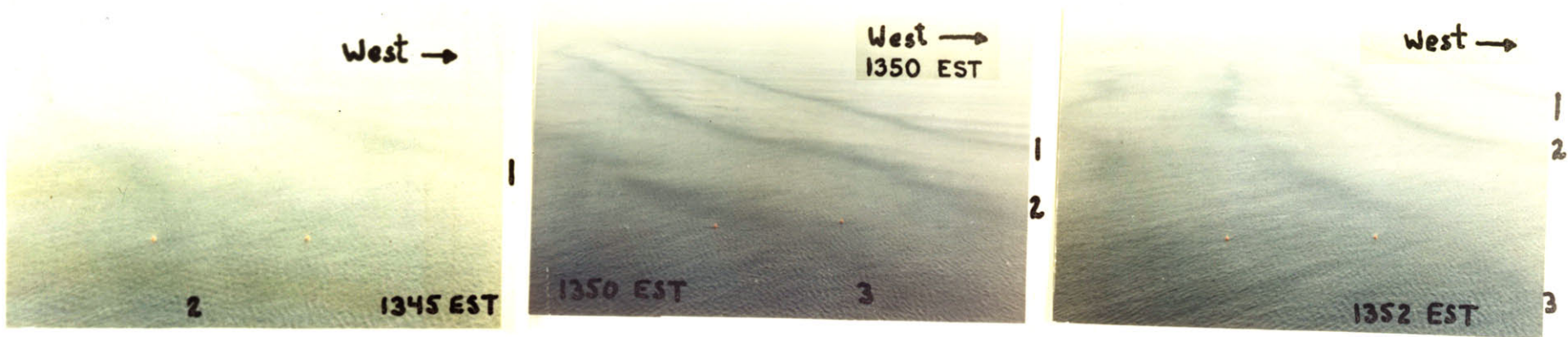
they rolled to and fro, but they remained virtually motionless during the intervals of slightly rippled or calm water. The narrow bands are similar to bands of ruffles on smooth water (Shand, 1953; Pickard, 1961; Frassetto, 1964) and do not represent slicks of calm water in a wind rippled sea (Ewing, 1950; Lafond, 1962). Both types of surface features have been associated with near surface short period internal waves.

1. Spectral Analysis

Estimates of the spectra of the high frequency temperature fluctuations recorded between July 25 and July 27, 1967 (figures 11 and 12) were computed by a method described by Welch (1967) and Bingham et al (1967). The time series was divided into five sections; each section was multiplied by a window; a modified periodogram was computed for each section; Bartlett-Daniell estimates of the spectral density function were formed from the periodograms.

The normalized, complex, finite Fourier transform of a series of N temperature measurements is

$$A_k = \frac{1}{N} \sum_{i=0}^{N-1} T_i^* e^{(-2\pi j) i \frac{k}{N}} \qquad i, k = 0,1,2,3,\cdots,(N-1) \quad (2)$$
$$j = \sqrt{-1}$$

where T_i^* is the ith temperature measurement. Equation (2) can

be written as

$$A_k = a_k + j b_{k+1} \qquad k = 0, 2, 4, \cdots, (N-2) \qquad (3a)$$

or $\qquad A_k = a_n + j b_n \qquad n = 0, 1, 2, 3, \cdots, \frac{N}{2} - 1 \qquad (3b)$

where (a_n, b_n) are the nth Fourier coefficients of the time series. The raw periodogram of a discrete time series is the sequence of values $(a_n^2 + b_n^2)/\Delta f$ that are attached to the frequencies, or harmonics, $f_n = \frac{n}{N \Delta t}$. The fundamental resolution separating the harmonics is

$$\Delta f = \frac{1}{N \Delta t} \qquad (4)$$

The zeroth harmonic is the D.C. component or mean value of the series. The direct computation of the Fourier coefficients was preformed by a Fortran IV subroutine (IBM, 1967) which utilizes the Fast Fourier Transform (Cooley and Tukey, 1965) and which limits the number of data values to a power of two. Each periodogram ordinate describes the energy within the band $f_n \pm \frac{\Delta f}{2}$ with two degrees of freedom. The periodogram is a poor estimate of the spectrum (Hinich and Clay, 1968). Daniell estimates (Madden, 1963) with ν degrees of freedom were formed by averaging $\nu/2$ periodogram ordinates. This smoothing decreases the resolution between harmonics, but increases the

stability of the estimates. The stability was further increased

by (ensemble) averaging the Daniell estimates from each of the

sections.

Each section was multiplied, before taking the Fast Fourier

Transform, with a window that slowly tapered the time series

towards zero at both ends. A window suggested by Welch (1967)

was selected. This data window is very close in shape to the

Hanning (Blackman and Tukey, 1958) or cosine arch spectral win-

dow, and defined by

$$W_i = 1 - \left[\frac{i - \frac{N-1}{2}}{\frac{N+1}{2}} \right]^2 \qquad (5)$$

Using the initiation of the high frequency temperature

fluctuations as origins for the individual sections, the tem-

perature record was divided into five 8-hours, 32-minutes (i.e.,

1024 values) segments such that 3-hours, 1-minute occurred

prior to the origin. Each member was adjusted to zero mean

and zero linear trend by (1) removing the mean value from the

entire section, (2) subdividing the section into 17 segments

of 30-minutes length beginning with the third data point, (3)

removing the mean value from each segment, and (4) removing

the mean value from the zero linear trend data together with

the first and last two data values. The trend in the variance

had not been altered. Each section was then multiplied by

equation (5) and the modified Fourier coefficients were computed for the sequence of values $(W_i \cdot T_i)$, where T_i represents the adjusted temperature series. The amplitude of the nth harmonic of the modified periodogram, denoted by P_n, was computed from

$$P_n = \frac{1}{U} \cdot \frac{(\bar{a}_n^2 + \bar{b}_n^2)}{\Delta f} \qquad n = 1, 2, 3, \cdots, \frac{N}{2}-1 \qquad (6)$$

where $(\bar{a}_n, \bar{b}_n)$ are modified Fourier coefficients of the series $(W_i \cdot T_i)$. U is a dimensionless correction factor introduced to compensate for the data window and is defined by (Welch, 1967)

$$U = \frac{1}{N} \sum_{i=0}^{N-1} W_i^2 \qquad (7)$$

Bartlett-Daniell estimates of 30 degrees of freedom were obtained in the following manner: (1) Daniell estimates of 6 degrees of freedom were computed for each of the five sections.

$$P_n^6 = \left(P_{n-1} + P_n + P_{n+1} \right)/3.0 \qquad n = 3, 5, 8, \cdots \qquad (8a)$$

The superscript designates the number of degrees of freedom. A re-indexing of the harmonic number yields the equivalent relation

$$P_n^6 \longleftrightarrow P_q^6 \qquad (8b)$$

$$n = 3, 5, 8, \cdots \qquad q = 1, 2, 3, \cdots$$

(2) Bartlett-Daniell spectral density estimates of 30 degrees of freedom were formed by ensemble averaging the estimates P_q^6, i.e.,

$$P_q^{30} = \frac{1}{5} \sum_1^5 P_q^6 \qquad (8c)$$

Spectral density estimates of 60 and 120 degrees of freedom were then formed by averaging the ordinates P_q^{30} in blocks of two or four.

Spectral density estimates of 60 degrees of freedom occurring in the interval 146-minutes to 2.16-minutes are shown in a contour diagram (figure 15) in which depth and frequency are coordinates and lines are drawn through points of equal spectral density. The amplitude of the high frequency temperature fluctuations is largest at 10 m below the surface, decreases in both directions, and has periods between six and eight minutes. This is not surprising since the Calcomp plots of the observations indicated a similar result. The valley occurring between the two 15-$({}^o C)^2$/cpm peaks is observed with greater resolution in a contour diagram (figure 16a) and spectral density function (figure 16b) drawn from estimates of 30 degrees of freedom. The two peaks, possibly reflecting
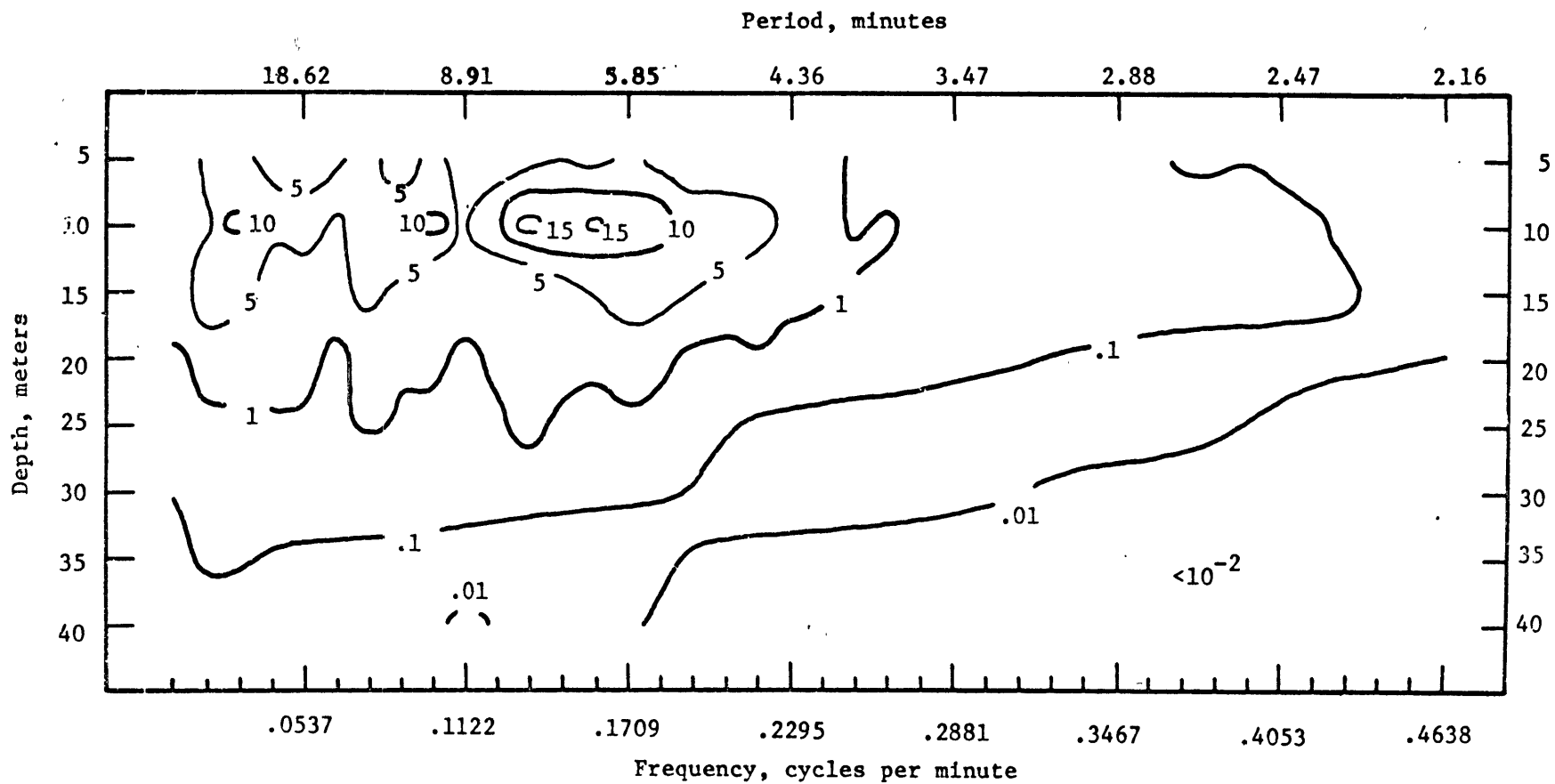
Figure 15. Contours of spectral density, $(^\circ C)^2/cpm$, of the temperature fluctuations. ($\nu = 60$; $\Delta f = .01170$ cpm)
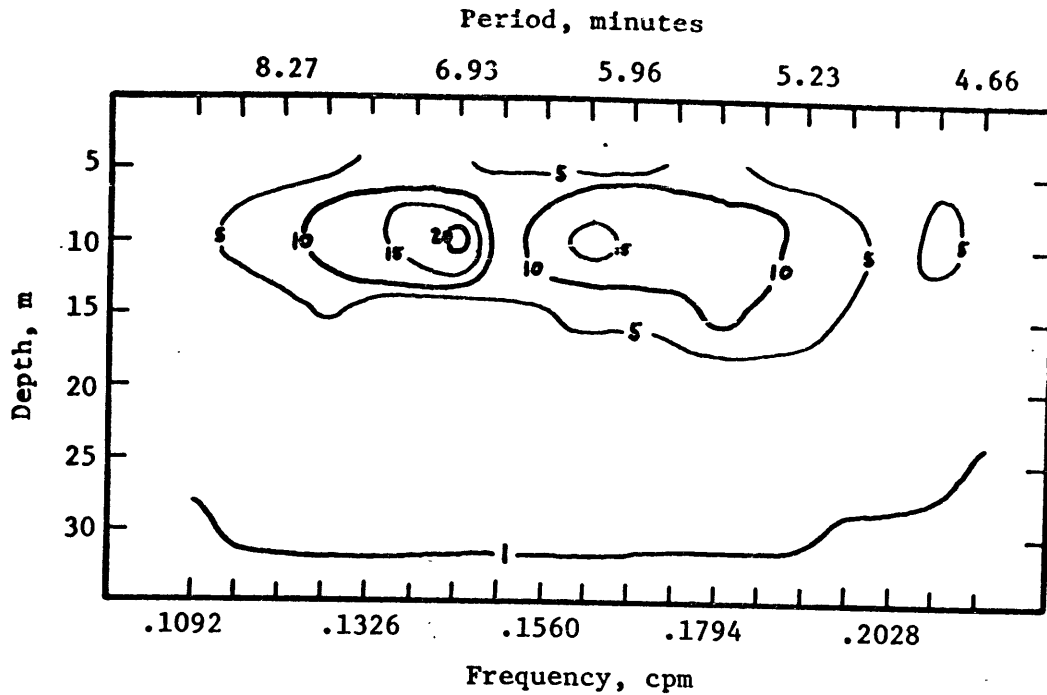
Figure 16a. Contours of equal power density, $(^\circ C)^2$/cpm, on a frequency-depth plot of the high frequency temperature fluctuations. Each estimate has 30 degrees of freedom.
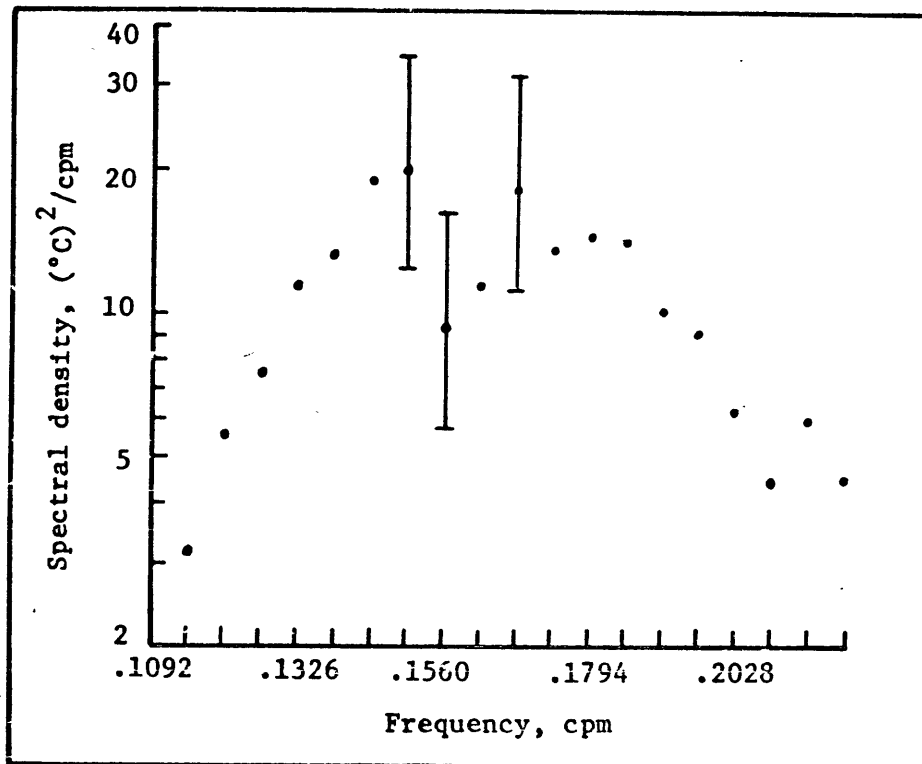
Figure 16b. Power density spectrum of the high frequency temperature fluctuations at the 10m level. Each pair of horizontal bars represent the 95% confidence limit.
(ν = 30; Δf = .00585 cpm)

the frequency change with time noted in figure 11, are not

significant relative to each other at the 95% confidence

level (figure 16b). For a particular confidence level, a

peak in the spectral density function is considered signi-

ficant and therefore representative of a sinusoid if the vari-

ation of the peak as defined by the χ^2 (chi-square) distri-

bution does not overlap the variation of the estimate of the

closest spectral trough. The 95% confidence limits for this

case are .62 and 1.73 (Munk et al, 1959) of each spectral

estimate, which means that there is a 95% probability that the

true value of the spectral density lies within the stated

proportions of the measured value. In figure 16b a pair of

horizontal bars represent the 95% confidence limits of the

estimate.

Dispersion of the high frequency temperature fluctuations

was investigated from the time rate of change of the spectrum.

Five independent, successive 32-minute groups of ensemble

averaged spectral density estimates of 20 degrees of freedom at

10 m below the surface are shown on a contour diagram (figure 17)

in which frequency and time are coordinates. The first 32-

minute interval began at the initiation of the high frequency

fluctuations, and each interval was adjusted to zero mean and

zero linear trend by subdividing it into four equal sections.

Neither an increase nor a decrease in frequency with time can

be inferred from figure 17.

Period, minutes

Figure 17.
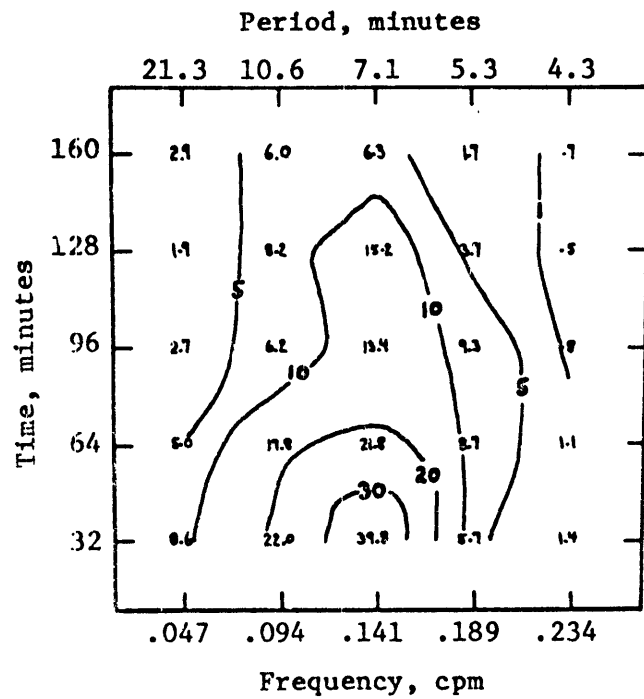
Contours of equal power density, $(°C)^2/cpm$, on a frequency-time plot for 160 minutes of (ensemble-averaged) measurements at 10m. The tick marks on the time axis indicate successive 32-minute intervals following the initiation of the high frequency fluctuations.

Spectral density estimates of 120 degrees of freedom were computed at depths from 5 m to 25 m below the surface (figure 18). For this case the 95% confidence limits are .795 and 1.30 of each estimate. Below 5 m, significant spectral density peaks are centered at periods of about 7 minutes. The rms vertical displacements of the temperature fluctuations occurring between 8.5 minutes and 5.3 minutes (the 6th, 7th, and 8th harmonics of figure 18) were evaluated from the conservation of temperature (Kraus, 1966).

$$\eta_{rms} = \frac{T_{rms}}{\left(\frac{dT}{dz}\right)_{mean}} \qquad (9)$$

The derivation of equation (9) assumes small amplitude motion in a resting fluid. The mean temperature gradient was evaluated using the average temperature of the five sections. Because the temperature in the thermocline decreases in a series of irregular steps (Cooper and Stommel, 1968), the computed value of the temperature gradient and the local value may be different. The apparent rms vertical displacements shown in Table 3 were greater than displacements arising from motion of the thermistors.

Period, minutes

9.40   4.47   2.93 2.18

95% confidence
limit

Legend

●	5m
○	10m
△	15m
×	20m
⊙	25m

Spectral density, $(°C)^2$/cpm

10

1

.1

.01

10

1

.1

.01

5   10   15   20

Harmonic number

$\nu = 120$
$\Delta f = .023436$ cpm
$f_1 = .012695$ cpm

Harmonic number	Frequency (cpm)
2	.036131
4	.083003
6	.129875
8	.176747
10	.223619
12	.270491
14	.317363
16	.364235
18	.411107
20	.457979

Ensemble averaged
spectral density estimates
of the temperature fluctua-
tions at five depths

Figure 18.

Table 3

ENSEMBLED AVERAGED STATISTICS
OF THE HIGH FREQUENCY TEMPERATURE FLUCTUATIONS

Depth	Mean temp.	Mean vertical temp. gradient	RMS temp.	RMS vertical displacement for periods between 5.3 and 8.5 minutes
m	°C	°C/m	°C	m
10	9.84	.670	.982	1.47
15	7.64	.349	.625	1.79
20	6.35	.215	.421	1.96
25	5.49	.130	.231	1.77
30	5.05	.073	.131	1.79

2. Coherency Analysis

The normalized complex coherency function is a measure of the degree of linear relationship between harmonic constituents at different locations. It is defined by (Madden, 1963; Davidson and Heirtzler, 1968)

$$R_{12}(n) = \frac{\Phi_{12}(n)}{\left[\Phi_{11}(n) \cdot \Phi_{22}(n)\right]^{\frac{1}{2}}} \qquad 1 \le n \le \frac{N}{2}-1 \qquad (10)$$

$\Phi_{12}(n)$ is the cross spectral density function between two time series designated as "1" and "2", and $\Phi_{11}(n)$ and $\Phi_{22}(n)$ are the

auto spectral density functions of each series. A substitution into equation (10) of the polar form of the cross and auto spectral functions, which are products of the complex Fourier transform (Robinson, 1967), yields

$$R_{12}(n) = ave \left\{ 1 e^{i \Theta_{12}(n)} \right\} \tag{11}$$

where ave $\{ \}$ means independent averages of successive values (e.g., Daniell estimates). The unit vector $1 e^{i \Theta_{12}(n)}$ represents the normalized Fourier transform of the nth constituent evaluated from the two time series. The relative phase between ·constituents is given by

$$\Theta_{12}(n) = \Theta_1(n) - \Theta_2(n) \tag{12}$$

The phase of a Fourier transform constituent is given by

$$\Theta = Tan^{-1} \left(\frac{b_n}{a_n} \right) \tag{13}$$

A complex coherency estimate of 2ν degrees of freedom is formed by a vector summation of 2ν consecutive unit vectors. The magnitude of the summation is called the coherence, and the coherence and phase estimates each have ν degrees of freedom. A coherence estimate of unity over a frequency interval results when the unit vectors have the same directions; a coherence of

zero occurs when the vectors have opposite directions and there-
fore sum to zero. To quote Madden (1963, p.19): "The heart
of the measure of coherency comes from the consistency of the
phase relationship between the two sets of data."

The vertical coherency of the high frequency temperature
fluctuations measured from July 25 to July 27, 1967 was com-
puted directly from the modified Fourier coefficients used in
the spectral analysis. For each section the phase difference
between the measurements at 15 m and the other depths was
calculated from equation (12). Daniell estimates of coherence
and phase with 5 degrees of freedom were computed by averaging
the unit vectors with a box car filter of width five, and an
ensemble average of the Daniell estimates yielded estimates
having 25 degrees of freedom.

With probability P, a phase angle ϕ lies between $\phi \pm \Delta\phi$
(Munk et al, 1959); values of $\Delta\phi$ for P = 95% are listed in
Table 4a. The confidence limits of a coherence estimate
(Table 4b) were interpolated from Groves and Hannan (1968, p.157).
Significant coherences are determined by the null hypothesis,
i.e., with probability P, (1 - P) · 100% of the coherence es-
timates will be greater than R^* when their true value happens
to be zero. R^* is defined by (Groves and Zetler, 1964)

$$R^* = \sqrt{1 - (1-P)^{\frac{2}{\nu-2}}} \tag{14}$$

A correlation exists between constituents in a frequency band
when the phase difference remains constant and coherence is
significant.

The vertical coherency of the high frequency temperature
fluctuations is shown in figure 19. Phase spectra have been
drawn only when the coherence was significant, and the dashed
horizontal line on the coherence plots indicates $R^* = .47$.
Only the first 60 of the 102 estimates are shown. For the re-
maining 42 estimates, the harmonic numbers associated with
coherence values greater than R^* are contained in Table 5.
From about 8 m to 30 m below the surface the harmonic consti-
tuents in the frequency interval .1133 cpm to .2010 cpm (8.8
minutes to 5.0 minutes) have significant vertical coherency at
the 95% confidence level. Zero phase difference occurs between
the constituents at 10 m, 20 m, 25 m, and 30 m below the sur-
face, and a uniform phase results throughout the thermocline
region when the confidence limit $\Delta\phi$ is applied to the phase
difference between those levels and the 15 m level. A different
transfer characteristic of the thermistor at 15 m could have
caused the 310° phase difference between the 15 m level and the
other levels. The vertical coherency analysis indicates that
the high frequency temperature fluctuations are coherent and in
phase in the region above the sill depth. The whole thermocline
heaves up and down in phase with periods from six to eight
minutes.

Table 4a

VALUES OF $\Delta\phi$ FOR DIFFERENT COHERENCES ($\nu = 25$, P = 95%)

Coherence	$\Delta\phi$
.5	66°
.6	45°
.7	32°
.8	23°

Table 4b

VALUES OF UPPER AND LOWER LIMITS FOR DIFFERENT COHERENCES
(P = 95%)

Coherence	(Coherence)$_{\text{upper limit}}$	(Coherence)$_{\text{lower limit}}$
.326	.600	0.0
.447	.693	0.0
.548	.76	0.1
.632	.81	.26
.707	.85	.37
.774	.88	.51
.837	.92	.62

Table 5

HARMONIC NUMBER OF COHERENCE ESTIMATES GREATER THAN .47

	Harmonic number of coherence estimates greater than .47
R_{31}	81
R_{32}	71, 81, 101
R_{34}	66, 75, 87, 97, 101
R_{35}	67, 71, 74, 79, 81, 82, 97, 98, 101
R_{36}	nil
R_{37}	nil
R_{38}	98
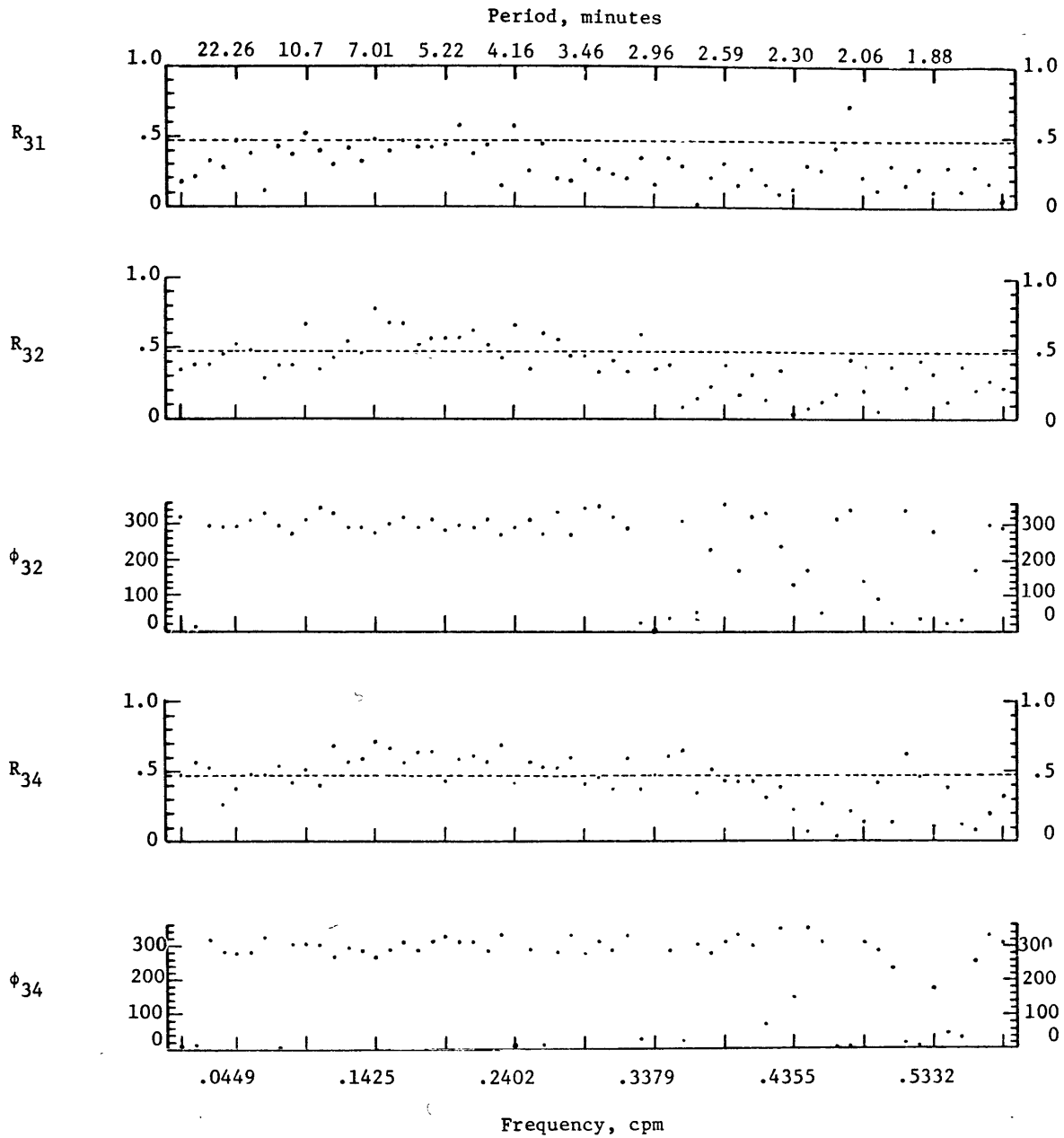
Figure 19. Ensemble averaged coherence and phase at different depths relative
to the temperature fluctuations at the 15m level.
(α = 25; Δf = .009765 cpm; f_1 = .005859 cpm)

Figure 17.   Ensemble averaged coherence and phase at different depths relative to the temperature fluctuations at the 15m level.
(α = 25; Δf = .009765 cpm; f_1 = .005859 cpm)

Internal Wave Model

The Eulerian equation describing the vertical velocity of small amplitude two-dimensional motion in a stably stratified Boussinesq fluid which has a basic velocity distribution $U(z)$ is (Bretherton, 1966)

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)^2 \left(w_{xx} + w_{zz}\right) - \left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\left(U_{zz}\, w_x\right) + N^2\, w_{xx} = 0 \tag{15}$$

The x-axis has been oriented in the direction of propagation and the vertical axis z is positive upwards. A possible solution is a sum of simple harmonic (progressive) waves. Let one such constituent of frequency $\sigma\left(= \dfrac{2\pi}{\text{period}}\right)$ and horizontal wavenumber $k\left(= \dfrac{2\pi}{\text{wavelength}} \right)$ be represented by the real part of

$$w(x,z,t) = W(z)\, e^{i(kx - \sigma t)} \tag{16}$$

where $W(z)$ is the amplitude of the vertical component of the perturbation velocity. Substitution of equation (16) into equation (15) yields

$$\frac{d^2 W}{dz^2} + \left[\frac{N^2 k^2}{(\sigma - kU)^2} + \frac{k\, U_{zz}}{(\sigma - kU)} - k^2 \right] W = 0 \tag{17}$$

Equation (17) and two vertical boundary conditions form an eigenvalue problem. Along a rigid horizontal bottom the vertical velocity equals zero.

$$W = 0 \quad \text{at} \quad z = 0 \tag{18}$$

At the free surface the total (atmospheric) pressure is constant, say zero.

$$P_{total} = 0 \tag{19}$$

Since the frequency of internal waves is much less than the frequency of surface waves with the same wavenumber, the free surface condition can be approximated as (Phillips, 1966, p.165)

$$W = 0 \quad z = H \tag{20}$$

When U = 0 at all depths, equation (17) reduces to

$$\frac{d^2 W}{dz^2} + \frac{k^2 \left(N^2 - \sigma^2\right)}{\sigma^2} W = 0 \tag{21}$$

In regions where $\sigma < N$ the function W(z) has an oscillating behavior. Internal gravity waves (in a resting medium) with vertical wavenumber γ are then described by w(x,z,t).

50.

$$\gamma^2 = \frac{k^2(N^2 - \sigma^2)}{\sigma^2} \tag{22}$$

Internal gravity waves are vertical oscillations which propagate horizontally and have maximum vertical displacements between the surface and the bottom. For each mode, σ is a monotonic increasing function of k. The amplitude of the vertical velocity of internal wave mode one contains a single maximum between the top and bottom, and the entire water column oscillates in phase. For this mode the horizontal velocity reverses direction once between the top and bottom.

Equation (17) was solved by a method.that was suggested by Haskell (1953). The continuous vertical distributions of velocity and density are represented by many horizontal layers of constant U and ρ_0. In a layer denoted by m, equation (17) becomes

$$\frac{d^2 W_m}{dz^2} - k^2 W_m = 0 \tag{23}$$

and
$$W_m = A_m e^{kz} + B_m e^{-kz} \tag{24}$$

In order that σ and k remain unchanged at each interface between successive layers, two conditions must be satisfied at each interface. The kinematic condition requires continuity of

the vertical displacement,

$$\frac{W_m}{(\sigma - k U_m)} = \frac{W_{m-1}}{(\sigma - k U_{m-1})}$$ (25)

The dynamic condition requires continuity of total pressure. To first order, the total pressure at the displaced position of a fluid particle is given by (Hines and Reddy, 1967)

$$p_{total} = p_0 + p + \eta \frac{dp_0}{dz}$$ (26)

where p_0 is the hydrostatic pressure which is continuous at an interface. The disturbance pressure (within a layer) p is evaluated from the horizontal component of the equation of motion, and to first order the vertical displacement η is given by

$$w = \frac{\partial \eta}{\partial t} + U \frac{\partial \eta}{\partial x}$$ (27)

The continuity of pressure at an interface is expressed by

$$\rho_m (\sigma - k U_m) \left(\frac{dW_m}{dz} - \frac{g k^2 W_m}{(\sigma - k U_m)^2} \right)$$
$$= \rho_{m-1} (\sigma - k U_{m-1}) \left(\frac{dW_{m-1}}{dz} - \frac{g k^2 W_{m-1}}{(\sigma - k U_{m-1})^2} \right)$$ (28)

Equations (25) and (28) reduce to the interfacial conditions for the case U = 0.

The interfacial conditions were cast in a matrix formulation suggested by Haskell (1953): The surface and bottom boundary conditions were related by a 2 x 2 matrix A_{ij}, and for free surface and rigid bottom the matrix element $A_{22} = 0$.

The matrix element A_{22} was computed from prescribed values of σ, k, U(z) and $\rho_o(z)$. Values of U(z) and $\rho_o(z)$ were evaluated in the middle of the layers. Twenty-three layers were used to approximate the vertical distributions of velocity and density. The solution is obtained by varying k until $A_{22} = 0$.

For $U(z) \neq 0$ the eigenfunction of mode one was computed for periods between six and eight minutes. The spectral and vertical coherency analysis has shown that for periods from six to eight minutes the temperature fluctuations recorded between 10 m and 30 m below the surface contained significant (at the 95% confidence level) vertical displacements which were coherent and had a uniform phase.

Figure 20 contains the vertical distribution of W (mode one) computed for the case U(z) = 0 and for the U(z) profile shown in the lower portion of that figure. The mean velocity profile U(z) was estimated from the speeds measured at 10.6 m, 25.8 m, and 42.6 m below the surface during the time of the large amplitude high frequency speed fluctuations. The directions corresponding to these measurements were parallel to the wave crests and therefore use of U(z) may be meaningless. These measurements were: U(10.6 m) = 40 cm/sec, U(25.8 m) = 15 cm/sec, and U(42.6 m) = 10 cm/sec. The density distribution used in the computation of W is also shown in

the lower portion of figure 20. Density was determined from the ensembled averaged values of mean temperature which had been computed in connection with the spectral analysis and from the salinity measurements made on July 28, 1967. The vertical distribution of the ratios of the rms vertical displacements (Table 3) to their maximum value have been plotted alongside the eigenfunction W. From 10 m to 30 m below the surface the rms vertical displacements of the temperature fluctuations and the eigenfunction computed for mode one with $U(z) \neq 0$ agree within 10%.

The horizontal wavelength, period and horizontal phase velocity corresponding to the eigenfunction shown in figure 20 are listed in the upper right portion of that figure. Table 6 contains additional eigenvalues computed for periods near 7-minutes. For the eigenvalues given in Table 6 the vertical distribution of the eigenfunction for mode one remained unchanged for the cases $U(z) = 0$ and $U(z) \neq 0$. In both cases the frequency increased with wavenumber. The constancy of the computed phase speed for 10% variations in wavelength and period is further (see figure 17) indication that these short period internal waves are non-dispersive.

The propagation speed computed for 7-minute period internal waves when $U(z) \neq 0$ was the same as the speed measured for individual narrow surface bands. The propagation speed of the high frequency temperature fluctuations measured with the towed thermistor also had the same magnitude as the computed speed. (The measurement of propagation speed from the temperature records obtained at buoys T1 and T2 was unsuccessful. Although the buoy spacing was less than one-half the computed wavelength, the buoys

were too close together to measure travel times, which were

smaller than those foreseen because of the large (and previously

unknown) tidal current. The thermistor cable suspended from the

third buoy, located 525 m east of buoy T2 failed to operate.)

The wavelength computed for 7-minute period internal waves when

$U(z) \neq 0$ was twice the distance separating the narrow surface

bands, suggesting that two narrow surface bands are associated

with each wavelength of the internal waves measured at Station T.

There is doubt concerning the direction of the mean current $U(z)$.

If the current is parallel to the wave crests then eigenfunction

calculations using $U(z)$ are meaningless. When $U(z) = 0$ the (theo-

retical) wavelength computed for 7-minute internal waves of mode

one agrees with the spacing measured between individual surface

bands but the observed speed of the bands does not agree with the

computed (theoretical) phase speed.

Table 6

EIGENVALUES COMPUTED FOR VELOCITY AND
DENSITY DISTRIBUTIONS SHOWN IN FIGURE 20

$U(z) = 0$			$U(z) \neq 0$		
L m	T min.	c cm/sec	L m	T min.	c cm/sec
			301	6.6	76.3
134	6.7	35.0	321	7.0	76.7
158	7.5	35.3	344	7.5	77.0
173	8.0	36.1	371	8.0	77.5

L = Horizontal wavelength
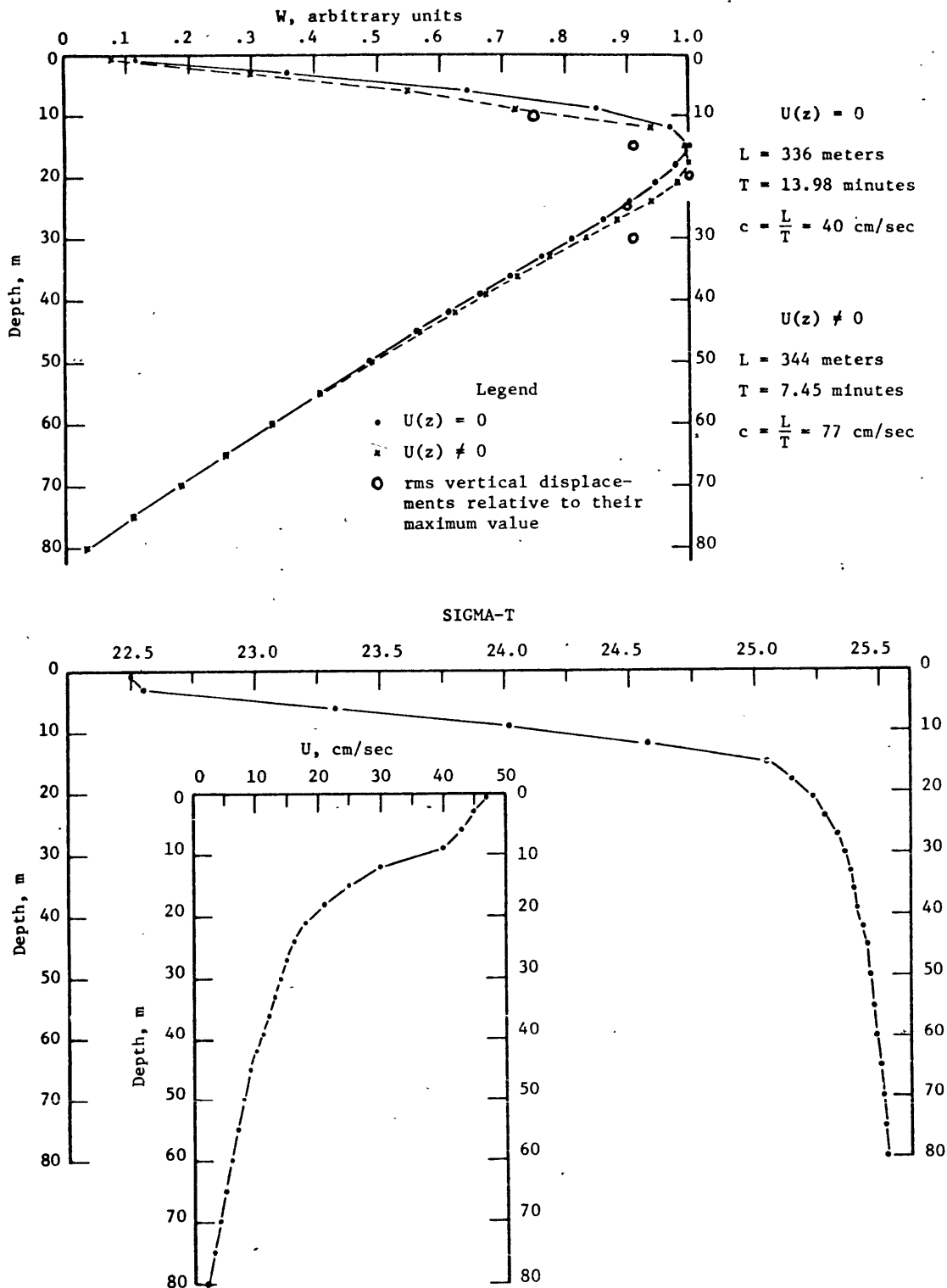T = Period
c = Horizontal phase velocity

Figure 20. Upper: Vertical distribution of the amplitude of the vertical velocity (in arbitrary units) for internal wave motion of mode one in a fluid at rest and in a fluid moving with the profile U(z). The eigenvalues corresponding to W are shown on the right. Lower: Vertical distribution of density (SIGMA-T) and velocity used in computing W.

In conclusion it must be said that there are many unan-
swered questions which arise from a study of the data. For
example, we would like to know what generates the 7-minute
waves. It is tempting to suppose that they are a result of
shearing instability in the tidal flow. At Station T the
Richardson number does drop almost to 1/4 for very short in-
tervals, but further upstream the Richardson number appears
to be somewhat higher, so that this speculation is inconclusive.
According to Miles (1961) and Howard (1961), a sufficient con-
dition for small amplitude stable motion in parallel stably
stratified inviscid flow is a Richardson number everywhere
greater than 1/4. A Richardson number was evaluated at Station
T for 10-minute averages between 1148 EST, July 14 and 2000
EST, July 16 (1967) using: (1) 10-minute temperature averages
calculated from the thermistor measurements at 15 m and 20 m
below the surface, (2) 10-minute averages of velocity recorded
at 10.6 m and 25.8 m, and (3) a single salinity measurement
made at 15 m and 20 m. Figure 21 contains a Calcomp plot of
10-minute averages of Richardson number, vertical shear, and
Brunt-Väisälä frequency. Variations of the Brunt-Väisälä fre-
quency were smaller than variations in shear. When the shear
decreased, the Richardson number increased. The Richardson
number, though always greater than 1/4, was less than unity
and as low as .3 prior to the initiation of the short period
internal waves.
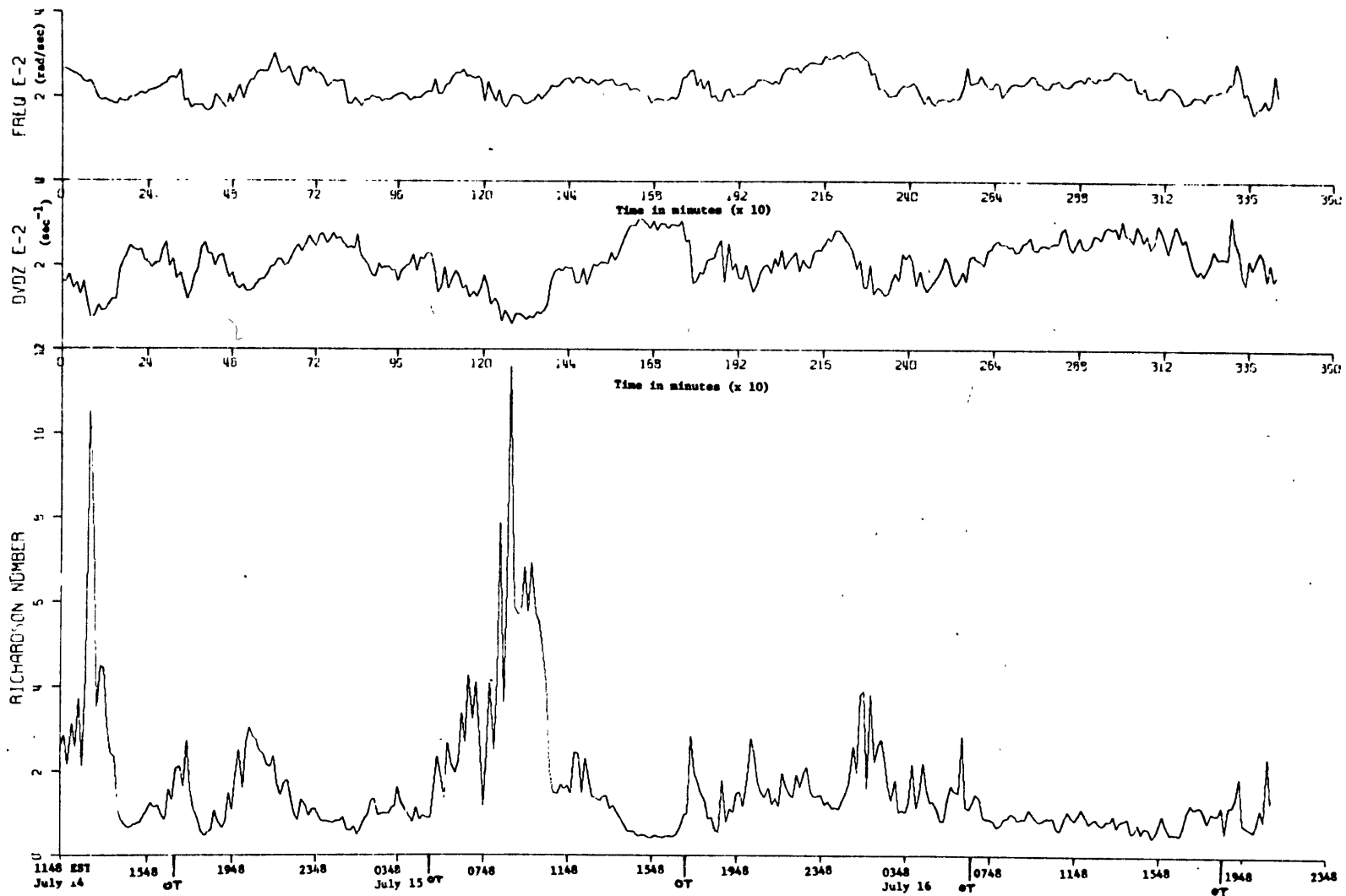
Figure 21. Ten minute averages of Richardson number (lower curve), vertical shear (middle curve) and Brunt-Väisälä frequency (upper curve). On the lower abscissa, a tick mark labelled OT is the onset time of a group of short period internal waves measured at Station T.

51.

Another feature of the phenomenon which is not clearly explicable is the actual mechanism of the banded structure. Internal waves are cellular and contain alternating zones of convergence and divergence (Eckart, 1961). Bands of calm water in a rippled sea (Lafond, 1962) and bands of ruffled water in a smooth sea (Pickard, 1961) have been associated with convergence zones of short period internal waves occurring near the surface.

It is difficult to say much definite about the abrupt rise in temperature accompanying the first wave. The kinematics is fairly clear. Figure 22 describes the vertical displacement occurring at the onset of the abrupt temperature rise (figure 12) and the speeds occurring before and after a group of short period waves (figure 8).
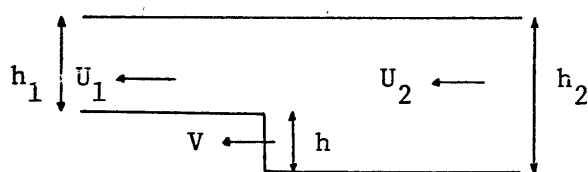
$h_1 = 15$ m $U_1 = 35$ cm/sec

$h_2 = 25$ m $U_2 = 45$ cm/sec

$h = 10$ m

Figure 22. Thermal front

The speed of the thermal front is

$$V = \frac{U_2 h_2 - U_1 h_1}{h} \tag{29}$$

Using the parameters listed in figure 22, $V = 60$ cm/sec, which
is the same order as the measured propagation speed of the abrupt
rise in temperature. The ultimate reason for flooding the sur-
face from the east with a layer of deeper thermal structure
is unknown.

Appendix A: Field Program

During the spring of 1965 equipment was gathered, a site selected, and a fishing vessel hired. The initial plan consisted of mooring a Richardson buoy with three nylon lines and recording on the buoy the temperature from thermistors placed along three multiconductor cables. The thermistor cables were to be anchored along the bottom at an angle from the buoy and buoyed vertically with subsurface floats. The first buoy launching took place at the end of June and was disastrous! The rented fishing boat was less than 35 feet in length and totally inadequate. For example, only two 1500 pound Stimson anchors could be loaded for fear of sinking the boat, and during the lowering of an anchor, the vessel nearly rolled over. At the beginning of July the project was in difficulty -- it was without a boat; two anchors had been lost; the buoy was shoaled at the Scituate Coast Guard Station; the equipment was stored in a garage in Scituate; and a 1500 pound Stimson anchor rested on the Scituate Town Pier against the wishes of the Harbormaster. Near the end of July a suitable vessel was hired. She was a 57-foot fishing dragger called the FRANCES ELIZABETH and was equipped with two winches, two windlasses, Loran, automatic pilot, fathometer, a knowledgeable skipper and a crew of two.

A single taut-wire mooring was used for the remainder of the 1965 summer when it became apparent that six lines extending at an angle from the buoy would inevitably result in chaos. Since internal waves had not previously been observed in Massachusetts Bay, their existence and not their directional properties would now be investigated. Wire cable was used to reduce the elasticity in the mooring and to keep each thermistor fixed in space. The thermistor cable hung from the buoy and was attached to the wire cable with swivel snap hooks.

A successful recording of temperature did not occur until late September due to failures of the electronic instruments. On that occasion four small surface floats, each having a net buoyancy of 100 pounds, buoyed up a wire cable and a thermistor cable attached to it. Recording took place on the fishing vessel stationed about 50 feet from the floats. The Richardson buoy had not been used because the mooring cable had parted near the anchor in mid-September. The buoy drifted towards the southwest and was retrieved by the Race Point (Massachusetts) Coast Guard when it was a mile offshore from their station.

The data collected in September, 1965 consisted of temperature measurements from nine depths sampled at 8-second intervals for a period of 2-1/2 hours. The data was plotted on the Calcomp plotter and fluctuations of 6-minute periods were observed in the upper 40 m.

In the spring of 1966 two additional temperature recording digitizers and two Richardson buoys were purchased. Each recorder was programmed to sample temperature at nine depths and pressure at one depth every two minutes. This sampling rate was chosen to obtain as long a series as possible (i.e., about eight days) with the minimum amount of aliasing. At the end of June, 1966 three temperature recording buoy systems were moored at Station T using the F/V FRANCES ELIZABETH. The buoys (A, B, and C) were located at the apexes of a right angled triangle (figure A1), although an equilateral triangular array had been planned. The buoys remained on station until their removal by the FRANCES ELIZABETH on September 1, 1966. The successes and failures in connection with the data obtained from the moored buoys during the 1966 summer have been summarized in Table A1.

Throughout the 1966 summer, data was collected from the MIT research vessel RR SHROCK. With it the digitizers were placed on (and taken off) the buoys; the buoy array was surveyed with magnetic compass, measured line, and sextant; water was collected in Van Dorn bottles for subsequent salinity determination at the Woods Hole Oceanographic Institution (WHOI); and BT's were run. On three separate occasions a Roberts current meter, on loan from the U.S. Naval Underwater Weapons Research and Engineering Station, Newport, R.I., failed to operate. Also, two flights were made over Station T in the WHOI heliocourier.
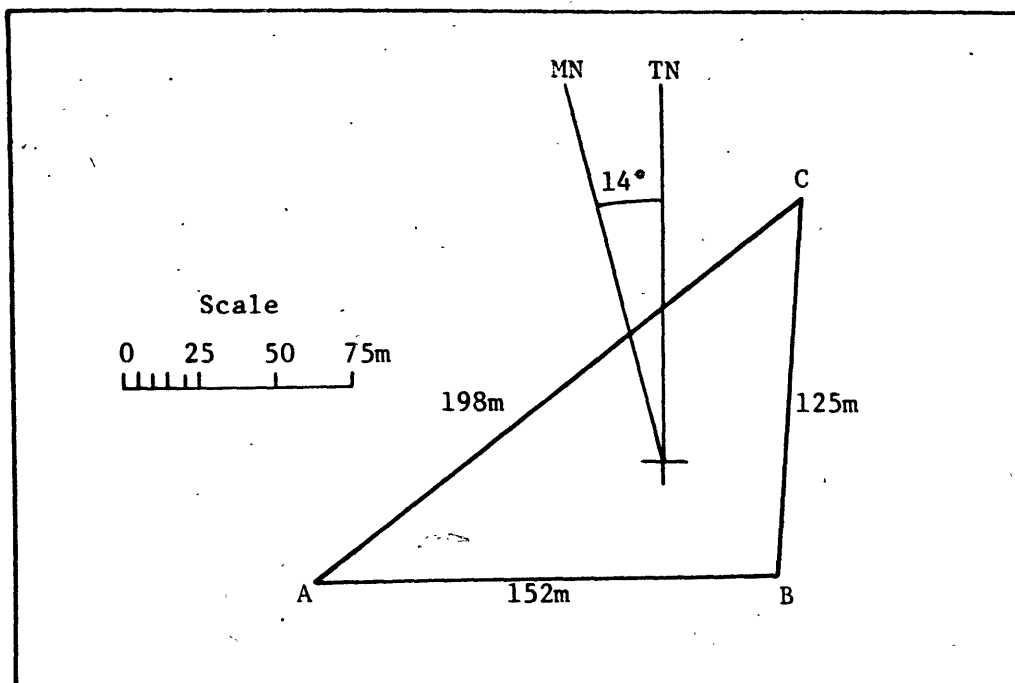
Figure A1. Location of buoys at Station T (1966)

Table A1.

STATION T MOORED BUOY RESULTS-1966

BUOY	EXPT 66-I	EXPT 66-II	EXPT 66-III
A	NO DATA Digitizer Failure	DATA July 28-Aug 3 Spurious Obs Aug 3-Aug 8	NO DATA Spurious Obs
B	NO DATA Digitizer Failure	NO DATA Digitizer Failure	NO DATA Severed Thermistor Cable
C	DATA July 6-July 9 Time Origin Lost	DATA July 28-Aug 3 Time Origin Lost	DATA Aug 24-Aug 30 Time Origin Lost

At the end of June, 1967 four moorings were established at Station T using the R/V RR SHROCK. Two of the three temperature recording buoy systems were set along a line perpendicular to Stellwagen Bank (figure A2). The fourth mooring contained three Geodyne current meters (Table A2). The buoys were removed in early September, but not before a fully instrumented temperature recording buoy system (buoy T1) was lost on the east side of Stellwagen Bank. Table A3 summarizes the data collected from the moorings.

The mooring system was launched by the anchor-first method. On arrival at Station T, the Richardson buoy with its chain bridle was lifted onto the water. Using the A-frame, the anchor was then gently lowered until the end of the wire cable came to a height about five feet above deck level. The mooring cable and an auxiliary winch-cable were connected through a pear-shaped ring. The thermistor cable was then attached to the wire cable and slid into the water by its own weight. The connector end of the thermistor cable was threaded through the center of the toroid and the cable fastened to the tension bridle. The pear-shaped ring was lowered to deck level and the chain leader from the buoy bridle was shackled to it. Half-inch (and larger sizes) round pin anchor shackles were used in the mooring. To disengage the vessel from the mooring the winch pulled in the wire cable a foot or two and a nylon stopper was secured to the ring. The winch released its hold on the mooring system and the auxiliary cable was removed. The nylon line was then cut

close to the ring dropping the anchor 1-2 m to the bottom.

Table A2

LOCATION OF VELOCITY MEASUREMENTS

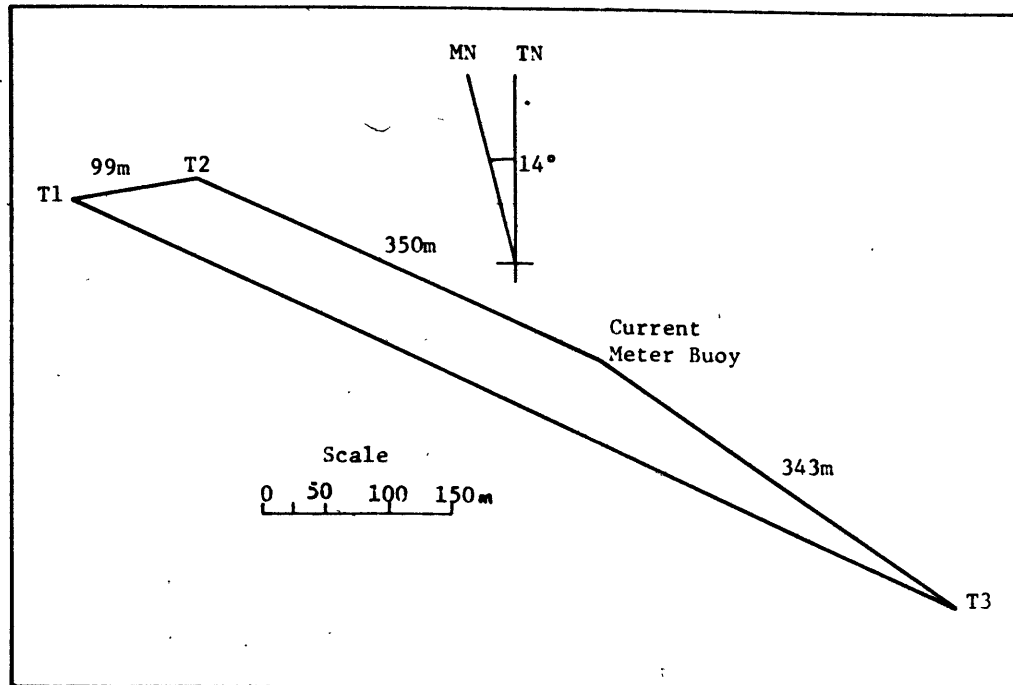| Date (1967)	EXPT	Location	Water depth m	Amount of slack m	Current meter Ser. no.	Depth m
July 13-17	67-2	42°16'32" N 70°24'54" W	82	4	H879 H880 H881	10.6 25.8 45.6
July 24-28	67-3	42°16'12" N 70°16'36" W	29	2	H879 H880 H881	15.2 7.6 22.8
August 22-25	67-4	42°16'32" N 70°08'30" W	61	3	H879 H880 H881	45.6 25.8 10.6

Figure A2. Location of buoys at Station T (1967)

Table A3. MOORED BUOY RESULTS-1967

BUOY	EXPT 67-II	EXPT 67-III	EXPT 67-IV

T1	DATA
	
	July 13–July 17
	
	Location:
	  Station T
		DATA
		
		July 25–July 28
		
		Location:
		  Station T
			DATA
			
			Aug 25–Aug 29
			
			Location:
			  Station T

T2	DATA
	
	July 13–July 17
	
	Location:
	  Station T
		DATA
		
		July 25–July 28
		
		Location:
		  Station T
			NO DATA
			
			Buoy Lost
			
			Location:
			  11 km East of
			  Stellwagen Bank

(3)
Current
Meters
	DATA
	
	July 13–July 17
	
	Location:
	  Station T
		DATA
		
		July 24–July 28
		
		Location:
		  Stellwagen Bank
			DATA
			
			Aug 22–Aug 25
			
			Location:
			  11 km East of
			  Stellwagen Bank

		Photographic survey over Station T, July 26	
EXPT 67-I failed because the buoys did not remain on station.			

Appendix B: Data Format

The format of the IBM-compatible magnetic tapes containing the temperature data is shown in Table B1. Each row or tape-record contains ten sequential measurements of a single sensor while a column of fourteen rows represents the fourteen sensors observed at a particular time. Uninterrupted time series belonging to various sensors are obtained by sorting the data as the tape is read into the computer, and a listing of the Fortran IV statements which preform this operation is given in Table B2. Prior to any type of analysis the temperature time series is scanned for obvious errors (i.e., temperature values greater than $20^{\circ}C$ or less than $2^{\circ}C$) which are replaced by a linear interpolation using the preceding value and the next value. For example, if $T_i < 2^{\circ}C$, then T_i becomes $(T_{i-1} + T_{i+1})/2.0$. These errors result from 'bad' spots (dirt, wrinkles) occurring on the original 4-inch magnetic tape or Kodak film, or because bits were destroyed during the processing of the data by Geodyne. The data collected from the tape recording digitizers contained fewer of these errors.

The format of the 7-track tapes containing the velocity measurements is shown in Table B3. Each row contains a 50-second average value of the compass, vane, direction (= compass - vane), normalized unit vector of compass and vane, speed, and the angle of inclination of the meter from the vertical.

The compass, vane, and direction represent the angle of the instrument in the earth's magnetic field, the direction of the current relative to the instrument, and the direction of the current relative to magnetic north, respectively. By convention, the current direction is specified as the direction toward which the current is flowing. The normalized unit vectors provide a measure of dispersion occurring among the individual compass and vane values, i.e., a 1.0000 indicates that all twenty elements of the 50-second sample were identical.

Table B1.

A LISTING OF A TEMPERATURE-TAPE

```
 9 23?.   4.81   4.81   4.81   4.81   4.81   4.81   4.81   4.81   4.81   4.81
10 240.   4.54   4.54   4.64   4.64   4.64   4.64   4.64   4.64   4.64   4.64
11  ?.    4.43   4.43   4.43   4.43   4.43   4.43   4.43   4.43   4.41   4.41
12  ?.    4.3?   4.39   4.39   4.3?   4.39 . 4.39   4.39   4.39   4.37   4.37
13  0.4995.004995.004997.004997.004995.004995.004995.004997.004997.00
14  0. 179.64 175.78 172.99 173.23 176.57 175.77 178.97 179.90 174.09 177.27
 1 24?.500?2.005002.005002.005002.005002.005002.005002.005002.005002.00
 2 24?.  14.12  14.12  14.11  14.12  14.13  14.16  14.16  14.19  14.16  14.22
 3 243.  21.23  21.23  21.23  21.23  21.23  21.23  21.23  21.23  21.23  21.23
 4 244.  13.55  13.29  13.13  13.04  13.06  12.78  12.90  12.80  12.72  12.66
 5 24?.   8.22   8.19   8.14   8.08   8.04   8.01   7.99   7.95   7.94   7.92
 6 24?.   7.45   7.45   7.45   7.45   7.45   7.43   7.41   7.39   7.35   7.33
 7 247.   5.80   5.82   5.86   5.92   5.96   6.00   6.04   6.05   6.08   6.08
 8 24?.   5.24   5.24   5.24   5.24   5.24   5.24   5.24   5.24   5.24   5.24
 9 24?.   4.81   4.81   4.81   4.81   4.81   4.81   4.81   4.81   4.81   4.81
10 25?.   4.64   4.64   4.64   4.65   4.66   4.66   4.68   4.69   4.69   4.66
11  ?.    4.41   4.41   4.41   4.41   4.41   4.41   4.41   4.41   4.41   4.41
12  ?.    4.37   4.37   4.37   4.37   4.37   4.37   4.37   4.37   4.37   4.37
13  0.4997.004997.004997.004997.004997.004997.004997.004997.004995.00
14  0. 179.14 175.10 173.34 172.85 172.63 172.70 172.74 174.17 173.55 180.45
 1 25?.500?2.005002.005002.005002.005002.005002.005002.005002.005002.00
 2 25?.  14.22  14.22  14.26  14.27  14.27  14.27  14.27  14.27  14.28  14.35
 3 25?.  21.23  21.24  21.24  21.24  21.24  21.24  21.24  21.25  21.25  21.25
 4 254.  12.57  12.50  12.49  12.53  12.75  13.16  13.33  13.19  13.07  13.01
 5 255.   7.90   7.88   7.86   7.88   7.94   8.03   8.06   8.03   7.99   7.97
 6 255.   7.29   7.29   7.31   7.33   7.33   7.41   7.43   7.41   7.37   7.39
 7 257.   6.11   6.13   6.15   6.15   6.17   6.21   6.21   6.17   6.13   6.11
 8 25?.   5.24   5.24   5.24   5.24   5.24   5.24   5.24   5.22   5.22   5.20
 9 25?.   4.81   4.81   4.81   4.81   4.81   4.81   4.81   4.81   4.81   4.79
10 26?.   4.65   4.56   4.66   4.65   4.66   4.66   4.66   4.65   4.64   4.64
11  ?.    4.43   4.43   4.43   4.43   4.43   4.43   4.43   4.43   4.43   4.43
12  ?.    4.37   4.37   4.37   4.37   4.37   4.37   4.37   4.37   4.35   4.35
13  0.4995.004997.004997.004997.004997.004997.004997.004997.004997.00
14  0. 180.35 179.28 180.27 172.94 175.42 175.42 175.94 177.08 179.21 173.09
 1 261.500?2.005002.005002.005002.005002.005002.005002.005002.005002.00
 2 26?.  14.35  14.35  14.35  14.35  14.35  14.37  14.40  14.40  14.40  14.40
 3 26?.  21.25  21.25  21.25  21.25  21.25  21.26  21.26  21.26  21.26  21.26
 4 26?.  12.93  12.87  12.93  12.98  13.12  13.25  13.16  12.99  12.80  12.62
 5 26?.   7.97   7.97   7.97   7.95   7.95   7.95   7.94   7.92   7.88   7.84
 6 266.   7.37   7.39   7.41   7.43   7.43   7.43   7.41   7.33   7.26   7.18
 7 267.   6.09   6.09   6.09   6.11   6.13   6.13   6.11   6.09   6.08   6.02
 8 26?.   5.20   5.20   5.20   5.20   5.20   5.20   5.20   5.20   5.20   5.20
```

LEGEND

A. Horizontal

Column	Remarks
1,2	Sensor number
3-7, inclusive	Sequential numbering of time series
8-77, inclusive	10 sequential measurements

B. Vertical

Sensor Number	Measurement	Depth (meters) 1966	1967
1	Reference resistor (5000 ohms)	Inside the digitizer	
2	Time (hours)	Inside the digitizer	
3	Temperature (°C)	Inside the digitizer	
4	Temperature (°C)	5	5
5	Temperature (°C)	11.1	10
6	Temperature (°C)	17.2	15
7	Temperature (°C)	23.3	20
8	Temperature (°C)	29.4	25
9	Temperature (°C)	35.5	30
10	Temperature (°C)	41.6	35
11	Temperature (°C)	47.7	40
12	Temperature (°C)	53.8	45
13	Reference resistor (5000 ohms)	59.9	50
14	Pressure (feet)	66.0	55

A LISTING OF THE STATEMENTS THAT SORT THE DATA

```
C     A TAPE RECORD CONTAINS 10 DATA POINTS
C     NCARD = 1 TAPE RECORD
C     SENSORS 4 THRU 8 ARE TO BE SORTED FROM THE TAPE
C     IN THE READ STATEMENTS (10,900), THE SUBSCRIPT I
CC    DETERMINES WHICH OF THE 14 SENSORS WILL BE SORTED
C     THE ARRAY TEMP(J) CONTAINS A TOTAL OF NPOINT VALUES
CC    BELONGING TO SUBSCRIPT I, NFIRST BEING THE FIRST VALUE.
CCC   THE FIRST VALUE IN TEMP(J) IS SUBSCRIPTED BY 1


      DATA IEND/'$$$$'/
      READ (5,600) NCARD
  600 FORMAT (I5)
C     SORTING OF THE PROBES
      DO 10 K=1,NCARD
      KK = K-1
      READ (10,900) ((DUMMY(I,J),J=1,10),I=1,3)
      READ (10,900) ((POINTS(I,J+KK*10),J=1,10),I=1,5)
      READ (10,900) ((DUMMY(I,J),J=1,10),I=1,6)
  900 FORMAT (7X,10F7.2)
   10 CONTINUE
C     ESTABLISHMENT OF THE TEMP ARRAY
    1 READ (5,901) NPOINT,NFIRST,NSENSE,(IHEAD(I),I=1,7)
  901 FORMAT (3I5,7A4)
      IF (IHEAD(1) .EQ. IEND) GO TO 100
      NLAST = NFIRST + NPOINT
      J=0
      DO 20 I=NFIRST,NLAST
      J=J+1
      TEMP(J) = POINTS(NSENSE,I)
   20 CONTINUE
```

*   *   *   *   *   *   *   *   *   *   *   *   *

EXAMPLE

```
1000 TEMPERATURE VALUES OF SENSORS 4,5,6,7, AND 8
ARE TO BE PLOTTED, AND THE FIRST VALUE OF SENSOR 4
BEGINS AT 500 DELT, 600 DELT FOR SENSOR 5, AND 700 DELT
FOR SENSORS 6,7, AND 8, DELT IS THE SAMPLING INTERVAL.
THE INPUT CARDS ARE
 170
1000   500     1   THERMISTOR 1 (5 METERS)
1000   600     2   THERMISTOR 2 (10 METERS)
1000   700     3   THERMISTOR 3 (15 METERS)
1000   700     4   THERMISTOR 4 (20 METERS)
1000   700     5   THERMISTOR 5 (25 METERS)
             $$$$
```
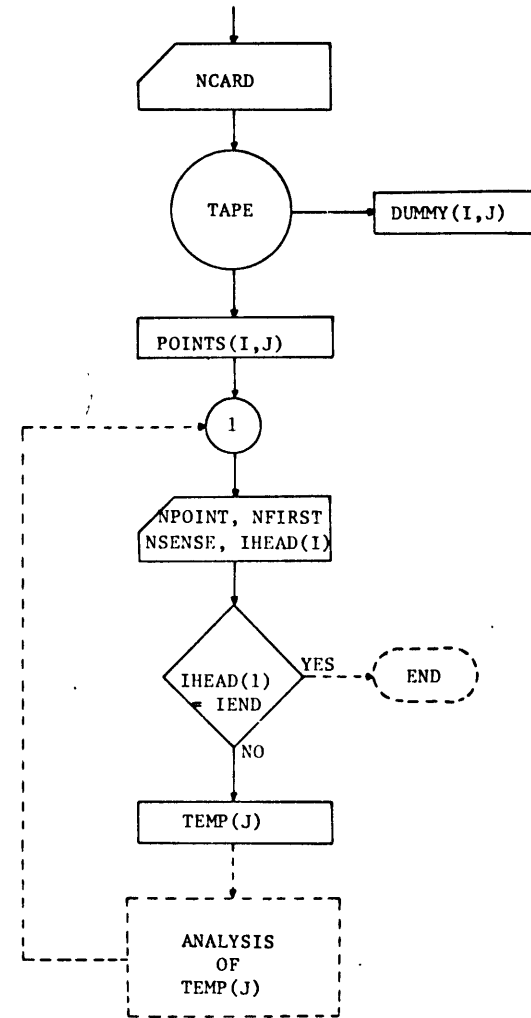
Table 83

A LISTING OF A CURRENT METER-TAPE

VALIDITY INDICATORS	INTERVAL NO.		COMPASS	VANE	DIRECTION	NORMALIZED UNIT VECTOR COMPASS	VANE	ROTOR SPEED IN KNOTS	INCLINOMETER READING IN DEGREES
**	101	AVERAGE	139	8	131	0.999	0.926	0.29	0
	102	AVERAGE	140	2	138	0.999	0.882	0.30	0
	103	AVERAGE	135	7	128	0.999	0.937	0.40	0
	104	AVERAGE	138	16	122	0.999	0.943	0.42	0
	105	AVERAGE	134	9	125	0.999	0.956	0.43	0
**	106	AVERAGE	132	11	121	0.998	0.986	0.43	0
	107	AVERAGE	132	7	125	0.999	0.970	0.40	0
	108 .	AVERAGE	137	13	124	0.998	0.947	0.28	0
	109	AVERAGE	136	10	126	0.999	0.928	0.42	0
	110	AVERAGE	142	24	118	1.000	0.913	0.42	0
**	111	AVERAGE	143	27	116	0.999	0.976	0.41	0
	112	AVERAGE	146	19	127	1.000	0.958	0.38	0
	113	AVERAGE	144	19	125	0.999	0.965	0.45	0
	114	AVERAGE	146	17	129	1.000	0.855	0.42	0
	115	AVERAGE	147	26	121	0.999	0.988	0.38	0
. **	116	AVERAGE	145	21	124	0.999	0.975	0.45	0
	117	AVERAGE	141	22	119	0.999	0.964	0.45	0
	118	AVERAGE	143	24	119	0.999	0.983	0.43	0
	119	AVERAGE	140	18	122	0.999	0.924	0.46	0

Appendix C: Temperature and Salinity Measurements at Station T

The mean value of a series of (ungrouped) temperature

measurements was computed from the formula

$$T_o = \frac{1}{N} \sum_{i=1}^{N} T_i$$ (C1)

where T_i is the value of the ith temperature measurement, N is

the total number of terms in the series[1], and for sequential

measurements $N\Delta t$ is the length of the series in time. The

overall variability of the measurements was described by the

standard deviation

$$SD = \left[\frac{\sum_{i=1}^{N} (T_i - T_o)^2}{N} \right]^{\frac{1}{2}}$$ (C2)

Vertical distributions of mean temperature and standard devia-

tion computed from the thermistor measurements obtained at

Station T in 1965 are shown in figure C1. Minima and Maxima

temperature values are given in Table C1. Bathythermographs

were lowered once or twice during each experiment to determine

the deep water temperature (Table C2).

Water, collected in eight 2-liter Van Dorn bottles spaced

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[1] The symbol N has also been used to represent the Brunt-Väisälä
frequency.
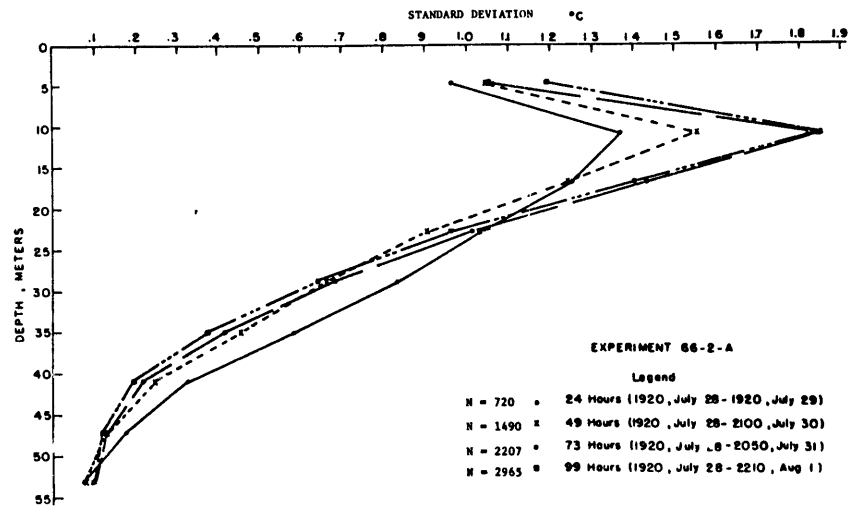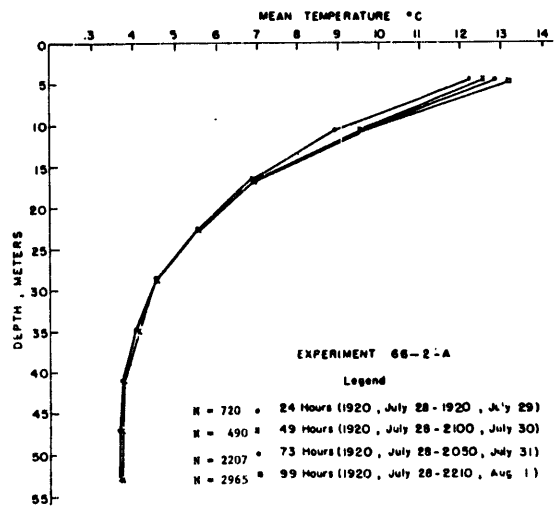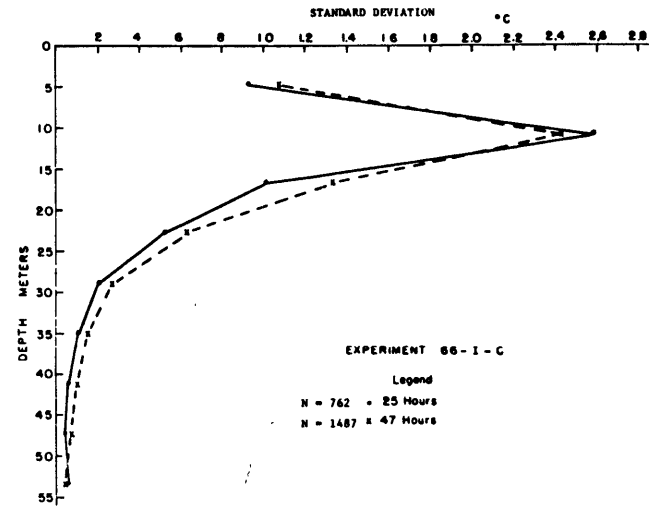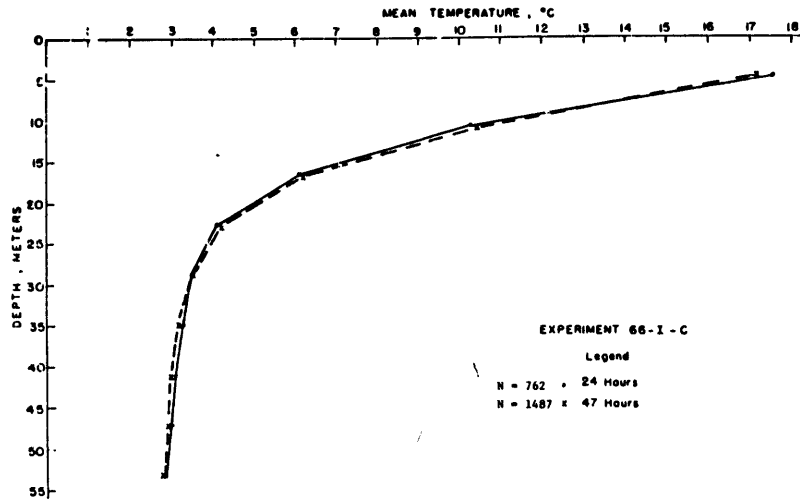
Figure C1

Table C2

AVERAGE TEMPERATURE DISTRIBUTION

Depth			Experiment				Therm. Tow
	66-I	66-II	66-III	67-II	67-III	67-IV	Sept. 19
m			Temperature (°C)				
0	18.1	17.0	15.9	14.0	18.0	17.7	14.5
50				3.9	3.9	4.0	4.4
55	3.0	3.8	4.0	3.85	3.85	3.9	4.1
60	3.0	3.8	4.0	3.85	3.8	3.9	4.0
65	3.0	3.6	3.9		3.75	3.9	4.0
70	2.9	3.5	3.8		3.75	3.85	4.0
75	2.9	3.4	3.6				4.0

Table C1

MINIMUM AND MAXIMUM TEMPERATURES

Depth	Expt 66-1-C		Expt 66-2-A	
	MIN	MAX	MIN	MAX
m	°C		°C	
5	13.87	18.88	10.54	16.09
11.1	5.70	18.35	6.43	14.62
17.2	4.18	16.29	5.20	13.85
23.3	3.02	9.94	3.97	12.26
29.4	2.11	6.00	3.64	9.79
35.5	2.86	4.55	3.69	7.11
41.6	2.75	3.53	3.54	6.29
47.7	2.75	3.16	3.49	5.01
54.8	2.68	2.99	3.47	4.37

Depth	Expt 67-2-T2		Expt 67-3-T2	
	MIN	MAX	MIN	MAX
m	°C		°C	
5	7.83	16.48	10.48	18.75
10	6.19	15.02	7.11	16.39
15	5.52	14.02	5.96	14.66
20	4.85	13.36	5.34	13.04
25	4.47	12.14	4.87	9.91
30	4.47	11.47	4.60	7.39
35	4.43	8.01	4.35	6.44
40	4.41	6.15	4.31	5.54
45	4.35	5.28	4.29	4.91

10 m apart, was analyzed for salinity by the conductivity method at WHOI. The salinity measurements are given in Table C3 and vertical distributions of the average salinity are shown in figures C2 and C3.

Density was computed directly from Knudsen's Formula (Lafond, 1951, p.91) using the mean values of temperature and salinity. The density gradient was computed from the formula

$$\left(\frac{d\rho_o}{dz}\right)_j = \frac{(\rho_o)_{j-1} - (\rho_o)_{j+1}}{2\,\Delta h} \qquad 2 \leq j \qquad (C3)$$

where subscript j represents a thermistor (e.g., j = 1 being the uppermost thermistor) and Δh is the spacing between thermistors. Gravity, calculated from the International Gravity Formula (Jacobs et al, 1959, p.91), was equal to 980.401 c.g.s. units.

The statistics of the temperature measurements recorded between July 25 and July 27 (49 hours) at buoys T1 and T2 are compared in Table C4.

Figures C4, C5, and C6 are Calcomp plots of the actual temperature measurements. The times of initiation of the high frequency temperature fluctuations are summarized in Table C5.

Table C3

SALINITY MEASUREMENTS AT STATION T

Date (1966) Depth (feet)	July 7	July 7	July 8	July 8 Salinity (%)	July 9	July 9	July 14
30	31.189	31.577	31.614	31.657	31.520	31.575	31.597
60		32.037	32.103	32.110	31.890	31.744	31.980
90	32.151	32.202	32.199	32.222	32.179		32.173
120		32.241	32.247	32.242	32.238	32.348	32.225
150	32.248	32.271			32.267		32.252
180		32.286	32.270	32.272		32.287	32.299
210		32.301			32.306	32.303	32.330
240					32.336		32.320

Date (1966) Depth (m)	Aug 11	Aug 11	Aug 23 Salinity (%)	Aug 23	Aug 24
10	31.940	31.942	31.679	31.660	31.872
20	32.112	32.158	31.962	32.049	32.126
30	32.180	32.175	32.110	32.156	32.173
40	32.194		32.163	32.186	32.220
50	32.215		32.189	32.217	
60	32.255		32.233	32.232	32.229
70	32.240		32.228	32.228	32.239
80	32.264			32.257	32.252

Date (1967 Depth (feet)	July 13	July 17	July 25 Salinity (%)	July 28	Aug 29
5	30.938	30.994	30.508	30.794	31.038
35	31.284	31.379	31.386	31.671	31.519
65	31.700	31.700	31.858	31.858	31.806
95	31.817	31.916	31.983	31.984	31.942
125	31.935	31.987	32.029		
155	32.022	32.023	32.067	32.037	32.030
185		32.025	32.112	32.048	
215	32.038	32.034	32.120		
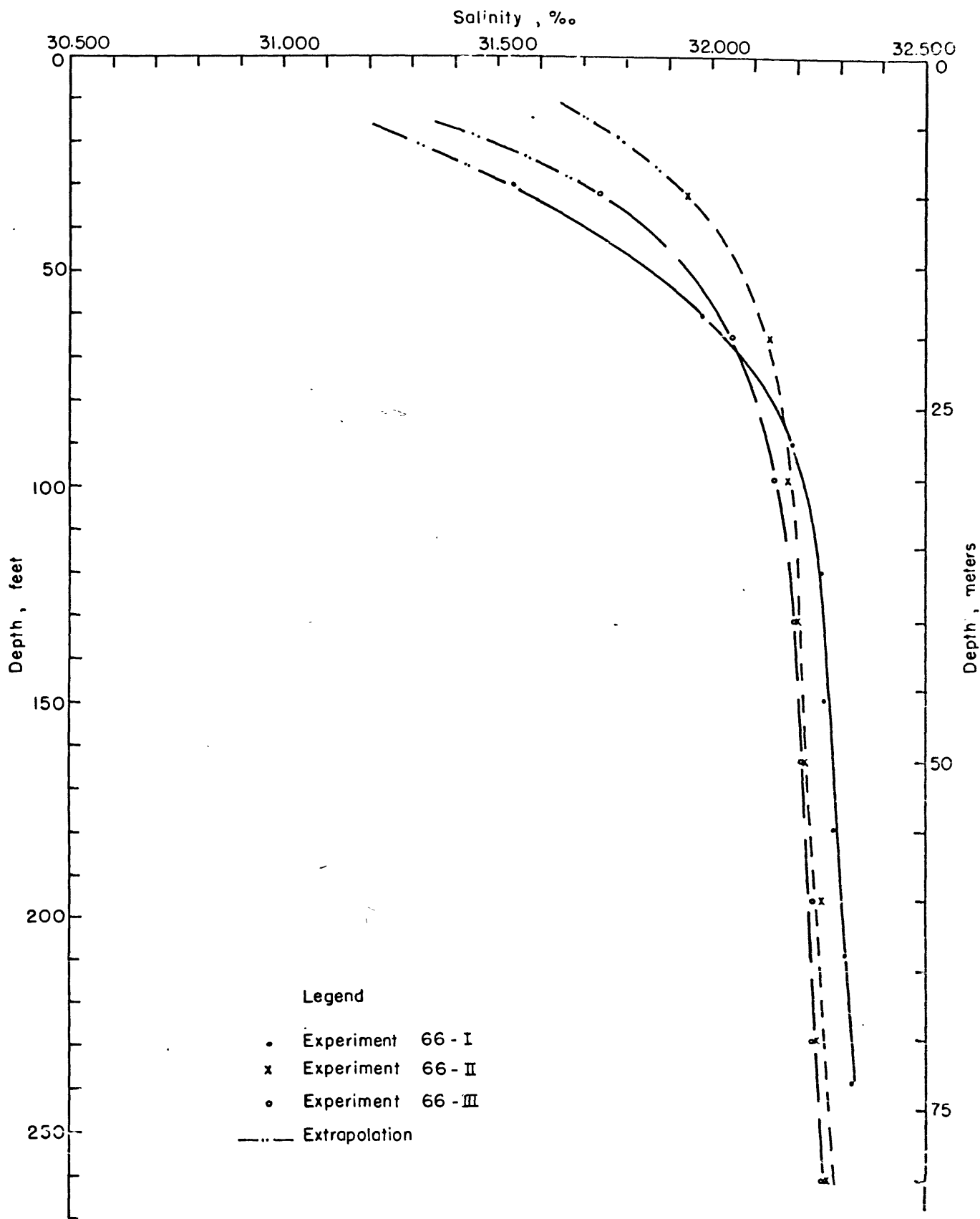
Figure C2.   Salinity Observations, 1966
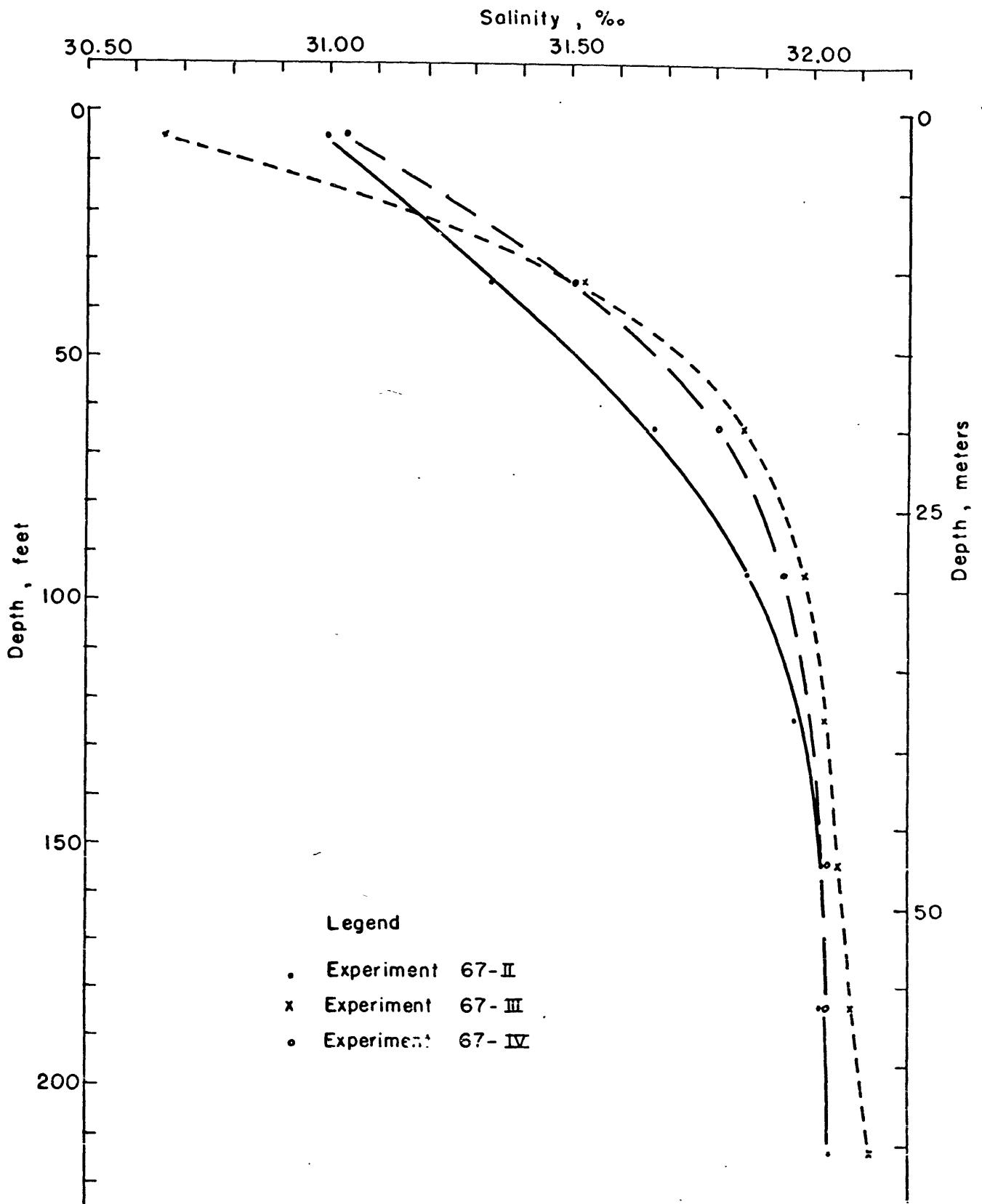
Figure 63.   Salinity Observations , 1967

Table C4

A COMPARISON OF 49 HOURS (N = 5900) OF MEASUREMENTS
RECORDED AT BUOYS T1 AND T2 DURING EXPT 67-3

Depth m	Buoy	Mean	SD	Max	Min	Brunt-Väisälä Frequency cpm
		Temperature (°C)				
5	T1	15.36	1.66	19.16	11.25	
	T2	13.91	1.64	18.75	10.48	
10	T1	10.71	1.66	18.08	7.77	.397
	T2	9.73	1.52	16.39	7.11	.371
15	T1	8.21	1.15	15.31	6.02	.281
	T2	7.69	0.96	14.66	5.96	.262
20	T1	6.70	0.69	13.36	5.48	.211
	T2	6.35	0.63	13.04	5.34	.200
25	T1	5.63	0.44	10.44	4.93	.159
	T2	5.46	0.41	9.91	4.87	.149
30	T1	5.13	0.30	7.59	4.60	.119
	T2	5.04	0.27	7.39	4.60	.112
35	T1	4.72	0.22	6.68	4.29	.0871
	T2	4.71	0.19	6.44	4.35	.0847
40	T1	4.57	0.14	5.90	4.29	.0704
	T2	4.52	0.12	5.54	4.31	.0697
45	T1	4.35	0.09	5.15	4.18	
	T2	4.41	0.08	4.91	4.29	
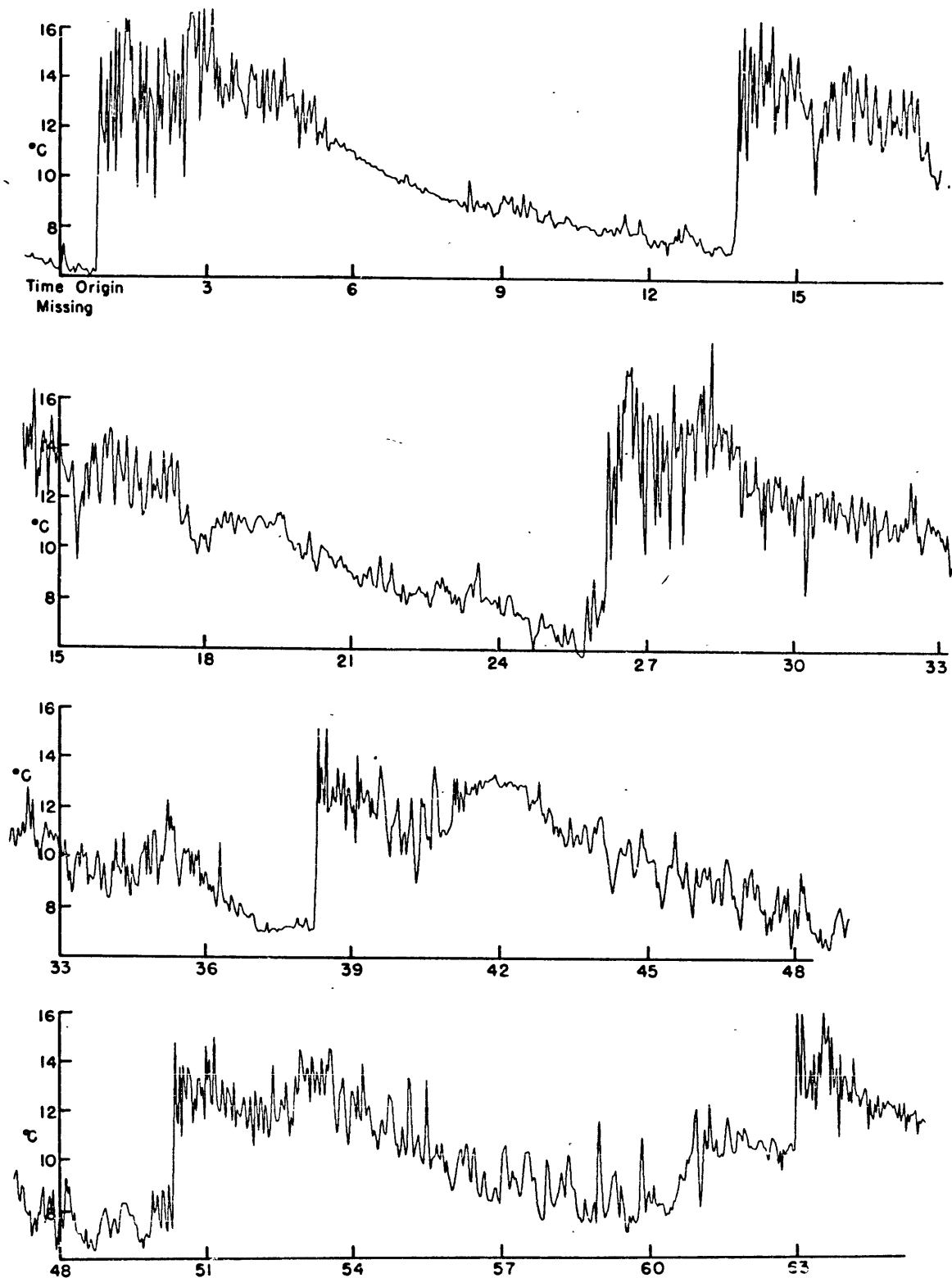
Figure C4 .   A Calcomp plot of 1950 temperature measurements at 11.1m
at Station T (buoy C) between July 6 and July 9, 1966.
Sampling interval was 2-minutes. The time origin is miss-
ing and the interval between tick marks on the abscissa is
3-hours.
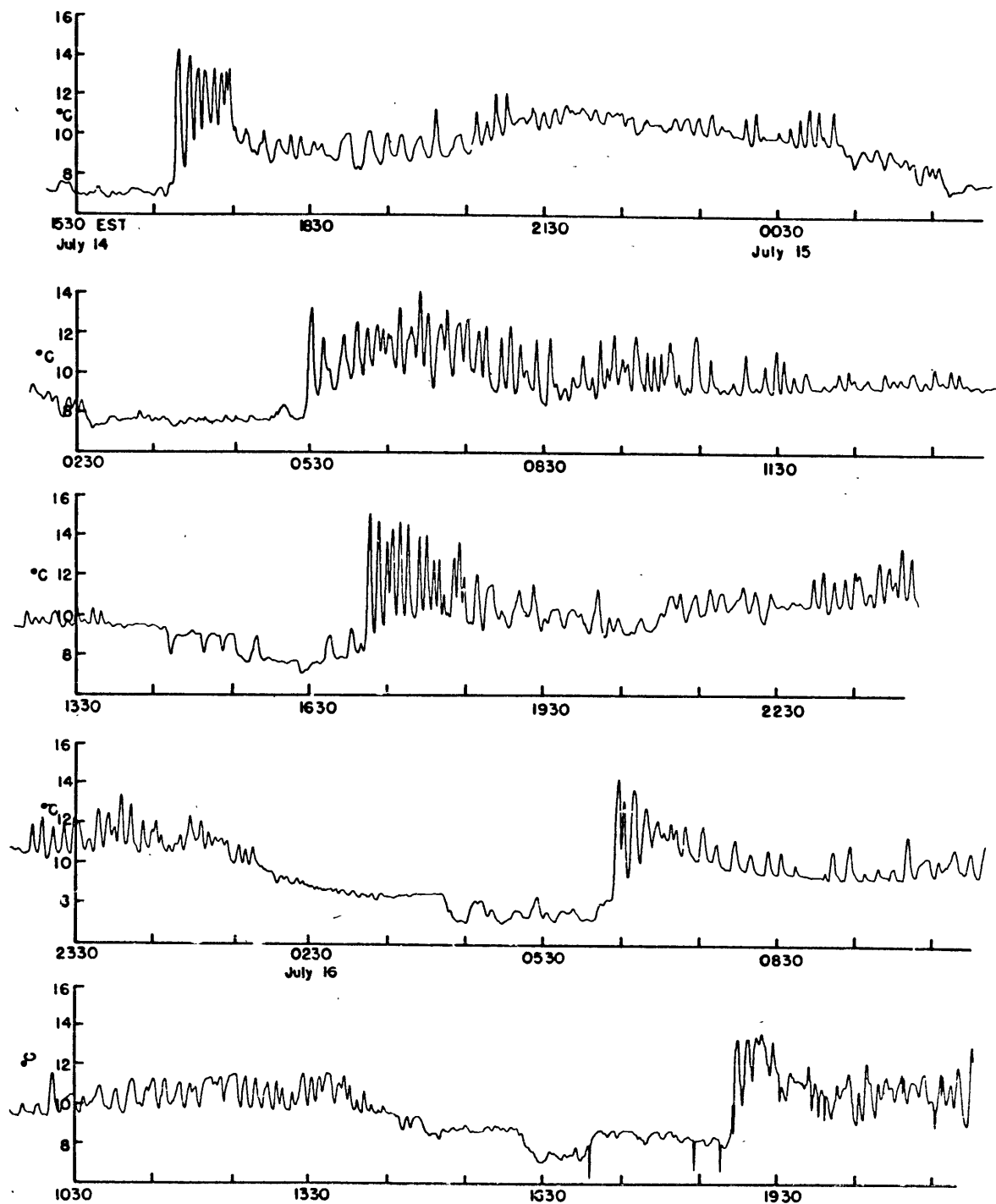
Figure CS . A Calcomp plot of 6500 temperature measurements at 10m
at Station T (buoy T2) between July 14 and July 17, 1967.
Sampling interval was 30-seconds.
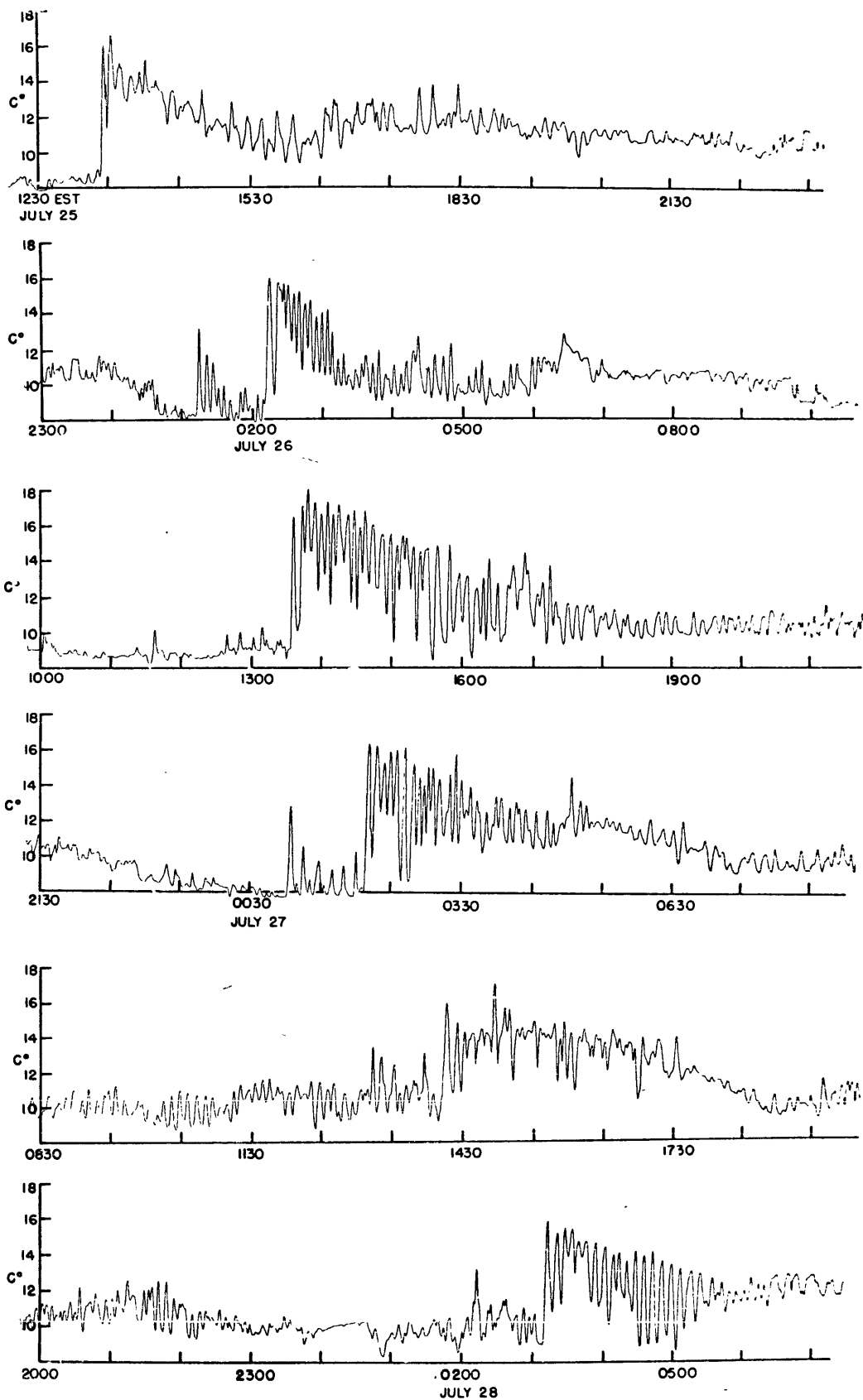
Figure C6 . A Calcomp plot of 7900 temperature measurements at 10m
at Station T (buoy T1) between July 25 and July 27, 1967.
Sampling interval was 30-seconds.

Table C5

TIMES OF INITIATION OF THE HIGH FREQUENCY TEMPERATURE FLUCTUATIONS

EXPT 66-2A			EXPT 67-2			EXPT 67-3	
EST	Date		EST	Date		EST	Date
1919	July 28		1645	July 14		1325	July 25
0752	29		0526	14		0212	26
1919	29		1713	15		1336	26
0731	30		0626	16		0209	27
2058	30		1855	16		1415	27
0909	31					0316	28
2054	31						
0927	August 1						
2212	1						
1026	2						

84.

Appendix D: Velocity Measurements

Bigelow (1927), using drift bottle measurements, des-

cribed the circulation in Massachusetts Bay as being dominated

entirely by tidal currents such that any non-tidal (geostrophic)

drift would be easily obscured. In the Gulf of Maine the tide

flows northward along Cape Cod but runs westward into Massachu-

setts Bay drawing southward around the northern tip of Cape Cod

and reaching a speed of 75 cm/sec in the narrow channel between

Provincetown and Stellwagen Bank. Haight (1942) has reported

that the maximum speed of a current rose measured on Stellwagen

Bank for 60 days in 1919 was 8 cm/sec. Although commercial

fisherman from Scituate speak of a strong current (about a knot)

on Stellwagen Bank, recent Coast and Geodetic Survey Tidal Cur-

rent Tables have indicated that the current on the Bank is

variable and too weak to measure.

The speed and direction measurements from each experiment

were organized into histograms (figures D1, D2, D3, D4, D5, and

D6). A histogram contains no reference to the frequency con-

stituents nor to the time sequence of the data, and it can be

interpreted if a large enough sample is used although its des-

cription becomes less reliable as the number of processes con-

tributing to the measurement increases. The speed histograms

at Station T are bimodal and the smaller high speed mode oc-

curring at the lower depths represents the resultant of tidal
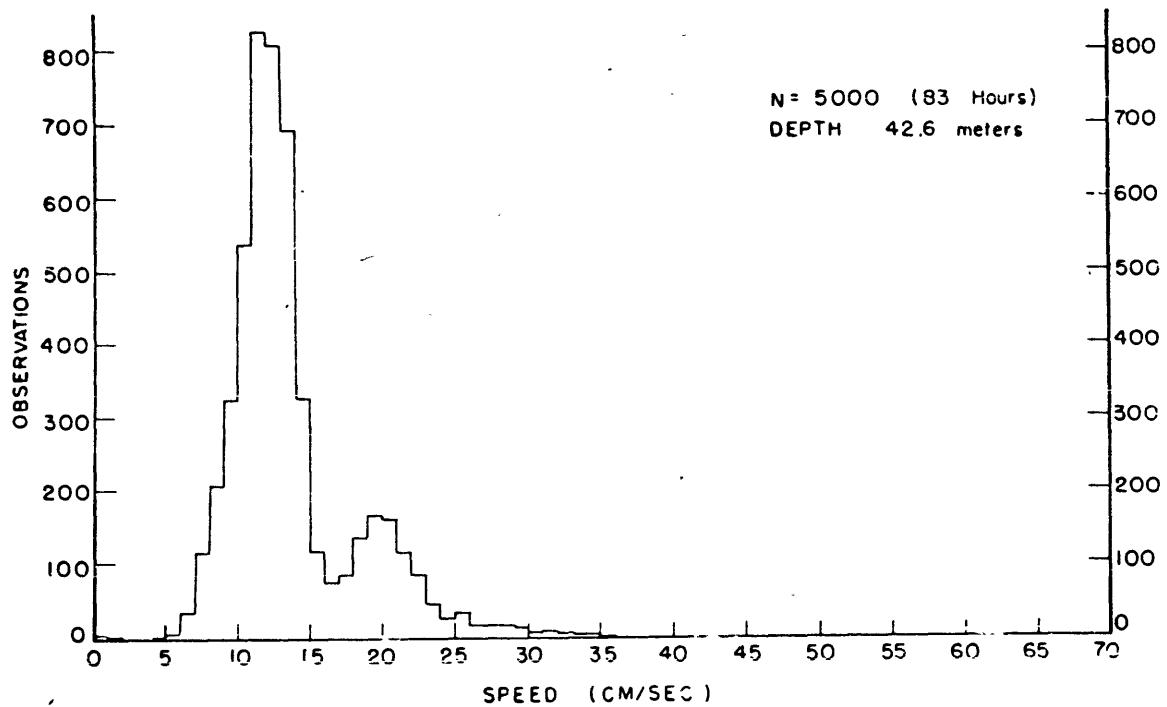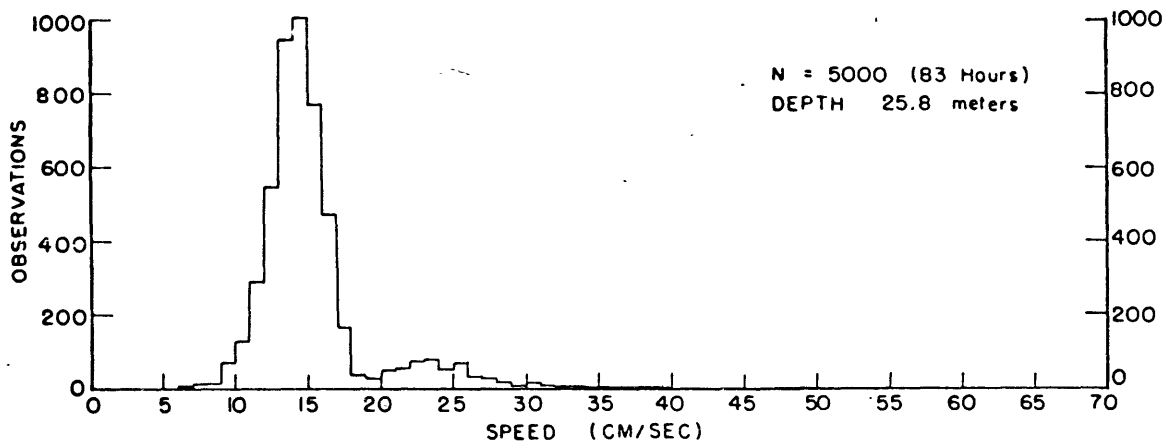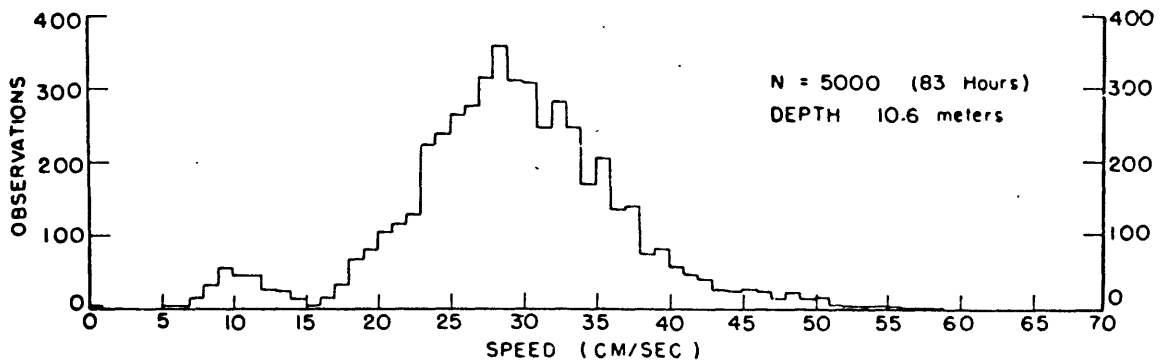
Figure D1.   SPEED HISTOGRAM (EXPT 67-2)
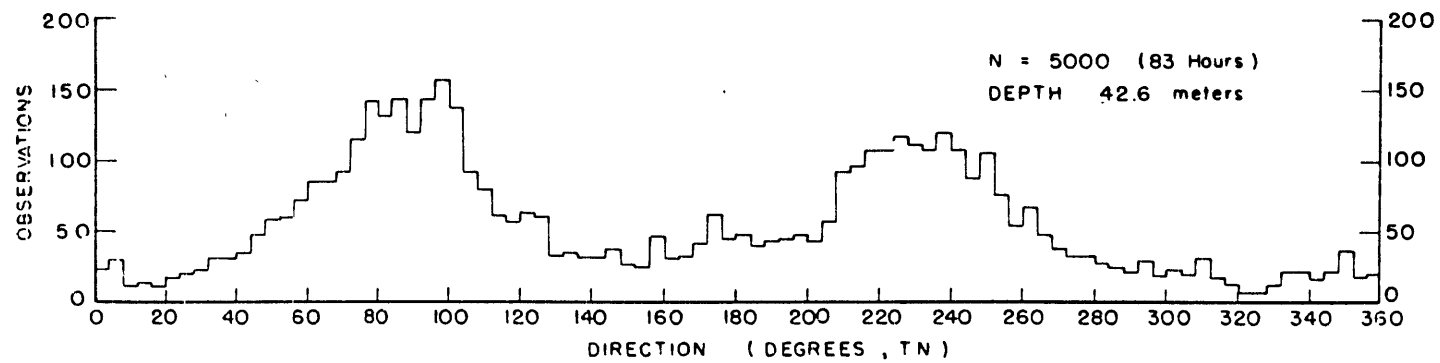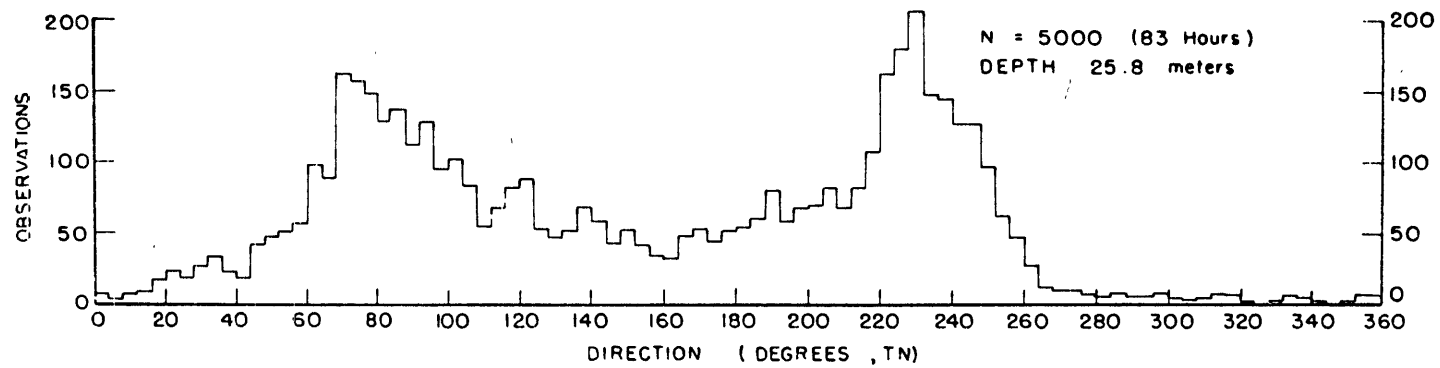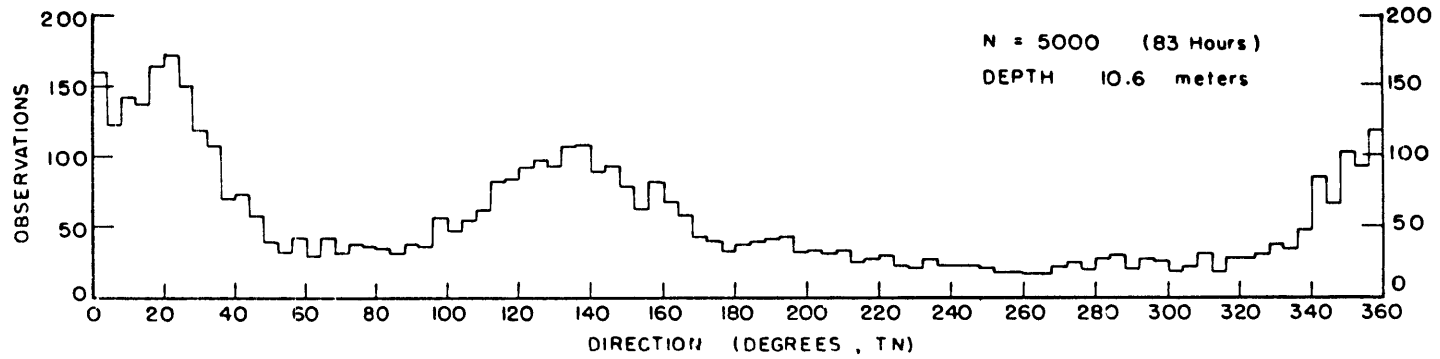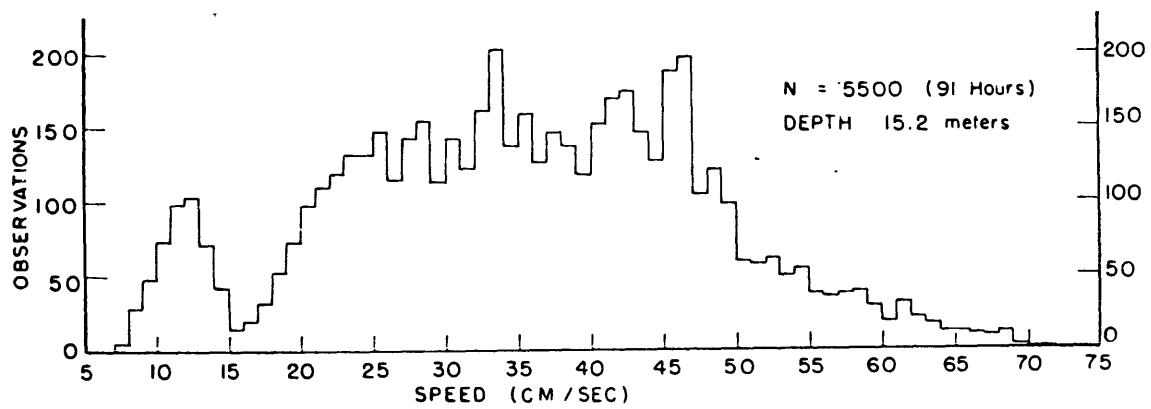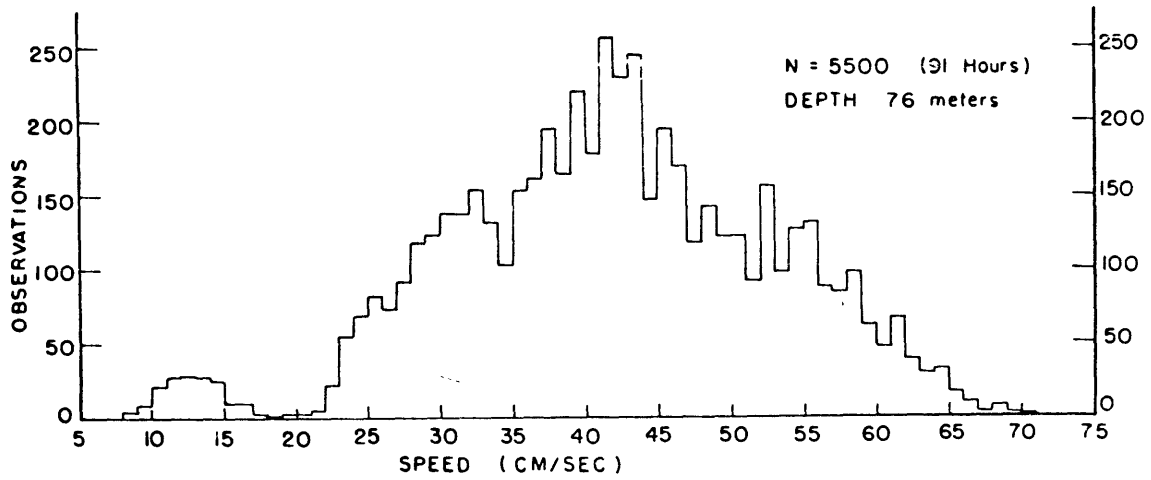
Figure 32.   DIRECTION HISTOGRAM (EXPT 67-2)

86.

Figure 33.   SPEED HISTOGRAM (EXPT 67-3)

N = 5500   (91 Hours)
DEPTH   7 6 meters

N = 5500   (91 Hours)
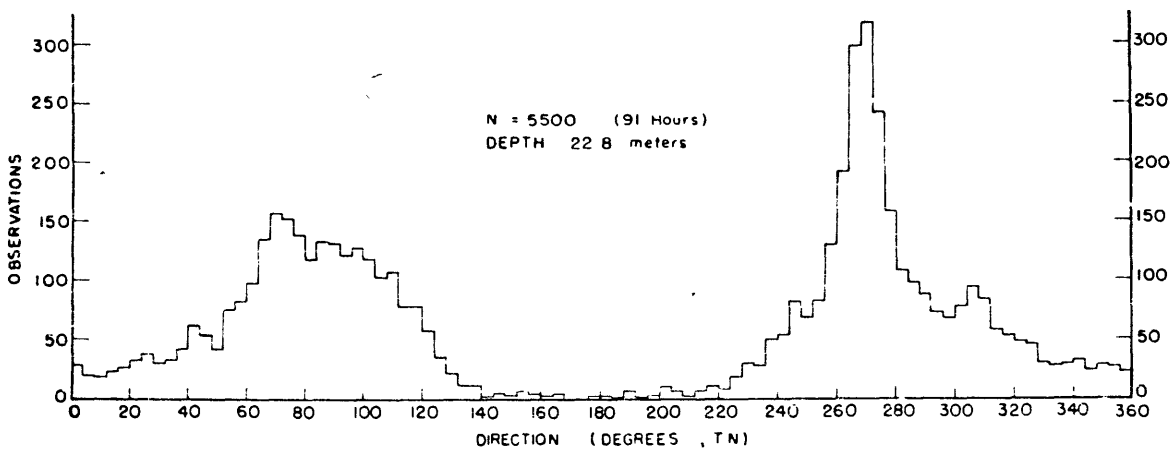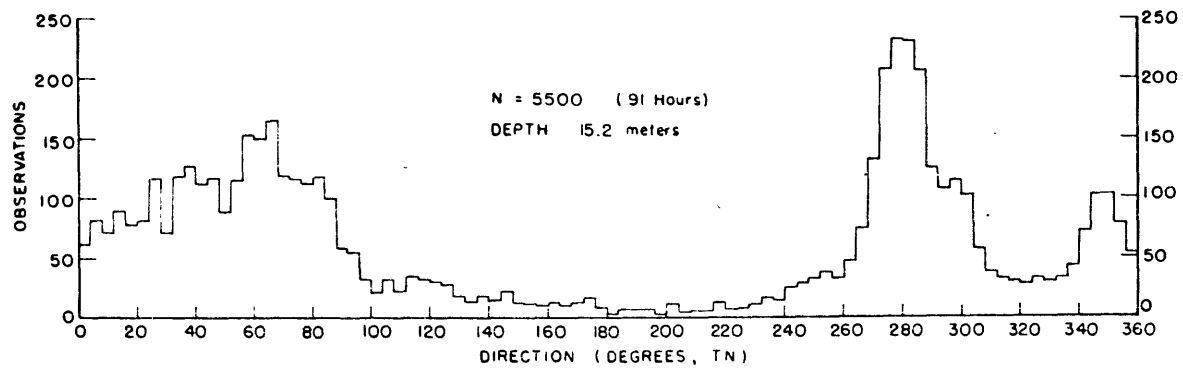DEPTH   15.2 meters

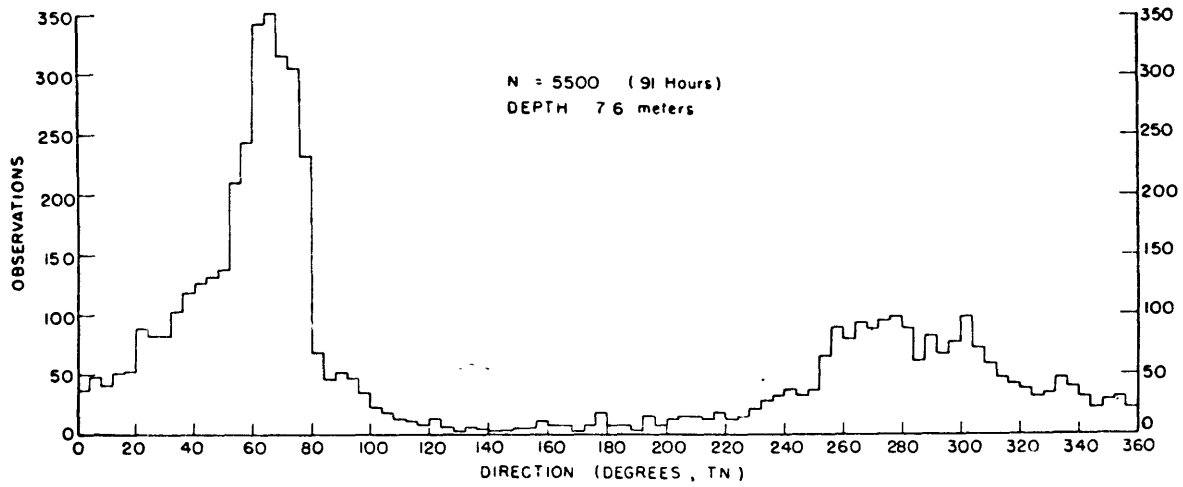N = 5500   (91 Hours)
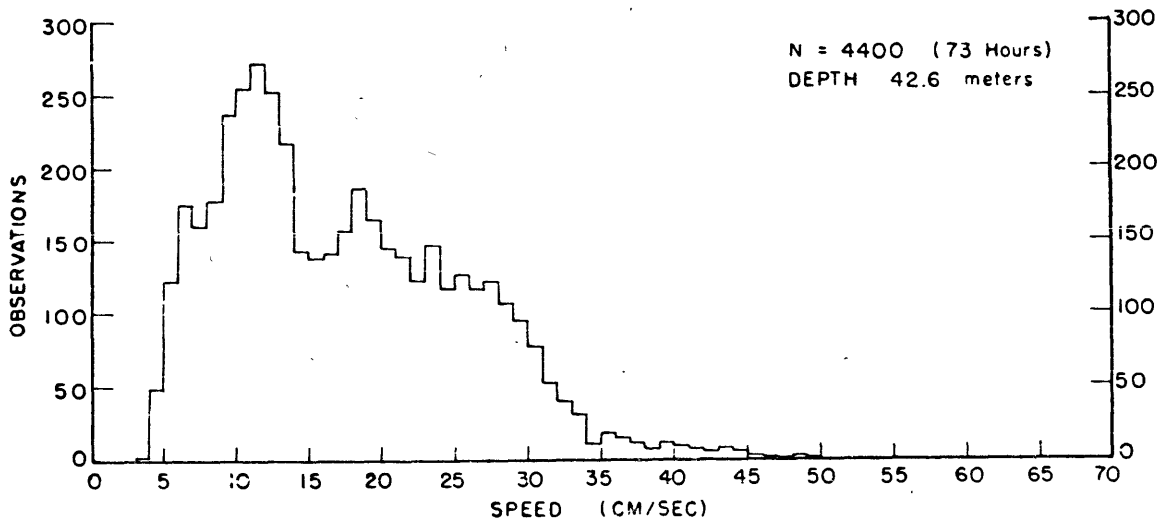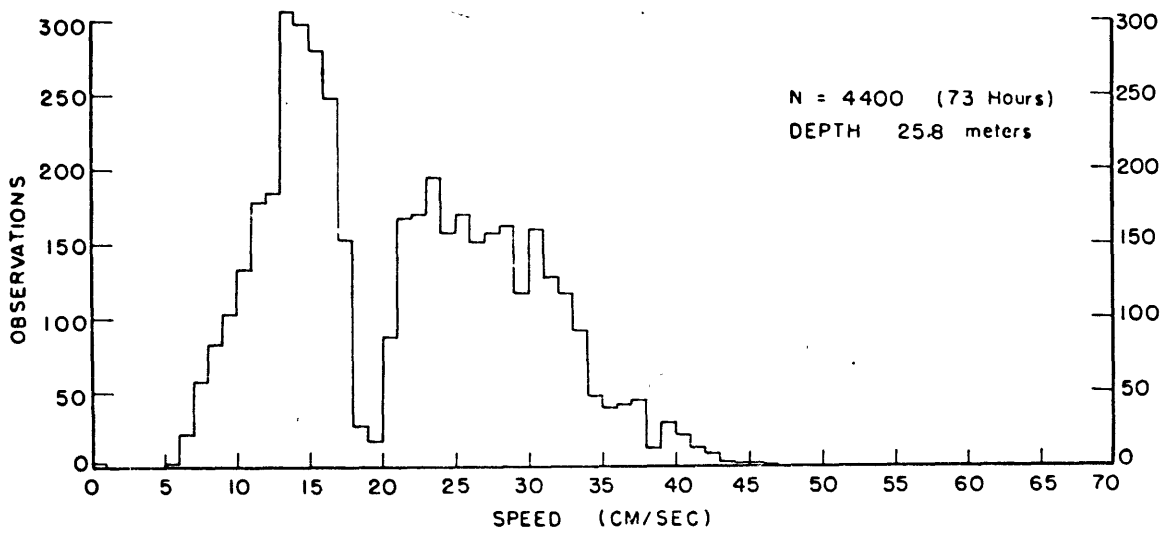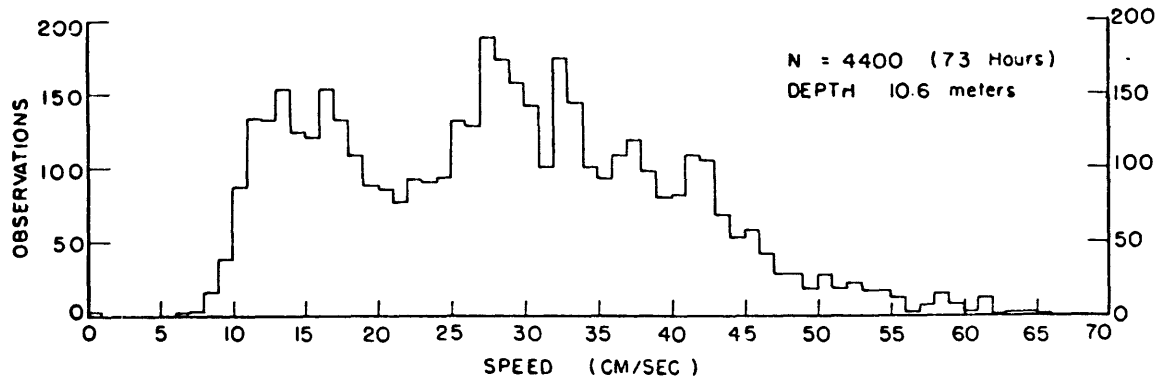DEPTH   22 8 meters
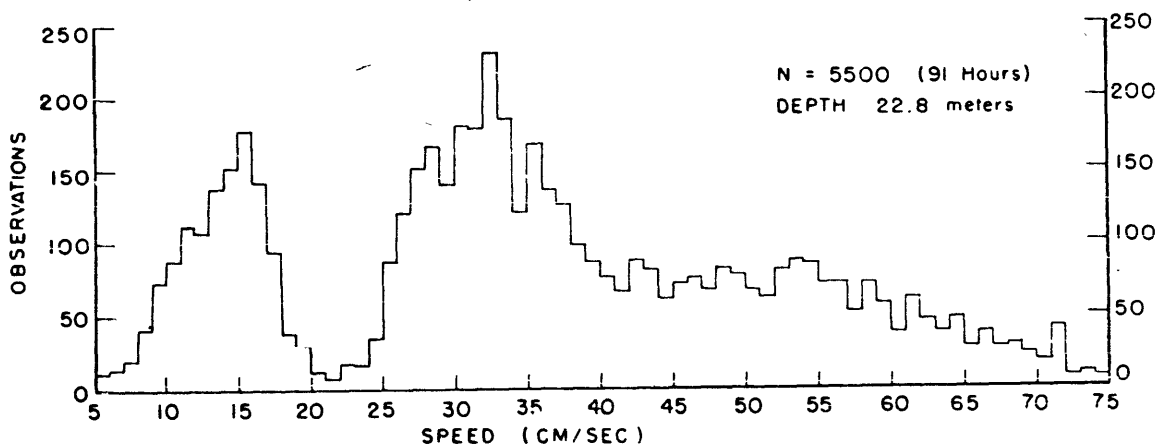
Figure 34   DIRECTION HISTOGRAM (EXPT 67-3)
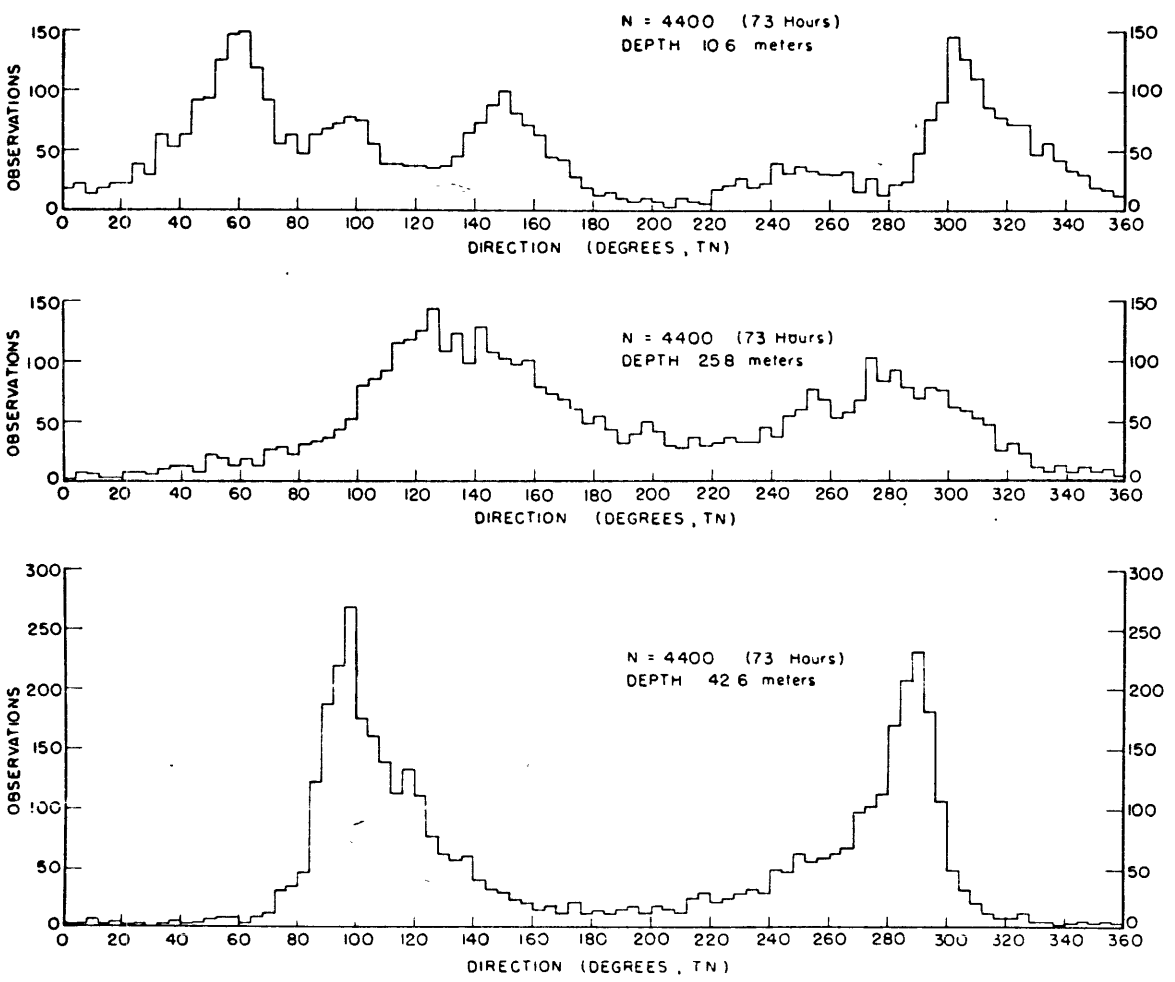
Figure D5.   SPEED HISTOGRAM   (EXPT 67-4)

Figure 36.   DIRECTION HISTOGRAM (EXPT 67-4)

current and internal wave motion. The large modes describe the tidal currents which are present all the time.

Direction histograms were plotted on a linear scale to eliminate the polar distortion inherent in a polar plot (Webster, 1964), and 0° refers to true north. The crest of Stellwagen Bank trends about 15° west of true north. In each of the three experiments the histogram of the lowermost instrument shows a back-and-forth type of motion suggestive of shallow water flood and ebb tidal currents. This is best exemplified at the mouth of Massachusetts Bay (EXPT 67-4) where two similar modes occur at 095° and 290°.

Figures D7 and D8 contain Calcomp plots of the speed measurements at Station T at 25.6 m and 42.8 m.

The time origin of the velocity data was destroyed by film exposure at the beginning of an experiment. Also, the film was all used before the current meters were taken from the water, removing any possibility of attaching a time origin at the termination of the experiment. It was assumed that the initiation of a distinct group of large amplitude high frequency speed fluctuations occurred simultaneously throughout the depth, as was seen in the temperature data. The difference in time in the arrival of the phenomena at the instruments arising from horizontal separation due to cable curvature is neglected because the maximum horizontal separation between the instruments was 4.5 m or a maximum of 2 to 3 minutes arrival time
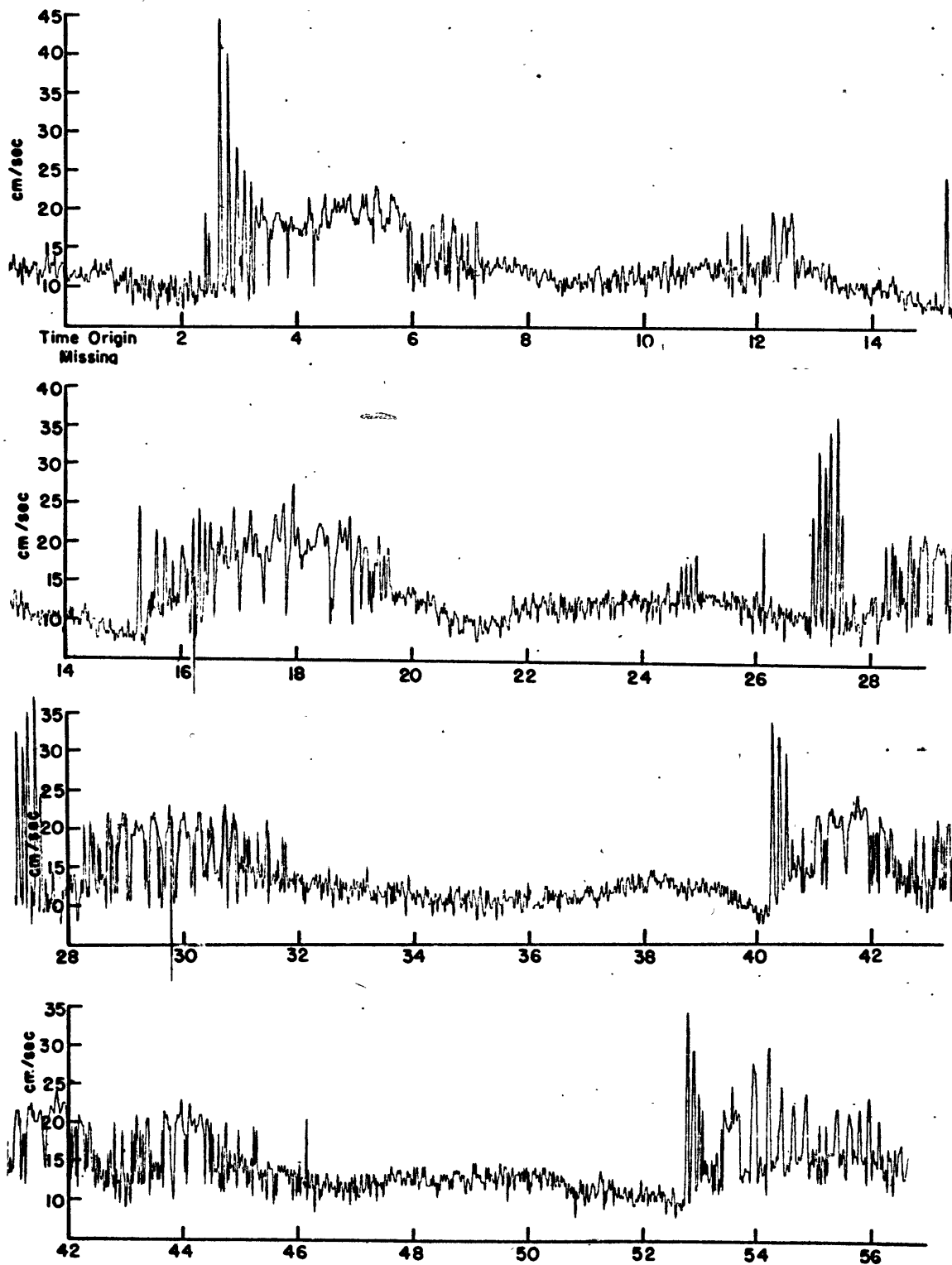
Figure 37 . A Calcomp plot of 3900 speed measurements at 42.6m at
Station T between July 13-17, 1967. The time origin is
missing and the interval between tick marks on the ab-
scissa is 2-hours.

Figure 18. A Calcomp plot of 3900 speed measurements at 25.8m at Station T between July 13-17, 1967. The time origin is missing and the interval between tick marks on the abscissa is 2-hours.
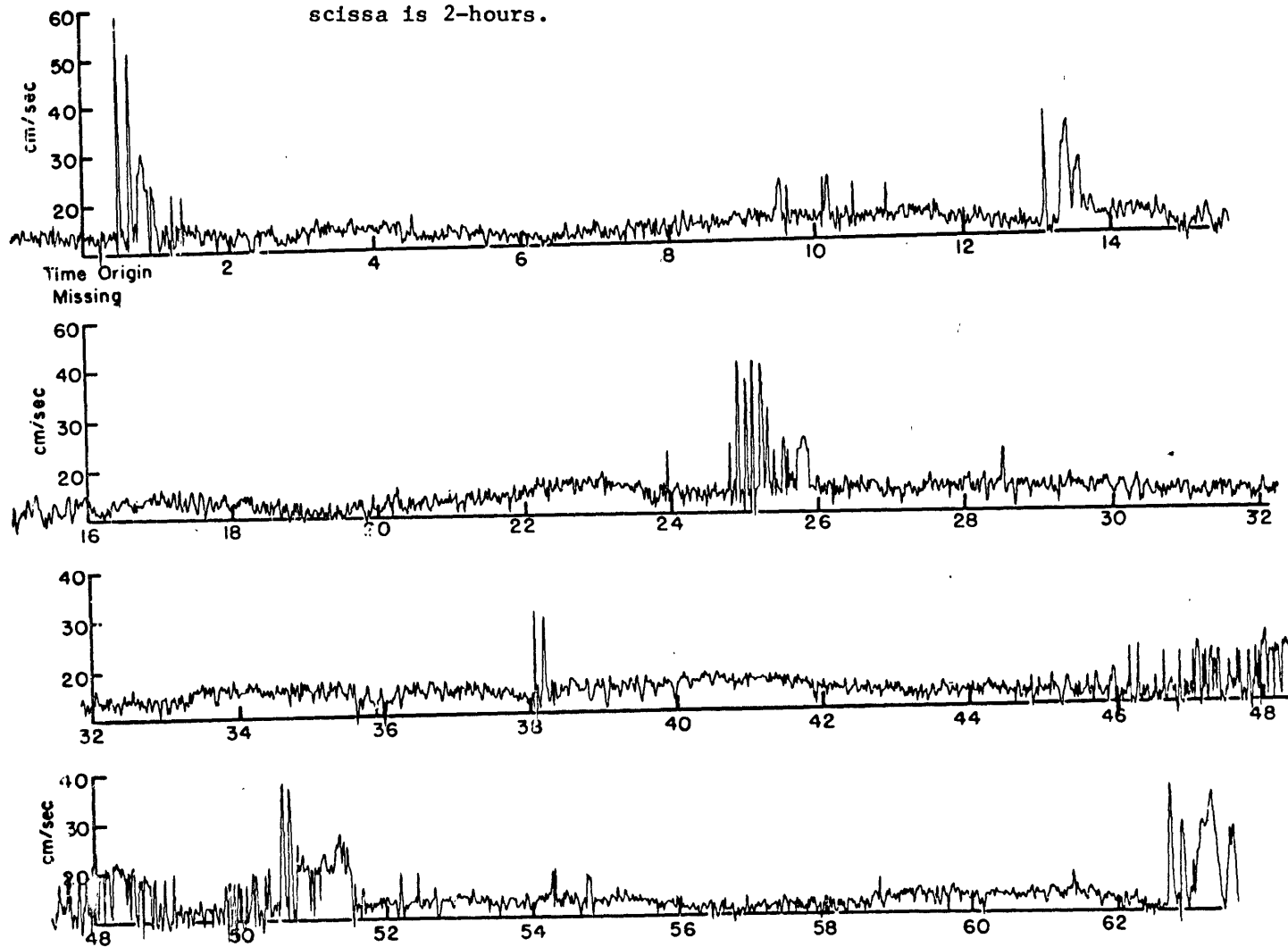
difference. A real-time origin for the data collected at Station T is based upon two features. The temperature measurements do not contain a well-defined abrupt increase in temperature until 1648 EST, July 14 which was 28 hours and 31 minutes after the temperature recording began; also, the speed records do not contain any distinct groups until about 28.5 hours have elapsed from the start of their data. All instruments had begun operating within three hours of one another. The second feature is that time intervals between the successive groups of large amplitude high frequency fluctuations in the speed records were similar to the time intervals between successive groups of short period internal waves. Therefore, the three sets of velocity measurements were aligned so that their first distinct group coincided in time with 1648 EST, July 14. Using 1648 EST, July 14 as a real-time reference, the length of data lost from film exposure was 128 minutes (H880) and 106 minutes (H879). The fact that three full turns of film around the supply spool corresponds to 126 minutes of recording time indicates that the choice of the simultaneous time for the data can be considered valid, since exposure of film at the beginning of an experiment usually arises from a hurried LOAD procedure. In the future, the instrument should be turned ON (prior to recording data) until the supply spool has completed three full turns.

Appendix E: Temperature Measurements from a Towed Thermistor

A thermistor, enclosed in a Braincom Navitherm assembly just above a V-fin, was towed at 16 m below the surface on the western side of Stellwagen Bank on September 19, 1967. The towing occurred from the stern of the R/V RR SHROCK. Temperature was recorded continuously on a Leeds-Northrup strip chart recorder. The time constant of the thermistor and its attachment was about .5 minutes; the vessel travelled at about 6.5 knots. Figure E1 portrays the pattern of towing and identifies the temperature sections. Navigation was accomplished by dead reckoning, by following bottom topography although the fathometer was inoperative 15% of the time, and with two small floats located approximately 12 km west of Stellwagen Bank. The small floats were sighted on only three occasions, and it is reasonable to assume that at any time the position error of the vessel was at least 3 km.
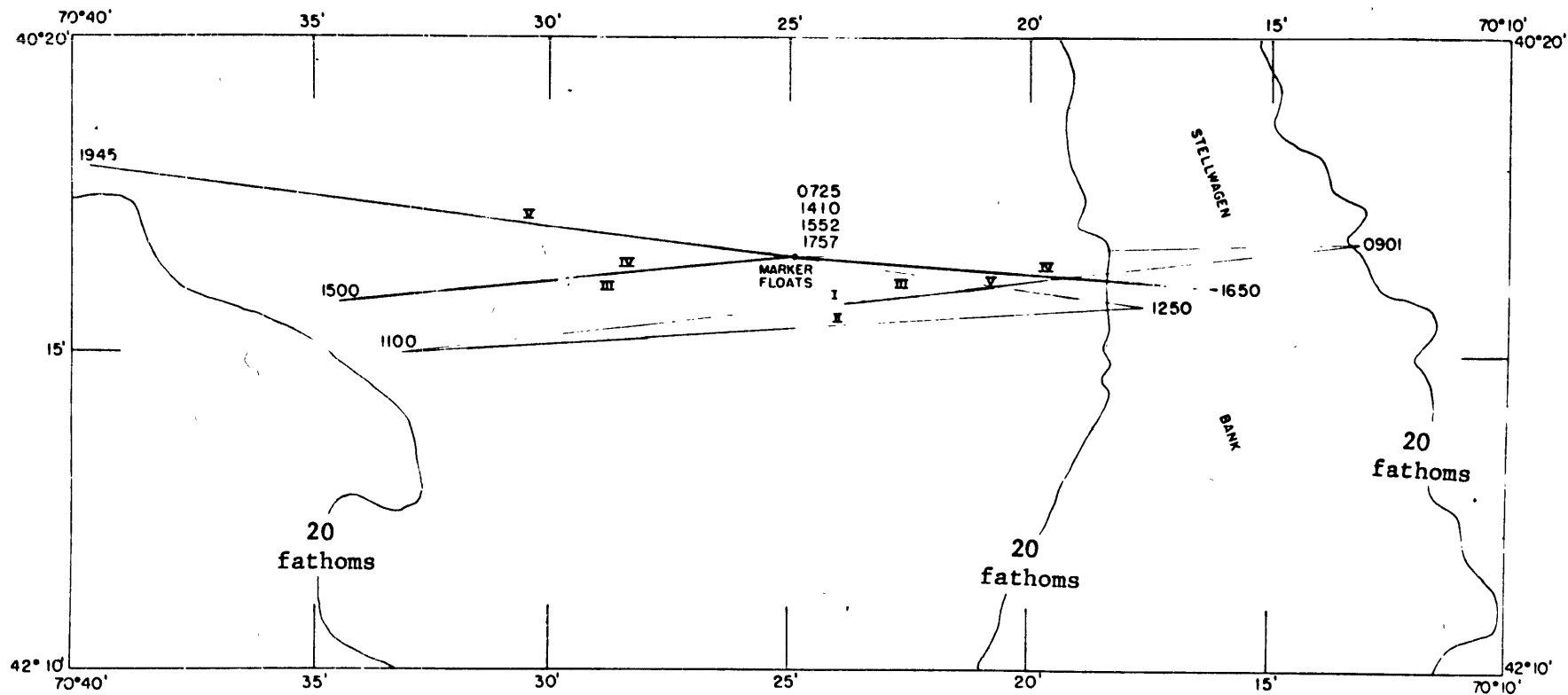
Figure E1. The pattern of thermistor towing on September 19, 1967, and the identification of thermal structure data sections. Times (EST) shown are when the vessel changed course and when the marker floats were passed.

Appendix F: Problems for Further Investigation

1. What is the mechanism generating the surface bands?

2. What is the mechanism generating regularly occurring
    groups of short period internal waves?

3. Are internal waves a sink for the ordinary tide?

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