

AN INVESTIGATION OF SOME
LONG-TERM OUTPUT FORECASTING TECHNIQUES
FOR THE MINERAL INDUSTRY

by

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ABSTRACT

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The logistic or Pearl-Reed growth curve has been used in the past to describe the secular trend of many economic time series, including those of domestic mineral production, and has often been extrapolated for purposes of long-term forecasting. Work done many years ago shows the parameters of the logistic to be unstable in the description of population data. The present study shows that the logistic is a suitable implicit model of mineral production from a limited area, but that the skew form of the curve is more suitable than the symmetrical form found in most of the literature. By fitting a number of both symmetrical and skew curves by the commonly used techniques and also by indirect least squares, it has been shown that in practice the curve fails to be sufficiently stable to be a useful model or a suitable forecasting tool for mineral production series of the United States.

Since domestic production of the major metals of which the United States is a net importer must depend on demand, a study of long-term demand forecasting was made by deriving an actual forecast for 1975. Lead consumption was chosen since it shows a declining postwar trend, hence is a greater test of technique than most other metal consumption series.

In attempting to set up an explicit model of lead consumption based on correlation techniques and resultant regression equations, the problem of secular trend is encountered. It is herein concluded that there is no satisfactory solution to this crucial problem, hence the method was abandoned.

Total demand forecasts are presented, based on relation of lead consumption to the Federal Reserve Board Index of Industrial Production, per capita consumption and future population estimates, and simple trend projections. These methods give a net forecast of 1.475 ± .125 million tons consumption of lead for 1975.

The second major type of forecast made is based on end-use analysis of lead consumption on an industry basis. By investigating the technology of applications of lead it is possible to evaluate trends in consumption on a more rational basis than simply using trend projections or pure estimates, although in some cases lack of data or information prohibits extensive investigation of the given demand sector, and estimates must be made. In general, statistics available were found to be inadequate as a basis for such a forecast.

Combining forecasts for all demand sectors, the total predicted consumption is 1,460,000 \pm 145,000 tons for 1975, in remarkably good agreement with the forecast in toto. It is concluded that although the end-use method has a severe drawback in the amount of time required, it is the best method for use in the mineral industry since it tends to inspire more confidence in the forecast and can be more clearly evaluated and readjusted as conditions change than in the case of the total forecast method. Furthermore, it allows one to investigate the relative importance of dissipative uses, important to the primary metal producer since these are the chief generators of markets for his products.

A brief discussion of the recovery of secondary lead outlines the problems involved and suggests that further study be initiated to establish more definitely such important variables as recycling time and recoverability factor for each lead use. For 1975, a total recovery of about 743,000 tons of secondary lead is indicated.

The market for new lead is seen to be 572,000 to 862,000 tons. Although considerable research is needed on the problems of determining future supply patterns, it is concluded that increasing world-wide industrialization will tend to increase the share of demand supplied by domestic producers of new metal.

Thesis Supervisor: Roland D. Parks

Title: Associate Professor of Mineral Industry

BIOGRAPHICAL NOTE

James Anthony Lawrence White is a native of Toronto, Ontario, where he attended the University of Toronto from 1951 to 1955, graduating with an engineering degree (BASc) in Mining Geology, honors standing in the graduating year. He also received the School Letter. Upon graduation he worked as a petroleum geologist for a short time, joining the staff of the Anaconda Company (Canada) Ltd. as exploration geologist in 1956. In September 1957 he enrolled at the Massachusetts Institute of Technology and was granted an SM degree in Geology and Geophysics in June 1958. The Institute awarded a tuition scholarship for 1958-59, and a second one for 1959-60, and the MIT Canadian Alumni Fund awarded a third scholarship in 1960. Application for the PhD degree was made for September 1960. Mr. White is a registered Professional Engineer in the Province of Ontario and a member of the Canadian Institute of Mining and Metallurgy.

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Mrs. J. Webster gave invaluable help in debugging the first IBM 704 program. Work with the computer was done at the Computation Center of the Massachusetts Institute of Technology.

Finally, I wish to express my indebtedness to my wife for her unfailing support and enthusiastic help in the preparation of this report.

AN INVESTIGATION OF SOME
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To guide their long-term planning, modern mining and exploration companies would like to have a dependable estimate of the future level of activity they might expect for their phase of the industry, the factor of most direct importance being the long-term level of domestic production of the mineral or metal being sought or produced. In 1951, S.G. Lasky¹ published a series of forecasts (reviewed in 1955²) for copper, lead, zinc, bituminous coal, and aluminum, based on the logistic curve. This curve was fitted to domestic production data, hence, extrapolated, gave a forecast thereof. Being highly condensed, the articles gave no indication of the method used for fitting the curves; the "least squares" technique was used for some, while others were fitted by the "method of selected points"³. (See Pearl, (1940) for an outline of these methods).

The logistic curve has been fitted to other series of economic data⁴, generally for the purpose of forecasting. As remarked by Davis (1941 a), "If the law of biological growth does hold, indeed, within statistical limits for the growth of industry, this is certainly a very important matter from the standpoint of predictive economics". The reference to the "law of biological growth" stems from the origin of the curve, since it was derived by Raymond Pearl, a biometrician, to describe the growth of laboratory animals over

1 Mineral Industry Futures Can Be Predicted: Eng. & Mining Journal, v. 152, no. 8, August 1951, pp.60-64.

2 Mineral Industry Futures Can Be Predicted II : Eng. & Mining Journal, v. 156, no. 9, September 1955.

3 S.G. Lasky, personal communication.

4 See for example Kuznets (1930), Prescott (1922), Putnam (1953), Stanley (1949), Bratt (1936).

time, and later the growth of populations of fruit flies in a limited environment. (Pearl, 1924)¹.

By far the greatest attention given the curve has been due to its use by Pearl (1924), and Pearl and Reed (1920) to describe population growth, and considerable controversy has raged over its usefulness. (See for example Bratt (1940) and (1958), Davis (1941 a), Davis (1958), Lehfeldt (1916), Lotka (1925), Tuttle (1957), Wilson and Puffer (1933)).

Unless there can be a deductive justification for assuming for an industry the habit of growth exhibited by the logistic, using it for forecasting is no more than simple trend extrapolation, which, as any textbook on economic statistics points out, is an untrustworthy method of prediction. Schumpeter (1930-31) suggests that trend lines through unanalyzed material are useless. While the author believes that the mineral industry's habit of growth should be well-described by the logistic, it is apparent from the literature that it has often been used for many series without adequate consideration of its meaning or the form of the curve best suited to the data at hand. The following is therefore a brief appraisal of the nature of mineral production history and an outline of the properties of the symmetrical logistic curve, the form used in all applications encountered, except some of those of Pearl.

GROWTH CHARACTERISTICS OF PRODUCTION OF A MINERAL COMMODITY

First, it can be shown that the growth of mineral commodity production started from a zero level, and increased at a high percentage rate in the early years as more deposits of the mineral were discovered, new treatment methods evolved, new uses for the product were discovered, and the population of consumers grew. Like so many cases of growth, the early years may well have shown an exponential rate of increasing

¹ It was actually derived earlier by Verhulst (1844) but received no attention in the intervening years.

production. That is, the rate of growth was dependent simply on the increasing size of the industry. It is during this period that the number of patents applied for on new industrial techniques reaches a peak. Prescott (1922) has aptly termed this the period of experimentation.

If the commodity can survive this period it becomes a part of the social fabric, accepted and expected by the consumer, and used in increasing quantity. This influence tends to overlap the period of experimentation, and the net rate of growth is the most rapid as new markets become well established and technological advance lowers costs and increases uses. The economics of scale can be effected as production enters this stage, and with an assured market the atmosphere for investment in the industry becomes more stable.

But the rate of growth cannot increase forever, and it will at some point pass through a maximum, beyond which those factors inimical to growth begin to grow in importance as the growth factors wane. It is patently possible that more than one maximum may be attained if some major factor of growth changes, as for example the bringing-in of the porphyry coppers changed the nature of the copper industry. And it likewise appears possible that the periods of growth and decay may not be symmetrical in form.

In some industries a plateau is established at a maximum performance level, and the plateau level can in theory be expected to persist indefinitely, perhaps growing with population to some degree. But in the mineral extractive industries there is an increasing pressure which tends to cause production decreases, and that is the pressure of decreasing reserves. It may be due to the approaching limit of absolute resources, or of economically recoverable resources in the light of substitution, and importation from foreign sources, but it must eventually lead to the death of the extractive industry within any given area, be it continent, nation, or deposit. Thus any established production level will be followed by a period of "declining growth rate" or of

2

decay. As old age is entered, the rate of growth of the cumulative industrial production will once again approach zero.

This sequence of history in the development of an industry based on a declining resource is typical. The multitude of factors which shape its detailed course, and cause the business cycles and seasonal changes which occur as fluctuations around this overall trend do not enter into the above discussion in any concrete way. It is the time characteristics of the overall trend of the data upon which interest centers here.

To state more specifically the nature of the trend which it is expected that cumulative mineral production will follow, the following may be observed.

- 1) The trend will start from a lower asymptote equal to zero, and increase at a geometric or near-geometric rate in the period when X (time) is small. It is thus concave upward.
- 2) The rate of growth per unit of time is to be proportional to two things: (i) the absolute size which the industry's production has attained at the beginning of the given unit of time and (ii) the amount of potential resources still unused, i.e. the resource base of the area in question. The importance of the second factor should increase with time. (There is the possibility of episodic growth, which may cause exceptions to this rule).

This second characteristic of the trend is the most important, since many curves will exhibit some or all of the others noted, but only the logistic includes this one, and it is the only major assumption which can be questioned. The beginning of a typical mineral's production history is marked by limited markets, difficulties of technology, capital shortages, and a limited number of operating mines. Each of these problems is reduced in magnitude as production continues for a few years so that the rate of growth tends to accelerate. For example, it is easier to increase production when there are ten producers, each with capital, manufacturing technology, experience, and operating mines, than when one mine alone must make the increase.

The effect of limited resources will be that of a damper on the increasing growth rate when further production increases become impossible for some operators due to lack of ore. The growth rate is therefore reduced by the factor of ultimate resources minus the amount already produced.

- 3) At some point in time the rate of growth will pass through a maximum and then continuously diminish. Again the possibility of epochal or cyclical growth may cause exceptions.
- 4) As X becomes large the rate of growth will approach zero, and the curve will approach an upper asymptote. If one is considering cumulative mineral production data, the upper asymptote can be seen to be representative of total ultimate recoverable resources, to which the cumulative production will eventually accrue.
- 5) The trend cannot turn back upon itself. Once a certain cumulative total has been reached the curve cannot go below that level, since this would be the equivalent of "negative production".
- 6) The first derivative of the cumulative production trend equation should yield a curve which will fit the typical form of annual production data. That is, it should be asymptotic to zero when time is very small, increase to a maximum, possibly establishing a plateau level of annual production, and then decrease to become asymptotic to zero when time is very great.

Assume an upper limit of growth equal to ultimate recoverable resources, (K), a constant greater than zero. Then when time (X) equals infinity, cumulative mineral production (y) should equal K , or $\frac{K}{1}$. When X equals negative infinity, y should equal zero, i.e. $\frac{K}{\infty}$. The denominator of the expression for growth of mineral production must therefore go to 1 when $X = \infty$, and to ∞ when $X = -\infty$. Furthermore, it must be a continuous function over this range and must be continuously decreasing as X increases (excluding for the moment the possibility of epochal growth). One function which fulfills these requirements is $1 + e^{-X}$ or $1 + 10^{-X}$, and the expression

for cumulative production can be written as

$$y = \frac{K}{1 + e^{-X}} .$$

This is the equation of the symmetrical logistic.

CHARACTERISTICS OF THE LOGISTIC CURVE

The more general form of the logistic, developed by Pearl (1930), is

$$y = \frac{K}{1 + Ce^{f(x)}} \quad (C > 0)$$

$$\text{or } y = \frac{K}{1 + e^{f(x)}}$$

When $f(x) = a_0 + a_1x$, the curve is S-shaped, and symmetrical about the point $(\frac{a_0}{a_1}, \frac{K}{2})$. It could be used to represent cumulative mineral production.

The first derivative, which generates the curve which should describe annual production, is given by:

$$\frac{dy}{dx} = \frac{-a_1y(K-y)}{K} .$$

This is a bell-shaped curve, symmetrical, asymptotic to zero when $x = \infty$ or $-\infty$, and with a maximum at $x = -\frac{a_0}{a_1}$. It also illustrates the fundamental growth characteristics of the logistic, for if K is allowed to go to infinity (i.e. there is no restriction on production due to limited reserves) $\frac{dy}{dx} = -a_1y$, which is the equation for geometric increase. Hence a_1 is the potential growth rate, damped by the pressure of an upper limit of growth, which grows in importance as time increases, as shown by $(K-y)$, the potential remaining for growth. Chart 1 illustrates the form of the curve.

When $f(x) = a_0 + a_1x + a_2x^2$, the value of the last term, which, in every form of the logistic, dominates the exponent as x becomes large, is the same for equal values of x , either positive or negative. Hence the curve is asymptotic to zero (or K) when $x = \pm\infty$. If a_2 is positive, the curve is asymptotic to zero, if negative, to K . It will be found to be symmetrical, and have a maximum (or minimum) at $x = -\frac{a_1}{2a_2}$. This form could then be used to represent annual mineral production series.

For all forms of the logistic in which the $f(x)$ in the exponent of e is third order or higher, the curve is no longer symmetrical, and the derivative curve, while still being asymptotic to zero or K at $x=\pm\infty$, can show one or more cycles of growth. Their properties are otherwise similar to the first and second order curves, with the important extension that for odd-ordered curves, if the coefficient of the highest power of x is negative the curve is asymptotic to zero at $x=-\infty$ and K at $x=+\infty$, while if it is positive, the curve is asymptotic to zero at $x=+\infty$. This form has not commonly been used in the literature because it is difficult to fit.

RESULTS OF FITTING THE SYMMETRICAL LOGISTIC TO ECONOMIC AND POPULATION DATA

A documentation of extensive experimentation, to which reference is all too seldom made, is the paper by Wilson and Puffer (1933), who fitted symmetrical logistics with $f(x) = a_0 + a_1 x$ to population data by the method of selected points and also by least squares. They found that it was impossible to obtain meaningful and consistent values of the parameters by any method used.

Bratt (1936) indicates a wide range of upper asymptotes for similar logistics fitted to steel industry growth.

Unpublished work by Uffen¹ found the parameters unstable when the curve was fitted to data of production from single mining camps, since adding one or two years of data changed the curve considerably.

White (1958) fitted a number of these curves to mineral production data by the most commonly used methods and found that the value of the upper asymptote varied widely, far beyond any reasonable estimates for mineral resources, depending upon the points fitted.

Lasky (1951, 1955) used the first and second forms of the curve described above to forecast mineral production data. Where the cumulative curve had passed the inflection point

¹ R.J.Uffen, Univ. of Western Ontario, personal communication.

he fitted the first form to the cumulative data, (e.g. his lead curve). If the inflection point had not been reached he fitted the same form to annual data. This of course gave a curve for annual production which could not show a decline even in infinite time, and is not the correct form of the curve to use. This may have been corrected in the second paper (1955) -no equations are given therein. For bituminous coal production, the second form described was fitted to annual data. The question of stability of the estimates of the parameters was not mentioned.

Thus in all cases for which data were presented, the estimation of the parameters of the symmetrical logistic was unsatisfactory. Therefore, the method of fitting by least squares was programmed for the IBM 704 electronic computer so that a large number of iterations of the approximation technique could be done rapidly. There is, however, good reason to expect that the growth period of mineral production will not often be symmetrical with the period of decline, for not only does the factor of diminishing resources cause the decline, but many other forces are also at work, some of which do not exist during the growth period. Deviations from the long-term trend, if assumed to be represented by the symmetrical logistic, cannot logically be expected to be symmetrically spaced in time on the basis of duration and/or magnitude, and this will lead to skewness. In other instances a technological discovery has obviously led to a new cycle of growth, e.g. the porphyry coppers changed the life cycle of the copper industry. Therefore the author has worked with the skew curve, in which the highest power of x in the exponent is the third, this being the simplest skewed form.

The greatest problem in fitting logistic curves is estimating the value of the upper asymptote, K . Two methods were tried in this study. The first is based on the fact that estimates of United States mineral reserves are available, and a range of values for K was used that corresponds to these estimates, the values chosen not being changed by the least

squares technique, as are the other parameters. The second method was an attempt to have the curve estimate the value of K, by getting a first approximation to it by the method outlined by Pearl (1940), and including this value with those of the other parameters in the least squares approximation. Because the United States mineral production series have not approached the upper asymptote closely, this technique did not converge to a solution, but since the appraisal of the rationality of the estimate of the asymptote which would result will be made on the basis of the known reserve data, the first method is essentially as good.

RESULTS OF FITTING THE SKEW LOGISTIC TO MINERAL PRODUCTION SERIES

Skew logistics have been fitted to series of statistics of cumulative United States production of copper, lead, and zinc from domestic ores.

Zinc

Since the only continuous series of United States zinc production data since first production are in terms of slab zinc, the current estimates of recoverable zinc were converted into terms of slab zinc, and combined with the total produced to date to estimate total recoverable zinc. Using the Paley Commission's estimate of recoverable zinc in measured, indicated, inferred and marginal ore classifications, a total of 51 million is reached. Curves fitted with K=30, 40, and 50 million tons are shown in charts 2 and 3, the curves of the latter being the first differences of the cumulative curves. For each curve, data from 1858 to 1955 were used, and the least squares technique iterated four times. The following points may be noted.

- 1) For K=30 million the curve falls off more rapidly than data to 1959 would justify.
- 2) For K=50 million, the curve rises too rapidly.
- 3) For K=40 million, the fit appears best, but the weak development of a second cycle of growth (possibly explainable

by the increase in use of zinc in diecasting since 1945) in the curve for $K=50$ million gives the latter curve the least root mean square deviation. Also, reserve estimates indicate 50 million tons as a more likely figure.

- 4) None of the curves fit the early history exactly. From 1858 to 1893 they lie below the data, from 1893 to 1912 above it, while no such deviations of the general economy are generally recognized (see for example the Babsonchart). The data depart somewhat from a true logistic shape.

To test the effect of possible errors in early statistics or the use of two or more noncontinuous series, data to 1890 inclusive were doubled, adding a total of 631,205 tons to actual cumulative production to date, and a curve fitted to data to 1958, with K equal to 40 million (chart 4). This changed the forecast for 1975 by 196,223 tons or almost one-half of the lower forecast figure, although the change in the data amounted to only 2.4% of cumulative production to 1958. The parameters are therefore concluded to be unstable with respect to such errors.

Since the value of the upper asymptote is not known precisely, the possibility of using a range of values in making a forecast was investigated. The range for forecasted 1975 production for $K=30$ million and for $K=50$ million is 74,687 to 627,721 tons, obviously too wide to be of any use. For $K=48$ million the predicted value for 1975 is 588,826 tons, i.e. a change of 4% in K , from 48 to 50 million, changes the forecast by 6.6%. Thus, the range which might be used for K is restricted to 2 or 3 million tons to have a reasonable range for the forecast, and this is a rather stringent limitation.

To see how many years of data are necessary to establish the growth pattern, data from 1858 to 1919 were used, with K equal to 30 and 50 million (chart 5). Although the fit is reasonable within the range of the data, it misses the later trend of the data entirely. Sixty-two years of production history is apparently not sufficient, in spite of the fact that the domestic industry was well-established by 1919.

Copper

Because the reserves of copper are even more open to question than those of zinc, due in part to the strong effect of a change in the cost-price ratio on the determination of ore limits in large low-grade mines and in part to the large tonnage of metal contained in a single porphyry copper, a wide range of values for the upper asymptote was used, 60 million to 160 million. (See charts 6 and 7). The curves vary widely in shape and the existence of a second growth cycle, which might be accredited to the production from the porphyry coppers, precludes the determination of any minimum RMS deviation as an estimate of the most suitable curve, since as K is increased, the second cycle dips further toward the low values of the Depression years. The use of a range of K values, as shown above, is not a satisfactory solution.

Using a statistical base of some hundred years or more, a few more years of data should not change the form of the curve by an important amount. The two curves for K=72 million tons were fitted to data to 1945 and 1951. Compared below are the resulting "forecasts" for the period 1952-1958, and the actual data.

Year	Forecast, Data to 1945 incl.	Data to 1951 incl.	Actual Production
1952	803060	953918	927365
1953	817874	978001	943391
1954	833265	1002144	834381
1955	849072	1026100	1007311
1956	865199	1049504	1117580
1957	881458	1072056	1081055
1958	897713	1093365	990000
Av. 1952-58	849663	1027012	985869

Six years more of data changed the "forecasted" average for the next 7 years by 21%, and reduced the error of the average by 95,063 tons or 9.6% of the actual production figure. This is of course a considerable change, and also indicates instability of the curve.

Lead

Estimates of lead reserves given by the Paley Commission and others correspond to cumulative discoveries of recoverable lead of 30 to 40 million tons, including sub-marginal material. However, when values of K from 30 to 40 million were used in fitting the skew logistic to data from 1821 to 1956 and 1870 to 1956, for all values greater than K=30 million the coefficient of x^3 obtained, even when the least squares approximation technique was iterated ten times, was positive. As explained above, this means that the curve is asymptotic to zero when $x = \infty$, hence it is not suitable to represent cumulative mineral production. This change is believed to be due to the extreme range between values for the 1920's and those for the 1930's, this effect dominating over the later data, so that in charts 8 and 9, the curves for both K=33 and K=40 million do not fit recent data well at all. For K=30 million, the fit is no better.

Since the curve for K=30 million is so nearly symmetrical, although data since about 1950 seem to be asymmetric, a computer program was written to fit the symmetrical curve by first estimating the parameters by the method of selected points, then using the least squares approximation technique. The value of K, after ten iterations, is 34.605 million. Other symmetrical curves fitted by the three-point method alone (White, 1958) gave an extremely wide range of values for K. As with the curve for K=30 million, postwar data is not well represented (see chart 10). Therefore it is concluded that neither the symmetrical logistic nor the skew form as fitted here gives a good description of the trend of lead production, since, aside from the form of the curve, the values obtained for K are too low. Production to 1959 is 28.5 million tons, measured and indicated ore 2.9 million tons, and inferred ore 2.7 million (U.S. Bureau of Mines, Bulletin 585). Also, since these estimates were made, Kennecott Copper has discovered a new major ore area in South-

east Missouri¹. The value of 33 million is obviously too low, and 34.6 million leaves no allowance for further discoveries. Indeed, these low values are the cause of the too-rapid decrease shown by the curves in postwar years, since the area below the annual curve must be equivalent to the value of K.

SUMMARY OF EXPERIMENTAL RESULTS WITH THE SKEW LOGISTIC

On a logical basis the logistic curve would seem to embody basic assumptions which are typical of the mineral extractive industry. Work with the symmetrical logistic has shown it to be unstable. Because of the complex of causes which condition growth and decay it would not appear likely that the two phases of the industry's life should be symmetrical, hence it was logical to fit the skew logistic and examine the results of experimental curves as forecasting tools. Within the range of the data, the curve gives a reasonable-looking graduation of the observations, but it was found to have the following failings which make it unsuitable for forecasting mineral production.

1. It is necessary to have a dependable estimate of ultimate recoverable resources. An error of 5% in this figure produces a correspondingly large error in the forecast, for industries in the growth phases typical of the United States base metals. Such estimates are not available.
2. Errors in reporting, or a somewhat different early history of the industry would make a considerable difference in the forecast over 100 years later.
3. The habit of growth of an industry may change drastically from that described by a logistic fitted up to the most recent years, and no indication of the change will be given by the curve.
4. The accumulation of only 5 or 6 more years of recent data can change the trend line more than appears desire-

¹ Dr. Julian W. Feiss, Geologist, Kennecott Copper Co.; personal communication.

able. It thus becomes impossible to say to what final year of data the curve should be fitted, and the curve is considered unstable.

5. The root mean square deviation is not a useful measure of the suitability of any particular value of K for the asymmetrical curve.

6. For some time series the equation derived by the least squares procedure yields a curve which fits the data used but does not give a logical form of curve beyond the range of the data.

CONCLUSIONS

On the assumption that the long-term production of a mineral commodity obeys simple laws with respect to its distribution over time, it is possible to represent it by a simple implicit model. One curve which has been used as such is the logistic of the first and second orders.

Experimental calculations show that short-cut methods of fitting this curve are not dependable. Further calculations by the author, and by Wilson and Puffer (1933) show that least squares estimates likewise may not give dependable or reasonable results.

Because of the complex nature of the factors influencing mineral production over time, the skew logistic was fitted to production series, by least squares, with discouraging results. Instability of the curve with respect to errors in the estimated resource base, errors in the statistical series, and changes in the number of observations fitted indicate that, like the symmetrical curve, the skew logistic, as applied in this study, is not a useful forecasting device.

DEMAND FORECAST FOR LEAD, 1975

The logistic curve has been shown to be useless as a tool for long-term prediction of mineral futures. Unless simple trend extrapolation is used, and this has always been deprecated by statisticians (see for example Croxton and Cowden (1955)), it is necessary to forecast domestic production by the more classical approach of forecasting demand first, then considering what percentage thereof will be met by new domestic metal, secondary metal, and imported metal. Since there can be no single study for all of the metals, each being a problem in itself, lead was chosen here because the declining United States production and consumption per capita make it a most interesting study and at the same time provide a better test of technique than if the trends were parallel to those of population growth or GNP, etc. By far the greatest effort has been spent on the demand forecast, with only an outline of the many problems of the supply situation being given.

A study of the technology of each end-use of lead was done, so far as possible within reasonable time limits. In some cases data is not available, in others the technology is so complex that assumptions must be used. In still other cases the available statistics of lead consumption are too poor to allow a rational forecast. Series published in the yearbooks of the American Bureau of Metal Statistics, (ABMS), including those by Lead Industries Association, (LIA), were used, occasionally supplemented by the United States Bureau of Mines, (USBM), data. Where these data showed a logical relationship to industrial indices, ratios were used to forecast. In other cases, trend projections tempered with judgment have been used. Occasionally, estimates have had to be used.

As a check on the end-use analysis, a total forecast has been made by several methods, and reference is also made to several forecasts in the literature.

Besides the many trade journals and reference books consulted, companies and associations were a major source of information both on technology and industry attitudes toward the use of lead. Some 50 companies and associations were contacted, nearly all of which replied.

GENERAL CONSIDERATIONS

Assumptions

It is assumed that the United States economy will continue at a high level of activity and although business cycles are to be expected, no major depression will occur. Likewise, it is presupposed that the nation will not become engaged in a full-scale "shooting war" in the next 15 years, but that the cold war will continue. Other necessary assumptions were made in each demand sector analysis, as noted.

Population was assumed to be between 205,907,000 and 242,880,000 by 1975, based on published forecasts from several sources¹.

Inventories

No statistics exist on annual lead inventory changes on an industry basis, hence they could not be taken into account. Since the LIA accounting, used most in this study, is based on sales of raw lead products to manufacturers, manufacturers' inventory changes would distort the consumption pattern. Probably the only period when they changed a considerable amount was at the time of the Korean War (chiefly 1950-1953).

Trend Lines

Unless otherwise stated, all trends were fitted by least squares, and charts show the trend solid over the data fitted, dashed for interpolations and extrapolations. Statistics used are in terms of lead content unless otherwise stated, and no allowance has been made for lead content of exported finished goods.

¹ Including Roos (1957), United Nations (1958), Census Bureau (Statistical Abstract, 1959), Brown (1957).

Price-Consumption Relationship

Chart 13 shows that the long-term increase in the use of metal per capita and the population increase dominate over the classical inverse relationship of price and consumption. It was also found that substitution for lead is mainly on the basis of technology rather than price, and where due to both a quantitative relationship cannot be evolved. Furthermore, there is little reason to expect that forecasts of metal prices are as dependable as the demand forecast, and the relationship once determined would be no direct aid in forecasting demand. For a discussion of the intricacies of the short-term price-consumption relationship see Green (1958).

CORRELATION METHODS IN LONG TERM FORECASTING

The possibility of deriving a mathematical model which given the values for the determining variables will generate a long-term forecast of lead demand (or supply) is attractive. Essentially the method rests on the derivation of regression equations relating the dependent variable, lead demand, to a series of independent variables, usually interrelated, such as population, GNP, spendable income, etc. The first problem therefore is to find logical relationships to factors for which forecasts are available. Such a relationship must exist between lead consumption and the federal Reserve Board Index of Industrial Production. Chart 14, the scatter diagram, shows that there is a close relationship, but that it is not linear, and chart 15 shows that this is due to diverging secular trends although the cycles are well correlated. This problem is often met in time series correlations, and lead illustrates it to good advantage. Several methods were attempted to eliminate trend (see Croxton and Cowden, 1955, Chapter 22 for a discussion of the methods commonly used). The linear correlation coefficient for the raw data is 0.854, which is low for series showing autocorrelation and with strong trends in more or less the same direction. Including time as a third linear variable is an oft-used device, but simply means that one is assuming a linear trend for each variable. Hence the only benefit is to forecast the cycles, the trend already being taken as linear, and in long-term work the cycles are not important relative to the trend. Furthermore, to investigate the consumption-Index relation with the time factor held constant (i.e. by partial correlation coefficients) is meaningless, since both Index and demand are correlated to the time function anyway.

If one correlates annual changes in the variables, absolute or percentage, the trend is usually eliminated, but the regression equation is of no use in forecasting, since

it does not dictate the level of the changes, i.e. the position of the trend, but only the annual changes.

Chart 16 shows how well per capita solder use and the FRB Index are correlated as percentage deviations from assumed trends, but once the trend is assumed the correlation is of little value.

Finally, the correlation of battery lead consumption and motor vehicle registrations affords an example of how this technique has little advantage over trend projection for long-term forecasting. Chart 17 is the scatter diagram. The linear correlation coefficient, r , equals 0.9106, partly due to parallelism of the trends. The forecast from the regression equation is for 591,427 tons consumption in 1975. If one compares this analysis to that given below under "Storage Batteries", he will find that there are other variables at work and other new fields of battery use which the simple application of correlation methods will not indicate alone. If he then proceeds to investigate battery technology, he will find that it is simpler and more easily understood by most persons interested in the results if the approach used below is used.

Because of the many problems involved in using the mathematical model for forecasting, the author prefers to use a more direct and probably more efficient technique, viz using ratios of series, or simple trend projections based on technological knowledge, or estimates made on the same basis.

TOTAL LEAD CONSUMPTION FORECAST

Lead Consumption and the FRB Index

Chart 18 shows the high correlation between the Federal Reserve Board Index of Industrial Production (Total Manufactures) and domestic consumption of lead over the period 1920-1959. While most economic time series show at least some correlation, due to the general influence of business cycles, this relationship is a logical one, since lead is used in many of the industrial products included in the Index. Changes in inventories and price of lead cause differences in relative amplitudes of the two series. (See Green, 1958).

Chart 15 shows that the trends of the two series are diverging. Because of this the coefficient of linear correlation, using the Pearson product-moment formula (see any statistical text for explanation), is only 0.8539. As explained above, the problem of trend in correlation is critical. The solution used in this case was to examine the ratio of consumption to the Index, or lead consumed per Index point. As shown in chart 18, three trends were fitted to the annual ratios. The second is a straight line fitted to the logarithms of the ratios.

Trend I assumes substitution for lead in constant absolute amounts over time.

Trend III, fitted to postwar data only, reflects the rapid loss of several markets which lead has suffered, and can be expected to give too low a forecast.

Trend II would logically be the best trend, since it assumes a constant percentage decrease in lead use per Index point. That is, as markets are lost, the lead industry reacts by increased research and development, strong selling campaigns, and probably ultimately reducing the price of lead, so that a percentage rate of decrease would be the approximate result. This trend has a smaller standard deviation than the straight line fitted to the same period. Combining

the projected consumption per Index point, 5,304 tons, with forecasts of the Index¹ from 255 to 298, the predicted lead consumption for 1975 is 1.35 to 1.58 million tons.

Per Capita Consumption

A total forecast can also be made by combining predicted per capita consumption and estimated population figures. Five linear trends were fitted to data for various periods, as shown in chart 19.

If all data since 1920 are used, the data of the Depression and World War II cause the trend to lie above the postwar cycles. Trend 2 is therefore considered more logical to use than trend 1, and gives a forecast of 1.47 to 1.73 million tons.

Trend 3 again shows the rapid loss of lead markets since the war, and gives a low forecast.

Trend 4 was fitted to show the effect of using a short series of data of which the first and last year fall below the general level, and it gives an even lower projected consumption than trend 3.

Finally, the fifth trend was fitted to show how adding three more years of data, all within a recession period, lowered trend 2, and the forecast from this trend is 1.59 to 1.88 million tons.

On the basis of the most logical trends, namely 2 and 5, the forecast by this method is 1.47 to 1.88 million tons consumption for 1975.

Total Consumption

One further method of forecasting total consumption by trend projection was used. Chart 20 shows a logarithmic trend fitted to total lead consumption. The forecast from this trend is 1.46 million tons for 1975.

Chart 21 shows three linear trends fitted to total consumption, with results similar to per capita trends. That fitted to data from 1920 to 1930 and 1946 to 1959 is the most logical to use, and gives a projection of 1.35 million tons.

¹ Sources: Brown and Hansen (1957), and Roos (1957).

Summary

The most satisfactory trends fitted are listed below, in order of preference on a logical basis.

Trend	Forecast (millions of tons)
St. line to logs of FRB Index -Lead Cons.Ratio 1920-1959	1.35-1.58
St. line to Per Capita Cons. 1920-30,1946-59	1.47-1.73
St. line to Total Lead Cons. 1920-30,1946-59	1.35
St. line logs of Tot. Lead Cons.1920-30,1946-59	1.46

Taking the average for each of the first two trends, the range defined is 1.35 to 1.60 million tons by 1975, or 1,475,000 \pm 125,000 tons, which can be considered the forecast derived by the overall forecasting method.

DEMAND FORECAST FOR LEAD
ON AN END-USE BASIS

The following is a detailed break-down of lead uses as reported by the LIA. Each use has been investigated in some detail and a forecast made for future demand from that use sector.

Storage Batteries

SLI batteries (starting, lighting, ignition) for automobiles annually employ 90% of the 300,000 to 350,000 tons of lead currently used in all batteries each year. The remaining 10% is widely distributed in use, industrial electric trucks being the second largest application. Some 80 to 90% of this lead is recycled as scrap over 1½ to 2½ years, hence this large market does not benefit the primary lead producer in proportion to its size.

Battery grids are made of hard or antimonial lead (7-12% antimony) and pasted with litharge, red lead, and metallic lead. About 60-70% of total battery weight is lead, and about half of this is hard lead and half oxides.

Chart 22 shows that although SLI battery shipments are increasing with time the amount of lead used is decreasing, less lead being used per battery. The discrepancy in the lead figures is due to incomplete coverage by LIA, hence ABMS data are used here. Since it includes antimony, a discount for this factor has been made in the forecast.

Three linear trends fitted to total battery lead consumption, and two fitted to per capita consumption (charts 22 and 23) give a range of forecasts from 275,000 to 797,000 tons per year by 1975 (table 1).

Table 1. Trend Lines Fitted to Lead Consumption in Batteries.

No.	Trend Equation	Period Fitted(incl.)	1975 Forecast
1	$Y=134.84 + 6.5251X$	1920-'29, '47-'59	493,700
2	$Y=107.78 + 6.9927X$	1920-'39, '47-'59	492,400
3	$Y=391.146 - 1.1742X$	1947-'59	326,600
4	$Y=2.26456 + 0.061310X$	1920-'39, '47-'59	580,000-797,000
5	$Y=7.02054 - 0.078992X$	1948-'59,	275,500-378,500

The most logical trends, 1 and 2, yield a prediction of 492,000 to 494,000 tons. The postwar trend, number 3, shows that the consumption pattern is changing. There is no reason to believe that using more complex trends will improve the forecast. The method used here is to investigate the techno-

logy of this application of lead, and attempt to appraise possible future developments therein that will affect lead consumption, relying in some places on trend projection as an aid in prediction.

Technology of Battery Use of Lead.

New machinery for casting lighter weight grids, combined with reduced antimony content and addition of small amounts of silver compounds and other metals which improve the corrosion resistance of the grids, has led to a battery with longer life and less lead content. Although no statistics are available, it is possible that some of the decline is due to increased buying of lower-priced, lighter weight replacement units. This possibility must be ignored here.

The use of lighter weight grids has offset a consumption increase due to 12-volt battery use, these units containing 2 to 3 pounds more than the 6-volt (Allen, 1960). Similarly, a lead content increase of about 1.5 pounds per unit due to decreased antimony content is overshadowed. It is expected that a further decrease of 1 pound per unit will be made, based on current research on lead oxide action, and that this, and the use of "compact" cars, whose 12-volt unit contains the same amount of lead as the average 6-volt, will offset the further consumption gains due to widespread use of the large 12-volt batteries. It is concluded that battery weight will stabilize at about the current level of 21 pounds per unit.

Battery replacement rate also influences consumption. Chart 24 shows that average battery life is increasing, since fewer are shipped per vehicle registered in recent years. This is due to improved quality of the lead and lead oxides, reduction in antimony, addition of other metals to grid metal, in small amounts, and improved auto electric systems, notably the voltage regulators. Unfortunately, it is difficult to foresee how much further the battery life can be increased. The trend appears to be flattening off between 40 and 45 percent replacement per year, and it is improbable

that the rapid improvement of the postwar years will continue at such a high rate.

Forecast.

Table 2 shows the importance of this factor in the forecast.

Table 2. Lead Use in SLI Batteries, in tons.

Replacement Rate	40%	45%	50%
Lead Content (lbs.)			
19	422,415	475,217	528,020
20	444,648	500,229	555,810
21	466,880	<u>525,240</u>	583,600
22	489,113	<u>550,252</u>	611,391
23	511,345	575,263	639,182

It is based on a forecasted registration of 111,162,000 motor vehicles in 1975¹, and indicates a forecast of 525,000 tons \pm 100,000. Allowing for antimony content, this is reduced to 504,000 tons \pm 96,000. Note that the trend projections favored as based on the most logical years predicted about 493,000 tons, or less antimony content 473,300 tons, a reasonably good agreement.

Marine Starting Batteries.

Although data are extremely meager, it is obvious that a large market for batteries is developing from the wide use of electric-starting outboard motors. A forecast for this use is very tenuous, since replacement rate and the growth rate of motors in use are neither well-established as yet. Since 1947, an average of some 320,000 motors have been added to the number in use each year. Of the 500,000 sold each year recently, 62% are over 15 horsepower, of which a large number are electric starting². If the growth rate

¹ Source: Straight line interpolation of forecasts from the Third Progress Report of the Highway Cost Allocation Study.

² Boating, 1959: A statistical report issued jointly by the Outboard Club of America and the National Association of Engine and Boat Manufacturers.

continues at 320,000 motors per year and the percentage of electrics increases from the current 42.6%¹ to close to 100%, about 7.5 million batteries might be in use by 1975.

Local firms claim that a battery lasts two seasons, so that some 3.75 million replacement units would be purchased per year by 1975. Since these are all premium grade batteries, lead content per unit is estimated at 23 pounds, less 4% antimony, and this market is seen to represent possible use of 41,300 tons of lead annually by 1975. This is a preliminary figure and should be reviewed as more data become available.

Motive Power Use of Batteries.

Batteries are used to drive industrial materials handling trucks, which account for about half of the industrial battery use (10% of total battery lead use). These lead-acid battery trucks have about 20% of the market, and although they are very versatile and desirable machines, their initial cost is twice that of a truck with an internal combustion engine. Assuming that they will not increase their share of the market, which is taken to grow at the same rate as the FRB Index of Total Manufactures, this use of lead will increase at about the same rate as SLI battery use. Since the 10% of total battery lead use is included in the SLI battery calculation, no change is made in the forecast. If the share of the market could be doubled, about 28,000 tons more would be used per year by 1975.

Current efforts to promote electric pleasure vehicles may be well timed. Able to run at up to 60 mph, with a range of 60 to 100 miles², these cars could be used for an estimated 50% of urban travel by 1975. Type of battery, lead content, and rate of replacement, as well as actual consumer

¹ Outboard Boating, July 1960.

² Evans Taylor, (April, 1959), Appendix, being a resume of articles appearing in Electrical World.

acceptance, are totally unknown¹. While the effect on lead use in tetraethyl of wide use of electric cars can be approximately calculated, any attempt to forecast the effect on battery use would be meaningless. This very large potential market should be kept in mind however.

One other battery-powered vehicle is in use, the electric golf cart, and is forecasted to consume, say, one to two thousand tons of lead annually by 1975.

Substitution.

The nickel-cadmium battery uses two strategic raw materials, (cadmium is produced in far too small amounts to satisfy the SLI battery market), it is currently extremely expensive (\$200-325 for an automotive type²), and like the nickel-iron battery, has different electrical characteristics from lead acid units, for which United States automobiles have been designed. Most experts concur that there is no substitute presently known for the lead-acid battery.

The total forecast for battery lead use is then as follows.

Automotive (SLI) Batteries	504,000 tons ± 96,000
Marine Starting Batteries	41,300 tons
Motive Power Use -Industrial	included in SLI batteries
-Automobiles	?
-Golf Carts	1,000 tons - 2,000
Total	<hr/> 546,800 tons ± 101,000

Approximate total demand is then 547,000 ± 100,000 tons of battery lead annually by 1975. It should be remarked that this wide range is the equivalent of allowing hundred percent errors or more in forecasts for several of the smaller uses, which is indeed unfortunate, but unavoidable. Note that trend projection alone of battery demand takes no account of the new uses, such as marine batteries.

¹ The aspiring manufacturers contacted ignored several requests for information.

² New York Times, April 11, 1960.

CONSTRUCTION

Three types of lead product are used in construction -pipes and extruded products such as traps and bends, sheet lead, and calking lead. Solder is not included in building statistics. Only the USBM gives separate accountings for all three products, and there are no data available to show the amount used in each of the two most important fields -residential and chemical plant construction. Chart 25 shows the available data on a per capita basis, LIA series grouping all three products, the ABMS separating calking.

The use of easily installed copper pipe, costing about one-half as much as lead (based on local delivered prices) and giving long and satisfactory service, accounts for the decline in the "Pipes, Traps, and Bends" category, although cast iron and plastic compete against lead traps and bends, and some bends are used in lead pipe plumbing only. This decline is expected to continue, and since so few years of data are available and a linear trend projection is negative by 1975, consumption is simply estimated to flatten off at 0.15 pounds per capita, because of continued use of lead pipe in chemical plant construction.

Sheet lead is used for roofing and flashings, lining shower pans, ornamental pools, etc., and vent pipe flashings in residential and especially in institutional building. It is more widely used for lining tanks, sinks and other equipment in chemical plants, especially where sulphuric acid is handled, but because of poor abrasion resistance, and a tendency to creep and sag, necessitating special construction techniques, rubber, ceramics, glass, monel, stainless steel, and plastics are strong competitors, although often higher in original cost, and this contributes to the downtrend in per capita sheet lead use. Galvanized steel has taken over much of the roofing market since it is much cheaper on first cost, and aluminum and copper, because of greater strength and also architectural trends, also compete strongly.

in houses. Although it seems that this type of joint could easily be supplanted, many years of effort by different interests have failed to find a substitute¹, and it is assumed that none will be found in the next 15 years. Also, the inroads of substitutes for lead, chiefly plastics, cement, and aluminum wool, having made little headway to date are assumed to remain unimportant, although widespread acceptance of these materials in building codes could change this.

Chart 26 shows a close correlation between the Construction Material Index (Roos, 1957) and calking use of lead, about 4,500 tons of lead being used per index point. On the basis of a forecast of the index of 23.0 for 1975, lead consumption for calking in 1975 is predicted at 103,500 tons.

Linear trends fitted to data for various periods yield a wide range of forecasts of per capita use. An extrapolation of the postwar trend falls close to the above forecast.

Total Construction Use Forecast.

The total forecast for lead use per year in "Pipes, Traps, and Bends", "Sheet Lead", and "Calking" is thus 133,500 ± 5,000 tons.

¹ H.E. Robertson, Cast Iron Soil Pipe Institute, letter to the author, April 8, 1960.

Oil Refining and Gasoline

Of great interest to the producer of new lead is the rapidly increasing, totally dissipative, consumption of lead tetraethyl fluid (TEL), used to increase the octane ratings and antiknock qualities of gasoline. Not only has the total number of vehicle miles driven per year increased, but until 1953 at least, TEL consumption per mile, and hence lead consumption, has also increased (see chart 27).

Higher speed travel, heavier automobiles, higher compression ratios, widespread winter use of automobiles, and the introduction of premium and even third grade fuels have accounted for increasing TEL use per mile driven. The cause of the downtrend since 1953 is a combination of circumstances. Although average gasoline octanes have been increasing, this has been accomplished by processing the gasoline in cracking, reforming, alkylation, and isomerization units to give it a higher octane before any TEL is added, since health laws have a limit on the amount that can be used (formerly 3 c.c., recently increased to 4), and, more important, each additional c.c. of TEL added increases the octane number by a smaller amount. At current levels, one further c.c. adds only 1 - 1.25 to the motor octane number (Oil and Gas Journal, v. 57, no. 40, September 28, 1959, p. 46). With a high rate of construction of processing plants, combined with an economic recession which has reduced the amount of the annual increment in vehicle miles driven, an overcapacity of processing equipment has resulted. Since this represents a capital investment it is economically better to run these plants at full or nearly full capacity, yielding high octane base fuel, than to add lead to unprocessed fuel. This is reflected in the decrease in lead content of regular grade gasoline from 2.28 c.c. of TEL in 1956 to as low as 1.48 in early 1959. This is far below the average of the last five years.

It is therefore concluded that the post-1953 down-trend will flatten off, due to a decrease in plant construction, a return to more normal demand levels, and an increasing demand for higher octane regular grade gasoline to satisfy the "thrifty" engines now advertised to run on regular grade. The Ethyl Corporation concurs that the trend will flatten off, and perhaps begin to increase, although at a rate lower than before World War II¹. A number of simple logistic curves fitted by the method of selected points indicate an asymptote between 572 and 590 pounds per million miles.

The use of diesel fuel-burning jet aircraft will decrease TEL use in aviation gasoline, which has been 12-13% of total domestic use and included in the statistics used here. Small car use may also decrease gasoline consumption, although lower price and cheaper operation will probably cause an increase in the number of miles driven per year, hence balance out the decrease. Sales of unleaded gasoline represent 2% of total sales at present and are not expected to increase over this¹.

Thus, the assumption that TEL use of lead per million miles driven will flatten off at about 572 to 590 pounds seems to be reasonable in the light of what is known about the industry.

Substitution.

There are two possibilities of decreasing lead use in TEL by substitution -one is by developing an automobile engine which does not require high test gasoline for fuel, the second is by developing a substitute for TEL in high test gasoline. The diesel engine has been restricted mainly to truck use due to poor acceleration characteristics, and the free piston engine and turbine will probably find similar applications. Although many other types of engine have been

¹ M.E. Collins, Manager, Market Analysis and Planning, letter to the author, dated April 22, 1960.

tried, no serious competitor has yet appeared for the internal combustion engines used in automobiles, which have also been steadily improving with time.

Yet one of the oldest competing mechanisms, the electric car, is being seriously promoted today, and with a top speed of 60 mph and a range of 100 miles for one model in production on the west coast¹, they could capture a large part of the market, although they would essentially be confined to urban travel. One manufacturer will offer a kit to convert from gasoline to electric in one day, and states that he is "confident that by the year 1970, fully 50% of the automobiles used in urban travel will be electrically propelled"¹. If, say, by 1975 this were the case, some 25% of total vehicle miles would be travelled by electrics. This will be taken into account in the forecast.

New additives for gasoline are not direct substitutes for TEL for the most part, but are used to give a greater increase in octane number per c.c. of TEL when used in conjunction with it. Such for example are Shell's TCP (tri-cresyl phosphate), Ethyl Corporation's AK-33X, a manganese additive, and Texaco's TLA (tertiary butyl acetate)². By increasing marginal use of TEL, they could actually increase lead use.

Both before and after TEL was discovered, many hundreds of other compounds were tested unsuccessfully. Iron carbonyl, developed as a TEL substitute during World War II had some success, but causes spark plug fouling and rapid engine wear, hence now has no market. Furthermore, the Ethyl Corporation has a strong position in the industry because of its size, and long term contracts.

It is therefore concluded that excepting the advent of electric cars, no decrease in the need for TEL will occur in the next 15 years.

¹ Evans Taylor, (April, 1959).

² Oil and Gas Journal, September 1959, p. 47.

Forecast.

Vehicle miles driven for 1975 are estimated¹ at 1,081,000 million. If lead consumption in TEL is taken at 572 to 590 pounds per million miles, the forecast for 1975 is an annual consumption of 309,200 to 313,950 tons.

If electric automobiles are assumed to account for 25 percent of all travel in 1975, the forecast is for 231,900 to 235,500 tons.

¹ Linear interpolation of predictions given by the Highway Cost Allocation Study, Third Progress Report.

CABLE COVERING

The postwar development of polyethylene-sheathed cable for communication and power transmission lines has cut deeply into what has previously been the second, sometimes third largest use of lead. Extruded over the wires, the sheathing protects them and their insulation against moisture and physical damage. The main advantages of polyethylene sheathing are: it eliminates the use of a strategic raw material; reduces cable weight by 50%, hence saves handling effort in installation and decreases shipping cost; it also allows longer duct and aerial runs, eliminating many costly splices, and reduces the need for heavy support wires; in most installations corrosion resistance is greater, reducing cable failures and maintenance costs¹. These advantages account for the widening gap between the miles of communication cable in service and the amount of lead used (chart 28). (Power transmission uses a relatively small amount of cable).

Chart 29 shows the production of exchange type cable for the Bell System in recent years.

Superimposed on this rapid substitution is the development of communication systems which employ much less or no cable, viz microwave and coaxial cable systems. While coaxial tubes are thus far usually lead-sheathed, only 90,442 miles were in use in 1958², and it is possible that polyethylene sheathing may eventually be used on these also. Microwave systems use no cable. Systems designed to use artificial satellites or the moon to reflect signals will also reduce cable use as they become perfected and applied.

It is concluded, that advances in extruding lead alloy cable coverings with better abrasion resistance and corrosion rates notwithstanding, the weight and stiffness of lead-sheathed cable will cost it the market, and that by 1975, only say 2,000 tons will be used annually for coaxial and other special cables.

¹ Western Electric Engineer, (October 1958), Cable sheath - a review from iron pipe to Stalpeth: Staff Report.

² Statistical Abstract, 1959.

Paint and Varnish

The most important lead compounds used in paint and varnish are white lead or basic carbonate of lead, red lead, and litharge. Chart 30 shows the data available on these uses¹.

Although white lead is reputed to be a good pigment, it has decreased in use due to its toxic nature, which has a cumulative effect, and the advent of cheaper, whiter pigments, principally zinc oxide and titanium oxide. Thus consumption of white lead decreased from 64,500 tons of contained lead in 1934 to 13,700 tons in 1958. The use of synthetic resin and water vehicles has tended to speed the decline in lead use, since it nullifies one of its advantages -the formation of lead soaps in natural oils, yielding a tough, elastic paint film. Pure white lead paints chalk badly, yellow, and hold dust and dirt firmly, when exposed to weathering tests².

Basic lead sulphate and leaded zinc oxide are less toxic white pigments, but in 1945, the last year of data available, the former use had declined from 13,435 tons in 1929 to 3,000 tons, while in 1958, 23,021 tons of leaded zinc oxide, containing 3,402 tons of lead were manufactured, a decline to one-third the lead so used in 1953¹. These compounds are thus far less important than white lead.

The use of lead in white pigments continues to diminish through 1958, although the rate of decrease is lessening. Linear and exponential trends fitted to the United States Bureau of Mines data indicate zero consumption by 1975. It is assumed here that some at least will continue to be used, and the amount is estimated at 5,000 to 10,000 tons of contained lead per year. This estimate is probably generous.

Some 3,223 tons of lead were used in 1958 in the form of litharge, for a drying agent in varnishes and oils. This use is projected as following a linear trend, fitted from

¹ United States Bureau of Mines, Minerals Yearbooks.

² Remington and Francis (1954).

1946 to 1958 inclusive, yielding a forecasted consumption of 3,186 tons for 1975. There is currently no reason to believe that the trend of this use will change.

In 1959, red lead accounted for as much lead use in paints as did white lead. It is an excellent, although relatively expensive primer for steel in construction, bridges, ships, railroad structures, highway equipment, etc. Because the stock of materials which require such painting is increasing with time, the use of red lead shows an increase, but rather slight, over the period of observation. The potential rate of increase for red lead has not been realized because of competition from less expensive primers, such as iron oxide, aluminum and cement-like alkaline paints, some of which claim to perform as well as red lead. Price and drying time apparently account for the supplanting of red lead as an auto body primer. Another trend which diminishes red lead consumption is the rapidly growing use of aluminum, both enamelled and natural, for highway signs, markers, railings, and even bridges. The use of steel for structural work in construction, in ship hulls, at least some railway equipment, and many other uses will probably not be challenged by aluminum for many years to come, and these require considerable amounts of primer.

The best available estimate for red lead is then a simple trend extrapolation. A linear trend was used here over the postwar period, and the forecast is for 22,400 tons for 1975.

Total lead consumption in white lead, red lead and litharge, as used in paint and varnish is then 30,500 to 35,500 tons annually by 1975.

AMMUNITION

Lead is no longer used in most military ammunition, the high consumption rate in World War II shown in chart 31 being for shortarms ammunition. Even in high-powered sporting rifles, steel-jacketed bullets are used to keep the shape of the slug intact during its travel. Most lead used in ammunition is for shot for sportsmen (about 5 tons for every ton of lead used for bullets in peacetime)¹.

Chart 12 shows a continuing decline in per capita use of lead for this sector since the war (LIA data). There is an uptrend since the war shown in chart 31 (ABMS data), and since the LIA accounting is incomplete, the latter is used here.

Part of the uptrend is due to the inclusion in the accounting of about 2,000 tons of lead shot annually, used for coating sheet steel². A further part is only apparent, due to distortion caused by the Korean conflict. While this would logically seem to have increased consumption in 1949 or 1950 to 1952, in fact it decreased it, and the return to more normal levels appears to be an uptrend. Since any future change in the per capita lead consumption pattern would be due to such intangibles as psychological change in the attitude toward hunting, or future major changes in game laws, simple trend projection has been used to forecast directly. The trends fitted are given in table 4.

Table 4. Trend of Lead Consumption for Ammunition.

No.	Trend Equation	Years Fitted	Forecast for 1975 (Tons x 10 ³)
1	$Y=0.65165-0.0013585X$	1920-1959	57.1-67.4
2	$Y=0.58600-0.0033507X$	1920-1959	35.8-42.2
3	$Y=0.59041-0.0038442X$	1920-'39, '46-'59	39.0-46.0
4	$Y=0.4599+0.002622X$	1946-1959	59.5-70.2

¹ Lead in Modern Industry, p. 126.

² D.M. Borcina, LIA, personal communication.

Trend 1 was fitted to show the distortion caused by World War II and is decidedly too high. Trend 4 shows the distortion of the Korean War and the apparent uptrend. Trends 2 and 3 are taken as closely representative of actual trend, the war years in the former being reduced to the 1939 level, and the forecast for this sector is 41,000 \pm 5,000 tons for 1975. This is about the current level of consumption, so no growth is forecasted.

The author cannot agree with the Paley Commission's forecast of a doubling of 1950 consumption, based on a projected doubling of military defense material needs, since lead is not used for most military arms.

COLORS

Lead chromates are used in primrose, yellow, orange, scarlet and red paint colors and are the most important yellow pigments in the paint and allied industries, as well as being indispensable in process color printing. Since they are inexpensive relative to other colors, and have been progressively improved with time, their position is stable in the industry, and competitors such as zinc yellow have been in existence long enough to have accomplished any substitution for lead chromates of which they are capable.

The main form of lead used in making colors is litharge, although other forms are bought and converted.

Chart 32 shows that per capita consumption over the period of available data is increasing. Unfortunately the period is short, and distorted by World War II and the heavy buying in the Korean War period, in order to build up inventories against possible controls on lead. (The paint industry suffered considerably from stringent controls on lead in World War II). Thus, although the linear least squares trend line appears high, the reduction in inventory in 1954 and ensuing years is not shown in the data, and actual consumption would show a more linear pattern. Trying a free-hand trend to take this into account gave a similar projection to that of the trend shown - 0.2795 pounds per capita for 1975. Although this is a short and distorted base on which to forecast, no other method but trend projection was found feasible, since paint manufacturers were not willing even to estimate future demand as a guide.

The forecast is then for 28,775 to 33,942 tons, or 29,000 to 34,000 tons.

PRINTING

The small amount of typemetal indicated as consumed each year is only a sample of the amount in use, since every large printer remelts his own scrapped type, and a single daily newspaper might handle more than 4,500 tons of metal annually, sending out only the dross (skimmings) for resmelting. Most of the amount included in the accounting is scrapped metal from small shops where remelting is uneconomic. Obviously then it is difficult to make any analysis of the continuing postwar decline in typemetal use shown in chart 33, since it could be due to a decline in the number of small print shops or the amount of business they do, or an increase in the number that remelt their own type, or changes in the technology of printing. Of these, the latter appears most important, for electronically-controlled photographic processes are rapidly being developed and applied in the industry, e.g. the Mergenthaler and Photon machines. Most of these new processes use no lead, and undoubtedly such methods will be the basis of the future printing industry. However, for small shops the rate of machinery replacement will probably be slow since new techniques must be learned, the replacement costs will often not be justified for the advantages gained on small job work, machinery installed in recent years will last a long time, and used machinery from larger shops changing to offset (photographic) methods will probably be available at low prices. The trend of per capita use is therefore projected as declining at a lower rate than over the period 1947-1959, since a great deal of lithographic equipment was put into use from 1945 to 1950, and since 1951 the trend has flattened off somewhat. Although an extremely short base of data from which to work, a linear trend fitted from 1951 to 1959 is projected for the forecast, yielding a figure for 1975 of 24,750 to 29,200 tons annually. Because of the large amount of runaround scrap involved, this forecast must be regarded as somewhat provisional.

CERAMICS

Fields of use for lead which are grouped under the heading of ceramics include leaded crystal and other fine glass ware, glazes for artware, sanitary ware, wall tiles, china, etc., television picture tubes, and frits for porcelain enamel. Leaded crystal may contain as much as 80% lead by weight, and television tubes contain up to 30%. This latter category probably accounts for most of the postwar increase in ceramics use of lead shown in chart 34, and the index of production of television sets is shown for comparison, although no separate data exist for this use of lead alone.

The application of greatest potential for lead in the ceramics field is in porcelain enamelling. It was widely used some 25 years ago, but as in paints, its toxic qualities cost it the market. Today however the glazing material is "fritted" to form a granular silicate compound which is then fused to form the porcelain, hence dangerous powdered lead compounds are not handled much. Because of their low melting points (975°F) compared to the usual enamels (1450-1550°F) and their better performance than newly-developed low temperature enamels, (i.e. thinner coatings, fast application, more chip-resistant, fewer rejects, and generally more "foolproof" to apply), lead-base frits, usually about 45% lead, are becoming more popular, and in fact will probably be the major type used on all enamelled aluminum since firing aluminum even for a short time at 1450-1550°F would warp it badly and possibly melt it. The industry estimates that by 1961 about 500 tons of lead will be used¹. Potential is great for a major increase as enamelled aluminum increases its share of the construction market, being used for primary and secondary doors and window frames, eaves troughs and downpipes, interior wall panels, and perhaps most important curtain wall sheathing, which is only in its infancy. Of course enamelled aluminum will also

¹ Iron Age, August 8, 1959, p. 46

find uses in other fields, such as for highway markers, fences, venetian blinds, etc.

Adding several-fold to this potential is the expectation that lead-base frits will be used exclusively for enamelling aluminized steel¹, a sandwich of the two metals with many of the qualities of both, and this material, a relatively new product, could be used in place of either in many applications. It is less expensive and stronger than pure aluminum.

In porcelainizing steel, the gauge and grade of steel often must be chosen to withstand the high temperature of enamelling rather than on the basis of useage. This means that a further large increment in ceramic use of lead could occur if lead-base, low temperature enamels are accepted as a means of using cheaper grades and lighter weight steel in appliances, cabinets, construction, etc., and there seems to be a reasonably good chance that this will happen.

To forecast this sector on the basis of the available statistics is of little value, since they reflect chiefly the pattern of growth of use in television tubes, so far as can be discovered. As early as 1948 the ceramics industry predicted three-fold growth from 1948 to 1973 for architectural porcelain and a doubling for major home appliances². Rosenzweig (1957) estimates a 300 percent increase from 1954 to 1965 for aluminum use in construction, of which a considerable amount may be enamelled. A study conducted by the Porcelain Enamel Institute found that use of enamelled aluminum has grown from 3.5 million square feet in 1956 to an estimated 23 million in 1961, a 7.5-fold increase of which building accounted for about 65 percent³. The use of lead for porcelain enamelling is thus conservatively forecasted as tripling by 1975 to an annual consumption of about 50,000 tons.

1 Iron Age, loc. sit.

2 Ceramics Industry, June, 1948.

3 Modern Metals, March 1958, v. 14, no. 2, p. 72-73.

RAILROADS

Aside from storage batteries used for air conditioners, lighting, etc., the railways "consume" lead in journal bearings, mainly on freight car trucks, and this is the metal reported under this sector.

Each car has eight bearings of either solid, cartridge, or roller type, the first being used on almost all of the approximately two million freight cars in use in the United States (American Railway Car Institute, 1958). These solid bearings weigh about 30 pounds each, about 18% of this being babbitt, or bearing metal. For every hundred pounds of bearings used, some four pounds of babbitt is worn away and lost. Since they are purchased under toll arrangement the rest is recovered as scrap. With an average life of 3.3 years, some five million bearings are replaced annually, accounting for 10,000 to 15,000 tons of lead per year¹.

Chart 35 shows that there is a decline in the consumption of lead by railroads, and a decline in railroad ton-miles of freight hauled, but that lead used per ton-mile hauled is decreasing. Better lubrication, slightly increased tons per average car loading, and the advent of a small number of roller and sleeve bearing-equipped cars apparently account for the decline.

A logarithmic trend fitted to consumption per ton-mile (chart 36) indicates a lead consumption of 10.117 pounds per million ton-miles by 1975. A series of forecasts of the future rail freight volume that can be expected were made. Chart 37 shows a linear trend fitted to ton-miles, with a projected level of 634 billion for 1975. Chart 38 shows a linear trend fitted to per capita freight volume, yielding a volume for 1975 of 585 billion. A forecast made by the American Association of Railroads, and described as "probably

¹ G.T. Butcher, Product Manager-Bearings, American Brake Shoe Co., Railroad Products Division; letter to the author dated March 25, 1960.

unduly optimistic" is for 1,019 billion ton-miles¹, and assumes that the railways will hold 45% total intercity ton-miles. From the trend of percentage held it appears that this is indeed optimistic, and if 30% were held, some 680 billion ton-miles would be hauled. A compromise figure of 800 billion ton-miles is used here, about 35% retention of traffic.

Combining lead consumption per ton-mile with this figure, the forecast would be for 4,000 tons of lead per year for 1975. A logarithmic trend fitted to annual lead consumption (chart 39) projects a level of 3,200 tons for 1975, in good agreement.

Two developments will reduce the lead consumption. The first is the trend to roller bearings (see chart 35), which is expected to grow at about 1% of ownership per year or more². If, say, 20,000 roller bearing-equipped cars were built each year, 15 years would see about 300,000 in operation. (Presumably few would be retired from service in 15 years, the largest percentage of cars now in use being over 30 years old). This would reduce lead consumption by about 15%, since the number of cars in service is not expected to increase much (Chart 40 shows that freight carried per car can be increased by better speeds and car control).

Because of their high initial cost, roller bearings will probably be installed only on a small percentage of old cars. However, a second bearing development is the long-life cartridge bearing. This will last some 9 years without replacement, and contains about one-eighth as much lead as the solid bearing. Since, economically, this bearing appears to have considerable advantages over solid bearings, it is assumed that all conversions within 10 years or so will be to this type of bearing, some 2,000 of which are so far in service³. Thus apparent

¹ J.E. Monroe, Vice-President, letter to the author, dated March 11, 1960.

² Butcher, *ibid.*

³ A.F. Kaminskis, Engineering Supervisor, Cleveland Graphite Bronze Co.; letter to the author, dated March 30, 1960.

lead use will be reduced to one-twenty-fourth of the above-indicated amount, or in other words this demand sector is expected to be inconsequential by 1975. Of course the reduction will really be only apparent, since the part of the solid bearing returned for scrap actually accounts for much of the "consumption", and the cartridge bearing, from which no scrap return is expected, will also wear away in use.

AUTOMOBILE MANUFACTURING

The trend in total lead used in auto manufacturing and in apparent lead used per auto is shown in chart 41. There is probably a considerable amount of lead used and not included in the accounting since LIA reports only solder. Exclusive of the battery however, solder is the most important use of lead in an automobile. Body solder is used to fill and smooth outside welded seams and sheet metal wrinkles. The engine and heater radiators are soldered, as well as parts of the electrical system.

Unfortunately, data collected from various sources show little agreement on lead use per auto, the LIA figure of currently 3.5 pounds being lowest, the General Motors Corp. figure of 11 pounds¹ being highest. The best documented estimates are between 5 and 8 pounds², of which about 3 or 4 pounds are in body solder. The LIA discrepancy is probably due to the copper-lead bearing metal used on main and camshaft bearings, and also to some terne plate used. Since it is the only continuous series of data it was used here, and indicates that the recently decreasing trend has now tapered off at about 3-3.5 pounds per auto. The decrease is apparently due to better sheet metal forming and body style changes, probably also in some part to using clip connections in the electrical system.

The amount of solder used could be cut by one third if aluminum radiators, such as on the 1960 Chevrolet Corvette, or air-cooled engines, as on the Corvair, become widely used in domestic automobiles. Furthermore, cheaper substitutes have been sought for body solder for several years. It is assumed here that neither of these two developments will occur, and

¹J.M. Campbell, General Motors Research Staff, LIA Annual Meeting, 1956.

²Chrysler Corp. and Ford Motor Co., letters to the author.

that the LIA accounting will continue to show about 3-3.5 pounds of lead being used per auto.

The Automobile Manufacturers' Association has kindly compiled a number of trade forecasts for 1975 auto production¹, ranging from 7.7 to 11.5 million cars. These forecasts imply that they are for total sales, and that about 10% may be imports. Using a car-equivalence of 1.5 for trucks, as was done by the Paley Commission, the forecasted truck production of 1.7 million per year by 1975 is equivalent to 2.5 million automobiles. Therefore, a forecast figure of 10 million car-equivalents is used here for total domestic production, as a compromise figure among the many forecasts.

Combined with a lead consumption per unit of 3-3.5 pounds, the forecast for 1975 is 16,250 ± 1,250 tons annual consumption in automobiles.

¹ Letter to author, January 25, 1960.

CANS

A small tonnage of lead is used annually for sheet metal for cans, generally terne plate, but the largest amount is used for solder for side seams. Chart 12 shows that the per capita consumption for this sector has been declining since 1953, due to several changes in manufacturing technology, not a declining use of cans, since chart 42 shows this to be increasing.

The first change, begun in 1953, was the substitution of adhesive or cement for side seams on quart motor oil and citrus concentrate cans, and it is now the standard sealer for these containers. When a can is subjected to internal pressure, such as during processing of food in the can, or in beer, soft drink, and aerosol cans, or external pressure, as in vacuum packs, cement is not feasible, nor can it be used in containers for acid foods such as juices, tomatoes, fish. Thus there are few further applications of importance to be expected -lard, wax, paint, and a few other materials which do not use large numbers of cans.

A second change was a reduction in "Side Seam Allowance" (SSA) on cans, in conjunction with an increase in lead content of the solder to 98%, which increased creep resistance of the seam joint. SSA was reduced from 0.365 inches to 0.260 inches. Currently, SSA is being further reduced to 0.200 inches, which will further reduce solder used per can¹.

The use of cans should grow about as fast as population, except for some special fields, notably aerosols, and beer cans, which are expected to grow more rapidly. Projecting a linear trend fitted to per capita consumption of lead in cans indicates no consumption in 1975 due to the continuing recent

1 J.M.Heinen, Manager, Packaging Engineering, Continental Can Co.; letter to the author, dated June 6, 1960.

downtrend, so that trend projection is no help for this sector. Calculating a percentage decrease in lead use due to reduced SSA, weighted on the basis of decrease per type of can and number of each used, consumption per capita will be decreased to 59% of current level in the next few years. Using the average consumption from 1957-1959, 0.116 pounds, this would be reduced to 0.0684 pounds per capita, indicating a consumption of 7,042 to 8,306 tons, or 7,000 to 8,300 tons for 1975.

Two factors currently important could reduce this use to almost nothing. The first is the growing use of aluminum containers, which have many advantages over tin-plate cans but are still relatively expensive for most uses. A study by A.D. Little Inc. in 1958 indicated that aluminum may have 20 percent of the market by 1968, and extensive investigation by the author indicates that this is certainly a well-founded possibility and that 30 percent may be taken by 1975. These containers will be cemented, welded, or seamless, and use no lead. Since some of this growth will be in citrus concentrate containers and motor oil cans (already in use in aluminum), the forecast for lead is reduced here by 25 percent, to 5,200 to 6,200 tons, or 5,700 ± 500 tons. The extra growth expected in aerosol, soft drink and beer cans is not expected to benefit lead, since aluminum lends itself well to these containers and is already in use for some of them.

The second factor that could decrease lead use is the development of an economic method of welding light tin-plate. Much work has been done on this, and there appears to be a good chance of early success¹. Of course the use of lead would then be considerably curtailed, and the forecast of 5,700±500 tons is therefore considered to represent a maximum consumption to be expected.

¹ Heinen, loc. cit.

NEW USES FOR LEAD

Perhaps the most sensational new use of lead is for radiation shielding. Gamma radiation is not stopped by shielding but the energy level is reduced, and one speaks of "half-space", i.e. the distance necessary to reduce radiation of one energy level to one-half that level. The half-space varies with the density of the material, hence lead makes an excellent shielding, especially where space is at a premium, as in submarine nuclear power plants, experimental reactors in crowded quarters, shipping containers for radioactive materials, etc. In fixed reactors, concrete, water or simply air can often be used more cheaply than lead, and since the concrete also plays a structural role, it is the most commonly used shielding for such reactors.

It is estimated that the approximately 100 fixed reactors in the United States take from 10 to 1,000 tons of lead for shielding¹. No average was given, but since many are probably relatively small units, the figure of 500 tons is generous. Dr. C.E. Crompton, associated technical director of the National Lead Company estimated that by 1965 between 5,000 and 10,000 tons of lead will be used annually for reactor shielding, 1,000 tons for isotopes, and 2,000 tons for reprocessing nuclear fuel elements². If 10,000 tons of lead were used per year for reactor shielding, this would mean construction of about 20 reactors each year, which seems an excessively high rate of consumption to be maintained, especially since current estimates of nuclear plants for electric power are for a slow rate of construction over the next 10 years³. If one includes the shielding and containers, etc. needed for the "hot laboratories" as well, 10,000 tons per year by 1975 may be a reason-

¹ Steel, May 4, 1959.

² Steel, April 29, 1957.

³ U.M. Staekler, United States Atomic Energy Commission; Northern Miner, April 7, 1960.

able estimate. It should be remembered that once installed there is no need to replace the lead.

Lead will probably be used exclusively in shielding reactors on naval vessels, since space is an important consideration. The Nautilus used about 225 tons¹, the Savannah some 500 tons². As of May , 1959, 33 nuclear-powered submarines and 3 surface ships were either approved, begun, or launched by the United States. If 20 or 30 ships, including merchantmen are built per year with nuclear power plants, 10,000 to 15,000 tons of lead may be used annually.

It is assumed that nuclear-powered automobiles will not be developed in the next 15 years, and that "A-planes" will not be important in number. Also, if the general public begins constructing bomb shelters, it is probable that concrete will be used for shielding.

The estimate of lead consumption for shielding in 1975 is then 20,000 to 25,000 tons, and for isotopes and other such uses, perhaps an additional 5,000 tons.

Sound Attenuation.

Plastic and rubber sheeting containing 70 to 90 percent lead by weight has been used to stop transmittal of sound through walls, partitions, aircraft cabin walls (the DC-7 and DC-8 use this material). This use has "a potential running into the tens of thousands of tons per year"³. It is, however, impossible to evaluate possible future consumption at this early date.

Alloys.

The LIA is expending considerable effort to develop new alloys of lead which have greater strength and creep resistance.

¹ D.M. Borcina, LIA, personal communication.

² Steel, May 4, 1959

³ R.L. Ziegfeld, LIA, letter to the author, dated June 8, 1960.

Successful results could open up many new fields of use, but again it is impossible to estimate how much new consumption could result.

Plastic Stabilizers.

A variety of lead compounds are used to stabilize vinyl plastics. They are economical, efficient, and in some cases the only suitable stabilizers. No statistics are available on the amount used, but R.L. Ziegfeld estimates it at around 5,000 or 6,000 tons annually.

From 1945 to 1955 the annual consumption of vinyl plastics increased four-fold¹. It could easily double again by 1975, and a market for 10,000 to 15,000 tons of lead annually would exist.

Other New Uses.

Many lead compounds are coming into use in ceramics for the electronics industry, but the tonnage involved is limited. Use in enamelling is discussed above (see "Ceramics").

Antivibration pads for building foundations, a lead-epoxy resin "plastic lead", leaded lubricants for heavy machinery, and new lead coatings are all recent developments, none of which are expected to require large tonnages of lead.

¹ Dutch Boy Handbook on Stabilizers for Vinyl Resins:
National Lead Co., 1956.

MISCELLANEOUS USES

The following uses all together account for only about 2% of current consumption. Detailed forecasts were not made, but the uses were considered from the point of view of potential, and most have limited growth possibilities. The list below gives the classification, 1959 consumption (LIA data), and estimated 1975 consumption. See also chart 12a.

Classification	Consumption (Tons)	
	1959	Estimated 1975
Insecticides	4325	4000-5000
Foil	4600 (ABMS 1958)	5500-6000
Steel and Wire	5095	9500-11000
Collapsible Tubes	1115	1000-2000
Rubber and Hose	2320	2000
Coatings	1670	1000-2000
Brass Manufacturing ¹	4085	3000-4000
Lead Headed Nails	1740	2000
Seals	305	500
Abrasives	<u>330</u>	<u>200-400</u>
Total	25,585	28,200-34,900

¹ There is a wide discrepancy between LIA and USBM data for Brass Manufacturing, for which no explanation could be given. LIA data were used.

UNCLASSIFIED

In 1959 unclassified consumption on the basis of LIA accounting amounted to 14.2% of total consumption, or 116,925 tons. In the past it has averaged 15.2%, varying from 15.3% in 1957 to 9.6% in 1949. It was found impossible to relate the amount of unclassified use to any other variable, as might be expected, nor would it be correct to fit a trend to the pattern, since there is no logical reason to do so, and also since ABMS data was used for some classifications, giving a different trend from LIA data, so that the percentage will change as the trends are projected.

Taking the average, 15.2%, of total consumption, the unclassified category is expected to be about 192,964 tons in 1975.

UNDERSTATEMENT OF CONSUMPTION

The LIA data totals show a discrepancy from USBM data totals, and the LIA, because it does not cover all lead consumers, adds the difference as an "Understatement of Consumption". The classifications of "Batteries", "Ammunition", "Brass Manufacturing", and "Cable Covering" account for most of the discrepancy each year. For the first two, ABMS data, across which the understatement has already been distributed, has been used, while cable covering is expected to be such a small use that the understatement should also have become very small. In checking the forecast in the future, it should therefore be checked against LIA data, but the total should be less than the adjusted LIA total by the difference between the USBM and LIA accountings for brass manufacturing.

TOTAL CONSUMPTION FORECAST ON AN END-USE BASIS

The total forecast of domestic lead consumption for 1975 can now be assembled from the forecasts of the various demand sectors, as listed below. A "D" indicates a dissipa-

tive use, an "S" one from which considerable secondary lead will probably be recovered.

Storage Batteries	547,000 ± 100,000	Tons	S
Construction: Pipes, Traps, & Bends	16,800 ± 1,400		S
Sheet Lead	15,700 ± 1,300		S
Calking	103,500 ±		S
Oil Refining and Gasoline	311,600 ± 2,400		D
Cable Covering	2,000		S
Paint and Varnish: White Lead	7,500 ± 2,500		D
Litharge	3,200		D
Red Lead	22,400		D
Ammunition	41,000 ± 5,000		D
Colors	31,400 ± 2,500		D
Printing	22,500		S
Ceramics	50,000		D
Railroads	0		S
Automobiles	16,200 ± 1,200		D
Cans	5,700 ± 500		D
Miscellaneous Uses	31,500 ± 3,500		-
New Uses: Radiation Shielding	27,500 ± 2,500		S
Plastic Lead	1,500 ± 500		D
Plastic Stabilizers	12,500 ± 2,500		D
Others	?		-
Unclassified(15.2% of total of above)	<u>192,964 ± 19,122</u>		-
Total Consumption Forecast	1,462,464 ± 144,922	Tons	
or about	1,460,000 ± 145,000	Tons	
or	1,315,000 to 1,605,000	Tons	

Comparing this forecast to the result of total consumption forecasting, 1,475,000 ± 125,000 tons, the agreement is surprisingly good, in fact excellent, and both forecasts were derived entirely independently, with no counter-checking, and no addition of the various demand sector forecasts until all were completed. Neither was any final adjustment made.

This agreement is taken to indicate that the projection of overall demand is, in this case, a more efficient method than end-use analysis, and gives acceptable results. Rosenzweig (1957) found a similar agreement in using both methods for forecasting aluminum demand to 1965. Adams (1951) likewise found that mechanical forecasts for employment worked better than forecasts based on models. It must be remembered that these are only three investigations, and that there are many cases where such methods have failed.

The case for electric automobiles is one which deserves special mention, because the potential lead demand is so great. Should these autos actually become popular, the forecast must certainly be revised, at which time both battery consumption and tetraethyl use will be reevaluated. Since sufficient information is already available, the approximate effect on tetraethyl use has already been calculated. If, under the gross assumptions made about battery consumption, the forecast were altered, it would have to be increased by some 1.5 million tons!

SECONDARY LEAD SUPPLY

There is in the United States a vast pool of metal in use. In 1959, the USBM estimated this for lead at 5 million tons (Bulletin 585). From this pool, each year more lead is returned for reuse than is mined domestically, (see chart 43). Battery lead usually accounts for 65-70% of this, and recovery from cable coverings, babbitt (bearing metals) and type metal accounted for another 15% in 1958¹. Other scrap processed is not clearly related to any particular industry (e.g. solder, soft lead), and the above sources consistently supply about 80% of total lead recovered, so that they can be used to represent the total.

The greatest problems in forecasting secondary metal supply are those of estimating how much lead is potentially recoverable and how long it takes for the lead in service to be returned and recovered. Estimates used here are as follows.

¹ Minerals Yearbook, 1958

Scrap Source	(Merrill, 1950)	Recycling Time ¹
	Recoverability Factor	
Storage batteries	85%	2-3 years
Cable covering	90	25 (minimum)
Building	15	30
Bearing metal	60	8-10
Type metal	90	3-5
Other	32	?

Thus, by estimating the amount of metal to be put into use for each of the sectors noted, the given number of years previous to the forecast data, the 1975 supply of secondary lead can be approximated. There is an unknown factor in the amount of the supply which will actually be used, the balance being added to the supply of metal in use, but it is not possible to forecast this factor, and it is assumed that the entire estimated supply will be consumed.

On this basis, secondary lead supply in 1975 is expected to be as follows.

Storage batteries	432,500 tons (not including antimony)
Cable covering	123,000
Type metal	24,000
Babbitt metal	9,500
Building	15,500

Assuming that the first four uses will supply 81.4% of total, (the average of the last 19 years), secondary lead should supply about 743,000 tons of the projected 1,460,000 tons of lead demanded from all sources in 1975, or about 50%. Currently, some 34% is secondary lead, and the large increase in recovery from cable covering accounts for most of the additional 16% expected.

THE MARKET FOR NEW METAL

It is impossible to give a concrete forecast of how the market for about 717,000 tons of new metal expected for 1975 will be divided between domestic and foreign suppliers. At present the former account for half as much as the latter,

¹ R.H.Mote, USBM, letter to Prof. R.D.Parks, May 26, 1950.

and if this relationship persists, projected domestic production will be only about 177,000 tons by 1975. It is confidently expected however that continuing industrialization in the exporting countries and increased competition from other consumer markets will decrease the amount of foreign lead available to the United States, and that either subsidies will be offered to encourage domestic production, or a natural price increase as demand increases relative to supply will allow the domestic industry to at least persist at close to its current production level, if not increase it. The apparent shortage of reserves is likewise expected to persist for many years, eventually becoming an actual shortage which will cause domestic production to decline.

These comments are of course tentative and preliminary, and suggest the need for an unbiased study in depth of future world trade in mineral raw materials.

T A B L E O F C O N T E N T S

INTRODUCTION

Outline

PART A - THE LOGISTIC CURVE

GENERAL DISCUSSION

The Nature of Mineral Production Statistics

Long-Term Forecasting

Explicit Models

Implicit Models

The Present Study

SELECTING THE MATHEMATICAL CURVE TYPE

Logistic Types, and the Results to be Expected
from Them

The First Order Logistic

The Second Order Logistic

The Third Order Logistic

THE CHOICE OF LOGISTIC CURVES

Factors Promoting the Growth of an Industry

Factors Retarding the Growth of an Industry

FITTING THE LOGISTIC CURVES

Three-Point Method

Pearl's Method

Discussion

Improving the Initial Estimate by Least Squares

EXPERIMENTATION WITH THE SKEW LOGISTIC

Zinc-Skew Logistic

Resources

Curve Results

Copper-Skew Logistic

Resources

Curve Results

T A B L E O F C O N T E N T S (Cont'd)

Lead-Skew Logistic

Resources

Curve Results

The Definition of the the Upper Asymptote, K.

SUMMARY OF EXPERIMENTAL RESULTS WITH THE SKEW LOGISTIC
CONCLUSIONS

PART B - DEMAND FORECAST FOR LEAD

INTRODUCTION

GENERAL CONSIDERATIONS

Assumptions - General

Population

Inventories

Trend Lines

Price

Substitution

SOME COMMENTS ON THE QUALITY OF AVAILABLE LEAD

CONSUMPTION STATISTICS

Consumption Statistics Available

Errors in LIA Statistics

Errors in ABMS Data

Errors of Generalization

Distortions Due to Inventories

Distortions Due to Exports

CORRELATION METHODS IN LONG-TERM FORECASTING

TOTAL LEAD CONSUMPTION FORECAST

Lead Consumption and the FRB Index

Per Capita Consumption

Total Consumption

Summary

CONSUMPTION FORECAST ON AN INDUSTRY DEMAND BASIS

Storage Batteries (A General Example)

Construction

Oil Refining and Gasoline

Cable Covering

T A B L E O F C O N T E N T S (Cont'd)

Paint and Varnish

Ammunition

Colors

Printing

Ceramics

Railroads

Automobile Manufacturing

Cans

Miscellaneous Uses

New Uses for Lead

Unclassified

Understatement of Consumption

TOTAL CONSUMPTION FORECAST ON AN END-USE BASIS

DISCUSSION OF THE END-USE ANALYSIS

THE USEFULNESS OF END-USE ANALYSIS FOR FORECASTING

FORECASTS OF LEAD CONSUMPTION FROM OTHER SOURCES

STATISTICS OF SECONDARY LEAD CONSUMPTION

General Considerations

Problems

An Example

Estimate of Secondary Metal Recovery in 1975

The Market for New Metal

STATISTICAL APPENDIX

IBM 704 PROGRAMS

BIBLIOGRAPHY

CHARTS

1. Typical Logistic Curve (Symmetrical)
- 1a. Typical Logistic Curve (Symmetrical-logarithmic)
2. Cumulative United States Production of Zinc (With Logistics for K=30, 40, and 50 Million Tons)
3. Annual United States Production of Zinc (With Logistics for K=30, 40, and 50 Million Tons)
4. Annual United States Production of Zinc (With Logistics for K=40 Million Tons)(Perturbed Data)
5. Annual United States Production of Zinc (Early Data Only)
6. Cumulative United States Production of Copper (With Logistics for K=50,72,72, and 160 Million Tons)
7. Logistic Curves for Annual United States Copper Production (K=50,72,72, and 160 Million Tons)
8. Cumulative United States Production of Lead (With Logistics for K=30, 33, and 40 Million Tons)
9. Annual United States Consumption of Lead (With Logistics for K=30, 33, and 40 Million Tons)
10. Symmetrical Logistic Fitted to Annual United States Lead Production.
11. Annual United States Lead Consumption (ABMS Data)
12. Annual United States Per Capita Consumption of Lead by Industries (LIA Data)
- 12a. Annual United States Consumption of Lead-Minor Uses (LIA Data)
13. Scatter Diagram -Lead Consumption and Deflated Price
14. Scatter Diagram -Lead Consumption and the FRB Index
15. Lead Consumption and the FRB Index
16. FRB Index and Per Capita Solder Use of Lead as Percent Deviation from Trend and in Standard Deviations
17. Scatter Diagram -Motor Vehicle Registrations and Battery Lead
18. Lead Consumption / FRB Index Ratio
19. Annual United States Consumption of Lead Per Capita (Trends 1-5)

20. Annual United States Lead Consumption (Logarithmic Trend)
21. Annual United States Consumption (Trends 1, 2 and 3)
22. Consumption of Lead in Batteries
23. Pounds of Battery Lead Used Per Capita
24. SLI Batteries Shipped Per Auto Registered
25. Per Capita Consumption of Lead in Building
26. Consumption of Lead for Calking
27. Consumption of Lead in Tetraethyl
28. Cable Covering and Communications
29. Sheathed Cable Production for the Bell System
30. Consumption of Lead in Paints and Varnish
31. Per Capita Consumption of Lead in Ammunition
32. Per Capita Consumption of Lead in Colors
33. Per Capita Consumption of Lead in Typemetal
34. Consumption of Lead in Ceramics
35. Consumption of Lead by Railways
36. Pounds of Lead Consumed Per Million Ton-Miles
37. Millions of Ton-Miles of Railway Freight Carried
38. Ton-Miles of Rail Freight Carried Per Capita
39. Consumption of Lead by Railways, and Trend
40. Freight Carried Per Car
41. Consumption of Lead in Auto Production
42. Shipments of Metal Cans in Terms of Short Tons of Steel
43. The Division of Lead Supply
44. Trends of Secondary Lead Recovery
45. Battery Lead Consumption and Recovery

TABLES

Page

1. Trends Fitted to Lead Consumption-FRB Index Ratio, and Forecasts.
2. Trend Lines of Per Capita Consumption of Lead.
3. Trend Lines Fitted to Lead Consumption in Batteries, and Forecasts.
4. Lead Use in SLI Batteries in tons.
5. United States Motor Vehicle Mileage Driven
6. Relation of Lead-Sheathed Cable to Total Cable Produced by Western Electric Company.
7. Trend of Lead Consumption for Ammunition.
8. Population Gross National Product, and Freight Traffic in the United States.
9. Conversion of Railway Freight Cars from Solid Bearings to Roller Bearings.
10. Steel Consumed in Metal Cans, 1958.
11. Unclassified Consumption as a Percentage of Total Classified Consumption.

P A R T A

THE LOGISTIC CURVE

INTRODUCTION

Nothing can be more important to the mineral industry than a guide to the future position of mineral commodities in the industrial world. On that position rest many decisions on important topics, from tariff and trade problems to the questions of exploration technology, so that as in most other phases of business, experts have attempted to predict the future demand, supply, price, technology, etc. of the mineral commodities. It is not the aim of this work to review the predictions of others and their apparent degree of success or failure, but to illustrate, by producing yet another forecast, some of the methods that can be used and the weaknesses inherent in them. These weaknesses notwithstanding, this work was done to produce a concrete forecast as its prime goal, and to evaluate the methodology which one might encounter in forecasts of mineral futures.

* * *

For the domestic producer of a mineral commodity perhaps the most important forecast would appear to be for this year's or next year's demand for the mineral or metal and the price that will be obtained, but intelligent management realizes that it is just as important to know the long-term prospects as well, and long-term planning helps a company to understand and to weather short-term economic disturbances without disruption of its operations.

Furthermore, in mining, a long foreseeable future of production is required to justify the high cost of preproduction work and of replacing large treatment units, and this necessitates not only sufficient ore reserves but a market for the product. The producer knows that in most cases his product competes on an open market and that if demand is falling, the high-cost producers can be expected to be eliminated. On the other hand, a sharply increasing demand ought to encourage exploration for and development of new ore deposits, as well as increased production from established

mines. Demand is the key to mineral industry futures. To forecast prices or production without forecasting demand does not seem logical, (but given special assumptions, it can, of course, be done). What confuses the domestic production picture is that demand for many mineral commodities is not satisfied from one supply source. Mineral trade is strongly international to begin with, and the supply of metals in use is a vast reserve from which large amounts of scrap are returned every year. Since it is of most interest to the domestic producer to know the prospects for domestic mine production of the commodity in question, much work has been directed toward forecasting this directly, especially since this is the only rapid method of forecasting the future position of the domestic mining industry. On the basis of work described below, the author has concluded that there is no short-cut way to do this and that total domestic demand must first be evaluated. The question of the percentages of the demand which will be satisfied by domestic production, imports and scrap is a subject for a whole series of studies, and is mentioned only briefly in what follows.

Outline

It was decided that for the purposes of the mining industry, such as consideration of plant expenditures, property valuations, etc. a forecast for 15 years hence would be most useful, and this period was further enhanced by the fact that 1975 is also the target date of the widely-known Paley Report, hence of many other critical comments and reports, which in all give some basis for comparison of results. Using a forecast period of 15 years precludes any attempt to forecast the phase of the business cycle, and thereby allows some generalization in the result, which must be visualized as an attempt to predict only the level of the secular trend, about which the business cycle fluctuates.

Part A of this work is an evaluation of an implicit model, the logistic curve, assumed to represent mineral production from a given limited area over time, as a technique

for forecasting domestic mine production, using the base metals as examples. The method has been used by Lasky (1951, 1955) and Hubbert (1958) for mineral production forecasting, and by Kuznets (1933), Prescott, (1922) and many others for other industrial series.

Part B concedes the difficulties of forecasting domestic production without first deriving a prediction of demand, and is an exposition of some techniques in common use among forecasters, which were tested by the author in an attempt to forecast the domestic demand for lead from all sources in 1975. Briefly, these include 1) forecasting of demand in toto by : trend projection; on a per capita basis; ratios with other series, and 2) the division of lead demand into sectors with forecasts of each sector by the above methods combined with technical knowledge of the sector in question. The usefulness of simple and multiple correlation techniques was also investigated. Results obtained by the several methods have been compared among themselves as well as with several other forecasts noted in the literature.

PART A

If one can understand the factors which are important in controlling a reaction, and their relationship to one another, he will be able to predict the future course of that reaction, given the values of the important factors. This fact applies no less in economics, and here too it is an important test of a theory that it be able to forecast in greater or less detail the future behavior of the reaction which it seeks to explain. Thus by a causal analysis of a given sector of the economy, the economist will seek to determine the most important variables, i.e. those which will explain the greatest part of the behavior of the system, and the relation between them. If he then proceeds to combine these variables and an estimate of their future worth, he is said to be attempting to forecast by an explicit method. The ultimate refinement of this approach is the combination of the factors involved, using the methods of econometrics, into a series of interrelated equations which, supplied with suitable values will generate a satisfactory facsimile of the observed phenomenon. Such a system of equations is referred to as a "model".

At the other extreme of economic forecasting is a method of estimating the future commonly referred to as a "seat of the pants" forecast, in which the individual consciously or unconsciously makes a prediction of the future based on his past experience and his educated guess as to what changes may be expected in the important variables in the future.

Somewhere between these two extremes lies a method commonly known as a "naive" method or an implicit technique, though it is generally less so than the "seat of the pants" method. One such method is the practice of determining some simple expression which seems to describe the overall pattern of the past performance of the phenomenon, and which has implicitly within it assumptions typical of this past performance. Because of its simplicity the function can then be

extrapolated into the future, to yield a forecast. The function can be termed an "implicit model".

GENERAL DISCUSSION

The Nature of Mineral Production Statistics

Mineral production statistics constitute a time series, that is the values are distributed with respect to time rather than frequency, as in the case of a sample. Time series are often met within many sciences, for example in the study of electrical phenomena, but these are generally stationary time series, which means that the generative forces are constant, at least over the period of observation. Economic time series are dynamic; the factors generating the series are constantly changing with time, and it is this dynamic feature which makes economic data so difficult of interpretation, especially in the investigation of their cyclical behavior, for although harmonic analysis has been well established for stationary time series its application to economic series has met with dubious success.

However, some factors of an economic time series may be constant over long periods. This is often true of cyclical seasonal movements.

Classically, the economic make-up of a series of data such as annual mineral production has long been considered to consist of at least the following elements:

- 1) secular or long-term trend
- 2) cyclical movements
- 3) periodic movements
- 4) irregular movements

Two other major components are commonly recognized:

- 5) "long cycles" (Kondratieff, 1935)
- 6) secondary trends (Kuznets, 1930)

Any one of these components can be studied as an entity, or the series may be examined as a whole. The success with which any one component can be isolated from the others is, however, a source of considerable debate among economists,

and the difficulties of successful analysis of the business cycle are well known.

Long-Term Forecasting

If the long-term forecast is defined as one concerned with events more than five years hence, there is little hope of predicting the phase of the business cycle in which the economy will be found at the target date. Indeed short-term forecasting of the cycle is often unsuccessful. The aim of most long-term forecasting is a prognosis of the position of the secular trend of business, on which the business cycle will be superimposed.

There has not been in the past as much interest in long-term as in short-term forecasting, for several reasons. One of course is the possibility of rapid accumulation of profits should the forecaster successfully predict market activity consistently. Another is that many business activities are concerned chiefly with the short-term outlook, such as marketing and often annual budgeting. An industry which produces a non-durable good and in which machinery obsolescence comes rapidly is interested mainly in the short-term.

But in addition to these short-term considerations industry wishes to know what absolute growth it may expect to attain within say ten to twenty years, with an outlook toward expansion or contraction of productive facilities, long-term financing, diversification into more attractive fields, investments, and so on. Many methods have been tried in the search for a dependable forecast of secular trend, including both explicit and implicit models, and each method has at least some justification. Of course the explicit model will attempt to explain each of the major factors causing the growth phenomenon and to forecast its future as the sum of its parts. On the other hand, the proponent of the implicit model will argue that it will always be impossible to evaluate many of the qualitative factors which

influence the business level, and in fact even to know how to describe them is difficult, so that the best description of the overall economy will be one given by an implicit model, designed to include all the qualitative factors. Furthermore, an implicit model will usually be expected to give not only a forecast for some target date, but as well the intervening path of the trend, a decided advantage over most explicit forecasts.

Explicit Models

An explicit expression for each major component of the time series' behavior must be determined in the construction of this type of model. The expression may be non-quantitative in some cases (e.g. "Wars will disrupt the entire model") but will generally be capable of expression in one or more equations. A differential equation of the change of the component with respect to time might be one result. Often the expression will consist of an algebraic equation relating the component to some other factor, usually causative, and sometimes containing time as a second factor. For example, the number of tons of wheat harvested may be found to be equal to some constant (a weighting factor) times the accumulated annual rainfall, or a child's rate of growth may be a function of average number of calories of food consumed per day, and his age (the time factor).

Briefly, these relationships are usually determined by first postulating a series of reasonable relationships between cause and effect, and then investigating the nature of the relationships, usually by correlation methods. Of course spurious or "nonsense" correlations sometimes occur, and good judgement must temper the results, but even more insidious are correlations of doubtful value, such as that found between coronary attacks and cigarette smoking. The explanation may simply be that tense people tend toward both these things or it may actually be a truly causative relationship. Such results tend to cloud the issue, for they may or may not be valid for prediction.

When a series of relationships has been established and a preliminary set of weights assigned, the model, consisting of a set of related equations equal in number to the number of variables being considered, is tested as an entity, and any necessary adjustments of weights are made.

If the model is to be used to forecast, it will be necessary to know the future values of the component factors of the model, and this is the point of difficulty in forecasting with an explicit model. Of course, high and low estimates for various factors may tend to average out, and the overall result may be quite satisfactory. If the model has sought to allow for certain non-measurable, qualitative factors by difference there is no way to forecast these influences, which may be quantitatively important.

Implicit Models

These difficulties lead to consideration of implicit models, especially in forecasting work. The most obvious difference between these two types of model is clear from the names applied, but the most important difference may be overlooked. The implicit model does not try to explain on the basis of a simple equation the dynamic forces which shape the economic time series. Instead it makes certain assumptions about the typical behavior of the data, basing them on a priori reasoning with respect to the data itself and most important with respect to the nature of the industry whose record is to be forecast. Thus, all of the factors of economic development need not enter discreetly into the trend equation, since their effect is reflected in the data at hand, but the major causative factors of the industry's performance must be accounted for when choosing the trend equation, and its performance must be in accord with their effects.

In making a long-term forecast by the use of an implicit model, the most common approach is to forecast secular trend alone, although in some cases account is taken of the business cycle, long cycles, or secondary movements.

Secular trend may be thought of as representing the normal level of industry, about which short-term effects oscillate.

The Present Study

The study in hand is an investigation of the possibilities of forecasting United States mineral production on a long-term basis. As a number of efforts have been made in this direction in recent years, e.g. by S.G. Lasky (1951, 1955), the Paley Commission, (1952), the Twentieth Century Fund, it appears that there is considerable interest in such a forecast.

The first phase of the current study is an examination of the value of a forecast from an implicit model. It is proposed to show that the best expression for the trend of mineral production is the logistic or Pearl-Reed curve, and to evaluate the forecasting powers of the curve as an implicit model of secular trend.

The second phase will deal with more explicit models and the analysis of the major dynamic economic factors in the industry.

SELECTING THE MATHEMATICAL CURVE TYPE

There is no "ideal" trend type which will be satisfactory for all economic time series on an a priori basis. Nor is there any quantitative and exact means of establishing which curve should be used in a given case, although it is often obvious from the data what general type of curve, e.g. straight line, exponential, parabola, etc., is required to give a statistically good fit to the data. If the curve-fitting is to be anything more than a mechanical graduation of the data, and an actual implicit model of the system, then there must be not only the quantitative evidence of a statistically good fit, but as well the curve must exhibit those characteristics which are known to be typical of the industry in question. Thus, while there is a myriad of curves which have a "time shape" like that of mineral production series, only a few of them will resemble in their habit of growth the typical nature of mineral production history as

determined by the underlying factors of the mineral industry.

It is now to be shown what the growth characteristics of the mineral industry are.

First, it can be shown that the growth of mineral commodity production started from a zero level, and increased at a high percentage rate in the early years as more deposits of the mineral were discovered, new treatment methods evolved, new uses for the product were discovered, and the population of consumers grew. Like so many cases of growth, the early years may well have shown an exponential rate of increasing production. That is, the rate of growth was dependent simply on the increasing size of the industry. It is during this period that the number of patents applied for on new industrial techniques reaches a peak. Prescott (1922) has aptly termed this the period of experimentation.

If the commodity can survive this period it becomes a part of the social fabric, accepted and expected by the consumer, and used in increasing quantity. This influence tends to overlap the period of experimentation, and the net rate of growth is the most rapid as new markets become well established and technological advance lowers costs and increases uses. The economics of scale can be effected as production enters this stage, and with an assured market the atmosphere for investment in the industry becomes more stable.

But the rate of growth cannot increase forever, and it will at some point pass through a maximum, beyond which those factors inimical to growth begin to grow in importance as the growth factors wane. It is patently possible that more than one maximum may be attained if some major factor of growth changes, as for example the bringing-in of the porphyry coppers changed the nature of the copper industry. And it likewise appears possible that the periods of growth and decay may not be symmetrical in form.

In some industries a plateau is established at a maximum performance level, and the plateau level can in theory be expected to persist indefinitely, perhaps growing with

population to some degree. But in the mineral extractive industries there is an increasing pressure which tends to cause production decreases, and that is the pressure of decreasing reserves. It may be due to the approaching limit of absolute reserves, or of economically recoverable resources in the light of substitution, and importation from foreign sources, but it must eventually lead to the death of the extractive industry within any given area, be it continent, nation, or deposit. Thus any established production level will be followed by a period of "declining growth rate" or of decay. As old age is entered, the rate of growth of the cumulative industrial production will once again approach zero.

This sequence of history in the development of an industry based on a declining resource is typical. The multitude of factors which shape its detailed course, and cause the business cycles and seasonal changes which occur as fluctuations around this overall trend do not enter into the above discussion in any concrete way. It is the time characteristics of the overall trend of the data upon which interest centers here.

To state more specifically the nature of the trend which cumulative mineral production may be expected to follow, these characteristics may be observed:

- 1) The trend will start from a lower asymptote equal to zero, and increase at a geometric or near-geometric rate in the period when $X(\text{time})$ is small. It is thus concave upward.
- 2) The rate of growth per unit of time is to be proportional to two things: (i) the absolute size which the industry's production has attained at the beginning of the given unit of time and (ii) the amount of potential resources still unused, i.e. the resource base of the area in question. The importance of factor (ii) should increase with time. (There is the possibility of episodic growth, which may cause exceptions to this rule).

Characteristic number 2 is perhaps the most important for the curve, since many curves will exhibit some or all of

the others noted, but only the logistic includes this one, and it is the only major assumption which can be questioned. The beginning of a typical mineral's production history is marked by limited markets, difficulties of technology, capital shortages, and a limited number of operating mines. Each of these problems is reduced in magnitude as production continues for a few years so that the rate of growth tends to accelerate. For example, it is easier to increase production when there are ten producers, each with capital, manufacturing technology, experience, and operating mines, than when one mine alone must make the increase.

The effect of limited resources will be that of a damper on the increasing growth rate when further production increases become impossible for some operators due to lack of ore. The growth rate is therefore reduced by the factor of ultimate resources minus the amount already produced.

- 3) At some point in time the rate of growth will pass through a maximum and then continuously diminish. Again the possibility of epochal or cyclical growth may cause exceptions.
- 4) As X becomes large the rate of growth will approach zero, and the curve will approach an upper asymptote. If one is considering cumulative mineral production data, the upper asymptote can be seen to be representative of total ultimate recoverable resources, to which the cumulative production will eventually accrue.
- 5) The trend cannot turn back upon itself. Once a certain cumulative total has been reached the curve cannot go below that level, since this would be the equivalent of "negative production".
- 6) The first derivative of the cumulative production trend equation should yield a curve which will fit the typical form of annual production data. That is, it should be asymptotic to zero when time is very small, increase to a maximum, possibly establishing a plateau level of annual production, and then decrease to become asymptotic to zero when time is very great.

Assume an upper limit of growth equal to ultimate recoverable resources, (K), a constant greater than zero. Then when time (X) equals infinity, cumulative mineral production (y) should equal K, or $\frac{K}{1}$. When X equals negative infinity, y should equal zero, i.e. $\frac{K}{\infty}$. The denominator of the expression for growth of mineral production must therefore go to 1 when $X=\infty$, and to ∞ when $X=-\infty$. Furthermore, it must be a continuous function over this range and must be continuously decreasing as X increases (excluding for the moment the possibility of epochal growth). One function which fulfills these requirements is $1+e^{-X}$ or $1+10^{-X}$, and the expression for cumulative production can be written as

$$y = \frac{K}{1 + e^{-X}} \quad .$$

This expression approaches 0 and K asymptotically, and its first derivative has the characteristics noted above to be typical of annual mineral production data, which is of course the "rate curve" for cumulative production.

This equation has been termed the logistic, or is sometimes known as the formula for the Pearl-Reed curve, after the two men who earlier derived it on an empirical basis to fit biological growth data. Pearl felt that it demonstrated a fundamental biological law which controlled growth, since his graduation of a wide variety of growth data series was excellent.

The form first derived by Pearl and Reed in 1920 was

$$y = \frac{be^{aX}}{1 + me^{aX}} \quad , \quad \text{where X represents time.}$$

This formula was the same as that derived by Verhulst (1844) nearly seventy-five years previously. Working with this form of the curve, Pearl found it too restricted, and developed a more general formula:

$$y = \frac{b}{\frac{1}{e^{aX}} + m} = \frac{K}{1 + Ce^{Ka'X}} \quad (\text{Pearl, 1930})$$

where $K = \frac{b}{m}$, $C = \frac{1}{m}$ and $Ka' = -a$, where K is greater than zero.

Then $\frac{dy}{dX} = -a' \cdot y \cdot (K-y)$,

i.e. the rate of change of y with respect to time X varies directly as the size already attained at that time (y) and the remaining potential for growth ($K-y$). Since these factors vary with time, the term ($-a'$) may be replaced by a function of time $f(X)$, and

$$\frac{dy}{dX} = f(X) \cdot y \cdot (K-y)$$

$$\frac{dy}{y(K-y)} = f(X) dX$$

$$\text{or } \frac{K-y}{Cy} = e^{-K \int f(X) dX}$$

$$\text{so that } y = \frac{K}{1 + Ce^{-K \int f(X) dX}} = \frac{K}{1 + Ce^{f(x)}}$$

where $f(x) = -K \int f(X) dX$, and C is greater than zero.

Where growth starts not from zero but from some level above zero, Pearl advocates using an augmented formula

$$y = d + \frac{K}{1 + Ce^{f(x)}}$$

where d is the lower asymptote. There seems little merit to using this additional constant since the data can always be adjusted to the non-zero asymptote.¹

Logistic Types, and The Results to be Expected From Them

The equation for the generalized logistic may be written as: $y = \frac{K}{1 + e^{a_0 + a_1 x + \dots + a_n x^n}}$ (e^{a_0} is equivalent to C)

i.e., where the exponent of e is a polynomial in x of degree n . Generally, x is taken to represent time, the independent variable (Pearl, 1924).

If the number of variables in the equation is equal to the number of observations, the curve will pass through each of the observed points, with deviation equal to zero if the points are distributed in the form of a possible logistic. But it then fails to be the generalized expression which is

¹ See especially Wilson and Puffer (1933)

desired as a representation of the secular trend of the data. Conversely, too few degrees of freedom will not allow the curve to follow the data in even a general way, e.g. a straight line will not adequately describe a curvilinear set of data points. Some compromise is necessary. On the one hand, the equation must be restricted enough to approximate the trend of the data, on the other, it must be sufficiently free that it can approach the shape of the trend. The settlement of this question is indeed important in the case of the logistic, as will now be shown.

For convenience, the type of logistic curve will hereafter be designated on the basis of the degree of the polynomial in x . For example, the equations

$$y = \frac{K}{1 + e^{a_0 + a_1 x}} \quad \text{and} \quad y = \frac{K}{1 + e^{a_0 + a_1 x + a_2 x^2 + a_3 x^3}}$$

are the first order and third order logististics respectively in this terminology.

The First Order Logistic.

This is the simplest form of the logistic, and its formula is:

$$y = \frac{K}{1 + e^{a_0 + a_1 x}} \quad (1)$$

This is the form evolved by Pearl and Reed, and is also found in their works in the form

$$y = \frac{K}{1 + Ce^{a_1 x}} \quad (2)$$

which is of course equivalent, where $C = e^{a_0}$; hence must be positive.

The first derivative with respect to x represents the slope of the curve, or the unit rate of change of y per unit of time, and is

$$\frac{dy}{dx} = \frac{-K \cdot e^{a_0 + a_1 x} \cdot a_1}{(1 + e^{a_0 + a_1 x})^2} \quad (3)$$

Since $K = y + ye^{a_0 + a_1 x}$, $(K - y) = ye^{a_0 + a_1 x}$. (4)

Writing (3) as $\frac{dy}{dx} = \frac{K}{1 + e^{a_0 + a_1 x}} \cdot e^{a_0 + a_1 x} \cdot \frac{-a_1}{1 + e^{a_0 + a_1 x}}$

and substituting from (1) and (4)

$$\begin{aligned}\frac{dy}{dx} &= y \cdot \frac{K-y}{y} \cdot (-a_1) \cdot \frac{y}{K} \\ &= \frac{-a_1 y(K-y)}{K}\end{aligned}\quad (5)$$

Then $\frac{dy}{dx} \cdot \frac{K}{y(K-y)} = -a_1$, or $\frac{dy}{\frac{y(K-y)}{K}} = -a_1 dx$

Now since a_1 represents the derivative of the $f(x)$, in this case a first order function, it can be replaced by the derivative of the general $f(x)$, i.e. $f'(x)$, so that

$$\frac{dy}{\frac{y(K-y)}{K}} = f'(x) dx \quad \text{and} \quad \frac{dy}{dx} = \frac{y(K-y)}{K} \cdot f'(x)$$

Since y and K are both positive, the logistic thus has its maxima or minima when $f'(x)=0$, and also when $y=0$ or K .

In the first order logistic, $f'(x)=a_1$. If $a_1=0$, the formula no longer contains x and is not a function of time, but simply gives one discrete value of y for one value for the constants K and a_0 . Hence the first order logistic has a maximum or minimum only when $y=0$ or $y=K$.

It is obvious from (1) that the sign of a_1 is of great importance in the formula, and it will later be shown that in all forms of the logistic curve the sign of a_n is indicative of the general shape of the curve.

Consider first that a_1 is negative, since this is the usual case, and allow x to vary from $-\infty$ to $+\infty$.

When $x=0$, $y = \frac{K}{1+e^{a_0}}$. This is a constant term, beyond whose value y will monotonically increase for any positive value of x , since some amount ($a_1 x$) will be subtracted from a_0 , decreasing the value of $e^{a_0 + a_1 x}$ and hence increasing the value of y . When $x=\infty$, $y=K$, since $e^{-\infty}=0$. Because the slopes at $y=0$ and $y=K$ are equal to zero, the curve must be asymptotic to these two limits at $x=-\infty$ and $x=\infty$, and because x is always increasing there must be an inflection point between $x=-\infty$ and $x=\infty$. Inflection points occur where $\frac{d^2 y}{dx^2} = 0$, or fails to exist, since at such points the sign of the second derivative changes.

From the form
$$\frac{dy}{dx} = \frac{-a_1 y(K-y)}{K} \quad (5)$$

$$\begin{aligned} \frac{d^2y}{dx^2} &= -\frac{a_1}{K} \left[y(-1)\frac{dy}{dx} + (K-y)\frac{dy}{dx} \right] \\ &= -\frac{a_1}{K} \left(K \cdot \frac{dy}{dx} - 2y \cdot \frac{dy}{dx} \right) \end{aligned}$$

Having assumed a_1 negative, the sign of the second derivative will be positive so long as the expression in brackets is positive. This expression will be positive when K is greater than $2y$, since K , y , and $\frac{dy}{dx}$ are always positive, and will equal zero when K equals $2y$. Hence the inflection point occurs at $y = \frac{K}{2}$. For this value of y , $e^{a_0 + a_1 x}$ must equal 1 or $x = \frac{-a_0}{a_1}$. Since the inflection point occurs midway between $y=0$ and $y=K$, the curve is then seen to be symmetrical about the point $(\frac{-a_0}{a_1}, \frac{K}{2})$, and the first derivative curve must have a maximum at $x = \frac{-a_0}{a_1}$, where its derivative equals zero. The rate of change of y at this point is, by substitution,

$$\frac{dy}{dx} = -\frac{a_1 K}{4} .$$

One further property of the curve, and perhaps the most important one in many ways, is that the rate of growth depends upon (i) the absolute size already obtained (ii) the potential remaining for growth. In the early stages of growth, rapid advance is made and no pressure is felt from the ultimate limits of growth available, hence early growth is essentially geometric. If, in

$$\frac{dy}{dx} = \frac{-a_1 y(K-y)}{K} \quad (5)$$

we let K approach infinity, in the limit

$$\frac{dy}{dx} = -a_1 y, \text{ which is the equation for}$$

geometric increase and is dependent on the size of y obtained. Hence a_1 is the potential rate of increase, which becomes damped as the pressure of a limited value of K is exerted, as shown by $(K-y)$, the potential remaining for growth.

It should be noted that y cannot equal zero when

x equals zero, since the value of y is then given by

$$y = \frac{K}{1 + e^{a_0}} \quad \cdot \quad \text{Now if the curve is taken}$$

to be asymptotic to zero when time (x) equals zero, it is obvious that a_0 must be equal to infinity. While this does not violate the nature of the curve, it makes the curve incalculable for finite values of x. What is generally done is to shift the origin of time. For example, if it were shifted to the inflection point, where $x = -\frac{a_0}{a_1}$, then a_0 becomes equal to zero and the equation is

$$y = \frac{K}{1 + e^{a_1 x}}$$

Unless time is then counted as both positive and negative this means that all data before the origin are ignored when the equation of the curve is calculated.

When $x = -\infty$, y is seen to be equal to

$$\frac{K}{1 + e^{a_0 + a_1(-\infty)}} = \frac{K}{1 + e^{-\infty}} = 0 \quad , \quad \text{this regardless of the}$$

value of a_0 , but still taking a_1 negative.

In practice it is impossible to choose the exact mid-point of the curve until after it has been fitted. Hence some convenient point, other than $y = 0$, is usually chosen as origin, and negative values of x are used. The possible range of x is then $-\infty \leq x \leq +\infty$.

It was stated above that the sign of a_n is important in determining the shape of the logistic. Consider now a_1 positive, $-\infty \leq x \leq +\infty$.

When $x \doteq -\infty$,

$$f(x) \doteq -\infty$$

$$e^{f(x)} \doteq 0$$

and $y \doteq K$

When $x \doteq +\infty$,

$$f(x) \doteq +\infty$$

$$e^{f(x)} \doteq +\infty$$

and $y \doteq 0$

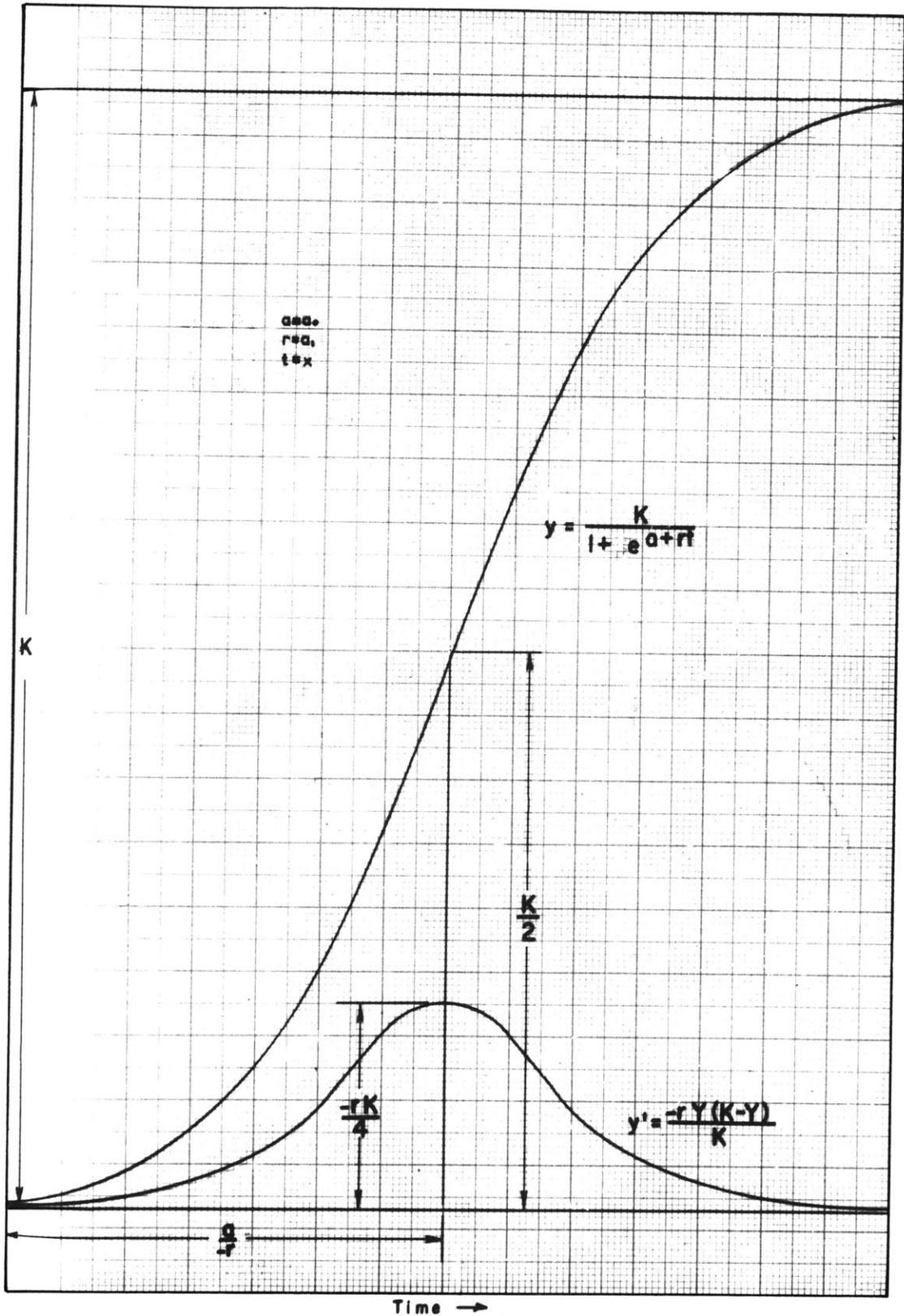


CHART I. Typical Logistic Curve. (Symmetrical)

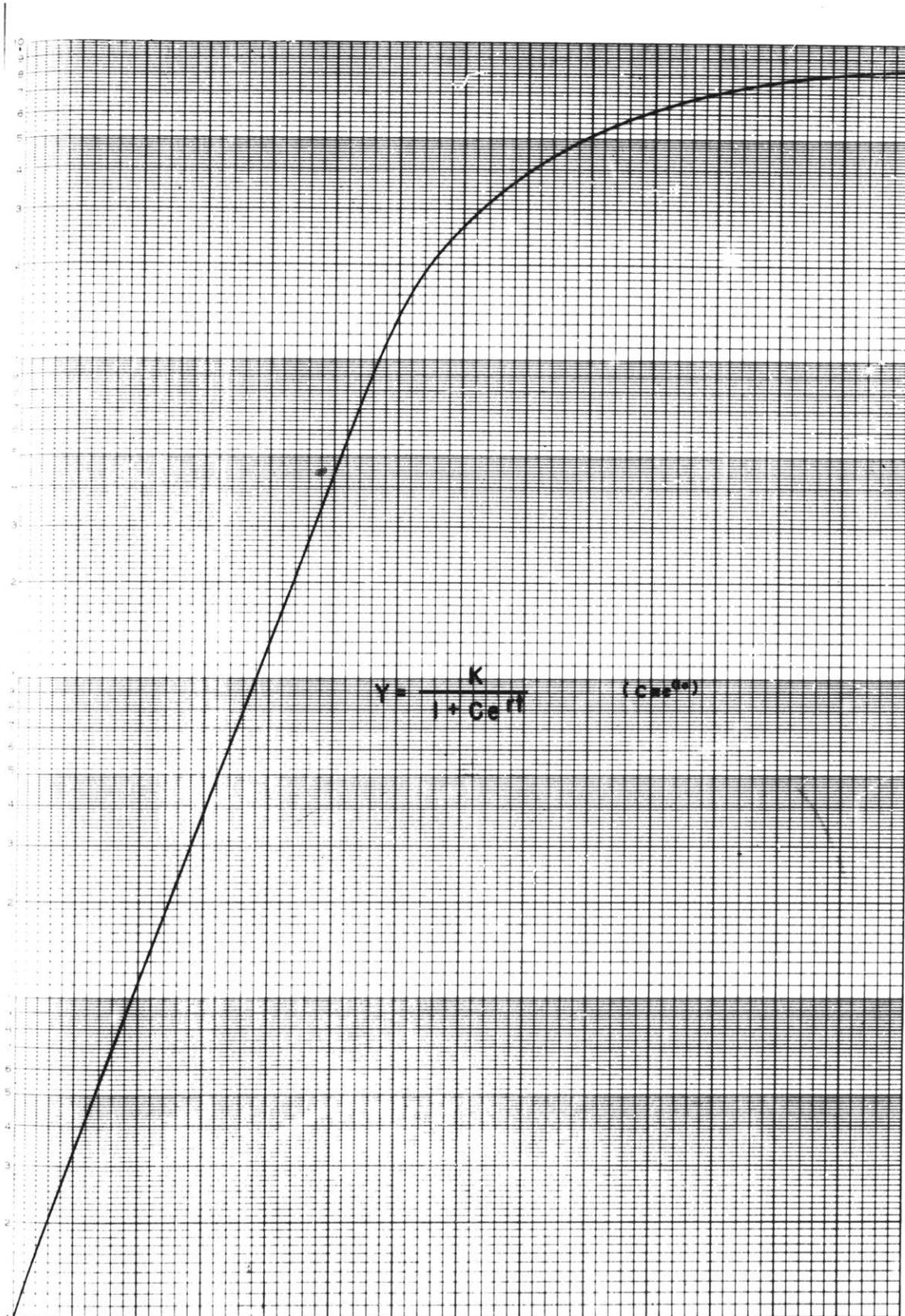


CHART 1a. Typical Logistic Growth Curve.

Hence when a_1 is positive the curve is one of decay and not growth, and always has a negative (or zero) slope.

Chart 1, after Pearl, summarizes the features of the logistic curve of the first order with a_1 negative. The curve for a_1 positive is the mirror image of this across the line $x=\infty$. Chart 1a shows the shape of the curve on semi-logarithmic paper.

The Second Order Logistic.

This form is the simplest of the second major class of logistics, in which n is even. It differs from the case where n is odd, in that the value of the n^{th} term in the exponential (and of all even powers of x) will be the same for equal positive and negative values of x . The curve is thus asymptotic to zero or K at $x = \pm\infty$, since the term of highest order will always dominate for large values of x . In the equation

$$y = \frac{K}{1 + e^{a_0 + a_1x + a_2x^2}} \quad (1)$$

when a_2 is negative, as

$$\begin{aligned} x &\doteq \pm\infty, \\ f(x) &\doteq -\infty \\ e^{f(x)} &\doteq 0 \end{aligned}$$

and $y \doteq K$,

hence the curve is asymptotic to K at $x = \pm\infty$.

when a_2 is positive, as

$$\begin{aligned} x &\doteq \pm\infty, \\ f(x) &\doteq +\infty \\ e^{f(x)} &\doteq +\infty \end{aligned}$$

and $y \doteq 0$,

hence the curve is asymptotic to 0 at $x = \pm\infty$.

The first derivative of the second order curve is given by: $\frac{dy}{dx} = \frac{K}{y(K-y)} \cdot f'(x)$, as shown in the discussion of the first order curve. Any maxima or minima of the logistic curve will thus occur where $f'(x)=0$, (or $y=0$ or K).

In the second order curve

$$f'(x) = a_1 + 2a_2x .$$

This will be equal to zero only when $a_1 = -2a_2x$, and since for any given logistic a_1 and a_2 are fixed in value, there can be only one maximum or minimum between the asymptotes.

This will occur at $x = -\frac{a_1}{2a_2}$. If an increment in x is defined as z , for the curve to be symmetrical,

$$y\left(-\frac{a_1}{2a_2} + z\right) \text{ should be equal to } y\left(-\frac{a_1}{2a_2} - z\right)$$

or, in the logistic formula

$$a_0 + a_1\left(-\frac{a_1}{2a_2} + z\right) + a_2\left(-\frac{a_1}{2a_2} + z\right)^2 = a_0 + a_1\left(-\frac{a_1}{2a_2} - z\right) + a_2\left(-\frac{a_1}{2a_2} - z\right)^2$$

$$\text{or } a_1z + a_2\left(\frac{a_1^2}{4a_2^2} - \frac{a_1}{a_2}z + z^2\right) = -a_1z + a_2\left(\frac{a_1^2}{4a_2^2} + \frac{a_1}{a_2}z + z^2\right)$$

$$\text{or } a_1z - a_1z = -a_1z + a_1z, \text{ q.e.d.}$$

The second order logistic thus describes a single symmetrical cycle, including the periods of growth and decay, in which the phenomenon reaches a maximum size and then declines. It will be found to be bell-shaped, symmetrical, and asymptotic to $y=0$ or K at its extremes. However, if the coefficient a_2 is small with respect to the other coefficients, there tends to be a flat peak rather than a bell-shape, although the curve is still symmetrical.

The second order logistic might be used therefore to represent annual mineral production.

The Third Order Logistic.

Although belonging to the same major class of odd-ordered logistics as the first curve discussed, the third order curve is included in this discussion because of the importance it assumes in the discussion to follow, and also to illustrate the properties of higher order odd curves, otherwise termed skew logistics.

The curve is represented by the equation

$$y = \frac{K}{1 + e^{a_0 + a_1x + a_2x^2 + a_3x^3}} .$$

It will display the same asymptotic qualities as the first order curve, under the various assumptions as to the sign of a_3 and the range of x . The interest in this form of the equation lies in the fact that the inflection point need no longer be at $\frac{K}{2}$, and that the curve is no longer symmetrical.

The maxima and minima of the logistic were shown to occur where $y=0$, $y=K$ or $f'(x)=0$. For the third order logistic

$$f'(x) = a_1 + 2a_2x + 3a_3x^2 .$$

The values of x for which this quadratic expression equals zero are given by

$$x = \frac{-2a_2 \pm \sqrt{4a_2^2 - 12a_1a_3}}{6a_3} .$$

When $(4a_2^2)$ is less than $(12a_1a_3)$, the roots are imaginary. Since the values of the curve which are of interest are all real, this case is the one in which the curve has no maximum or minimum in the range of real numbers.

When $(4a_2^2)$ is equal to $(12a_1a_3)$ the roots are real and equal and the curve will have one maximum or minimum. It then describes one cycle of growth, but is not necessarily symmetrical.

When $(4a_2^2)$ is greater than $(12a_1a_3)$ the roots are real and unequal and the curve has two maxima or minima, describing cyclical or epochal growth.

The points of inflection of the curve will occur when the second derivative of the curve equals zero. By differentiating the first derivative in the form

$$\frac{dy}{dx} = \frac{f'(x) \cdot y \cdot (K-y)}{K} , \quad \text{the second}$$

derivative is found to be:

$$\frac{d^2y}{dx^2} = \left[f''(x) + \{f'(x)^2\} \left(\frac{K-2y}{K} \right) \right] y \left(\frac{K-y}{K} \right) .$$

Setting this equal to zero, making substitutions in the result, and transposing terms, the values of x for which the second derivative equals zero are found from

$$y = \frac{K}{2} - \frac{K}{2} \cdot \frac{f''}{(f')^2}$$

For the third order logistic

$$f' = a_1 + 2a_2x + 3a_3x^2$$

$$f'' = 2a_2 + 6a_3x.$$

For a_n greater than 0, when $x = -\infty$, $y=0$. For a_n less than 0, when $x = +\infty$, $y=0$, when $x = -\infty$, $y=K$. Thus for fitting cumulative production data, a_n must be less than 0.

THE CHOICE OF LOGISTIC CURVES

All of the forms of the logistic curve described above have at some time or other been fitted to economic data. But the first and second order curves have been shown to give symmetrical curves whose inflection point, (or maximum in the second order case) is the point of symmetry. In applying these curves to economic time series, it is then necessary to accept the theory that the pattern of decay of the series will be exactly symmetrical with its growth phase. There appears to be no logical basis for this assumption. Consider the major growth and decay factors which this implicit model seeks to explain.

Factors Promoting the Growth of an Industry.

The major part of industrial growth is due to a few factors, which can be classified as demand, ultimate resources, technology, and political climate, as suggested by Lasky (1951). An examination of each of these groups with reference to the mineral industry shows that each consists of many factors. For example, demand is a product of at least the following factors:

1. Population size and quality.
2. Spendable Income -a measure of purchasing power.
3. Gross National Product, a measure of national well-being.
4. Net imports.
5. Relative real costs and prices compared to those of possible substitutes.
6. Stockpiling by government agencies.

Ultimate resources depend on:

1. The natural endowment of the area in question.

2. The concentration and availability of the deposits.
3. Ability to find these deposits.
4. The use of conservation in mining.
5. Technological progress in the latter two fields.
6. The economics of the industry, i.e. marginal cost of production v.s. selling price (see for example Davis, 1958).

The technology of an industry advances at an irregular rate. The well-known business cycle theory of Schumpeter is based on the fact that innovations come in waves. Kuznets (1930) has shown that the first few innovations in a new industry lead to a series of related innovations, which in turn open up new areas of interest and lead to further progress. But as time progresses there is less and less room for improvement in the industry and the rate of technological advance slows down. It may not always keep pace with demand, but often new technology will create new uses for a material and hence greater demand for it. Also as noted above, the technology of ore search, mining and milling has an important effect on the growth of the mineral industry.

The political climate is of positive or negative value in fostering growth of an industry, and international politics as well as regional are involved, as for example in the aircraft industry and the related aluminum industry, and in the uranium industry.

One further classification should be added to the four outlined above, and that is the question of capital available for investment. Demand for a product will make the industry involved more attractive to capital, but if the overall money situation is tight or there are other attractive investments in sight, growth may be retarded by lack of funds.

Factors Retarding the Growth of an Industry.

It is impossible to clearly separate acceleratory and deceleratory forces in the growth cycle of an industry, for with time an encouraging factor may become a discouraging one, as for example ore reserves or the rate of technical progress. But some factors are typical only of a slackening

of growth. These include retardation of a fast-growing industry by slower-growing complementary ones, the pressure of low-priced imports, and the decrease in the relative volume of available capital. To these must be added the effects of declines in the factors which promote growth.

All the factors of retardation mentioned are quite clear, except perhaps that of decrease in the relative amount of available capital. As an industry grows large and stable, profits of the size available to the original entrepreneur are no longer to be realized, and it becomes more difficult to secure capital. Furthermore, the large size of the activity means that a capital investment would have to be proportionately larger to be as significant as a smaller investment in the pioneer days of the venture.

To return now to the question at hand, can it be assumed that the growth cycle of industry, notably the mineral industry, is symmetrical? Clearly there is no apparent reason why the factors discussed above should make it so, and it would be surprising if they did achieve the degree of symmetry assumed in fitting the symmetrical logistics. (The existence of cyclical and irregular movements along the growth trend is something quite different from the present problem, and should not be confused with it). The possibility of skew growth is decidedly high over the entire growth cycle, so to fit a symmetrical curve to the data is to ignore this strong possibility. Thus there is no apparent justification for using the first order logistic to describe cumulative mineral production or the second order to describe annual production, and the simplest form of any interest is the third order, the skew curve.

This is not to say that the choice of the skew logistic curve as the best curve to represent secular trend will explain the causes of economic growth and decay. It simply results from a series of assumptions made about the behavior of the data on the basis of a consideration of the major events of growth and decay. The form of the equation illus-

trates the assumptions made, but it says nothing about the causes of the growth and decay of the series.

FITTING THE LOGISTIC CURVES¹

The simple symmetrical logistic has been fitted in many different ways, since the usual procedure of least squares fitting cannot be used, the constants not entering the equation linearly. But since the curve which minimizes the sum of the squares of the deviations is generally considered to be the "best fitting" curve, an indirect method of least squares fitting has been used by some authors.

This method consists of first deriving a preliminary estimate of the parameters of the curve, then expanding the expression in a Taylor series for y in terms of the initial estimates plus their correction terms. By truncating the series at the first order term the expression is linearized and can then be fitted by least squares. This assumes that the correction terms are small with respect to the initial estimates. If they are not, the series diverges and other estimates must be used.

The many ways of fitting the curve consist of methods of establishing the initial estimates. Often in practice the results of this step have not been corrected by the rather lengthy least squares procedure. Nair (1954) has described several of the less frequent methods of estimating two or all three parameters. The most commonly used method is probably the three-point method.

Three-Point Method.

Since there are three parameters in the equation of the simple logistic, the substitution of values for three points typical of the data will yield estimates for the parameters as follows.

Choose three points equally spaced in time and preferably distributed over the entire range of the data. Let these

¹ For a detailed discussion see White (1958).

be $(0, y_0)$, (x_1, y_1) , and (x_2, y_2) , with $(x_2 - x_1) = (x_1 - x_0) = n$. Both n and the origin year are chosen freely. Then if the logistic equation is written as

$$y = \frac{K}{1 + e^{a_0 + a_1 x}} \quad (e^{a_0} = c),$$

$$a_0 + a_1 \cdot 0 = \ln((K - y_0)/y_0)$$

$$a_0 + a_1 \cdot n = \ln((K - y_1)/y_1)$$

$$a_0 + a_1 \cdot 2n = \ln((K - y_2)/y_2)$$

from which
$$K = \frac{2y_0y_1y_2 - y_1^2(y_0 + y_2)}{y_0y_2 - y_1^2}$$

$$a_0 = \ln \frac{K - y_0}{y_0}$$

$$a_1 = \frac{1}{n} \ln \frac{y_0 (K - y_1)}{y_1 (K - y_0)}$$

This method can be extended to fit any form of the logistic. For the third order equation, five points are necessary and the above relationships become:

$$y_1^4 y_3^4 (K - y_0)(K - y_2)^6 (K - y_4) = y_0 y_2^6 y_4 (K - y_1)^4 (K - y_3)^4$$

$$a_0 = \ln \frac{K - y_0}{y_0}$$

$$a_1 = \frac{18B_1 - 9B_2 + 2B_3}{6x_1}, \quad a_2 = \frac{4B_2 - 5B_1 - B_3}{2x_1^2}, \quad a_3 = \frac{B_3 + 3B_1 - 3B_2}{6x_1^3}$$

where B_1 , B_2 and B_3 are defined as:

$$B_1 = \ln \frac{K - y_1}{y_1} - \ln \frac{K - y_0}{y_0}, \quad B_2 = \ln \frac{K - y_2}{y_2} - \ln \frac{K - y_0}{y_0},$$

$$B_3 = \ln \frac{K - y_3}{y_3} - \ln \frac{K - y_0}{y_0} \quad (\text{Pearl, 1924, p. 577})$$

Since the expression for K contains powers up to and including the eighth, it must be solved by Newton's method or other approximating techniques, and all real positive roots examined. It is thus much more difficult to apply this method (and all other methods) to the third order curve, and this in part explains why this form of the curve is little used.

Pearl (1930) has suggested the following method of fitting the curve.

Pearl's Method.

Writing the expression for the logistic as

$$y = \frac{K}{1 + e^{a_0 + a_1 x}}$$

it is easily shown that

$$\ln \frac{K-y}{y} = a_0 + a_1 x$$

i.e. $\ln \frac{K-y}{y}$ is a straight line function of time (x). Thus if some basis exists for estimating K, this function can be fitted by a straight line to give estimates of a_0 and a_1 . The degree of curvilinearity shown by the plot of $\ln \frac{K-y}{y}$ is a measure of how well the value of K has been chosen, or of how skew the data is. If no value of K gives values which lie close to a straight line, then a third order polynomial can be fitted to the logs of the values of $\frac{K-y}{y}$ to give estimates of a_0 , a_1 , a_2 and a_3 in the function of x, and a skew logistic used. It is important to note however that there is then no clue as to whether the chosen value of K is the best initial estimate, since the value which gives the closest approach to linearity may not be the best if the data are really skew.

This method is the basis for the several nomographs in the statistical literature that are used to fit the logistic (Rasor, 1949) and for the logistic graph paper available from the Codex Book Co. Inc., Norwood, Mass. Unless it is definitely known that the data are not skew, the value of K giving the straightest line cannot be counted upon to be the best one. In experimenting with the curve of cumulative zinc production, the value of K giving the straightest line was equal to the total produced to date.

Discussion.

The three-point method of fitting the simple logistic is the one found in standard treatises, and most commonly used in the literature (see for example Kuznets (1930), Lasky (1951 & 1955)). Extensive experimentation was carried out by White (1958) who showed that the results are extremely unstable, and that the value of K, the ultimate recoverable

resources and hence the upper asymptote, may vary beyond all reason. Fitting cumulative petroleum production data, White got upper asymptotes of 47.9, 52.3, 69.3 and 96.0 billion barrels, none of which agree with current estimates of ultimate recoverable petroleum. Further calculations by the author resulted in numerous estimates of K for other mineral production series which were often entirely untenable, being lower than the amount already produced or unbelievably high. For lead, values obtained ranged from 19.4 million tons (already passed in 1934) to 65.2 million tons (about double cumulative production to date).

Bratt (1936) gives similar results in trying to forecast the growth of the steel industry using logistic curves fitted by the three-point method.

An attempt was made to fit the second order symmetrical logistic to annual production data for lead and zinc, using the three-point method with least squares improvement. Although estimates of the parameters by the three-point method were used which covered a wide range, convergence was found to be extremely slow, even for the most reasonable-looking estimates of K, and even 10 iterations failed to make the values converge. This problem was likewise noted by Wilson and Puffer (1933) in working with the first order curve.

The most complete appraisal of the simple logistic curve of which the author is aware, and also, unfortunately, one of the works to which reference is least often made, is that of Wilson and Puffer (1933), contemporary biometricians of Pearl and Reed. They showed that while the curve occasionally yields "a tolerably good fit to the census enumerations" which they used for data, it often yielded upper asymptotes which were negative or infinite in finite time, both cases of course being untenable. Not only does this apply to curves fitted by the short-cut methods, (i.e. the three-point method, the probability paper method (see Croxton and Cowden 1955, and White, 1958), Pearl's method, Yule's method (Yule, 1925)),

but by laborious hand calculations, Wilson and Puffer also fitted 16 curves to population data by indirect least squares, by minimizing absolute and in some cases relative residuals, with unsatisfactory results, including negative and infinite upper asymptotes, and values of the standard error larger than the value of the asymptote. Clearly, this study shows that there is no satisfactory way of determining the constants of the simple logistic which will give stable results.

Unpublished work by Uffen¹ was attempted to fit the logistic to mineral production data from single mines or mining areas, and similarly resulted in unstable estimates of the parameters.

Having determined that the nature of mineral production will probably lead to a skew curve, the author has attempted to fit the skew logistic by first approximating the logistic parameters by Pearl's method, then improving the estimates by least squares, using an electronic computer (IBM 704). Up to 10 iterations of the least squares technique were used in testing, but 4 iterations were found to be sufficient in almost every case. The technique used was somewhat similar to the method of steepest descent in that a series of values of K was used and that yielding the minimum RMSD was to be considered the most suitable. Using another approach, when one value of K was chosen and K was included in the least squares reevaluations no convergence of the parameters could be obtained, the changes in K being much greater than those in the other parameters.

Improving the Initial Estimates by Least Squares².

Consider the case of a single variable x , and let the correction to x_0 be h , where $h \ll x_0$ and x_0 is the initial estimate for x . Then

$$f(x) = f(x_0+h) = f(x_0) + hf'(x_0) + \frac{h^2}{2}f''(x_0) + \dots$$

¹ R.J. Uffen, Head, Dept. of Geophysics, University of Western Ontario, personal communication, 1960.

² For a detailed explanation for the skew logistic see White (1958) or the Appendix.

This series can be truncated after the term in $f'(x_0)$ as an approximation to the $f(x)$. It is then a linear expression and least squares can be used for fitting, the approximation being improved by iteration, using the improved value of the initial estimate on each successive iteration.

The same method can be used for functions of more than one variable. This necessitates forming more normal equations for the least squares solution, and the work rapidly becomes excessive, especially if more than one iteration is used. Fortunately this type of problem can be programmed for a digital computer, as has been done in this study.

The least squares method of improving the initial estimates can be applied to any or all of the parameters, although the sum of squares being minimized will not be exactly that of the deviations from the actual logistic if all parameters are not included. Nevertheless this practice is statistically acceptable.

Note that when Pearl's method of fitting is used it is assumed that a good estimate of the upper limit of growth is known. In the case of mineral production this means that a dependable estimate of ultimate recoverable reserves exists. This is the case only within broad limits for most mineral resources, lead being perhaps the best known, but many experts have prepared such estimates. Thus if one places considerable trust in a given estimate he would use this figure as the upper limit of growth, fit the curve approximately, then improve all the parameters but K by least squares.

One of the difficulties in this approach lies in the fact that K represents not currently known reserves but ultimate recoverable resources, a much more difficult figure to estimate.

When there is little basis for assuming a value of K , some method like the three-point method seems preferable to a series of guesses. The value of K may or may not be subjected to change by the least squares procedure. In this case, since mineral reserve data gives some indication of the value which K should assume, a series of values were assumed for K ,

the other parameters estimated by Pearl's method, and improved by successive iterations of the least squares approximation method outlined above.

EXPERIMENTATION WITH THE SKEW LOGISTIC

On the hypothesis that the rate of growth of the mineral industry depends on (i) the absolute size already attained by the industry's production and (ii) the potential for growth remaining, the logistic curve should be a good implicit model. This does not say however that it is a practical method of forecasting to fit a logistic and extrapolate it to the desired date of forecast.

In order to test the reliability of extrapolation of the logistic, several experimental fits were made, using several iterations of the technique described in the appendix, with the results described below.

Zinc - Skew Logistic

Resources. Since Pearl's method was used to fit the skew logistic, without reevaluation of K, it was important to have a reliable estimate of the resources of the United States. The Paley Commission (1952) reported estimates of reserves in 1944 of 21.9 million tons of zinc, of which 11.7 was inferred; in 1950, 21.2 million tons of zinc. This estimate was still considered valid in 1952. About 80% of this zinc was reported as recoverable. These estimates did not include marginal deposits, and the Commission estimated that a further 20 million tons exists in such deposits. Of this, some 2.85 million tons of metal exist in deposits of all grades in the Tristate district; 450,000 tons of zinc is inferred in the ores of the Gossan Lead Belt, grading 0.6 to 0.7% Zn; 1.9 million tons (inferred) at Jerome, Arizona, grading 1.9% Zn; and some 1.5 million tons in low grade slag dumps which carry 5 to 6% Zn.

In the 1957 Minerals Yearbook (p. 1320) the United States Bureau of Mines reported 13,485,000 tons of zinc content in ores in the United States, measured and indicated. Some 85% of this is recoverable.

Thus the ultimate recoverable resources of zinc in the United States should be approximated by adding together the cumulative mine production to the date of the reserve estimate, plus measured, indicated, and inferred reserves of zinc metal. However mine production figures prior to 1905 are unavailable, as are total smelter output data, and the tonnage of primary material used in zinc dust and pigments cannot be located prior to 1921. Since it will be shown later that it is important to have a homogeneous statistical series it was thus decided to use cumulative United States smelter production of slab zinc from domestic ores, which series extends back to 1858, when 20 tons of zinc were produced at Lehigh Zinc in Bethlehem, Pennsylvania to mark the establishment of a domestic zinc industry. By comparing mine production and the above series a ratio was established by which to adjust the ore reserve data so as to give in essence the "reserves of slab zinc". Note that this assumes no future change in the ratio of zinc dust and pigments to slab production. Any change is expected to be insignificant compared to the uncertainty of the reserve data.

Thus the Paley Commission estimate of reserves in 1952 yields the following estimate of ultimate recoverable resources of zinc in the United States

Measured, indicated and inferred reserves	21,200,000
Marginal material	20,000,000
Times estimated recovery factor (80%)	32,960,000
Times "slab reserve" factor (.84)	27,686,400
Total unmined reserve as "slab"	27,686,400
Cumulative slab production to end 1952	<u>23,763,485</u>
Ultimate recoverable "slab zinc resources of the United States"	51,449,485

Since inferred reserves and marginal material make up over 50 percent of the reserves, 50,000,000 is taken as an upper estimate for K. Note that there is no provision for any large discoveries of new zinc. A deposit grading 8% Zn, considering 80% recovery, would have to contain some 16,000,000

tons to add one million tons of zinc to this reserve.

If a fairly complete series of estimates of zinc ore reserves over the years were available the rate of discovery could be worked out, and would be a help in estimating the possible future additions to ultimate recoverable resources. No such series appears available.

Other complications exist in the fact that not all zinc deposits will necessarily be mined -extensive substitution for zinc or low cost imported zinc could make them permanently uneconomic, and the lower grade deposits are especially vulnerable. Furthermore, many exploration experts feel that without striking advances in exploration techniques there will be few new major deposits of any of the base metals found in the United States in the future. Recent additions to the family of porphyry coppers seem to belie this attitude but it seems unlikely that the score of some 50 million tons of zinc metal discovered over the last 100 years will be matched in the next hundred years.

The figure of 50 million tons of zinc to be produced as slab zinc, as an ultimate goal, is possibly conservative but is useful for illustration and may be close to the truth.

Curve Results.

Three values of K have been used for illustration of results - 30×10^6 , 40×10^6 and 50×10^6 tons. The resulting curves, fitted with 4 iterations of the least squares technique, are shown in chart 2, along with actual data. Data used in fitting were from 1858 to 1955 inclusive. It can be seen that in each case the fit is good, and referring to the annual data, chart 3, the curves on which are the plots of the first differences of the cumulative curves, the trend traced out by the logistic seems to pass through the center of the major cycles in each case, but for K=30 million it falls off more rapidly than the data to 1959 would indicate likely, and for K=50 million it rises too rapidly. Note also that for K=50 million there appears to be a weak second cycle of growth starting around 1943. While use of zinc in

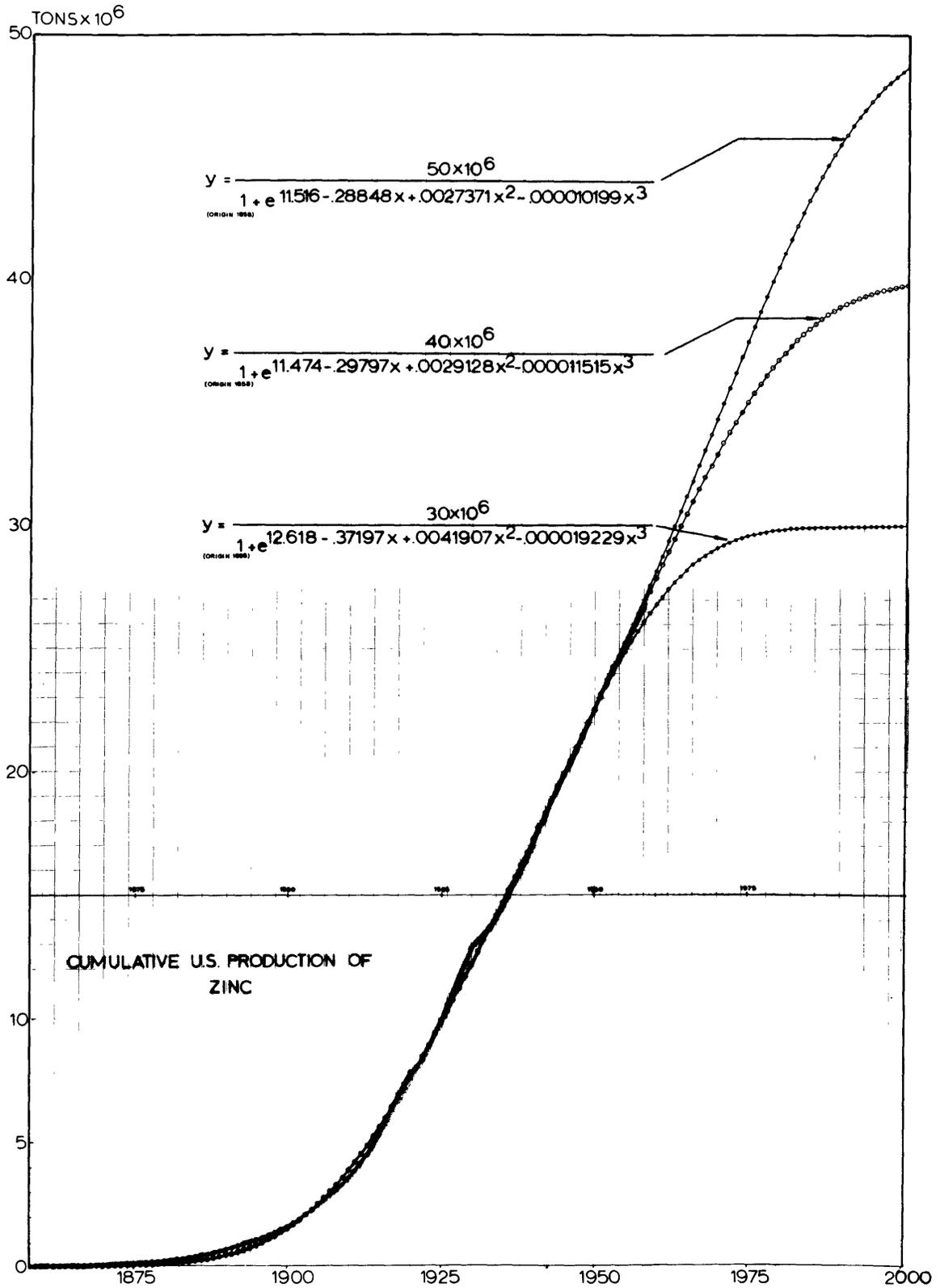


CHART 2

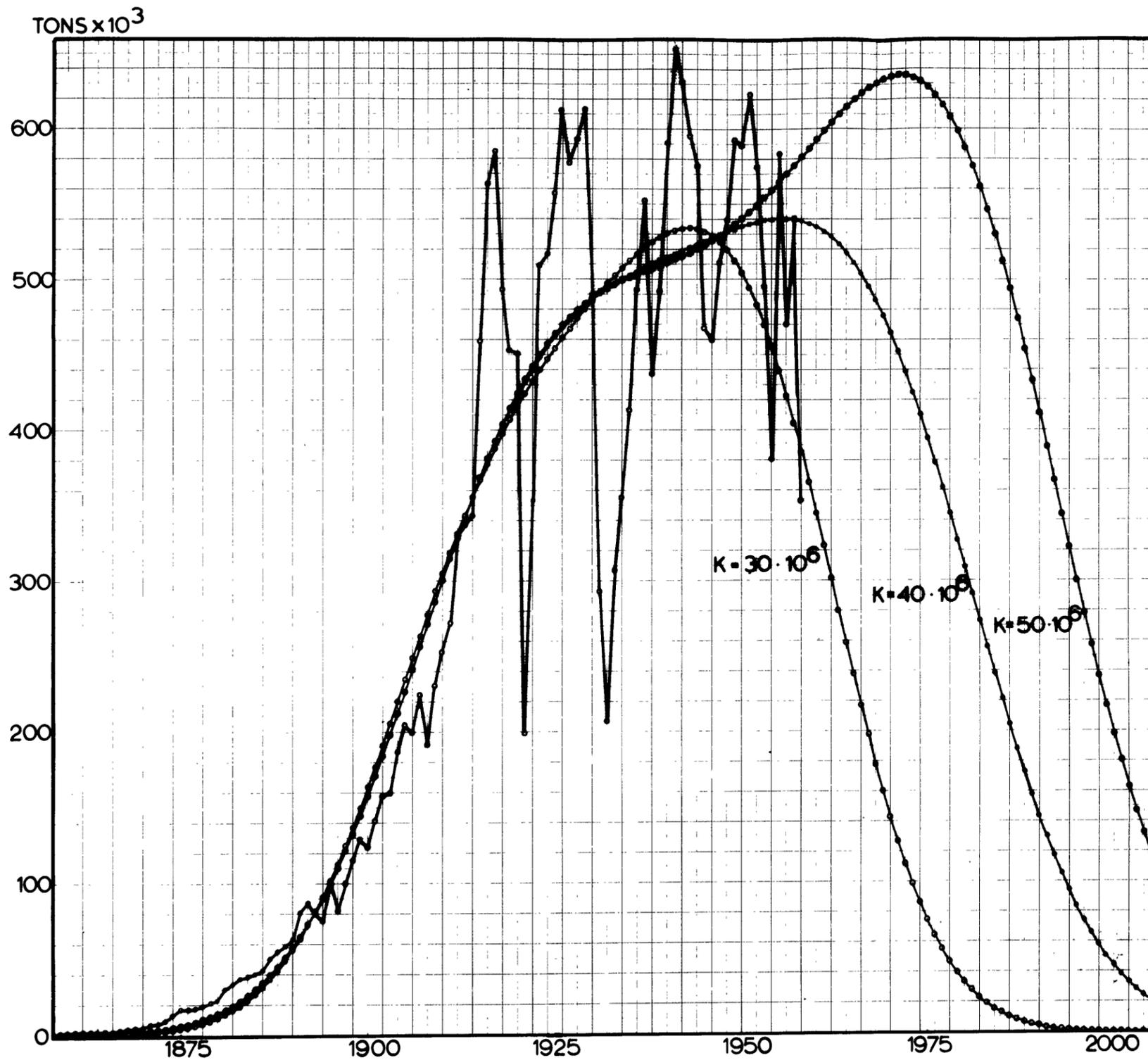


CHART 3. ANNUAL U.S. PRODUCTION OF ZINC

brass has declined somewhat from the war years, demand in those years was abnormally high, mainly for cartridge cases. Since 1943 however the use of zinc-base alloys for casting has increased from 70,000 - 80,000 tons per year to about 300,000 tons per year, and will probably continue to increase. It is thus close to the major use, galvanizing, in the tonnage used per year, and this may well account for the second growth surge.

No matter how large a scale is used, it usually appears that the fit of the logistic to cumulative data is good, since the process of integrating over the annual data is a smoothing technique. Hubbert¹ has suggested that it is more desirable to fit the annual data rather than the smoothed data. But this type of smoothing introduces no bias, and smoothing of raw data is an accepted procedure, providing no bias is introduced. (The commonly used moving average method of smoothing has this failing). For three reasons the annual data are not suitable. First, there is no way to estimate the upper asymptote of annual production rate. The industry may accelerate production to an overly high annual rate that will cause a sudden and precipitous drop when resources are rapidly exhausted, or it may husband its resources and use them at a low annual rate, but their eventual total will in either case be essentially the same. Second, in most cases of mineral production, if an upper limit of annual production is chosen, cyclical fluctuations will cause annual output to exceed it in some years, and Pearl's method of estimating the parameters will not work, since it then involves the logarithms of negative numbers, (i.e. $\ln \frac{K-y}{y}$). Third, and a critical objection, the second order curve which would be used to fit annual data is symmetrical, and one must go to at least a fourth order curve to avoid this.

Lasky (1951) fitted first order logistic curves to annual production data for zinc, copper, and aluminum. Since

¹ Verbal communication

these curves approach an upper asymptote and do not decline from it, they cannot show the overall growth pattern, being unable to represent the inevitable period of decline.

Kuznets (1933) answered this difficulty by grafting together at some chosen point a positive and a negative logistic, for which there is no justification, for as Lasky has noted¹, "The data beyond the peak do not stand alone but are related to what went before". He has therefore used the second order curve for fitting annual coal production (1951) and possibly some of the other series in his 1955 work, although no equations are shown.

Note that the fit in the early years of production is not perfect. From 1858 to 1893 the curves lie decidedly below the data and from 1893 to 1912 they are decidedly above it, while the entire period showed no long deviations of this nature from the secular trend of general business. (See for example "Business Trends and Progress", presented by Ex-Cell-O Corporation, 1960 or White, 1958, p. 18). This indicates that the production series departs somewhat from true logistic growth. The curve for K=50 million tons comes closest to fitting, and has the least root mean square deviation of the three. The RMS deviation is given by:

$$\text{RMSD} = \sqrt{\frac{(Y \text{ observed} - Y \text{ calculated})^2}{n}}$$

when n is the number of data points fitted. For these curves this measure of goodness of fit yields:

$$K = 30 \times 10^6, \quad \text{RMSD} = 228085$$

$$K = 40 \times 10^6, \quad \text{RMSD} = 200053$$

$$K = 50 \times 10^6, \quad \text{RMSD} = 196743$$

Curves were fitted for values of K from 30 million to 50 million, progressing in steps of 2 million, and results were intermediate to the above.

While it appears from the decreasing RMSD's that using higher values of K would improve the fit, this decrease was found to be no guide to the best values of the parameters,

¹ Personal communication to the author, March 24, 1958.

although this was originally thought to have been a good criterion. Although copper is the most completely developed example,¹ the zinc curve for $K=50$ million shows the cause of failure. As the upper asymptote is increased, the curve tends to develop into a 2-cycle form, and the downbend allows a closer fit to the extreme lows of the Depression years, reducing the RMSD considerably because of the effect of squaring the deviations in the least squares technique.

Since the ultimate recoverable resource cannot be known very closely, it may be possible to use a reasonable range of values for it. The range of forecasted 1975 production for $K=30 \times 10^6$ and for $K=50 \times 10^6$ is 74,687 tons to 627,721 tons, or about a year's production at the current level, obviously too great to be of any use. The predicted value of annual zinc production in 1975 for $K=48$ million is 588,826 tons, for $K=50$ million 627,721 tons, a difference of 38,895 tons or 6.8% of the lower estimate. While such a small error in a 20 year forecast is within any reasonable tolerance, a knowledge of the United States ultimate zinc resource position within 2 million tons (the equivalent of 2 medium-sized ore bodies) is not likely to be claimed by any mineral expert, nor is twice or three times this amount of error of estimate out of the question. But 12 or 18% error of estimate in the forecast becomes quite serious, e.g. if the forecast is 600,000 tons, an 18% range means a forecast of about 500,000 to 700,000 tons.

Consider next the effect of errors in the early statistics. As mentioned above, data for domestic mine production and total smelter production from domestic ore are not available for early years, so that some forecasters have used series which include foreign ore, and it is also quite possible that the collection of the slab zinc production data was somewhat incomplete. This should not make a noticeable difference in the forecasts made some hundred years later. To test this point, the statistics for the years 1858 to 1890 inclusive were perturbed by doubling them, adding a

¹ See "Copper" for a wide range of K values

total of 631,205 tons to cumulative production. The resulting curve is shown in chart 4. K was chosen as equal to 30, 40 and 50 million, 40 million being shown, and the curve fitted to actual statistics is shown for comparison. In the case of the perturbed data, statistics were used up to and including those for 1958.

The discrepancy between the two curves is large and disconcerting. In 1975 they differ by 196,223 tons or almost 50% of the lower estimate. Thus, a change in cumulative production to 1958 of 2.4% due to possible distortions in the series of data used causes a serious change in the parameters of the curve yielding a large difference in the forecasts almost 100 years later. Since errors, especially in early statistics, are quite possible, and since in fact a different pattern of cyclical movements at the time of early production could legitimately cause such a difference in the data, it appears that the parameters are too unstable to give a dependable forecast.

A further test was made to get some idea of the time period needed before the habit of growth is established. Data from 1858 to 1919 inclusive were used and the results are shown in chart 5 for $K=30$ million, and 50 million. The fit is extremely good over the range of the data, and extremely poor beyond the data. It appears that the geometric habit of growth was so largely dominant up to about 1919 that the effects of limited resources and other factors which slow growth were not taken into account at all by the curve. In fact, whenever the series has not passed its inflection point, the curve is especially unstable (Wilson and Puffer, 1933). In this case, with a background of 57 years of data the logistic missed the actual pattern of future growth entirely, becoming decidedly off the trend immediately. In effect, what one is attempting is to forecast the level of the upper asymptote, which may not be approached for over a hundred years. When the data are known over much of the eventual course of the curve, this forecast is not so dif-

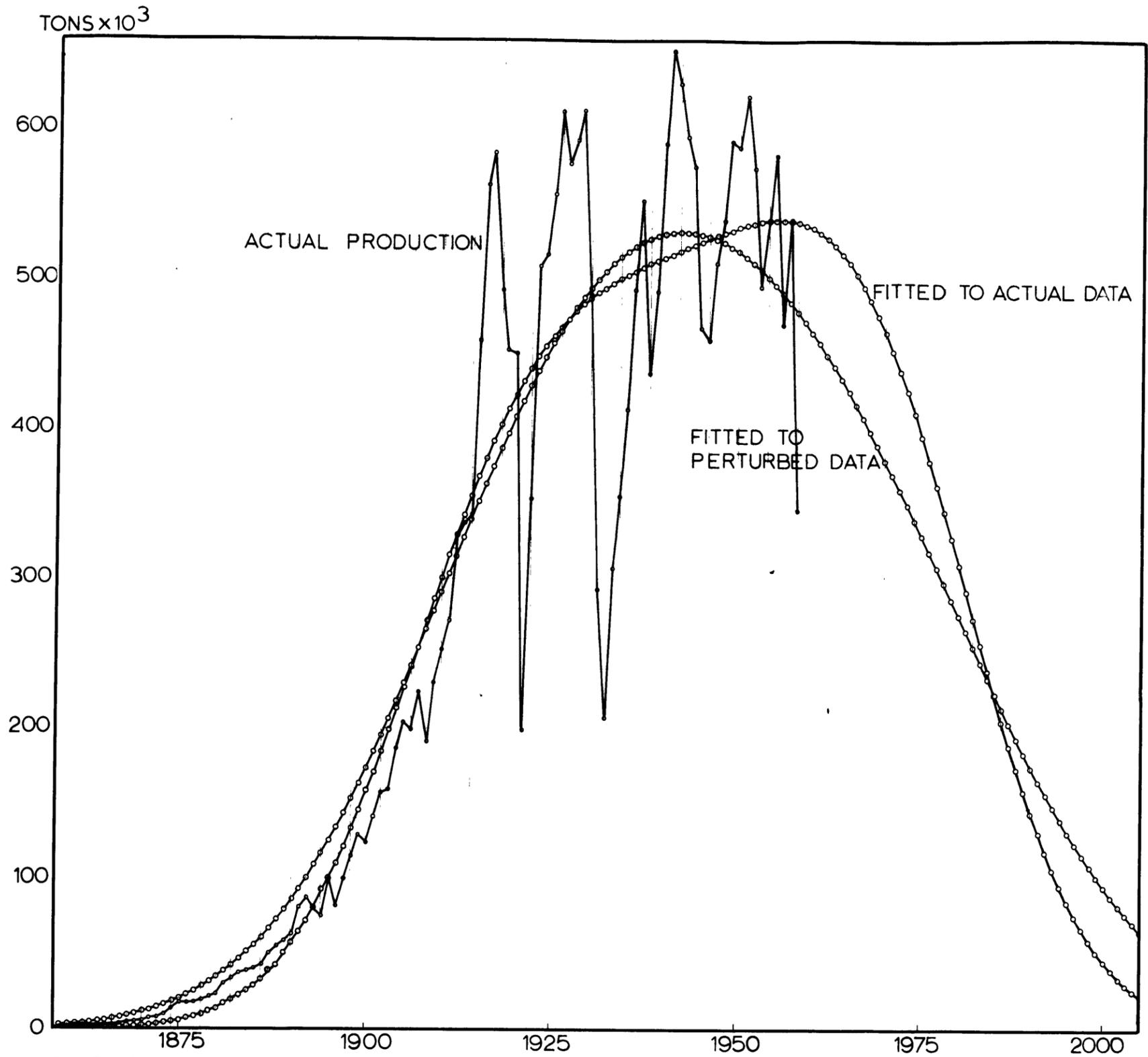


CHART 4. ANNUAL U.S. PRODUCTION OF ZINC (LOGISTIC CURVES FOR K=40 MILLION.)

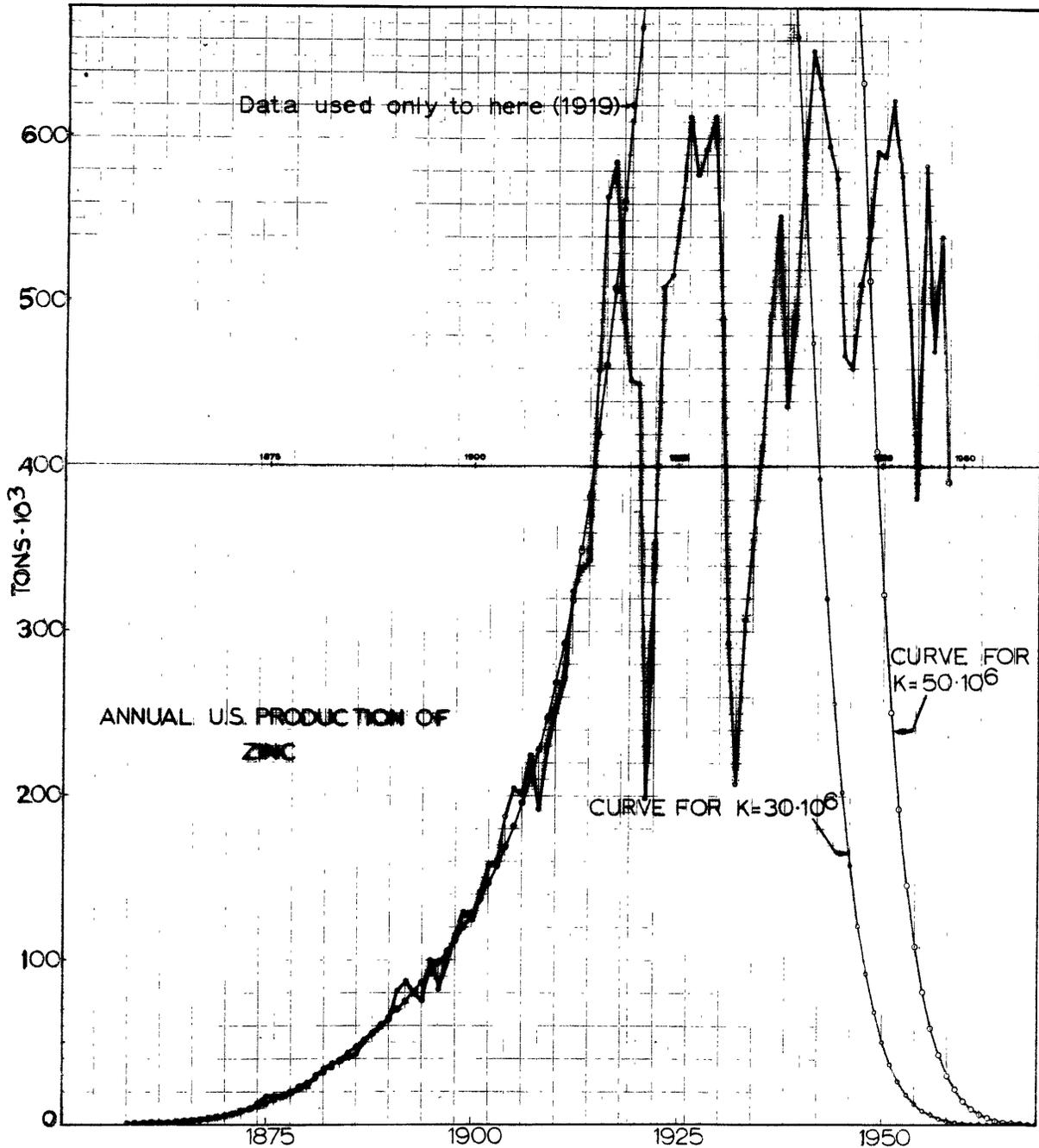


CHART 5 ANNUAL U.S. PRODUCTION OF ZINC - EARLY DATA ONLY

ficult. Hence Pearl could get good fits to his biological data quite easily. The use of the non-symmetrical logistics makes the attempt more difficult since even if the data extend beyond the inflection point, another such point in a new cycle may still appear. In mineral production forecasting one would therefore want to have data which go beyond the inflection point, and as well be satisfied that there is little or no chance of another cycle of growth.

To summarize the results obtained from the skew logistic fitted to cumulative zinc production data, the following may be stated.

1. The value of the upper asymptote (K) is very important in determining the shape of the curve, even within the range of the data.
2. A 4% change in the value of K produced a change of 6.6% in the forecast for 1975.
3. As with any secular trend, cyclical movements may cause "poor" forecasts for the short or medium term.
4. Disparities in the data used are more important than their magnitude might suggest, since differences in the early data produce quite different parameter values in the logistic equation, yielding a significant change in forecasts.
5. Although data for a long period of time may be obtainable, the effect of diminution of reserves may not be established in the trend, and the logistic does not describe adequately the future growth period when this factor becomes important.

Copper - Skew Logistic

Resources.

The volume of United States copper resources is even more open to question than that of zinc, for the large tonnages of metal contained in a single deposit, and the non-selective mining methods in common use make the price-cost ratio important in determining the cut-off which in turn changes the volume of material classified as ore. Furthermore,

the recent discovery of several new porphyry copper deposits makes it clear that they are not all found, as some experts have believed.

In hearing number 38 before the subcommittee on public lands of the eightieth congress, 1948, reserves as of January 1, 1944 were estimated at 20 million tons of recoverable copper, of which 70% was in the porphyry coppers. This figure included only "currently available" reserves, and made no attempt to predict ultimate reserves. Another 10 million tons of copper was fairly well known but not then economic. The chances of doubling or trebling the 20 million tons of reserves were considered excellent.

The Federal Trade Commission report on the copper industry, 1947, stated the United States reserves to be 29 million tons of copper as of January 1, 1945.

Year	Tons of Recov. Cu (90% Recov.)	Price	"Life"	At annual Prod. rate of	Reference
1931	18.5 x 10 ⁶	9¢	31yrs.	600,000	Barbour
1931	18.8	9	31	600,000	Rawles
1934	18.9	10	32	600,000	Barbour
1935	16.0	10	22	750,000	Leith & Liddell
1935	23.5	12	32	750,000	Leith & Liddell
1935	17.6	10	-	-	Joralemon, Lynch & Leith
	4.2 inf.	10	-	-	"
	4.3	Uncertain	-	-	"
1936	23.7	12.5	33	725,000	Joralemon et al
1944	20.0	13	33	725,000	Cannon et al
1944	30.0	13	25	800,000	"
1944	20.0	then current	-	-	Pehrson
1945	29.2	13	36	800,000	Fed. Trade Comm.
1948	15.0	-	-	-	U.S. Bur. of Mines
1952	25.0	-	-	-	Paley Commission
1953	26.0	-	-	-	Weise

The above table of reserve estimates for the United States is after the Paley Commission report, (table III, p. 144, volume 3), with additions.

R.G. Weise, in an unpublished tabulation of world copper resources, gave United States reserves as 26 million tons circa 1953.

By adding to the above reserve estimates the total amount of copper produced to the given year, an estimate of cumulative discoveries to date is obtained. For the year 1945, the cumulative discoveries amounted to 61.6 million tons, based on the reserves reported by the Federal Trade Commission. For 1953, cumulative discoveries were about 65.2 million tons. The cumulative discoveries for the above years are shown in chart 6.

The problem lies in appraising the potential for further discoveries of copper ore. From 1931 to 1953 some 23 million tons of copper have been discovered or developed as reserves. Some of this has come from long-known deposits, such as Butte, and similar vein deposits, where only a fraction of the inferred tonnage is classified as reserves at any given time. This would hardly be called new discovery. Other copper is discovered as a by-product of lead-zinc exploration, and the discovery rate is a function of lead-zinc activity.

If the additions to reserves were all considered new discovery, the rate over the past 22 years is then about one million tons of copper per year, or an orebody the equivalent of White Pine, Michigan every three years. While this is an admirably high rate of discovery, it is an open question as to how long it could continue.

The values of K used in this study have been spread over a large range because of this difficulty, but this does not solve the problem, for as shown in the case of zinc, a fairly small change in K changes the forecast a measureable amount. Thus any prediction made on the basis of the logistic would have to be narrowly qualified with respect to the assumption of ultimate recoverable resources.

Curve Results.

For a K value of 72 million tons, two curves are shown

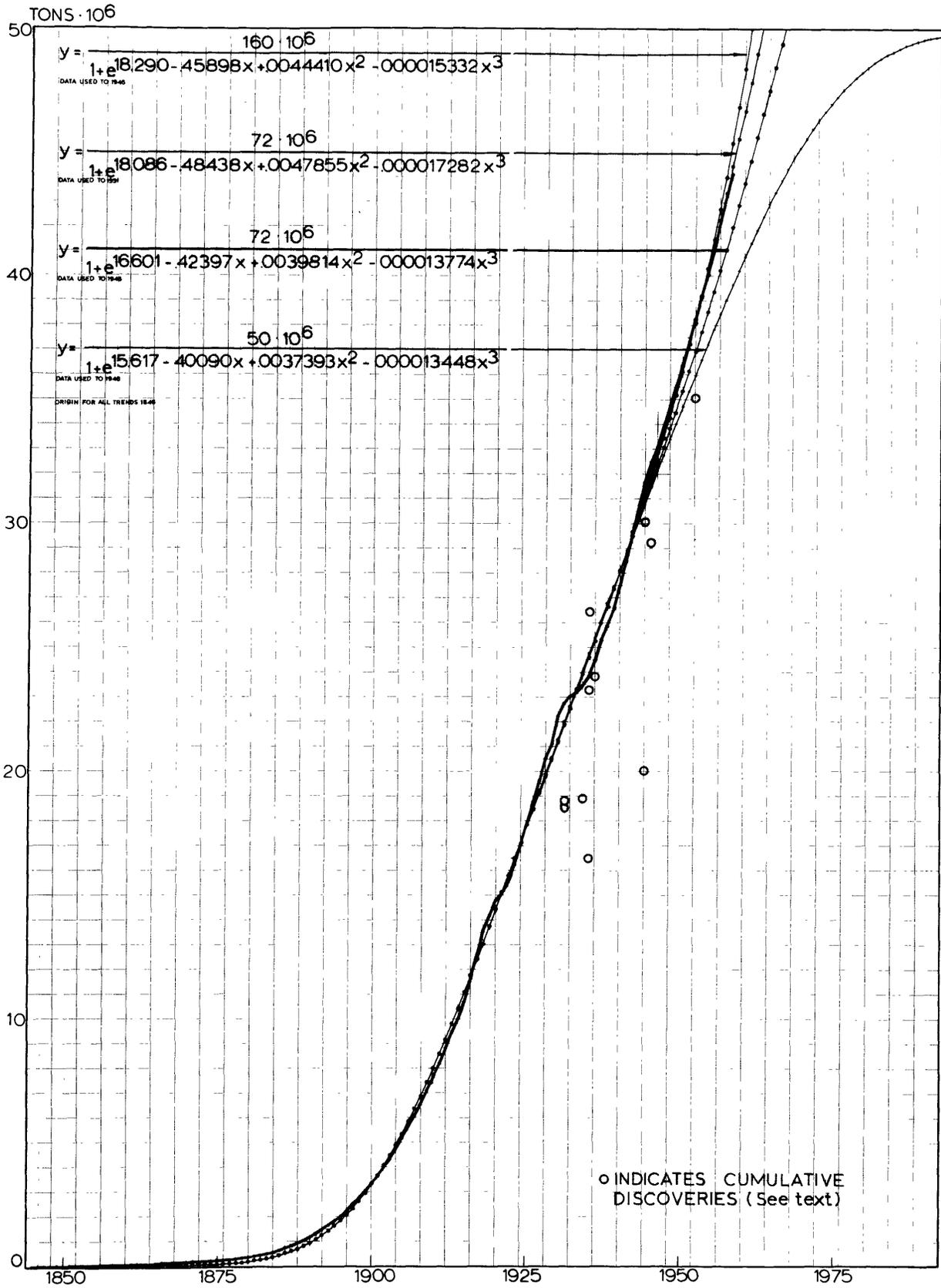


CHART 6 CUMULATIVE U.S. COPPER PRODUCTION

in each of charts 6 and 7, one fitted to data to 1945, the other to data to 1951. Compared below are the resulting forecasts for the period 1952-1958, and the actual figures of smelter production from domestic ores.

Year	Forecast, Data to 1945 incl.	Data to 1951 incl.	Actual Production
1952	803060	953918	927365
1953	817874	978001	943391
1954	833265	1002144	834381
1955	849072	1026100	1007311
1956	865199	1049504	1117580
1957	881458	1072056	1081055
1958	897713	1093365	990000
Av. 1952-58	849663	1027012	985869

Six years more of data changed the "forecasted" average for the next 7 years by 21%.

For the curve fitted to 1945, the forecast for the period 1946-1958 is compared to actual production in the table below.

Year	Forecasted Production	Actual Production
1946	730305	599656
1947	740220	862872
1948	751064	842477
1949	762835	757931
1950	775448	911352
1951	788883	930774
1952	803060	927365
1953	817874	943391
1954	833265	834381
1955	849072	1007311
1956	865199	1117580
1957	881458	1081055
1958	897713	990000
Av. 1946-1958	807415	908165

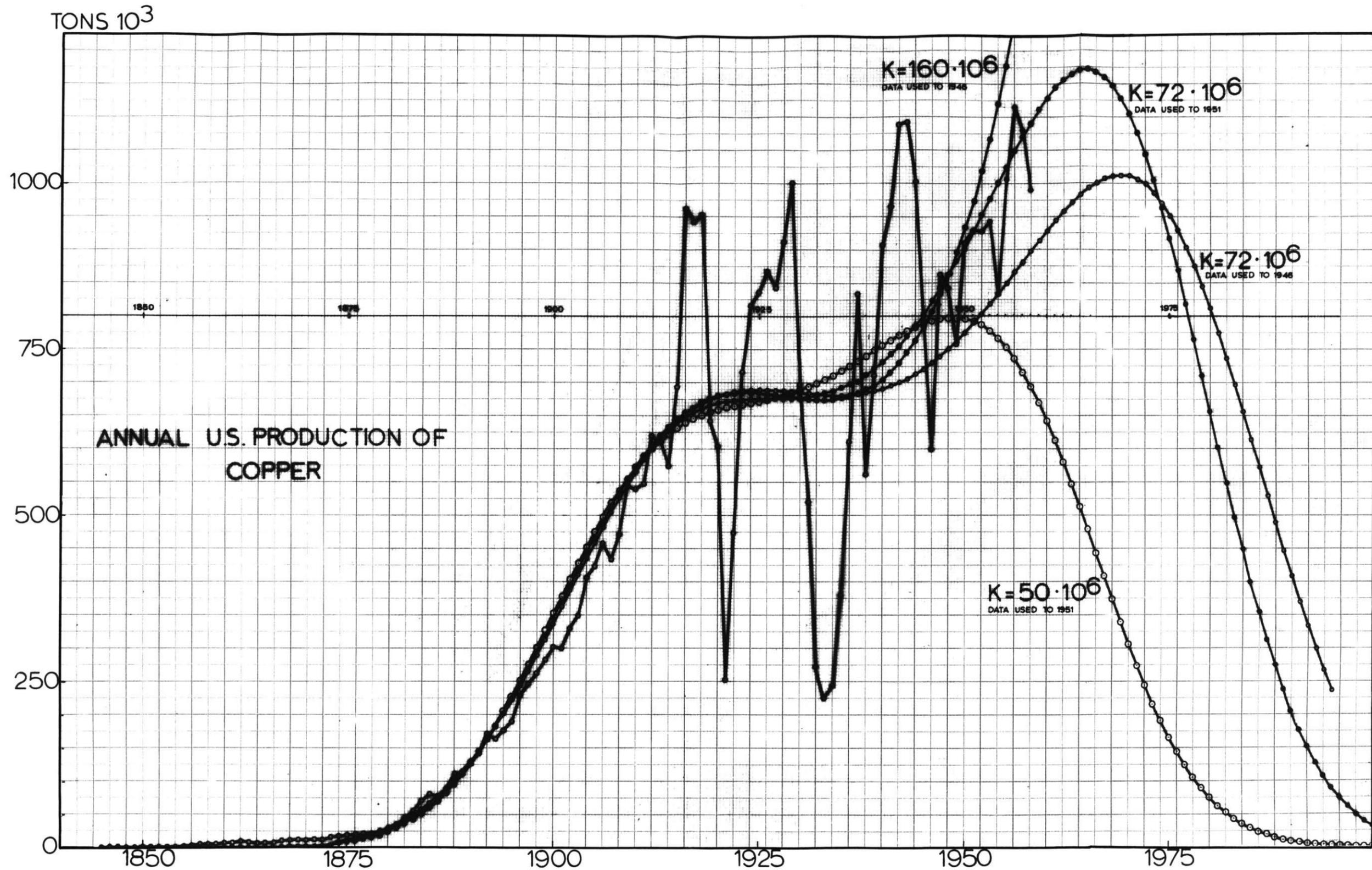


CHART 7. LOGISTIC CURVES FOR ANNUAL U.S. COPPER PRODUCTION.

Thus the curve fitted up to 1946 is low throughout the "forecast" years, while the second curve, fitted up to 1951, exceeds actual production, although by only a small amount. Indeed if an error in the seven year average of some 40,000 tons, or about 4.2% were typical of the curve's predicting ability it would be an excellent tool. Unfortunately, such accuracy is coincidental. If an attempt is being made to forecast 15 years ahead, on the basis of data covering almost 100 years, there seems to be no justification for using a curve so strongly influenced by the last few data points. Using it, the forecast would have to be revised every one or two years.

For comparison, curves are shown for values of K equal to 50 million and 160 million tons. Note the increasing importance of the second cycle as the upper limit of growth is increased, the curve reaching down toward the low values of the Depression years. This tends to reduce the RMSD, so that proceeding from $K = 50$ million to $K = 160$ million tons, no minimum RMSD was reached, although the curves can be seen to become less and less suitable as trends, especially if data up to date are considered.

It should be mentioned that for $K = 72$ million tons the second cycle of growth begins at about the time the porphyry coppers first became important producers, and this is a very encouraging fact. On examining curves for higher values of K however, it will be noted that the new cycle moves off to the right and is clearly a statistical effect of the low production figures of the post-World War I slump and the Depression.

The following conclusions may be drawn from the results of fitting skew logistic curves to cumulative copper production data.

1. If the upper asymptote is not known rather accurately, the use of a range to cover reasonable expectations is of no help, since the freedom of the curve allows it to change its shape considerably, even within the range of

the data.

2. Since the curve changes considerably upon addition of only five or six new data points, it is too unstable to use for forecasting.

Lead - Skew Logistic

Resources.

In 1934, Leith and Liddell estimated that there was 3,302,158 short tons of lead in the ground, corresponding to cumulative discoveries of about 22.4 million tons, assuming 85% recovery. According to the Paley Commission, in 1944 there was 7.75 million tons of lead in the ground, of which 5.15 million was inferred, and in 1950, 8.34 million tons in the ground, with another possible 8.34 million tons in submarginal material. These figures correspond to cumulative discovery totals of 30.2 million and 40.0 million, assuming 85% recovery, and including submarginal material. They are regarded as only an indication of the minimum reserves.

The lead reserves of the United States then are apparently considerably smaller than those of copper and zinc. But these other industries do not enjoy the current high rate of scrap return that lead does - some 45%, compared to 27% for copper and only 14% for zinc, (American Metal Market, May 23, 1952), and some feel that fear of depletion of lead resources is not as intense as would seem justified at first glance. The important trend will be that of the ratio of dissipative uses to non-dissipative. Most important among the former is tetraethyl, growing rapidly throughout the world. But the decline of the primary lead mining industry has long been in progress in the United States, and this in turn causes a decline in exploration for and discovery of new deposits. Again the question of ultimate recoverable resources is a difficult one, but the range of possibilities seems much more limited than, say for copper. Accordingly, in this study, estimates for K from 30 million to 40 million in steps of one million were tried.

Curve Results.

Results of fitting logistic curves of the third order to cumulative lead production data were unsatisfactory. Varying K from 30 million to 40 million, data were used from 1870 to 1956 inclusive and four iterations were used for each curve fitted. The results for $K=30$ million, 33 million, and 40 million are shown in charts 8 and 9, Note the close approach to symmetry of the curves for $K=30$ and 33 million, and the poor fit they give to the recent data, showing too sharp a decrease from the peak of annual production, represented by the year 1929 when 672,498 tons of lead were smelted from domestic ores. The curves for $K=33$ and 40 million are typical of all curves obtained for values of K greater than 30 million. It is of course impossible to have negative production, and although a minimum RMSD was obtained for $K=33$ million, that curve cannot be taken to represent the long-term trend because of the eventual negative values. As explained above, the positive third power term in X of the exponent of e accounts for the down-turn in the cumulative curves.

Similar results were obtained when data from 1821 to 1958 inclusive were used. Also, further iteration of the approximation routine indicated that convergence had been reached.

Nor can it be argued that perhaps 30 million is the most likely figure for ultimate resources, since the 28.5 million tons produced to 1959 would leave only about 5 years resources at current production rates, and greater reserves than 1.5 million tons are in sight. The United States Bureau of Mines reports measured and indicated reserves of 2.9 million tons, inferred reserves of 2.7 million. (Bulletin 585). Also, since this estimate, a new major ore area has been located by Kennecott Copper in Southeast Missouri.

On the other hand, chart 9 shows that the curve resulting for $K = 30$ million is very nearly symmetrical, although it does not fit the data since about 1948 well at all. It can be argued that for the method of fitting used,

MILLIONS OF TONS

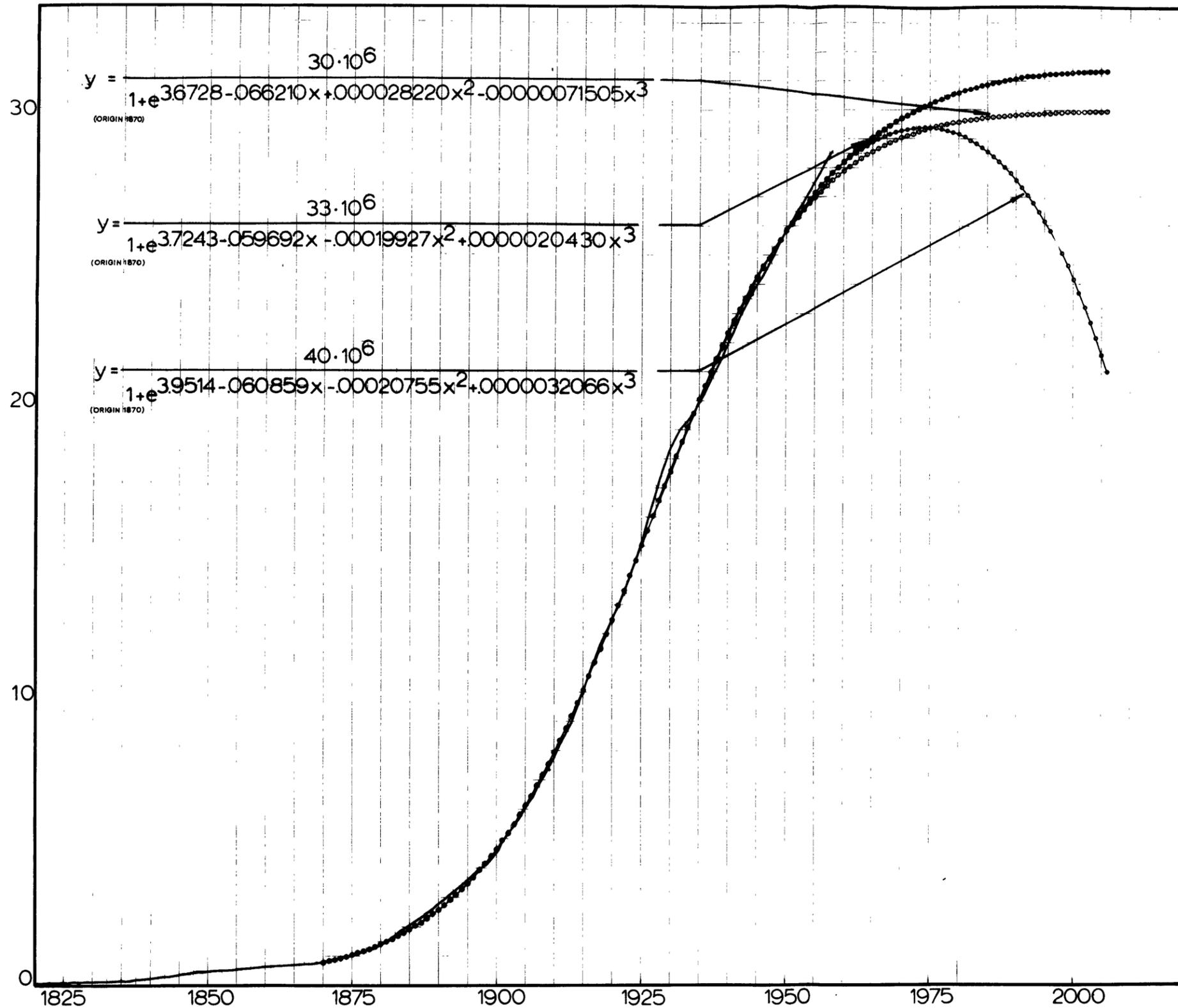


CHART 8. CUMULATIVE U.S. PRODUCTION OF LEAD.

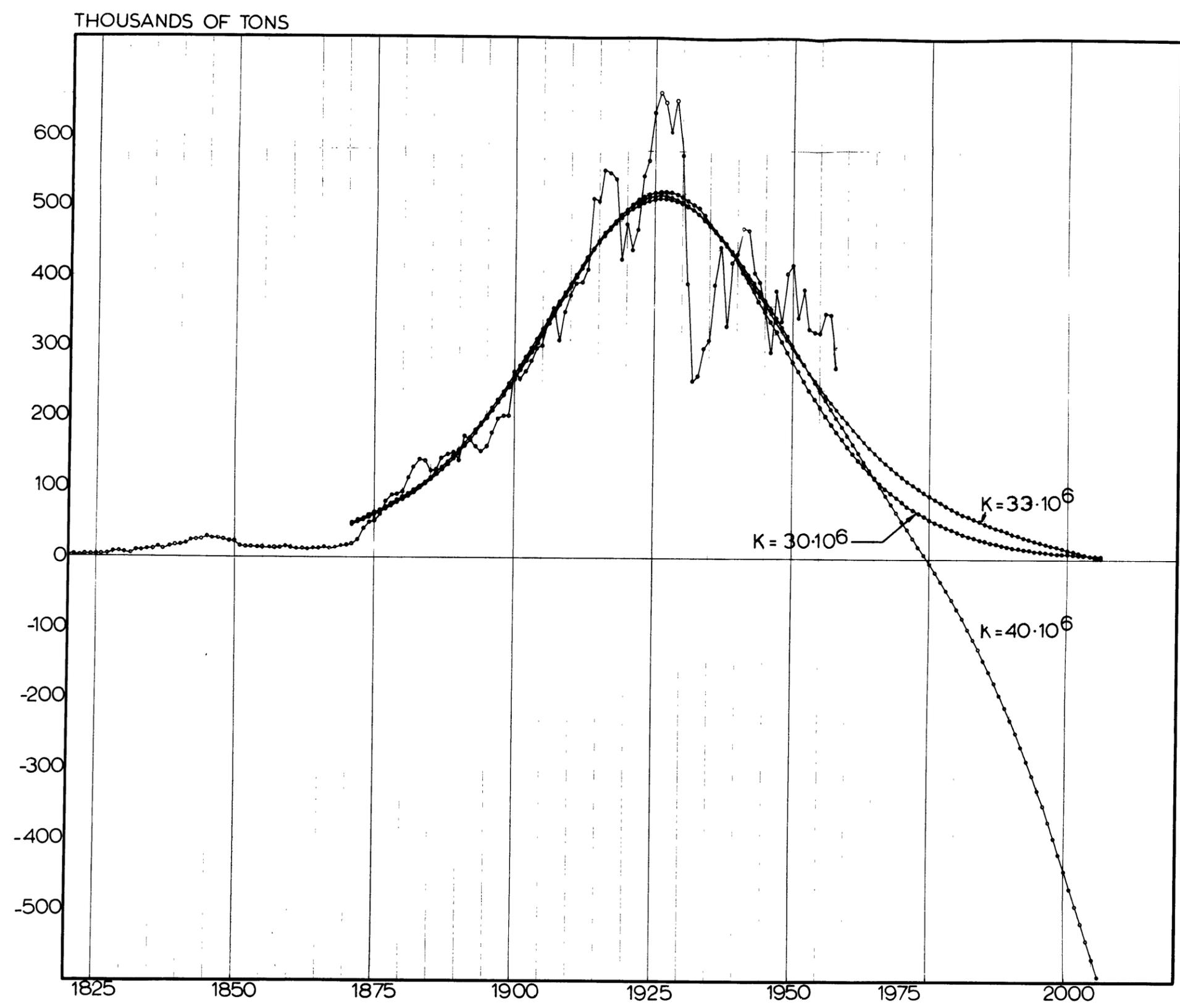


CHART 9. ANNUAL U.S. PRODUCTION OF LEAD.

the curve with K equal to 30 million comes closest to being symmetrical, and that the growth habit is actually symmetrical, so the extra degrees of freedom allow the curve to behave badly. Therefore, a curve was fitted to cumulative data from 1821 to 1958 using the three point method for a first approximation to K, and improving all the parameters by 10 iterations. The final value obtained for K is 34,605,000, and the equation and curve are shown in chart 10. In this case the curve is too high in the early years and probably shows too rapid a decrease in future years. Note that if the fit were better for the early years, the decline would be even steeper, since the curve is symmetrical.

It is the author's opinion that the actual production data reflect a rapid decrease from pre-Depression levels which was as much a matter of circumstance as of pressure from declining reserves, and that the post-World War II data show a considerable recovery from the steep rate of decline, so that a skewness of growth results. This is the cause of the poor fit of the symmetrical curve. Experimentation with weighted least squares was not carried out, since there was no apparent basis on which to weight the observations (cf. Wilson and Puffer, 1933, p. 302).

The cause of the positive value of the third power term of the exponent in the skew curve has also been interpreted to be due to the extreme range of values from the highs of the 1920's to the lows of the 1930's, this effect dominating over the data of the 1940's and 1950's. Therefore, this method of fitting the logistic to lead data is not applicable.

Several first order curves were fitted to this data earlier (see White, 1958), but since this method is unstable, no importance is attached to them as predictive tools. The best curves obtained were similar to those published by Lasky (1951).

From the results obtained in working with lead data it is concluded that with some time series which exhibit wide

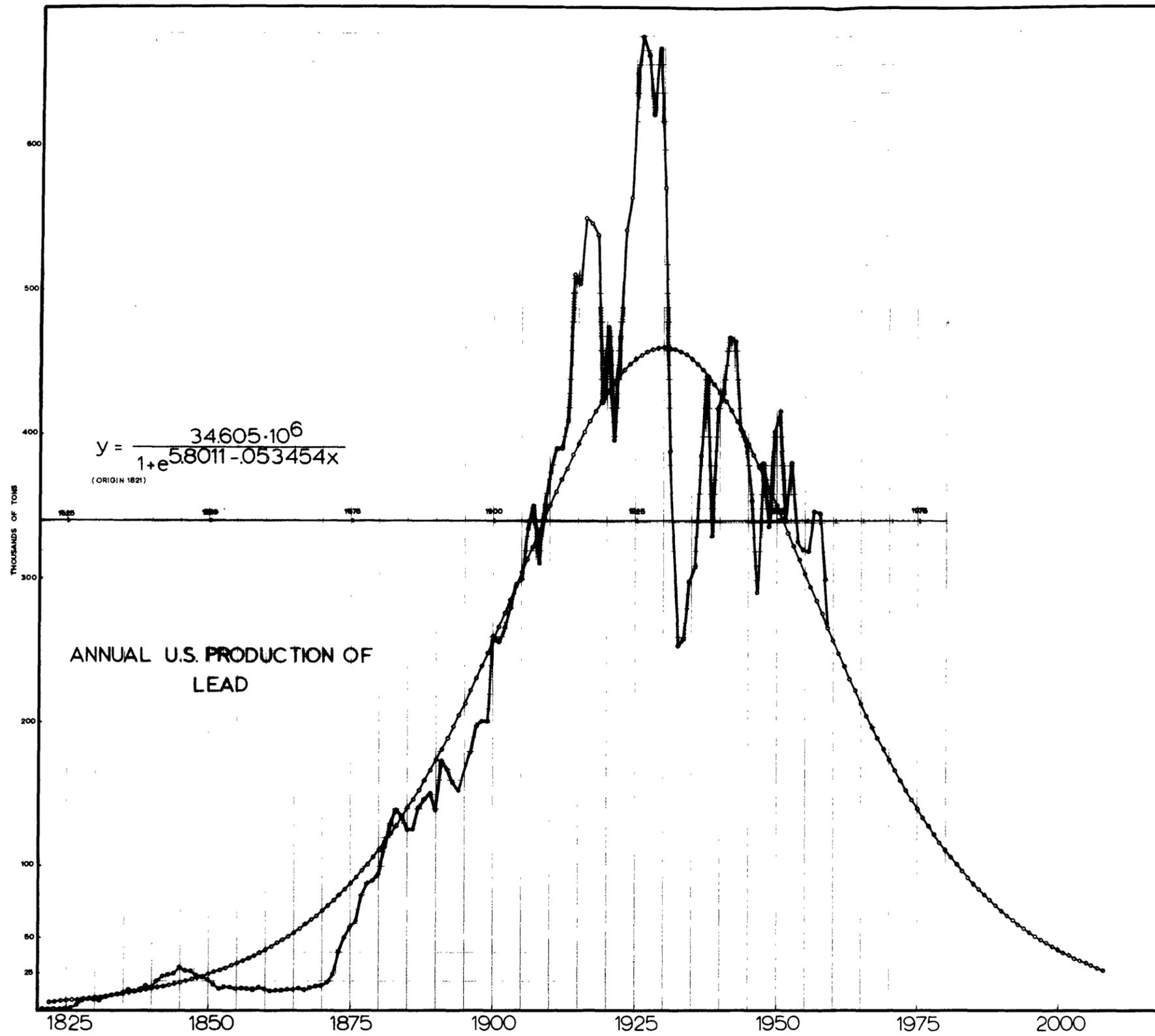


CHART 10. SYMMETRICAL LOGISTIC FITTED TO ANNUAL U.S. LEAD PRODUCTION.

fluctuations, the type of logistic curve which results from a least squares fit may not be suitable to represent the overall trend, and unless restrictions are placed on the parameters the third order form of the curve may give an asymmetrical growth-decline curve.

Similar conclusions were reached by Wilson and Puffer (1930) working with the first order curve, since in many cases they obtained negative asymptotes for their curves, i.e. declining curves resulted.

The Definition of the Upper Asymptote, K

In each of the foregoing sections, certain values have had to be used for K, the upper asymptote; in the case of cumulative mineral production the proper value is that of ultimate recoverable resources of the mineral or metal in question, and values have been used in this study which are derived from estimates of the nation's ore reserves, including measured, indicated, and inferred ore, but with only small allowances for new discoveries. Interestingly enough, first order logistic curves fitted by the three point method often yield values approximating these.

But to assume that ultimate discoveries of copper, lead, and zinc will amount to only a few more million tons is not in accord with the record of discovery over the last three or four decades, and gives little credit to the rapidly developing science of ore-finding. More than that, it assumes that the two or three years' reserves carried in many mines is their ultimate reserve rather than the amount of ore needed for current production. Swanson (1960) found for example that from 1945 to 1957, some 60% of the new lead and zinc ore discovered in the world came as additions to reserves in the operating mines.

It would seem then that higher values of the upper asymptote should have been used in the experimental fitting. This was tried for copper and lead with poor results from the curve, but even with better results there is the problem that unless one accepts the current reserve figures it becomes a matter of guessing what figure to use for ultimate resources.

As shown above, the allowable error in K is too small to give a well-founded curve under these conditions.

SUMMARY OF EXPERIMENTAL RESULTS WITH THE SKEW LOGISTIC

On a logical basis the logistic curve would seem to embody basic assumptions which are typical of the mineral extractive industry. Work with the symmetrical logistic has shown it to be unstable. Because of the complex of causes which condition growth and decay it would not appear likely that the two phases of the industry's life should be symmetrical, hence it was logical to fit the skew logistic and examine the results of experimental curves as forecasting tools. Within the range of the data, the curve gives a reasonable-looking graduation of the observations, but it was found to have the following failings which make it unsuitable for forecasting mineral production.

1. It is necessary to have a dependable estimate of ultimate recoverable resources. An error of 5% in this figure produces a considerably larger error in the forecast, for industries in the growth phases typical of the United States base metals. Such estimates are not available.
2. Errors in reporting, or a somewhat different early history of the industry would make a considerable difference in the forecast over 100 years later.
3. The habit of growth of an industry may change drastically from that described by a logistic fitted up to the most recent years, and no indication of the change will be given by the curve.
4. The accumulation of only 5 or 6 more years of recent data can change the trend line more than appears desirable. It thus becomes impossible to say to what final year of data the curve should be fitted, and the curve is considered unstable.
5. The root mean square deviation is not a useful measure of the suitability of any particular value of K for the asymmetrical curve.

6. For some time series the equation derived by the least squares procedure yields a curve which fits the data used but does not give a logical form of curve beyond the range of the data.

CONCLUSIONS

On the assumption that the long-term production of a mineral commodity obeys simple laws with respect to its distribution over time, it is possible to represent it by a simple implicit model. One curve which has been used as such is the logistic of the first and second orders.

Experimental calculations show that short-cut methods of fitting this curve are not dependable. Further calculations by the author, and by Wilson and Puffer (1930) show that least squares estimates likewise may not give dependable or reasonable results.

Because of the complex nature of the factors influencing mineral production over time, the skew logistic was fitted to production series, by least squares, with discouraging results. Instability of the curve with respect to errors in the estimated resource base, errors in the statistical series, and changes in the number of observations fitted indicate that, like the symmetrical curve, the skew logistic, as applied in this study, is not a useful forecasting device.

P A R T B

DEMAND FORECAST FOR LEAD

PART B

INTRODUCTION

In the previous part of this study it was concluded that the logistic curve, as an implicit model of mineral production in the United States, fails to be a useful forecasting device. One of the chief attributes of this approach to forecasting is that it could be used to predict domestic production directly. Any other method, except that of simple trend projection, must lead first to an evaluation of domestic demand for the forecast target year, and then an attempt to determine the percentages of the demand which will be satisfied by each of the three major sources of most metals (including copper, lead, and zinc) viz, domestic production of new metal, imports, and secondary metal or scrap. Each facet of this approach is a major study in itself, and while the general methodology will be similar for each metal studied, no general formula for forecasting, such as the logistic, can be evolved. The forecast of demand requires analysis of each sector thereof, a knowledge of current technology, and a review of foreseeable changes. The amount of scrap which may be expected can be calculated from a knowledge of the stock of metal in use in each major scrap-generating form, the time-lag of return, and the percentage return from each use. Of course, the amount of metal in use must be referred to the target date, taking the time-lag of return into account. Firm statistics on these factors are not available, and considerable research is necessary before they can be determined. The amount of metal which will be imported is a function of many variables, such intangibles as governmental policy on bartering, quotas, tariffs, and subsidies, the position of United States currency in the world, and the rate of industrialization in those foreign countries which now export mineral raw materials and in those which may become competitive markets of the United States for these

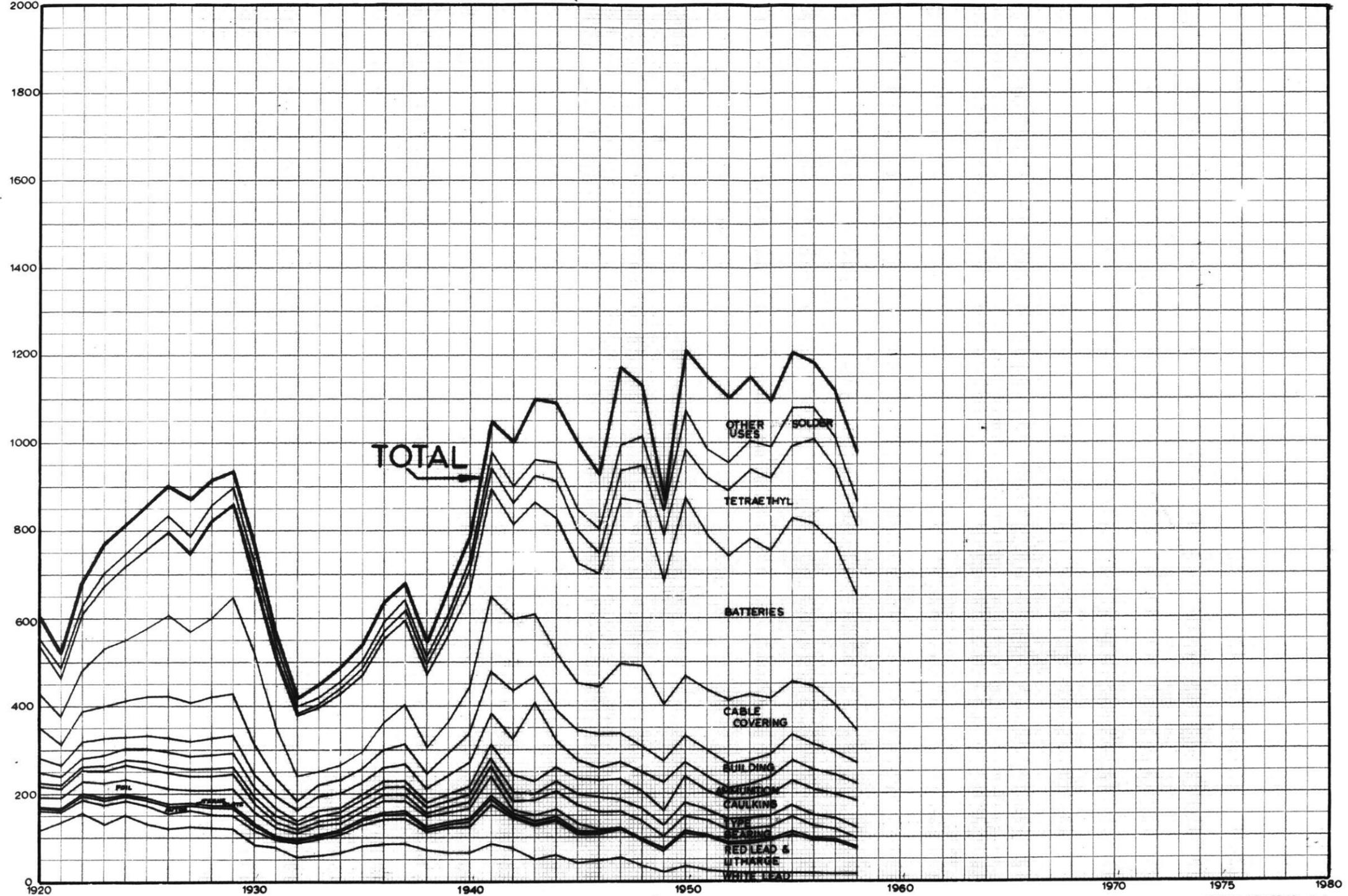
raw materials. Domestic production of new metal is not an independent variable but is logically related to conditions affecting imports, lower-priced foreign metal causing contraction of output, as well as a great number of other variables, and failing a simple model, is complicated to predict.

Unless one is willing to accept published estimates of demand for future years, which are often given with no explanation of methodology, or the barest description, he must begin the study of future United States metal production with an analysis of future domestic consumption. This is the aim of Part B. Time does not permit the complete analysis of the supply situation, so that the discussion thereof found at the end of Part B is only an outline of the areas which require further study and an attempt to approximate the amount by which each source may satisfy the predicted demand.

Because a demand analysis requires considerable detail it was necessary to choose a single mineral commodity for study. One of the most unique in its position in the United States today is lead, for in an expanding economy its consumption is failing to keep pace with national growth, while at the same time domestic production, as shown in Part A, is actually declining. Thus the many economic indicators used by business do not show a correlation of trend with lead, and it provides a more severe test of forecasting techniques than other metals that might have been chosen. Furthermore, the data available on lead consumption, while far from perfect, are much more complete than those for copper or zinc. Charts 11 and 12 show the consumption data reported by the American Bureau of Metal Statistics and Lead Industries Association respectively.

It is important to point out that a purely statistical analysis will not give dependable results. The method used in this study has required considerable acquaintance with the technology of the lead-using industries in order to make reasonable assumptions about their future demands for lead.

TONS $\times 10^3$



ENGINEERING, 1934-35, TO THE HALF INCH
WITH JEROME STEIN, FROM DRAWING OF
MADONNA, 1934-35, TO THE HALF INCH
DRAWING OF MADONNA, 1934-35, TO THE HALF INCH

CHART 11. ANNUAL U.S. LEAD CONSUMPTION

SOURCE: AMERICAN BUR. OF METAL STAT. YEARBOOKS

KEUFFEL & ESSER CO. N. Y.

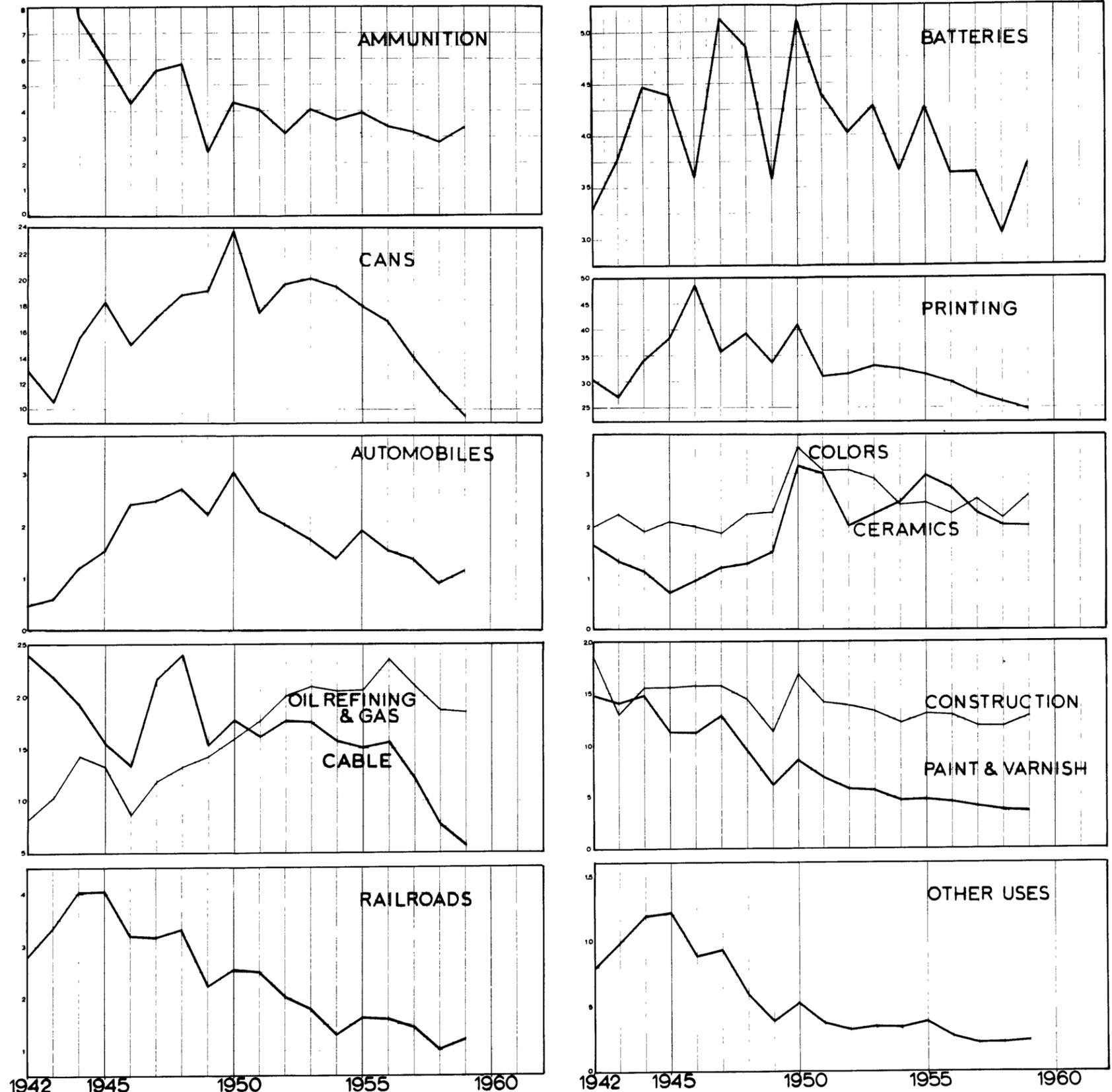


CHART 12. ANNUAL U.S. PER CAPITA CONSUMPTION OF LEAD BY INDUSTRIES (In pounds) SOURCE: LEAD INDUSTRIES ASSOCIATION

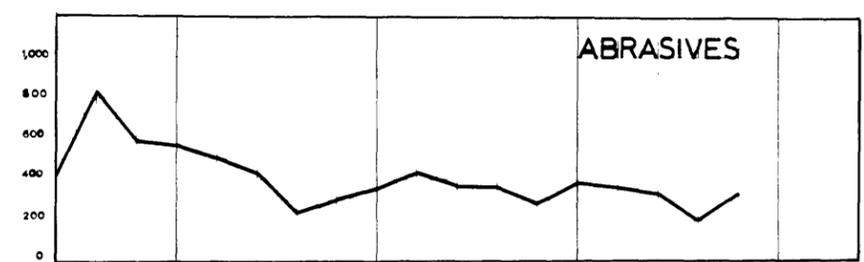
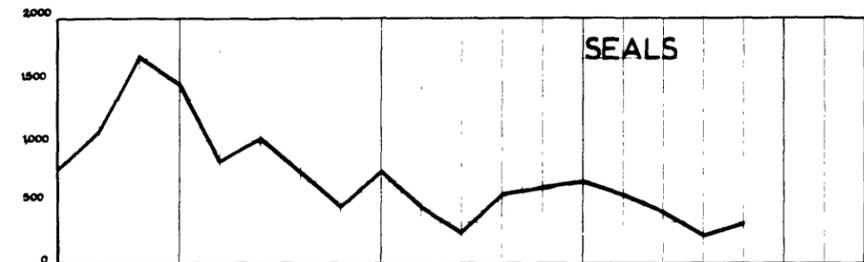
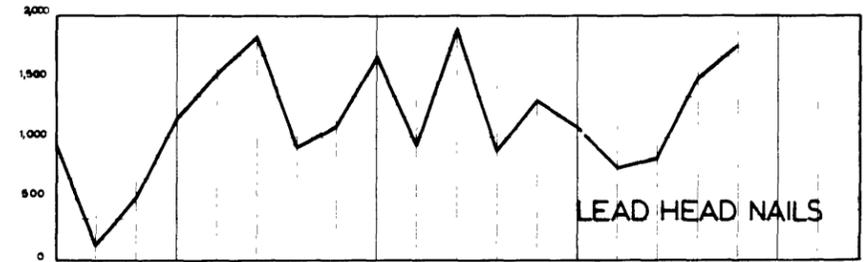
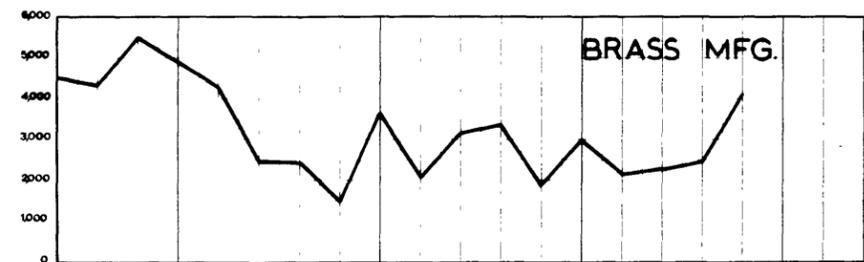
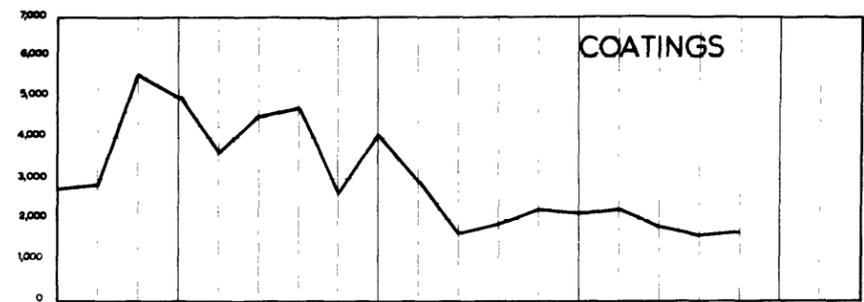
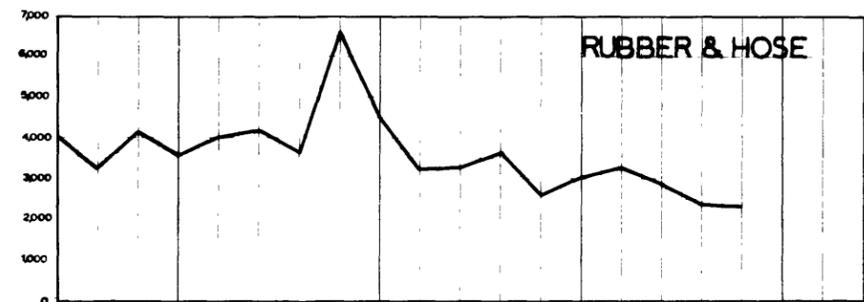
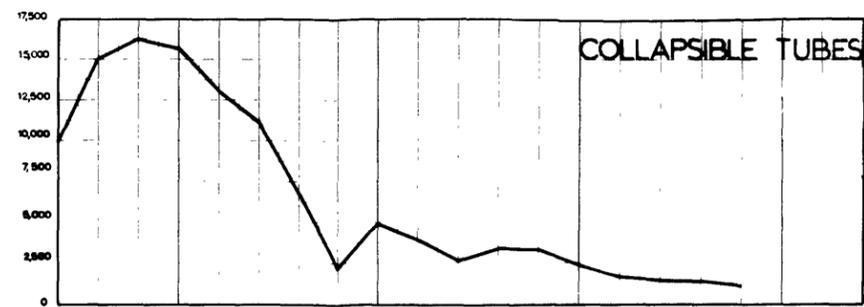
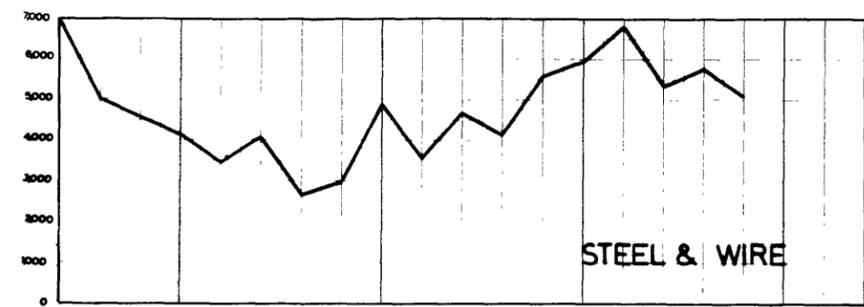
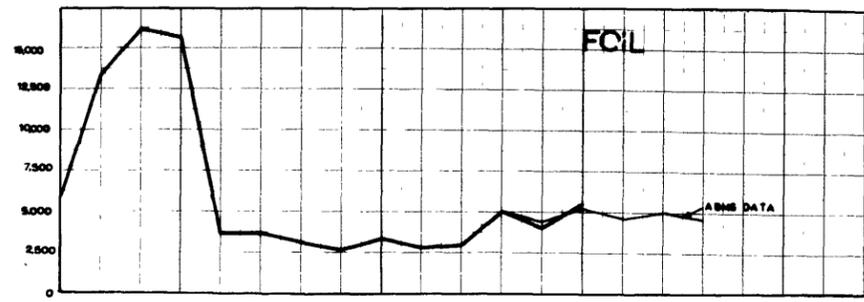


CHART 12a. ANNUAL U.S. CONSUMPTION OF LEAD - MINOR USES (in short tons) SOURCE OF DATA: LEAD INDUSTRIES ASSOCIATION

For example, the excellent correlation that exists between lead use for cable covering and the total miles of wire in telephone systems of the United States could not be of use in projecting lead demand for this application, since aluminum-polyethylene sheaths are supplanting lead.

The type of study used as the basis for the lead forecast is an end-use analysis. The dominant factors in the current level of lead demand in each major demand sector have been investigated in a quantitative fashion. The actual forecast relies in part upon the availability of authoritative projections of the level of the dominant factors, thus in fact relegating part of the responsibility for the forecast to the experts in each of several major use fields. This technique is justified, because it is obviously more appropriate to accept the estimate of the Highway Cost Allocation Study of Congress for the number of automobiles to be registered in 1975 than to attempt to make one's own forecast if this is not his specific field, and the principle applies to each use for lead.

As a check on the end-use analysis, which tends to give a low forecast because of difficulties in appraising the potential of new developments, a total forecast has been made by several methods. Reference has also been made to several published forecasts made in recent years.

GENERAL CONSIDERATIONS

Assumptions

As in every forecast, certain general assumptions must be made. The first and most important is that the American economy will continue at a high level of prosperity. This does not mean that the business cycle is presumed to have disappeared, but only that no major depression is assumed. Likewise, it is presupposed that the nation will not be engaged in actual war over the next 15 years, although continuance of the cold war is expected. Other necessary assumptions are described under the appropriate demand sector.

Population

In forecasts made on a per capita basis, a range of estimates has been used for the United States population not including armed forces overseas (forecasts including the latter were reduced by one million). The lowest is that of the Census Bureau, 205,907,000, the highest that of the United Nations, 242,880,000. About 220,000,000 seems to be the figure most often indicated. (See Appendix).

Inventories

No account has been taken of changes of inventories in this study, mainly because no detailed statistics exist. It must be assumed, although in some years, such as 1950, incorrectly, that the amount by which stocks on hand change from year-end to year-end is unimportant for the purpose of this study.

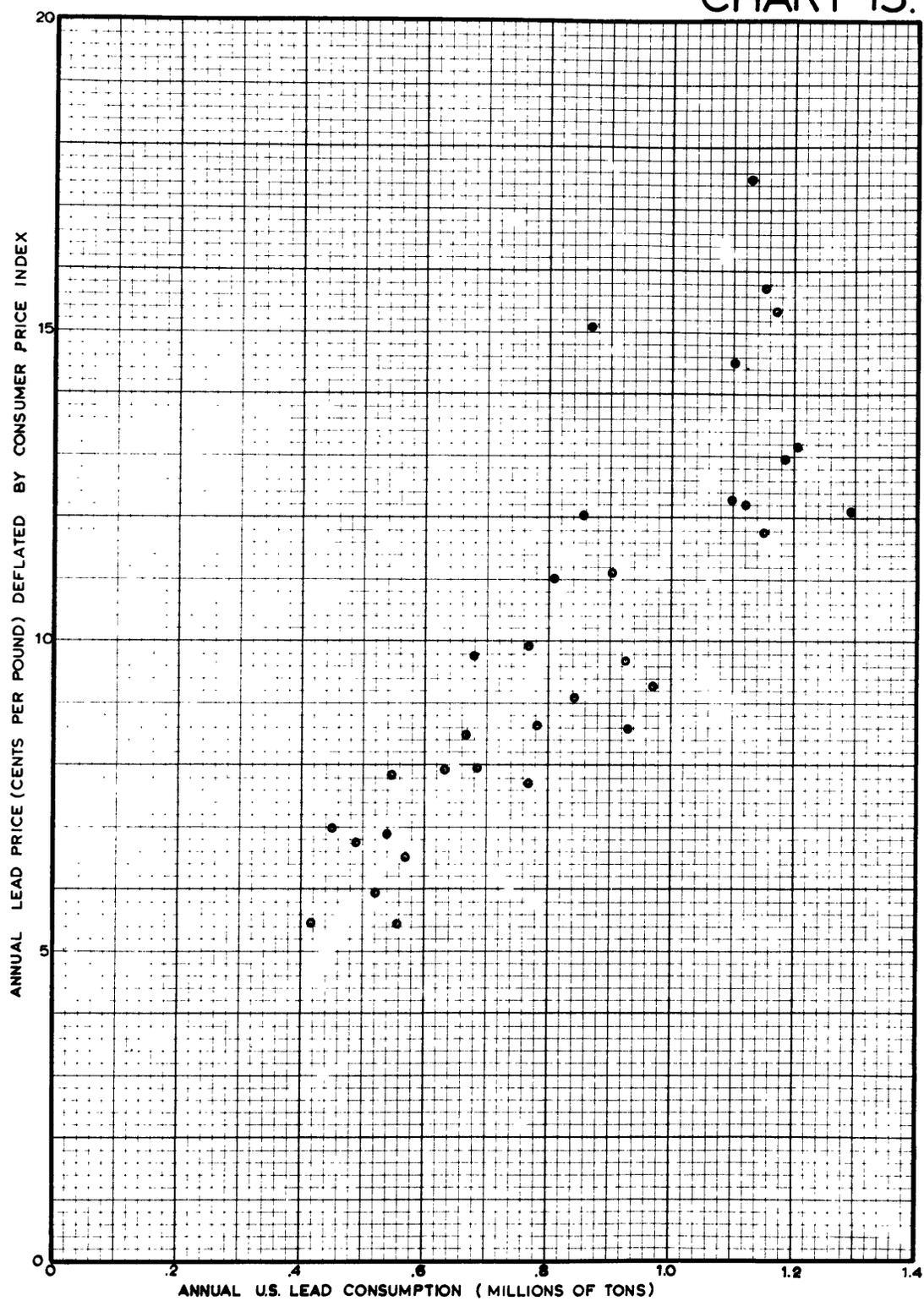
Trend Lines

In all cases where trends were fitted, the method of least squares has been used. In charts showing the trends, the line is solid over the years fitted, dashed otherwise, with a few exceptions as noted. Data used are given in the Appendix, and are in terms of lead content except as noted.

Price

Classically, consumption is an inverse function of price, and plotting one against the other should indicate a demand curve, sloping downward to the right, so that the higher the price the less is demanded. Chart 13 shows the historical data for lead, prices being annual averages as published in Metal Statistics, deflated by the Consumer Price Index to constant dollars; production data that of the American Bureau of Metal Statistics. That the trend slopes upward to the right is at first encounter surprising, but on consideration is entirely logical, since an increasingly industrialized society requires more metal of almost every kind than previously, and an increasing number of consumers adds to this trend as the population grows. These factors outweigh that of price at the levels at which lead has sold.

CHART 13.



Where lead has been replaced by other materials, it will be shown later that it is generally due to technology rather than price.

On the shorter term basis, an excellent summary by Green (1958), and another by Hendricks (1960) indicate that the complex interaction of demand, producers' and consumers' stocks, politics, and international reactions on the price structure is such that the derivation of even a short-term demand curve becomes difficult. Furthermore, should a long-term curve be obtained, it is no help in forecasting future lead demand, since price is probably more difficult to predict than demand. While several experts (e.g. Shea (1950) and Swanson (1960)) conclude that the real price of lead will continue to rise, this is also expected for the other metals and their relative costs will probably not change importantly. The prices of other materials which compete with lead are likewise difficult to predict. Thus, price has not been considered in this study beyond the above analysis.

Substitution

Linked to the question of price is that of substitution for lead by other materials. Unlike the competition of aluminum with copper in electrical applications or with steel in construction the cost "per unit of performance" is not calculable for most uses of lead, and the basis of replacement is most often technological rather than purely economic. In addition, where lead is suffering from competition, a host of competing products may be involved, as for example in chemical plant construction. Where substitution is important, it has been discussed in available detail under the appropriate demand sector, but it must be noted that cost data on competing products was not always available and that it usually requires an expert's skill in any given field to evaluate the actual cost of each competing product per unit of service.

On the other hand, lead may penetrate markets held by other materials. Unless one includes the growing use of

tetraethyl as a form of substitution for high-octane "cracked" gasoline, or the use of lead-base frits in enamelling, which was a market held by lead twenty-five years ago, lost, and now being regained from the other types of enamel developed, no case of lead being substituted for other materials in important amounts was encountered. Lead and concrete may be said to be competing for use as gamma radiation shielding in reactors.

SOME COMMENTS ON THE QUALITY OF AVAILABLE
LEAD CONSUMPTION STATISTICS

Any forecast is only as good as the data on which it is based. A major factor in favor of the forecast of lead demand in toto is the difficulty of obtaining dependable end use data, and the near impossibility of locating continuous long-term series which can be clearly and logically analysed for demand trends and which still have a high degree of accuracy. In the end use analysis of lead demand which follows, two categories indicate the amount of consumption unaccounted for - "Unclassified", and "Estimated Understatement"¹, but within each end use classification there is further error, which is more difficult to quantify. Also, if it were practical, the data should be collected in a different manner.

Consumption Statistics Available

Two main sets of statistics of domestic lead consumption exist. The first is that of the American Bureau of Metal Statistics (ABMS) (See Appendix), which is in part on a product basis, e.g. "White Lead", in part on an industry basis e.g. "Building", and is available from 1919. This series corresponds mainly to that of the United States Bureau of Mines, (USBM), except that some further distribution of unclassified consumption has been attempted, and in some divisions, several Bureau of Mines classifications are combined. The USBM accounting system changed in 1948, hence long series of data are not available from this source. Because of the diverse uses of such products as bearing metal and litharge, the product part of this series of data was rejected after extensive testing, mainly by regression analysis, as too general to forecast. As will be seen, some of the

¹ The classification herein called "Other Uses" includes consumption divided among the several end uses of such slight magnitude that they are singly unimportant for forecasting purposes. These currently account for a total of only 2% of total annual demand.

industry series were used of necessity.

The second set of statistics is the one used for the most part in this study; it is an attempt, begun in 1942 by Lead Industries Association, (LIA), to divide domestic consumption into sectors on an industry basis (see Appendix). Important difficulties exist. Data is collected on the basis of reports to the Association by manufacturers of lead raw materials, such as bearing metals, litharge, etc., and classified by the type of manufacturer of finished or semi-finished material to which the raw material was sold. The "Ceramics" series provides an example of the resulting difficulties. Litharge assigned to this series may have been purchased to be used in enamelling aluminum, making glass, or glazing ceramics, and each of these uses shows a different trend. Different indices are more closely related to growth in one or another of these applications than in the others, but it is impossible to separate the trends, and it is necessary to forecast the ceramics use as a whole. The generalizations which have had to be made are felt to be more serious than would be the further errors introduced in forecasting a somewhat larger number of sectors of use, but there is no choice beyond the level of subdivision used in this study, as based on currently available statistics.

The "Unclassified" series is ubiquitous, and has been dealt with here in a similar manner to any end use classification. It amounts to some 10-15% of total consumption.

Errors in LIA Statistics

Not all consumption is reported to the LIA, since some consumers are not members. By comparison with the USBM series, an estimate of understatement of demand is made, which amounts to 6-12% of total demand. In the ABMS statistics, this has been approximately distributed in the use classifications, and in some cases this procedure results in quite different trends than those of LIA. Where this is so, the fact is mentioned below and the most appropriate series is used. If an ABMS series is used, the LIA "Estimated

Understatement" is corrected by a corresponding amount.

It must also be noted that the ABMS includes in their accounting as lead the antimony content of hard lead and other alloys. The LIA does not follow this practice, and a correction has been made for this in calculations where it is important.

Such problems as duplications in accounting are considered by the Association, and must be assumed to have been eliminated in this study since it is impossible to appraise the magnitude of such errors.

The errors due to understatement, unclassified sales, and duplication are not equally distributed among the industry classifications. The accounting of consumption for oil refining and gasoline is much more reliable than that for batteries, because of the type and size of manufacturer to whom sales are made. It has been estimated¹ that while the former is essentially 100% correct, as much as 10% error could exist in the latter due to the many small shops which make and repair batteries and which are not reported.

Errors in ABMS Data

A glance at the statistical appendix shows that the ABMS data has some serious flaws. For example the classification of lead used in railway cars would be assumed to include journal bearing metal, but the accounting shows this use dimishing to nothing in 1938, while bearing metal is still used even in some new cars. Automobile production uses about 10,000 tons of lead annually, yet the ABMS data show a decline to 1,000 tons in 1943 and no further accounting after 1947.

Up to 1947 the lead content of lead ores used directly in manufacturing lead compounds was not included in ABMS data. A blanket statement also accompanies the statistics to the effect that "The theory of the above accounting is end-use, but inconsistencies occur".

¹Mr. David Borcina, Lead Industries Association, verbal communication, April 11, 1960.

Errors of Generalization

In cases where one major consuming factor uses most of a product, the error introduced into the forecast by attempting to handle each minor use separately is often more serious than basing the forecast on the one factor. For example, 90% of battery lead is "consumed" by the automotive industry, the rest being used in many different industrial applications, and a prediction based on automotive applications as representative of total use is a generalization which introduces some error. To try to compensate this type of error, the less important uses of lead within each demand sector have been considered from the point of view of potential growth, and where this is important the generalized forecast for the sector has been adjusted.

It is to be hoped of course that all the errors mentioned will tend to cancel each other. Even if they did, it would still be preferable to know more closely the actual differentiation of the uses of lead in most of the classifications considered. Failing this, the errors and limitations of the data should be kept in mind when evaluating the forecast.

Distortion Due to Inventories

While not really in the class of errors, the distortions due to the method of collecting data are summarized here, and may be seen in several of the series. Because consumption is recorded by LIA on the basis of raw material manufacturers' sales, for domestic use, changes in the inventory of raw material held by producers of finished or semi-finished products affect the apparent relation of consumption to any given index. The most striking example can be seen in many series at the start of the Korean War (1950) when producers with memories of the restrictions on raw material inventories of World War II bought large inventories and manufactured them for their own stockpile, and in 1951 and 1952 when it was realized that the strict controls would

not be reinstated and apparent consumption fell off as inventories were sold. This type of distortion makes it more difficult to recognize correlations of certain series.

Distortions Due to Exports

In some cases, products made from lead in the United States may be exported, e.g. some 200,000 batteries are exported annually, and the exact lead content thereof is not determined. The three reportings mentioned are described as statistics of domestic consumption of lead or consumption of lead in the United States, but make no allowance for the lead content of manufactured goods exported. Since this data is not available and the amount of lead involved is small and probably reasonably constant each year, no account has been taken of these exports in this study.

CORRELATION METHODS IN LONG-TERM FORECASTING

Having considered the use of an implicit model for forecasting mineral production, the next method to be examined is that of the explicit model, as described earlier. In that sense, the model which is derived can vary widely in type, and degree of refinement, being limited mainly by the available data and the time and experience of the forecaster. There is still much to be learned and developed in the theory of econometrics, which combines the theory of economics and the methodology of statistics, and this too imposes a practical limit.

It should be pointed out that the purpose of this study has been to find, if possible, a practical method of forecasting mineral production for the long term. The economic model was examined in that light, and when it was found that it adds more difficulties than are justified by the possible improvement of results, the idea of using a formal model was discarded. This was done at an early stage, since one of the greatest problems is simply the recurrent one of long-term trend, and this was met with at the outset.

* * *

By deciding on a logical basis what factors are important in determining lead demand in any given product, the principles of simple and/or multiple correlation can be used to derive an equation expressing lead demand in terms of these factors, and a coefficient which is indicative of how closely lead demand follows the movements of the factors. Any standard statistical text describes the mathematics involved. Having determined the correlation relationship or regression line, as it is termed, a forecast of lead demand can then be made by substituting in the equation reliable forecasts for the controlling factors. If a series of such equations are obtained and they are a set of simultaneous equations, the result is termed an econometric model, while if the equations are independent it is more properly referred to as a mathematical model (Beach,

1957).

Theil (1958), Klein (1953), and many others have devised advanced econometric models for various sectors of the economy as well as the economy as a whole. Because the problems encountered at a much simpler level in this study discouraged use of an econometric model, it is sufficient here to refer the interested reader to the works of the above authors, and to point out that the statistics available for lead consumption are not dependable enough to warrant the large amount of work involved, which would probably amount to several years, that forecasts which are available for the dominant factors might not work properly in combination with one another in the model, and that the record of forecasts made to date on the basis of econometric models has not been good, short-term forecasts being better in general than long-term. (See for example Adams, 1951).

An example of a mathematical model used to forecast metal consumption is that of Rosenzweig (1959), in a study done on aluminum. Consumption was related to GNP, to the FRB Index, to price, to a measure of possible substitution, and to time. This approach is much simpler than the econometric model, and results in a series of unrelated equations which are easily handled.

The first attempt at correlation analysis to construct a mathematical model for lead demand was to relate total consumption to some index. Since lead is used in such a variety of industries it should be correlated to the Federal Reserve Board Index of Industrial Production. The scatter diagram, chart 14, shows a fairly good correlation, but it is decidedly not linear. The problem immediately becomes one of choosing the form of relationship to use.

Examining the two series, chart 15, it is apparent that although the fluctuations or cycles are rather well correlated, (with the exception of the Korean War period), the trends are divergent, which accounts for the nonlinear

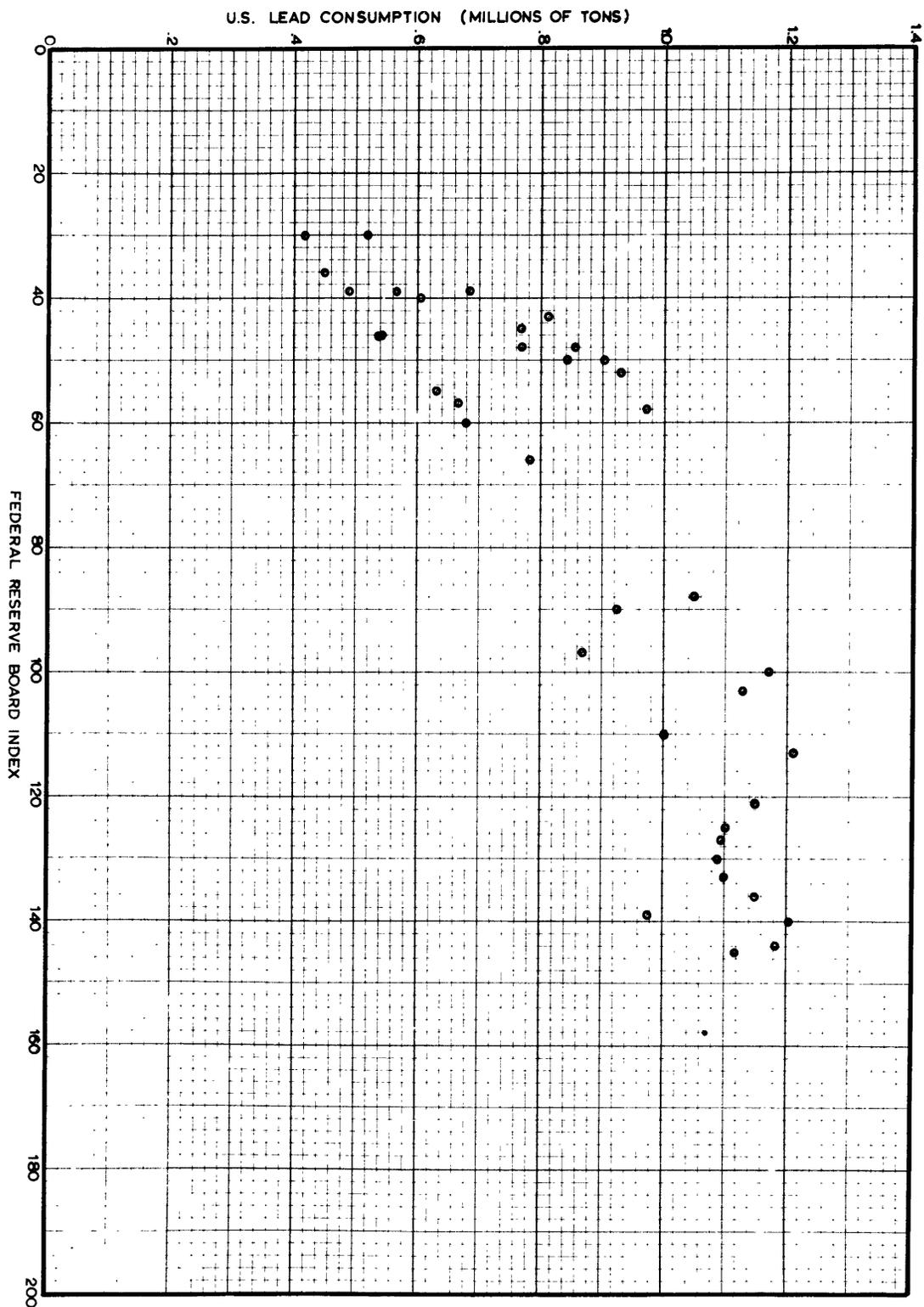


CHART 14.

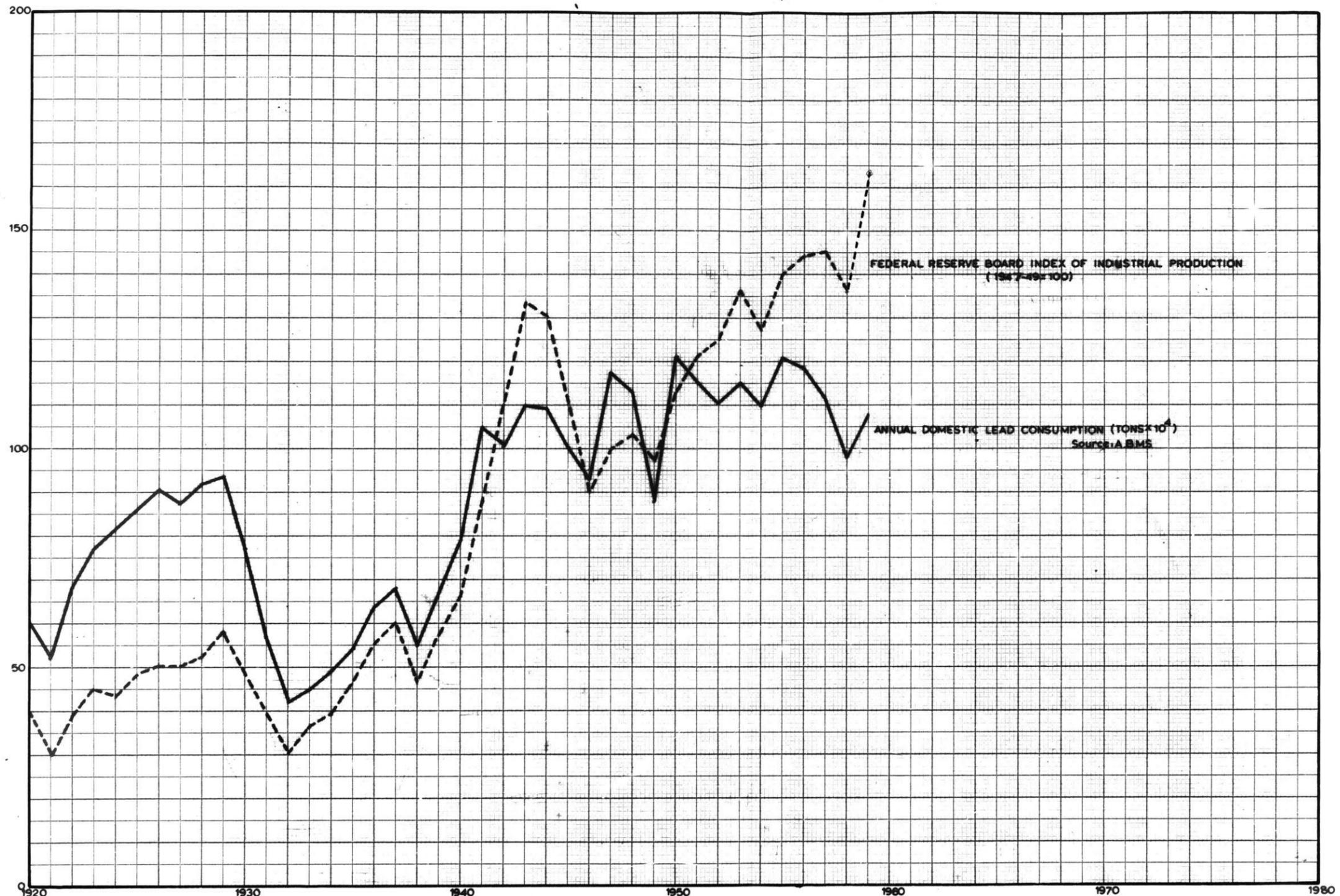


CHART 15. LEAD CONSUMPTION AND THE F.R.B. INDEX

KEUFFEL & ESSER CO. N. Y.

correlation. The problem can therefore be stated as one of how to deal with the trend. Croxton and Cowden (1955, chapter 22) give a good summary of some of the methods commonly used, and these were tried with various series of lead consumption data. Some of the results will be described here to outline the problems involved and explain why other methods were finally decided upon.

A linear correlation of total lead use (ABMS data from 1923 to 1955, not including 1941-1945) and the FRB Index of Total Manufactures gave a coefficient of 0.853940 using the Pearson product-moment formula. A multiple linear correlation including time as a variable was then considered, since the trends shown in chart 15 are not parallel. One of the great problems of correlating time series is that the trends influence the coefficient of correlation. If the trends are parallel the coefficient will be high even if the cycles are negatively correlated, or if they diverge the correlation will be low. What the inclusion of a linear time factor means is that a straight line is used to represent secular trend for each of the two series, so that any forecast based on the relationship derived assumes that the two straight line trends will continue. The forecasted value of the FRB Index must therefore lie on the straight line trend extended or distortion is introduced; one is not free to use the index values forecasted by experts in the field unless they fall on the trend. A logarithmic relationship might also be used but the same conditions apply. Furthermore there is rarely justification for using the logarithms of time, and if multiple nonlinear correlation is used the calculations become heavy, while the nonlinear relationships are difficult to justify.

By using partial correlation coefficients, the effect of each variable of a multiple correlation can be examined separately, and of course it would be of interest to examine the Index-consumption relationship with the time factor held constant. The coefficient of partial correlation,

$$r_{12.3} = 0.764$$

where x_1 = total lead consumption

x_2 = FRB Index

x_3 = time, which is held constant, should

indicate the correlation of the Index and lead consumption with the effect of time removed. But the device of holding the time variable constant is not effective, since there is an intercorrelation between time and the FRB Index ($r=0.854$) and time and lead use ($r=0.686$), hence it cannot really be held constant.

The correlation coefficients are also higher in value than they should be, due to autocorrelation or serial correlation of the variables, i.e. the values of the variables depend upon their previous values and are not independent observations (see Durbin and Watson (1951) for a statistic for testing for serial correlation and Ezekiel (1941) for a discussion of its effects). This problem is typical of most time series in which trend is important.

Upon examination, it was found that other methods of removing the trend were useful for investigating the degree of correlation existing but of no help directly in forecasting. For example, the third panel of chart 18 shows the absolute annual changes in the FRB Index and total lead consumed, and these show a high correlation, but having computed the regression relationship it is of no use in long-term forecasting of the level of total lead consumption since the relationship does not depend on the total value of the index and consumption but only on the amount of their annual changes. If it were possible to forecast the cycles of the Index over the long term, cycles in lead use could be forecasted on the basis of the relationship. Nonetheless, this is not the primary interest in this study.

If the fluctuations of one series are greater than those of the one to which it is being correlated, dividing both series by their standard deviation makes the amplitudes more nearly equal without affecting the correlation coef-

ficient. Also, the assumption implicitly made in the inclusion of time as a variable in multiple linear correlation is that absolute deviations from trend should be correlated, whereas relative or percentage deviations may be more suitable. Chart 16 shows the effectiveness of correlating per capita solder use of lead (ABMS data) and the FRB Index as percentage deviations from trend in terms of their standard deviations. (A logarithmic trend was used for the Index, a straight line for solder use). While this is helpful in illustrating that a correlation exists, like the method of using absolute deviations it does not help directly in forecasting.

It is concluded therefore that although multiple correlation is useful when two or more variables are important in determining the value of a third, the problem of secular trend and the accompanying autocorrelations and intercorrelations of the variables is a major deterrent to its use for time series.

To include time as a variable assumes a certain trend, usually linear, for the variable to be forecasted, so that the effect of the correlation is only to give some measure of the cycles, which not only are unimportant compared to the trend but can be forecasted no better over the long term for the other variables than for lead demand.

The author's conclusion may be compared with that of Rosenzweig (1959), who found that the trend was not sufficiently important to upset the forecast. This contradiction validates the opinion that lead consumption forecasting is an excellent test of techniques, since almost any general indicator commonly used in United States business shows a long-term uptrend, while lead shows a downtrend. That the GNP and aluminum consumption show parallel growth up till now is no proof of a causative relationship nor a guarantee that they will continue to do so. It is likewise not conclusive that business cycles occurring in both of two series mean that the series are really correlated since all sectors of the economy are usually affected by the cycles. On the

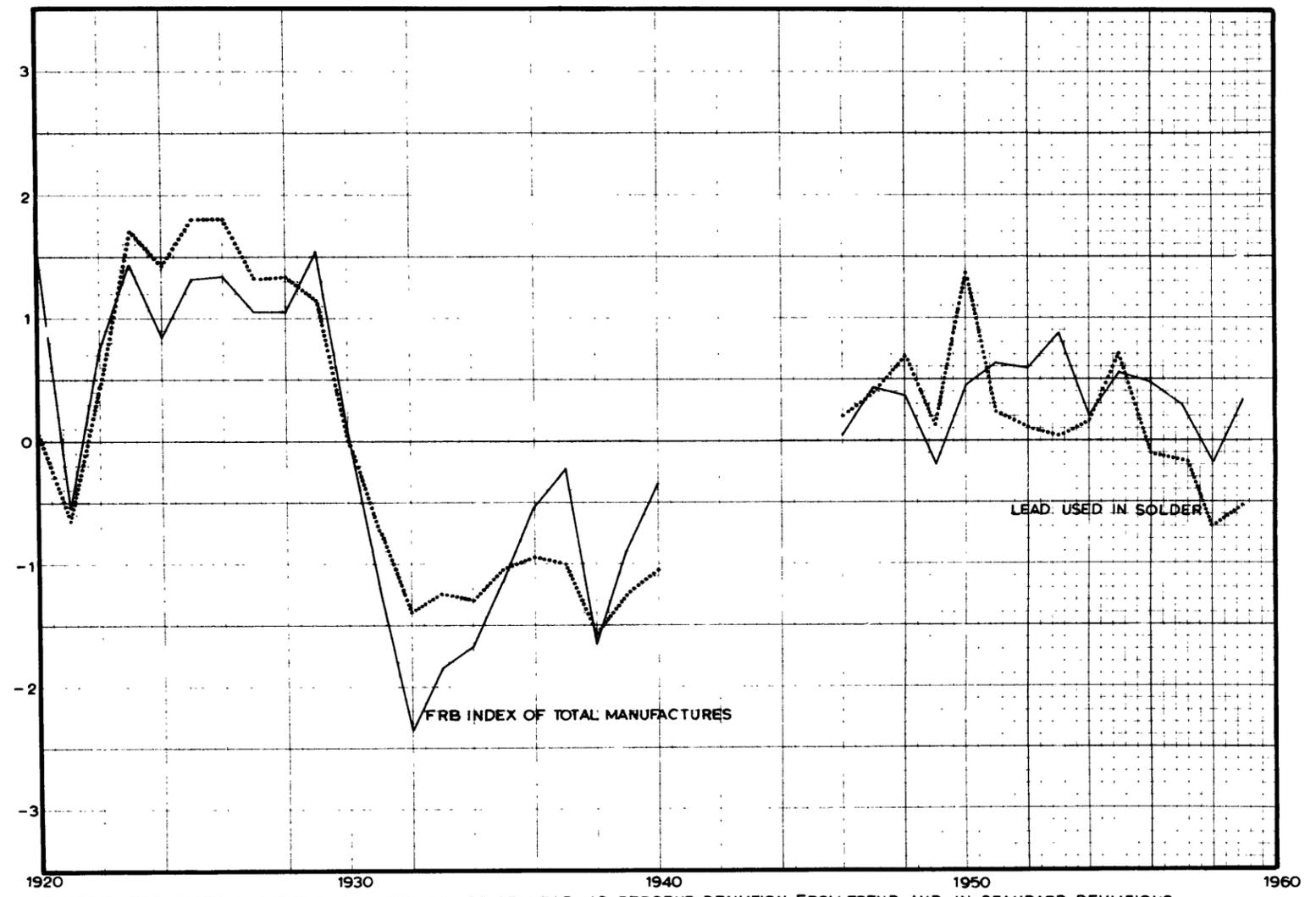


CHART 16. FRB INDEX AND PER CAPITA SOLDER USE OF LEAD AS PERCENT DEVIATION FROM TREND AND IN STANDARD DEVIATIONS

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other hand, some assumptions must be made as a basis on which to work.

Finally, one example can be given of the oversimplification which may result from using correlation techniques. Since 90% of the lead used in batteries is used in automotive types, this use of lead should be well correlated with auto-registrations, and the scatter diagram, chart 17, shows that this is so. The correlation coefficient equals 0.9106, and the equation is:

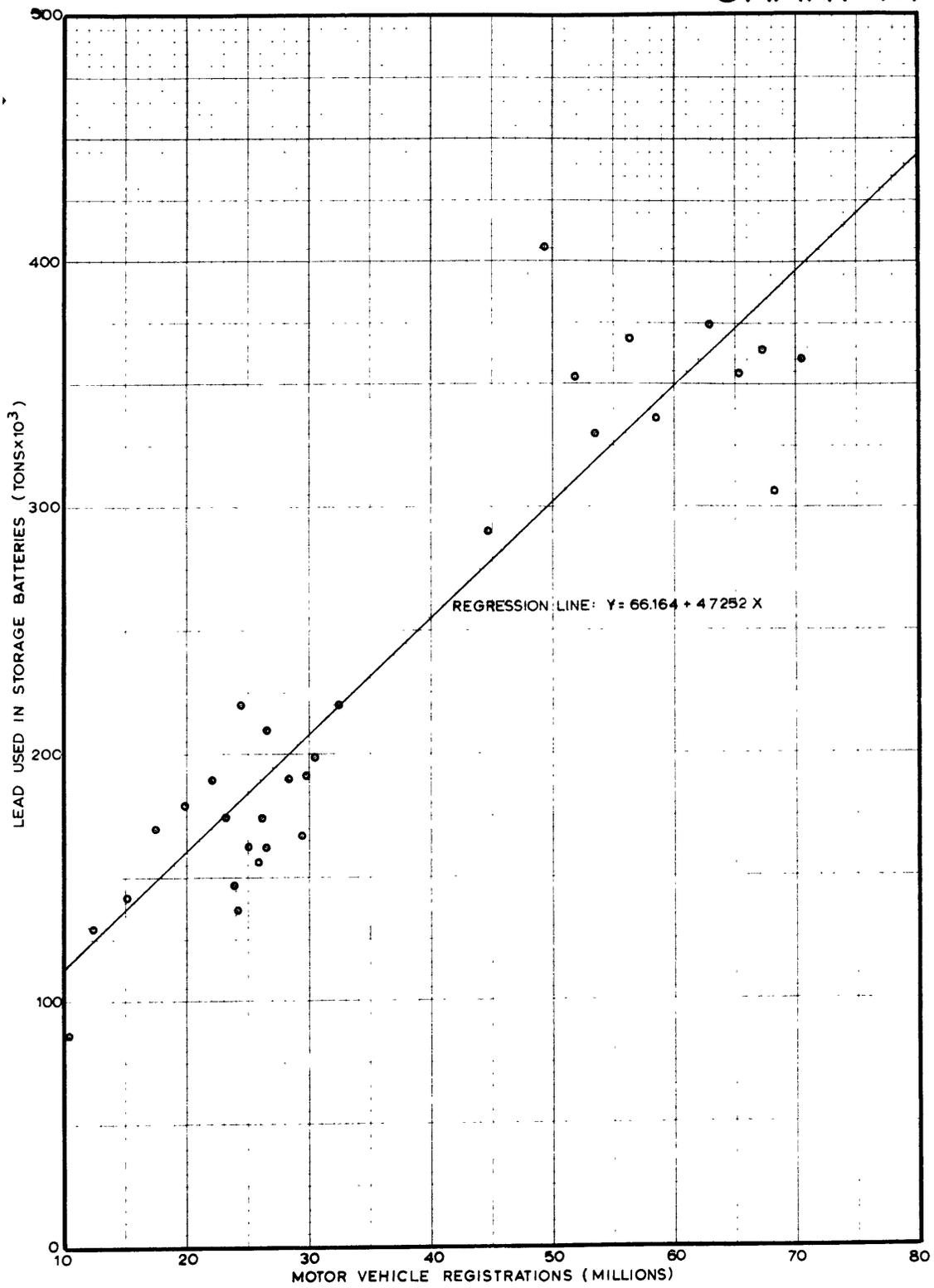
(Thousands of tons of battery lead consumed) =

$$66.164 + 4.7252 \text{ (Millions of motor vehicles reg.)}$$

The standard error of estimate is 38,850 tons. Using a forecasted value for automobile registrations of 111,162 thousand¹ in 1975 indicates a consumption of 591,427 tons of battery lead for that year. If this method of forecasting the consumption is now compared with that described in part B under Storage Batteries, the disadvantages and possibilities of inaccuracies of this statistical method are obvious. The greatest advantage is the ease of calculation, either by hand or on a computer, compared with the rather long time involved in collecting data for the other analysis. That the two forecasts fall rather close to one another is attributed to the fact that the number of vehicle registrations is indeed the most important quantity influencing battery lead use, but it is important to investigate the other possible variations such as changing weight per battery. Should two or more factors which vary simultaneously be found important, multiple regression could then be used, but the method used here seems as good, and possibly easier to understand.

¹ Source: Third Progress Report of the Highway Cost Allocation Study.

CHART 17.



TOTAL LEAD CONSUMPTION FORECAST

Lead Consumption and the F R B Index

Chart 18 shows that there is clearly a high correlation between the absolute annual changes of the FRB Index of Total Manufactures and those of consumption of lead over the period 1920-1959. This would be expected, since lead is used in so many manufactured articles and manufacturing processes included in the Index. Differences in relative amplitudes can be attributed for the most part to the effects of inventories and price on short-term demand, described by Green (1958), as mentioned above. But a comparison of the actual Index and total lead consumption (chart 15) shows that the long term trends are diverging. To take advantage of the close relationship noted, as a means of forecasting, some quantitative description of the relatively declining demand for lead as against the general index must be derived, and this was done by examining the ratio of total lead consumption to the Index, giving tons of lead consumed per FRB Index point (chart 18). The decline in this relationship since 1920, and since 1946 as well, is clear.

Three trend lines were fitted:

- I. A straight line, using data from 1920 to 1958, but excluding 1941 to 1945, since wartime uses disrupted the relationship.
- II. A straight line to the logarithms of the ratios, using the same data.
- III. A straight line, using data from 1946 to 1959 inclusive.

Results of these calculations are summarized in table 1, which also shows the values of the ratio for 1965, 1970 and 1975, and forecasts based on these ratios and the forecasted values of the FRB Index¹.

¹ Sources of Index forecasts are: Brown and Hansen, (1957), and Roos (1957).

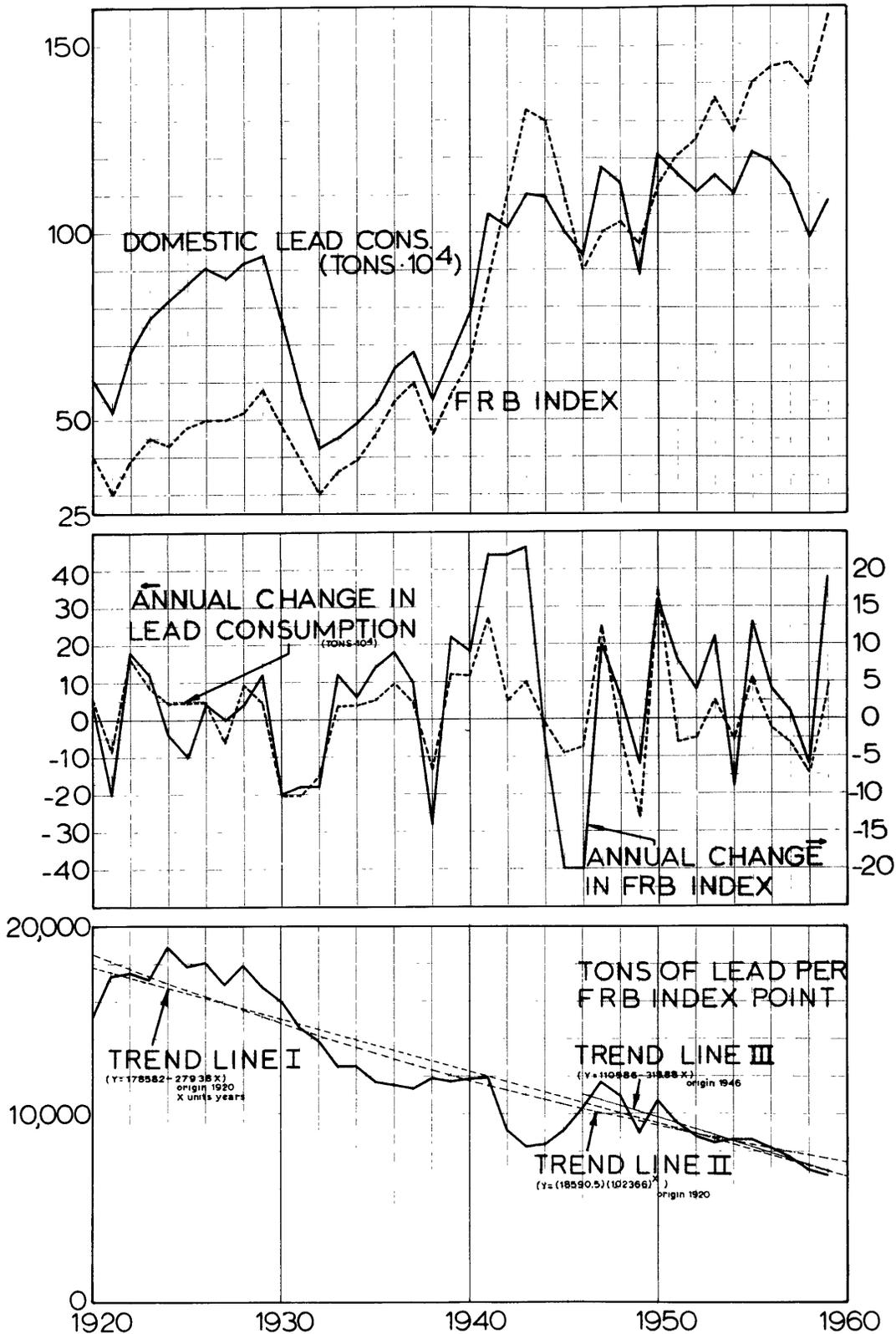


CHART 18. LEAD CONSUMPTION / FRB INDEX RATIO

Table 1. Trends Fitted To Lead Consumption - FRB Index Ratio, and Forecasts.

Trend Equation	Period Fitted	Standard Deviation	Forecasted 1965	Forecasted 1970	Ratio for 1975
I $y=17,858.2-279.38X$	1920-59	1214.8	5286.1	3889.2	2492.2
II $y=(18590.5)(1.02366)^X$	1920-59	1202.9	6494.3	5944.2	5303.7
III $y=11,098.6-318.88X$	1946-59	578.4	5039.9	3445.5	1851.1

	Forecasted FRB Index			Lead Demand 1965	Forecasted Demand (Tons per year)		
	1965	1970	1975		1970	1975	
I	197	225	255	1,041,362	876,070	635,511	
I	199	244	298	1,051,934	948,965	742,676	
II	197	225	255	1,279,377	1,337,445	1,352,433	
II	199	244	298	1,292,366	1,450,385	1,580,491	
III	197	225	255	992,860	775,237	472,023	
III	199	244	298	1,002,940	840,702	551,616	

y = Ratio of Tons of Lead Consumed to FRB Index of Total Manufacturers;

X = Time, for Equations I and II (year -1920), for Equation III (year -1946).

While the standard deviation of the trend fitted only to post-war data is least, it is felt that the high rate of market losses suffered by lead in this period will not be maintained in the future period, since the main force of introduction of new materials and technology originated during the war has been met over the past 14 years. This trend line would reasonably be expected then to give considerably too low a forecast by 1975, and probably by 1965 will already be too low, depending on the rate of substitution in markets which will be almost completely lost, e.g. cable covering.

A straight line trend indicates a change by a constant absolute amount per year. While this habit of change may persist for short periods, it is more likely that change will occur in many cases by constant relative amounts. This is to be expected in the case of lead use. What chart 18 indicates

is the rate of substitution for lead, and as more markets are lost, lead producers are attempting more and more strongly to retain the remaining ones, and find new applications, by strong selling, increased research and development, and probably ultimately by accepting a reduced price for their product. The rate of market loss is thus expected to be best approximated by the straight line to the logarithms of the ratios, trend II, and this in fact gives a satisfactory-looking curve, with a slightly smaller standard error than a simple straight line, and a forecast for 1975 of 1.35 to 1.58 million tons.

Per Capita Consumption

Another method of estimating future demand for lead is by combining per capita consumption and population levels. Many forecasts of the latter are available. Chart 19 shows a series of straight line trends fitted to per capita consumption of lead in pounds. Table 2 summarizes the trends obtained, the periods fitted, and the forecasted values for each trend using a population forecast range of 205,907,000 to 242,880,000. It was expected that all of these trends would not give "reasonable" projections, but each was fitted to illustrate a point.

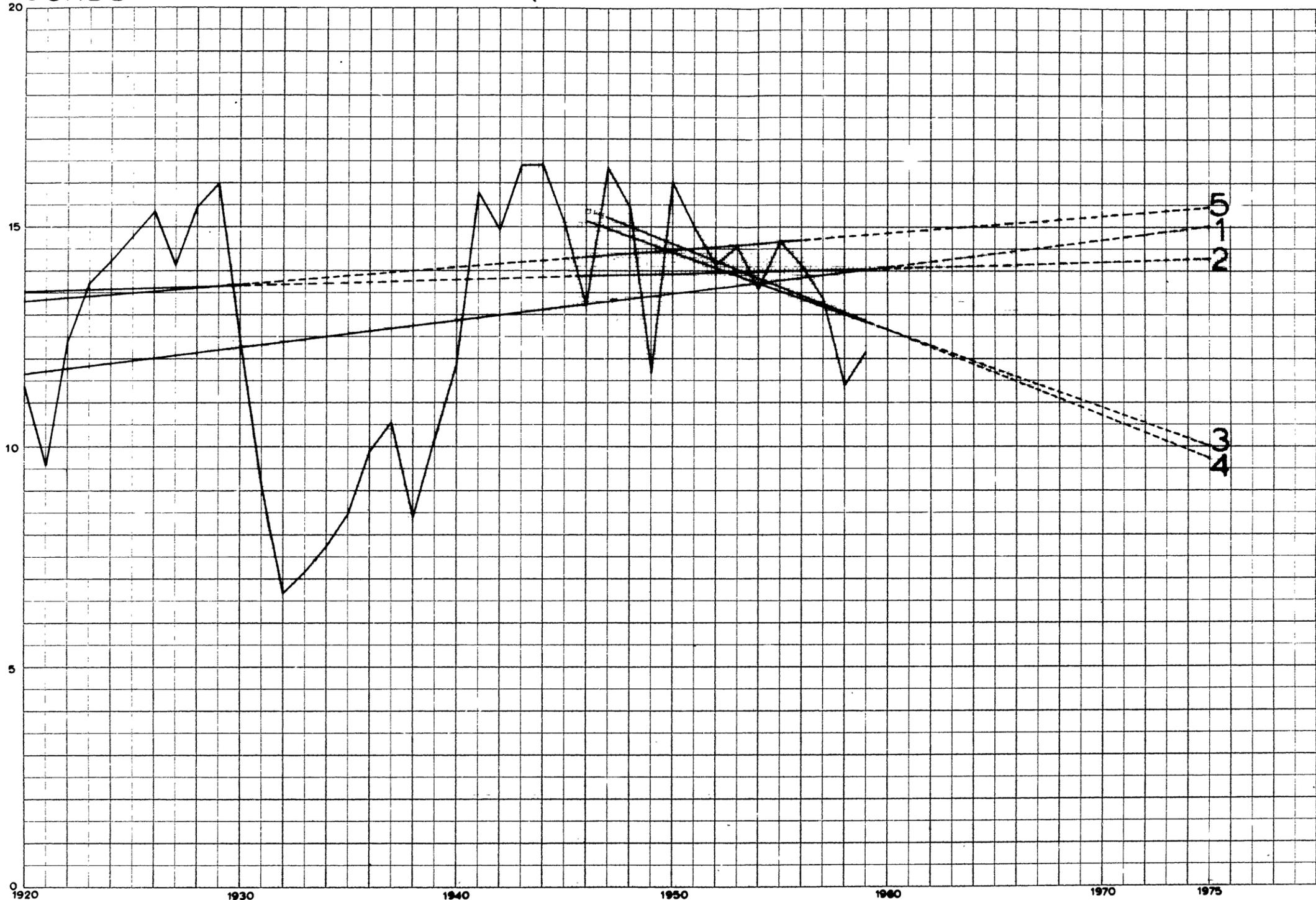
Table 2. Trend Lines of Per Capita Consumption of Lead

Period Fitted (Incl.)	Trend Function ^a	Projected Consumption in Pounds Per Capita	Projected Total Consumption
		1975	Maximum Minimum
1. 1920-1959	$Y=11.6535+0.0611313X$	15.016	1823543-1545950
2. 1920-30,46-59	$Y=13.5318+0.0135272X$	14,276	1733677-1469764
3. 1946-59	$Y=19.7285-0.176934X$	9.997	1214036-1029226
4. 1947-59	$Y=20.4866-0.195438X$	9.738	1182583-1002561
5. 1920-30,46-56	$Y=13.3076+0.0391749X$	15.462	1877705-1591867

^a Origin for all trends -1920; X units years.

Trend one does not pass through the middle of the post-war cycles yet yields the second highest forecast. This

POUNDS



ENCL. 1, 1948, 10 X 10 THE HALF INCH
WITH IN 1 6 3/4" SQUARE OF
KODAK SAFETY FILM
100% RELATIVE

CHART 19. ANNUAL U.S. CONSUMPTION OF LEAD PER CAPITA

shows the effect of both the Depression and the Second War, but the greater magnitude and length of the depression period as a major deviation from the trend tends to increase the slope of the line. This influence is exaggerated by the least squares technique, which weights the deviations by their square. However, since the large negative deviation is followed by the large positive deviation of the war period these tend to balance out, and the trend forecast figure is close to several others.

It is more reasonable to exclude the Depression and the Second World War in calculating the trend, especially since the level of consumption of the twenties lies at a level close to that of the post-war period. Trend ~~two~~ is therefore considered more suitable than trend one.

Fitting the trend to data only since 1945 uses only 14 data points for forecasting 16 -a much frowned-on practice. But it gives an excellent example of one possible effect of such practice, yielding a forecast which is too low. More important perhaps in this case it shows the alarming rate at which the established markets for lead have declined relatively since the war. (Any common measure of industrial development shows a positive uptrend, e.g. the GNP, or the Federal Reserve Board Index of Manufactures). It appears unlikely that such a high rate of substitution will continue in the future, since the introduction of wartime innovations wanes with time.

Trend four is likewise low, and was fitted to show the effect of having both first and last years of data fall in different phases of the cycle when a small number of data are used. In this case, since the lead shortage continued through 1946 this trend is also logically better than trend three, but is still unsatisfactory.

Finally, the fifth trend was fitted only up to 1956 to show that adding only three more years of data, all of which fall in a recession period, raised the trend and gave a forecast of demand 120,000 to 145,000 tons per year higher

than trend two, fitted otherwise to the same data.

On the basis of the most logical per capita trends, namely two and five, the range of forecast of lead demand by this method for 1975 is 1.47 to 1.88 million tons, which includes the forecast by trend one also.

Total Consumption

It is preferable to work in terms of per capita consumption, since the inevitable growth in population is certain to be an important factor in a total consumption trend. However, for purposes of comparison, straight line trends have been fitted:

(a) to the logarithms of total consumption. This is shown in chart 20. As explained above, the data for 1920 to 1930 and 1946 to 1959 are considered the most logical to use. This method gives the equation:

$$\log (\text{lead consumption in tons}) = 5.859337 + 0.00555483 (\text{year} - 1920)$$

or, in the exponential form,

$$(\text{lead consumption in tons}) = (723,331)(1.01287)^{(\text{year} - 1920)}$$

For 1975, the forecast from this curve is a total lead consumption of 1,461,683 tons, or approximately 1.46 million tons.

(b) to the actual data of total consumption: from 1920 to 1959; from 1920 to 1930 and 1946 to 1959; and from 1946 to 1959. These give the following predictions for 1975; trend two again being considered most suitable on the basis of the period fitted.

Data Used	Equation Derived	Forecast, 1975
1920-1959	$Y = 485,896 + 18507X$	1,503,781
1920-30 & 1946-59	$Y = 728,186 + 11391X$	1,354,723
1946-1959	$Y = 974,674 + 3758X$	1,181,364

The trends are shown in chart 21.

Summary

If all of the projections made in the above three sections, by comparison with the FRB Index, on a per capita

TONS

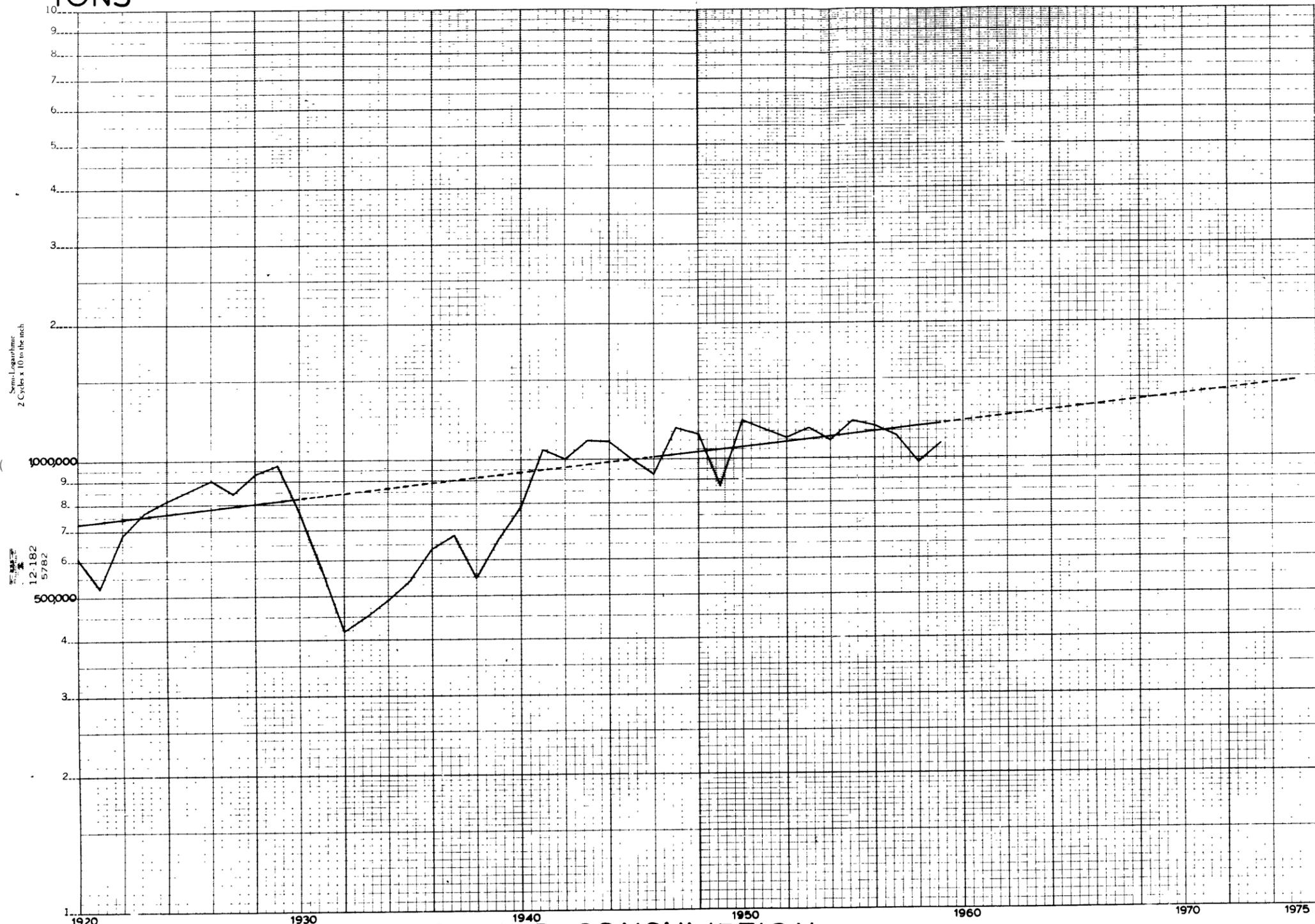


CHART 20. ANNUAL U.S. LEAD CONSUMPTION

TONS

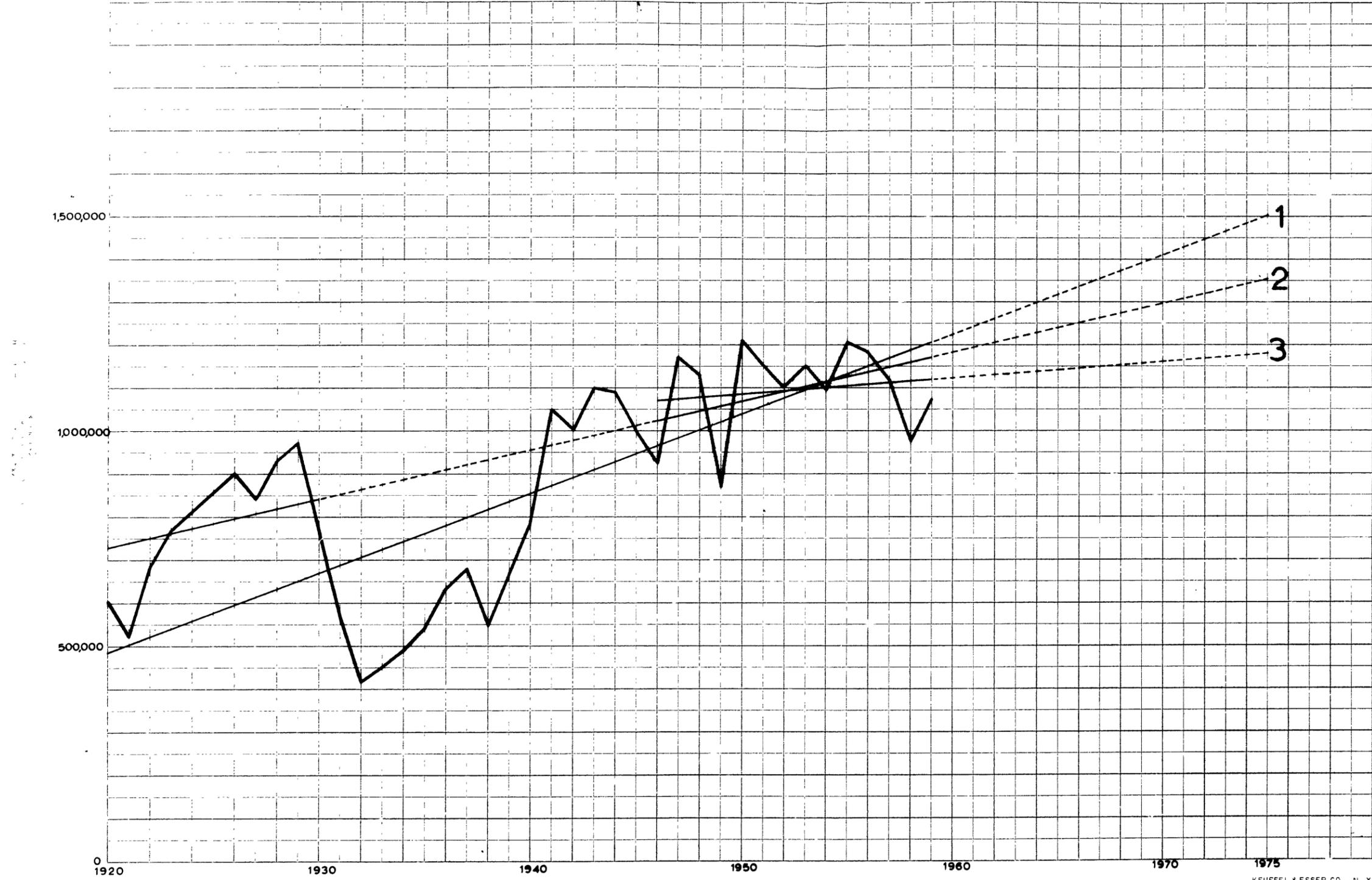


CHART 21. ANNUAL U.S. LEAD CONSUMPTION

basis, and on a direct trend basis, were to be accepted, they would define a range of demand for 1975 from 472,000 tons to 1.88 million tons. Yet any one of the trends or methods used can be justified to some extent, and to eliminate one or another, or indeed even to choose which trend type to use is a subjective decision based mainly on the wisdom and experience of the forecaster. Linear or logarithmic trends have been used here because for the period of available data they appear to describe the trend well enough that more complex curves are not needed. Furthermore the growth habit, i.e. by constant absolute or relative amounts, is easily understood and reasonably logical for the series in hand. But no absolute rule exists for choosing the type of trend to use to represent a series. Some texts, (e.g. Croxton and Cowden, 1955, p. 318) present a series of tests which are of some help in choosing the trend but cannot make the choice truly objective.

Likewise, in deciding which periods of data to use, there is a problem of choice. To project a trend based on only a few years' data to a target year 15 years hence is undesirable, yet in some cases older data does not seem relevant. In other cases the more recent data appears to be atypical and may have to be rejected. In every part of the study of lead demand the war period caused restrictions and distortions of the consumption pattern, and these years have been eliminated from the study. Shortages for one or two years after the war dictated that data up to 1946 or 1947 should be eliminated. Whether the depression years should also be eliminated, or which parts of the 1930 decade, is less clear, for while demand was lessened, there was not as great a distortion of the pattern of demand. For example, although less paint was purchased, lead was available to make that paint, while in World War II it was not, and substitutes were developed.

If data from 1930 to 1946 or 1947 are rejected, a gap of 16 or 17 years is made, and it then seems questionable to use data for the 1920's. But without these data, the forecast must be based on only ten to fifteen year's trend. These short term trends have yielded the lowest forecasts of lead demand in the above study, since the post-war years have seen a decline in lead demand due to innovations coming as an outgrowth of wartime research. The judgment of the forecaster now comes to bear, for he must decide if this decline will continue, diminish, flatten out, or even be reversed. By so doing, he decides what his answer should be, and then proceeds to obtain it! However, it is not really as bad as it looks, since the order of magnitude of future demand is probably reasonably apparent in most cases, and the trend lines chosen will be used to give a more concise forecast of that demand.

Thus, in the case of lead, a continuing decline to 472,000 tons does not appear reasonable because of the high post-war rate of substitution for lead and the trend yielding this forecast is rejected. This line of thought leads one to reject other trends based on atypical years, or to avoid illogical trend equations.

Of the trends used the most satisfactory ones are listed below, in order of preference on a logical basis:

Trend	Forecast (millions of tons)
St. line to logs of FRB Index -Lead Cons. Ratio, 1920-1959	1.35-1.58
St. line to Per Capita Cons. 1920-30,1946-59	1.47-1.73
St. line to Total Lead Cons. 1920-30,1946-59	1.35
St. line logs of Tot. Lead Cons. 1920-30,1946-59	1.46

Taking the average for each of the first two trends, the range defined is 1.35 to 1.60 mil. tons by 1975, or 1,475,000±125,000 tons, which can be considered the forecast derived by the overall forecasting method.

CONSUMPTION FORECAST ON AN INDUSTRY
DEMAND BASIS

By considering the history of lead consumption on an industry basis, the divisions of which are dictated by available statistical accountings, and examining in some detail the role of lead in each major use, a more logical base is evolved on which to build a total forecast of domestic consumption from all sources.

The greatest disadvantage of this method is the amount of time it requires, which is such that any forecast so evolved is a major undertaking, not like the possibilities which would exist had the logistic curve proved more useful. The greatest advantage is that the forecaster becomes acquainted with the possible causes of deviation of the forecast from future trends, and as these develop, the prediction can be reviewed on a logical basis. He also becomes aware of the probability of a disconcerting lack of information on some relatively obscure but important facet of the use of lead in any given field, which could upset his forecast.

While the Lead Industries Association has been most cooperative in this study, the source of much data has been other trade associations, and especially companies. Some of these deserve special credit for the efforts they have made to explain the field in question, while there are others which have given only superficial responses, reflected herein in lack of information on some phases of lead use. Over 50 companies and associations have been consulted, as well as numerous trade journals and other literature of the industries involved. Aside from annual review articles, responses included 46 books, pamphlets, reports and reprints, in addition to information included in letters.

Storage Batteries
(A General Example)

Storage batteries have for many years been the single largest consumer of lead, using generally about 50% of total consumption, and it is therefore important to investigate this sector as carefully as possible, since a small percentage error amounts to a considerable tonnage of lead.

The lead-acid wet cell storage battery is familiar today to everyone because of its use in automobiles. This single use consumed 90% of all lead used for batteries in 1959 (Allen, 1960). But batteries serve many other purposes, often as standby units in case of power failure. Telephone and telegraph systems use them in normal service, as do railroads, aircrafts, and boats, including small pleasure boats and submarines. Industrial haulage locomotives and electric handling trucks are important consumers. New uses include golf carts and commercial and pleasure cars, which could consume large amounts of lead in the future.

Briefly, the lead-acid battery cell consists of two lead plates suspended in sulphuric acid, the electrolyte, and produces 2 volts, so that for a 6-volt battery three cells are connected, for a 12-volt six, etc. The addition of more plates does not increase the voltage but the capacity (rated in ampere-hours). The plates are cast from hard lead, containing 7 to 12 % antimony, and small amounts of tin, arsenic, copper, cobalt, and silver. The interstices of the negative grid are pasted with litharge, or a litharge-metallic lead paste. Up to 25% red lead may be included in the paste in the positive cell, the remainder being litharge and fine metallic lead, called "black oxide" ¹.

Lead Use in Batteries.

While 60 to 70% of the total weight of a battery is lead, of which about 52% is antimonial, the remainder consists of lead oxides. In 1959, storage batteries used 127,000 tons

¹ Lead in Modern Industry, (1952).

of antimonial lead, 87,000 tons of litharge, 6,800 tons of red lead, and 108,000 tons of miscellaneous products, chiefly soft lead and black oxide, for a total of 328,000 tons of contained lead.

The term "consumption" is somewhat of a misnomer for this use of lead, since usually 80-90% of the lead in batteries is returned as scrap over a $1\frac{1}{2}$ to $2\frac{1}{2}$ year period. However, the statistics of consumption combine both new and scrap lead as one, and the forecast made here is for total use. This applies to all other uses discussed below as well. Cable covering, bearing metal, type metal, terne metal, and solder are also sources of scrap, and totalling, with batteries, 76% of lead use in 1950, account for a potential 60% scrap return in the lead industry (Merrill, 1950). In 1953, 40 to 45% was actually being recovered (American Metal Market, November 18, 1953).

Chart 22 shows the trend in lead use in storage batteries since 1920, (ABMS data) and since 1942 (LIA data). Note the decline since 1950. Also shown is the number of auto batteries shipped, or SLI batteries (Starting, Lighting, Ignition) which is increasing with time, and the resultant apparent pounds of lead per battery, (ABMS data divided by Batteries Shipped), which shows a decline. That such a decline should exist in spite of the increasing use of 12 volt batteries, which contain 2-3 pounds more lead than a standard 6 volt, is surprising, but will be explained below. Because of this decline, although forecasts are known for auto registrations, the tonnage of battery lead required is not simply a proportion thereof. Increasing service life per battery also complicates the relationship. New developments in battery uses add further considerations.

Statistical Difficulties.

Some of the problems of forecasting by simple trend projection have been outlined above in attempting an overall forecast of lead demand. Battery use illustrates these further, and it is hoped, also illustrates a method of counteracting at least some of them. However, there are statistical discre-

pancies in battery lead accounting.

From 1920 to 1958, the ABMS series shows a steady linear increase, with only a slight tendency to flatten off in recent years. Yet the LIA data indicate a decided flattening, and in fact a linear decline since 1950, ignoring cyclical fluctuations. While ABMS data credit antimony in hard lead as lead and LIA does not, this will not explain the difficulty. This discrepancy in data for the major use of lead illustrates, unhappily, the serious problem of obtaining reliable statistical information, as discussed above. The LIA accounting includes an item called "Underestimate of Consumption", based on comparison with the United States Bureau of Mines figures. This is distributed over the classifications used by the ABMS, so that their data are generally close to those of the Bureau of Mines. If the division of consumption into sectors were on the same basis, using the Bureau of Mines or ABMS data would of course eliminate this understatement, but only LIA reports solely on an industry-use basis. For storage batteries, cable covering, ammunition, gasoline, printing and several other uses, the estimates can be compared, and are found to give comparable trends for cable covering and gasoline, the printing and ammunition sectors being comparable until 1954, when LIA data fell below ABMS data, for which no reason can be found. For other uses, one or other set of data must be accepted, and since the LIA does not include all manufacturers, the Bureau of Mines data, as given by the ABMS, has been used for use sectors reported on an industry basis, such as batteries, and LIA data elsewhere. The "Underestimate of Consumption" has been reduced by proportionate amounts. The ABMS data has the advantage of being available over a longer period than LIA statistics.

Green (1960) has indicated that the Bureau of Mines estimates of lead consumed in batteries are probably too low by about 5%, as indicated by an underestimate of 622,000 replacement battery units in 1954 compared with actual Census figures for that year of 23,771,000 units. Since it is impos-

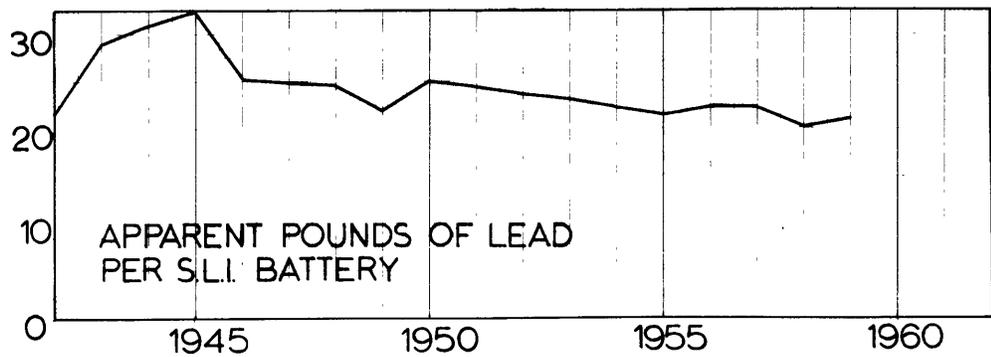
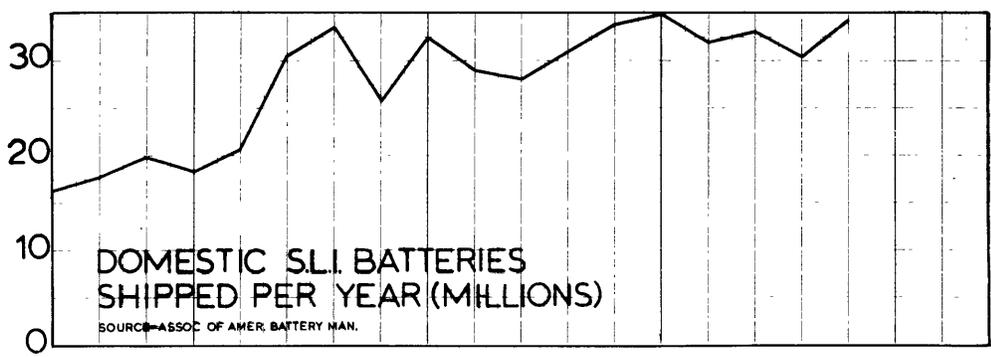
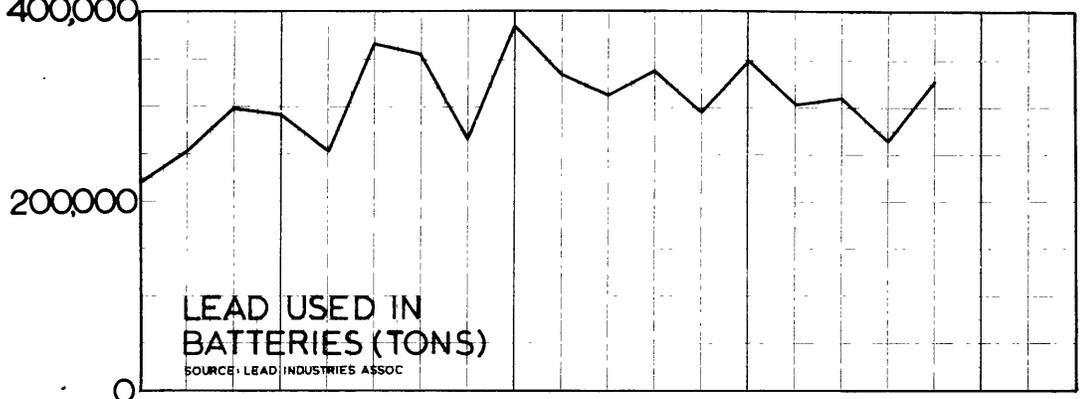
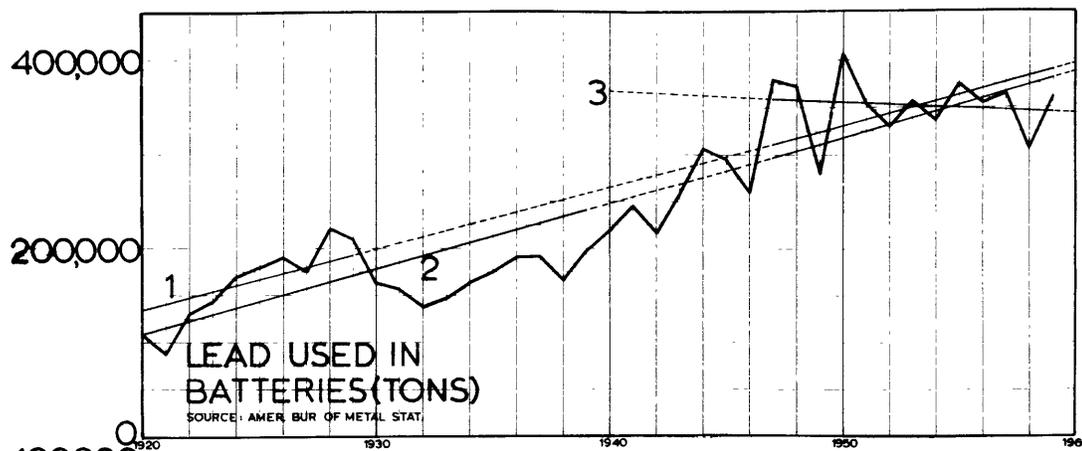


CHART 22. CONSUMPTION OF LEAD IN BATTERIES

sible to evaluate the suggested error in the Bureau of Mines data, it is assumed here that the error is essentially constant over time and since the forecast for 1975 will be judged on a relative basis it will not be of great importance. The data used for battery consumption of lead are those of the ABMS, the first panel of chart 22, and include some 10% antimony.

Trend Projections for Consumption.

If a straight line is fitted to lead use in batteries, it can be extrapolated to give a forecast for 1975. Three segments of data were so fitted, and the results are tabulated in table 3, trends 1, 2 and 3. Also, a trend can be fitted to per capita consumption and the projected figure combined with population forecasts. Chart 23 shows the trend of per capita battery lead consumption. The trends fitted, numbers 4 and 5, are also summarized in table 3.

Table 3. Trend Lines Fitted to Lead Consumption in Batteries.

No.	Trend Equation	Period Fitted(incl.)	1975 Forecast
1	$Y=134.84 + 6.5251X$	1920-'29, '47-'59	493,700
2	$Y=107.78 + 6.9927X$	1920-'39, '47-'59	492,400
3	$Y=391.146 - 1.1742X$	1947-'59	326,600
4	$Y=2.26456 + 0.061310X$	1920-'39, '47-'59	580,000-797,000
5	$Y=7.02054 - 0.078992X$	1948-'59	275,500-378,500

Over the entire period since 1920, trend line 2 appears to come closest to passing through the centers of the cycles of total use, and while 1958 and 1959 consumption data are below the trend, forecasts for 1960 of 380 to 385 thousand tons (Green, (1960), and Allen (1960)) fall nearly on this trend. The forecasts for trends 1 and 2 are essentially the same, some 493,000 tons. On the other hand, trend 3 shows a decline in consumption since the war, which is not reflected in either trend 1 or 2. The disparity is increased if straight line trends are fitted to per capita consumption.

Conclusions re Simple Trend Projection.

Clearly the demand pattern for lead in batteries has changed since 1946 or 1947, but unless it continues to change

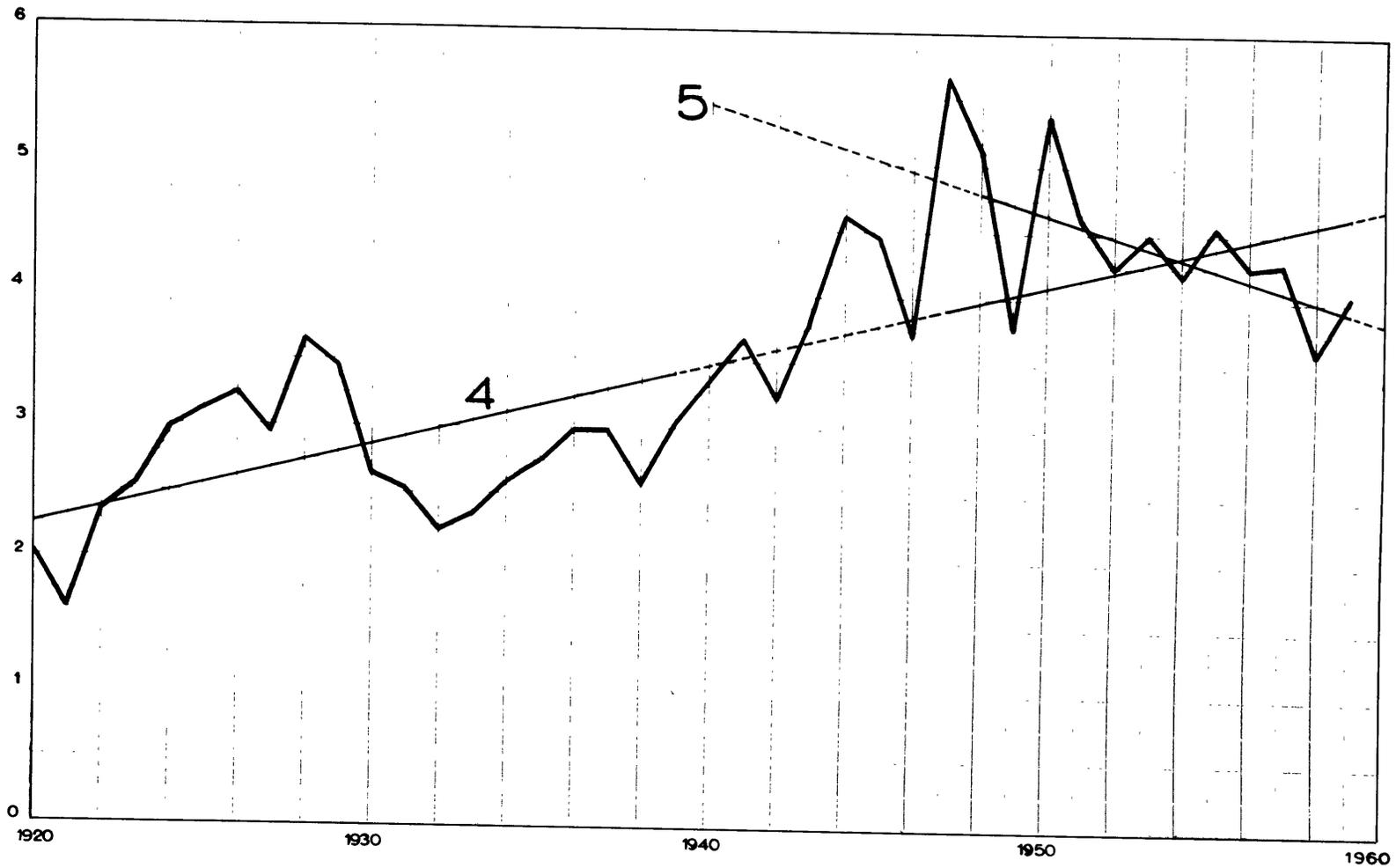


CHART 23. POUNDS OF BATTERY LEAD USED PER CAPITA.

in a similar way for the next 15 years, trends fitted to the post-war period will be too low, while trends fitted over the entire period are too high. If more complex curves are used to represent the trend, there seems to be no reason to assume that they will answer for the forecaster the question of how the pattern will change. It would be a safe but useless forecast to state that 1975 consumption will fall within the range defined by the 5 trends, i.e. 275,500 to 797,000 tons. Trend projection apparently fails to be of any direct use in forecasting demand for lead in batteries since in essence it assumes that the conditions that have held in the period to which the trend is fitted will continue to hold in the future. In this case, and others to be noted later, the post-war decline in lead demand makes this principle inapplicable, and the method of simple trend projection cannot adequately cope with the situation.

Batteries use the largest share of United States lead consumption each year. It may be noted in passing that if there has been an important change in this use since the war, it must affect the trend of total lead consumption. The method of prediction in toto actually assumes that the effect of a reduction in some uses will be nicely offset by an increase in others, giving continuity to the net effect, and it is indeed possible that this is the case, but combining the possible errors due to selection of the wrong trend type with the errors in this assumption can produce a large discrepancy.

An Attempted Solution to the Problem of Changing Consumption Patterns.

The most logical solution to the difficulty of a changing consumption pattern is also one of the most difficult to pursue -that is the investigation of the causes of the change and a prognosis of their future behavior. This cannot be carried out with respect to total demand alone, but must be considered for each major demand sector.

On the basis of admittedly limited knowledge of each sector of lead demand, the author has tried to select those

mathematical expressions for the trends of consumption that appear to be most reasonable, and these are not always the trends that look like the best-fitting ones. Hence the standard errors of the various trends were not calculated. In general, the result is that trends based solely on post-war data have often been rejected, or postulated as flattening-out in future years.

Technology of Battery Use of Lead.

Chart 22 shows the number of batteries shipped per year since 1942, and the apparent number of pounds of lead per battery, derived from lead consumption reported by the ABMS and the number of batteries shipped. According to Green (1960) and Allen (1960) industrial batteries consume an estimated 10% of the battery lead used each year. However, no attempt is made here to forecast industrial battery consumption separately and since it is estimated at a rather constant 10% of total use it does not distort the trend and can be included in it. Calculations on a more refined basis by the above authors confirm this, and the downtrend in pounds of lead per SLI battery is real and not just apparent.

By adding small amounts of other metals, especially silver compounds, to grid metal, and reducing antimony content, the corrosion rate of the grids has been reduced, and manufacturers have therefore developed machinery and methods for casting lighter weight grids without reducing battery life. This apparently accounts for the decline in lead content per battery. Only one other possible explanation is known, and it cannot be investigated closely because of lack of data. That is the quality of the replacement batteries sold. Essentially, the higher the capacity of the battery (and that depends on the number of plates) the higher the price. At present, 70% of the replacements sold are medium-priced, (18 to 20 pounds of lead each), 20% are low-priced, (14 to 16 pounds) and 10% are premium grade, (20 to 24 pounds)¹. Should these percentages change, as they might in an economic recession, the amount of lead per average

¹ Allen, (1960)

battery would change greatly. However, no series of data are available, and furthermore the forecast presented here assumes a continuing high level of economic activity, as has existed over the range of the data.

At the same time, several recent developments in battery technology have tended to increase the amount of lead consumed, but are not reflected in the trend. The first of these is the use of 12 volt batteries on essentially all United States automobiles produced since 1956. The excess of lead used over that in 6 volt batteries has been estimated at from 30% more (R.L.Ziegfeld) to 2-3 pounds more (Green, 1960) per battery on the average, the latter figure being substantiated by extensive calculations (Allen, 1960). It should be noted however that most 12 volt replacement units have less lead than the original equipment. The same source reports that the 12 volt batteries in "compact" cars have about the same average lead content as 6 volt batteries and that the foreign cars use about the same amount.

Over the past several years the antimony content of grid metal has been reduced from 9-13% by weight to less than 7½%, to improve the charge-retaining characteristics and corrosion resistance, at the same time increasing the lead content by 2 to 6%, or as much as 1½ pounds per unit.

It appears then that increasing use of lighter weight grids has more than offset these two developments. A further decrease in lead content may be in the offing if current research aimed at a more complete utilization of the active lead oxides is successful, and this could amount to a 3 or 4% reduction, or possibly one pound per battery within the next 4 or 5 years. There will be an increasing number of 12 volt batteries used however. In 1960, it is estimated that 54.7% of the total SLI batteries manufactured will be 12 volt units (Allen, 1960). Since the average age of cars over 1957-59 was 5.6 years (Auto Facts and Figures, 1959) and 12 volt systems have been widely used in new cars since 1956, this percentage should increase considerably over the

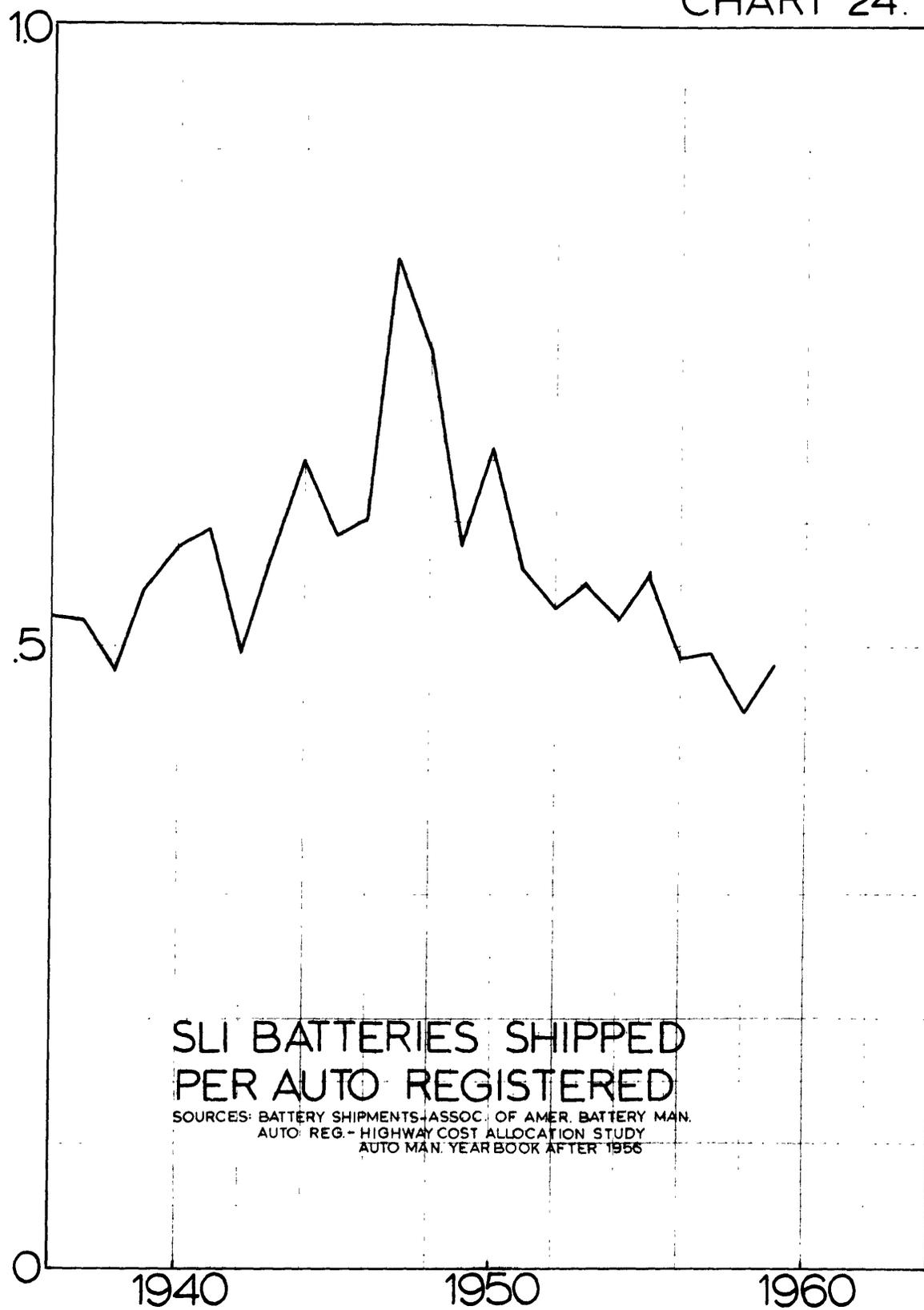
next 4 to 5 years, to approach 100%, and if a 12 volt battery contains about 3 pounds more lead than a 6 volt this should result in a slight overall increase in battery weight in spite of the postulated one pound decrease per unit. The advent of foreign cars and "compact" cars on the market will tend to reduce the number of larger batteries used to some extent, so it is reasonable to conclude that these changes will approximately cancel each other.

No opinion was ventured by manufacturers on possible further reductions in grid weight, and it must be assumed that this improvement has been completely implemented. The conclusion then is that the declining weight of lead per SLI battery will stabilize at about its current level, and the figure used in the forecast is 21 pounds.

Technology of Battery Manufacture.

Chart 24 illustrates the next problem to be considered -the increasing average length of service of the SLI battery, as shown by the declining number of batteries shipped per automobile registered. This decline may be accredited to the lead manufacturers themselves, since more uniform, higher quality lead and lead oxides account in part for longer battery life. Other developments such as reduced antimony content and the addition of silver compounds to the grid metal have improved corrosion resistance and battery life. Also contributing to longer battery life is improved design of auto electrical systems, notably the voltage regulators. Of course, this trend is important in forecasting lead demand, and unfortunately it is not possible to have any accurate idea of how far it might continue. To assume that it will continue at the rate of the last 10 years is not warranted, since several major developments have occurred in that period, and would indicate an extremely low replacement rate by 1975. Fitting a freehand curve to the trend since 1948 indicates a flattening-off at a rate between 40 and 45%. (Data for 1946 shows an abnormally low replacement rate due to shortages, while 1947 and 1948 are abnormally high due to

CHART 24.



replacement of low grade wartime batteries). This is obviously little more than a statistical guess at an important figure. To see how important an error in the figure would be, it can be noted that at an average battery content of 21 pounds of lead and predicted registration of some 110 million vehicles in 1975, a difference of 5% in the replacement rate is equivalent to 57,750 tons in the consumption figure. Using the ratio of battery sales to miles driven gives a similar result. A difference in average battery content of lead of one pound, at a 45% replacement rate would mean 24,750 tons. While neither the lead content nor the replacement rate are necessarily known within these margins of error, the two combined could total 82,500 tons, or probably 15% of the total forecasted demand for battery lead! The only consolation is that it is unlikely that the replacement rate will increase much beyond its present level, while the lead content per unit may increase slightly as the percentage of 12 volt replacement units increases, so that the two may tend to cancel one another.

Forecast for SLI Battery Use of Lead.

With these possibilities of error in mind, a forecast of demand for 1975 can be attempted. The predicted motor vehicle registration for 1975 is 111,162,000¹. Assuming a replacement rate of 45%, with an average lead content per battery of 21 pounds, this is equivalent to 50,023,000 batteries per year, containing 525,240 tons of lead in metal and oxide. Table 4 shows the variation in this figure for changes in each variable within what is considered the range of possibilities. Thus the forecast might be stated as 525,000 tons \pm 100,000. One other factor must be applied. Since ABMS data were used, and these include the antimony

¹ Straight line interpolation of forecasts from the Third Progress Report of the Highway Cost Allocation Study.

content of the battery metal, a reduction of the forecast of 4% must be made. (An average of 8% antimony in the hard

Table 4. Lead Use in SLI Batteries, in tons.

Replacement Rate	40%	45%	50%
Lead Content (Lbs.)			
19	422,415	475,217	528,020
20	444,648	500,229	555,810
21	466,880	<u>525,240</u>	583,600
22	489,113	550,252	611,391
23	511,345	575,263	639,182

lead, which makes up one half of battery lead use, the rest being oxides). The forecast is then 504,000 ± 96,000 tons. Note that the forecast by projection of the most logical trend was 493,000 tons, which, reduced by 4% is 473,280 tons, a reasonably good agreement.

Marine Starting Batteries¹.

A new market for lead-acid storage batteries has sprung up in the last few years due to increased boating activity and the advent of popular electric-starting outboard motors since about 1954. (Inboard marine engine batteries are classified under industrial batteries.) Most of these require a high-capacity battery, which therefore contains more lead than the predominant type of auto replacement sold. The rate of use of these marine units has not been established, nor are complete statistical data available on the use as a whole. The Outboard Boating Club of America (OBC) estimates that there are 5,845,000 outboard motors in use², of which 42.6% or 2,489,970 are electric starting³. In recent years over 500,000 motors have been sold per year, 62% over 15 horse-

¹ This discussion does not include electrically-propelled motors, which have been used in small numbers since 1930, and are not a large or potentially large market for lead because of limited consumer acceptance.

² Boating, 1959: A statistical report jointly presented by the OBC and the National Assoc. of Engine and Boat Mfrs.

³ Outboard Boating, July 1960.

power, and presumably a high percentage of these are electric starting. Growth in motors in use per year since 1947 has been at a rather steady rate of 320,000 per year, so that if this trend continues, about 11 million motors would be in use by 1975, and the percentage of electric starting units will probably increase as well, since average motor horsepower continues to increase, and more small boats are also using remote controls. The number of batteries in use by 1975 is therefore projected at about 7.5 million.

Battery life, based on estimates by local marine equipment renters, is two seasons. (There are no better data currently available). Thus by 1975, the replacement market will be some 3.75 million heavy-duty grade 12 volt batteries per year, and average battery weight will be greater than the 21 pounds used for SLI batteries. Using an estimated figure of 23 pounds, and a 4% reduction for antimony content, marine starting batteries are projected as consuming 41,300 tons of lead per year by 1975. This figure is obviously preliminary and should be reviewed as better data become available.

Motive Power Use of Lead-Acid Batteries.

Of the 10% of total battery lead consumption or approximately 30,000 tons per year that is estimated to be used in industrial batteries, about half of this amount is used for motive power, i.e. the power from the battery is used to drive the vehicle directly, in electric industrial trucks such as fork lift trucks, mine vehicles, and other materials handling units. It has been estimated (Taylor, 1959) that a potential of approximately 17,000 tons of lead per year is lost because of sales of propane and gasoline powered industrial trucks, and 2,220 tons because of alkaline battery sales. The same source points out that there are many features of modern electric trucks which should increase this market a great deal: safety, low maintenance, long life, low operating costs, high maneuverability and handling speeds. The high initial cost, roughly double that of internal combustion engine trucks when battery and charger are included,

is offset by a life three times longer and lower operating costs, while it can be avoided altogether by leasing the equipment. Yet currently, lead-acid battery powered trucks get only 20% of the total industrial truck market, missing an annual market for lead of 20,000 additional tons.

One unknown quantity in this application of batteries is the average rate of consumption. Another is the possible growth rate of the market, since it depends not only on industrial growth but also on the ability of electric trucks to substitute for other types. There is no reasonable way to forecast the long term pattern of substitution that the author has been able to discover, and this is typical of every case of substitution met in this demand study. Each manufacturer whose advice is sought endeavors to present his price data (if he will reveal it) and technical information in the manner most advantageous for his product, and will tend to be strongly optimistic about its future, so that nothing short of a full-scale market survey can give any indication of how well the product might fare on the market. Furthermore a strong promotional campaign can offset apparent technical or price disadvantages. Relative costs of raw materials or fuels might change over the forecast period to reverse a trend. The only practical solution is to make some basic simplifying assumptions and proceed with the forecast, hoping they are correct, or at least reasonable.

If 100% of the industrial truck market could be won for electric trucks there would be 5 times as great a lead consumption at current industrial useage level, or some 70,000 tons of lead might be used. Using the FRB Index of Manufactures as a measure of growth the potential market should be slightly less than double the current level by 1975. Assuming that since there is no essentially new factor at work in favor of electric trucks, they will only hold their fifth of the market, this use will grow about as fast as SLI battery use, with which the data were combined in the SLI forecast. Barring an extensive changeover to electric trucks, there would therefore appear to

be little possibility that this application will add much to the SLI forecast. If the share of the market doubled it would add about 28,000 tons to the battery lead forecast which in view of the range of error, and the lack of evidence to indicate increasing substitution by electric trucks, is not credited to the forecast made here.

A more important use of batteries for motive power would be widespread application in electric pleasure vehicles, such as those being delivered now on the west coast, (see the "Oil Refining and Gasoline" sector below), and in electric delivery trucks, already in use in Ohio. The batteries used range from 2 or 3 conventional ones in the smallest of the cars planned to a 2,200 pound one used in the delivery trucks¹. While manufacturers are probably overoptimistic, the chances of successful marketing do appear quite reasonable and it is necessary to consider the possible effect on battery use over the next 15 years. This is much easier said than done. In the section on oil refining and gasoline use of lead it is concluded that perhaps 50% of all urban travel or 25% of total travel will be done by electric vehicles in 1975. But just as in the forecast for SLI batteries it is necessary to know the weight of lead per battery and the replacement rate, or some such equivalent data, and of course at this early date it is not available². Furthermore, it is possible that selenium-celled batteries may be used for some of these vehicles, especially because of savings in weight.

If it is assumed that these cars will be used for half the urban travel by 1975, hence represent say 35% of total registration, that lead-acid batteries will be used, that four additional batteries will be used per car, and that these will last about one year, this market could be for 156 million batteries per year! This is an extreme speculation of course,

¹ Evans Taylor, (April, 1959).

² The aspiring manufacturer of electric automobiles contacted ignored the request for information of any kind.

but shows the potential to be enormous. In fact this very enormity is a limitation, since if 80% of the battery lead were returned as scrap, this use would require about 300,000 tons of new lead annually -about the level of total United States annual production.

Because of the speculative nature of this development, no consumption figure has been included in the forecast, but it should be kept in mind.

One other type of vehicle which is electrically powered may also be mentioned, the golf cart. Although the battery consumption rate is high -ten to twenty times that of an automobile (Taylor, 1959), it is doubtful if this market will become a large one because the cart is a luxury item and certainly not required or desired by a large percentage of players, its miles per day will be low, and the sport is seasonal. Perhaps one or two thousand tons per year might be consumed.

Substitution.

From time to time there is mention of the nickel-iron and nickel-cadmium batteries as substitutes for the lead-acid type for SLI applications. The nickel-cadmium unit uses two strategic metals, of which cadmium is produced only in small quantities, while the nickel-iron also uses a strategic metal. Costs of the raw materials are high. It is estimated that a nickel-cadmium auto battery today would cost \$200-325 (New York Times, April 11, 1960) and although it would outlast the car in which it was installed, the price is obviously not competitive. Also important is the fact that these batteries have different electrical characteristics from lead-acid cells, which make them less suitable for SLI operation. It is concluded here, reinforced by the opinion of several experts in this field that no substitute will displace any important amount of lead in battery useage over the next 15 years. The total forecast for lead "consumption" in all types of batteries in 1975 is therefore as follows.

Total Forecast.

Automotive (SLI) Batteries	504,000 ^{Tons} ± 96,000
Marine Starting Batteries	41,300
Motive Power Use -Industrial Batteries	incl. in Auto Batteries
-Automobiles	? ?
-Golf Carts	1000 - 2000
Total	<hr/> 546,800 ± 101,000 Tons

The forecast is therefore for 547,000 ± 100,000 tons of battery lead used per year by 1975. It should be noted that the wide range found necessary is the equivalent of a hundred percent error or more in forecasts for several of the smaller uses of lead, which is indeed unfortunate.

Construction

The application of lead in construction is an example of a number of uses combined in one demand sector of this study and thus differs from battery use. Three types of products constitute most of the consumption - pipes and extruded products, sheet lead, and calking lead. LIA statistics combine all three, ABMS separates calking, and the USBM reports all three separately. Aside from other areas of use these products are used in two major classes of construction, namely residential and chemical plant, but unfortunately no agency reports or can estimate the percentage entering each, nor could any study be found of the number of pounds of lead used in an average house. This difficulty reduces the level of confidence in the forecast.

Per capita use of lead is shown in chart 25 as reported by the LIA for "Construction", the ABMS for "Building" and "Calking" and the USBM for "Pipes, Traps, and Bends" and "Sheet Lead". (The USBM accounting procedure changed in 1948). The first is the sum of "Building" and "Calking" (approximately), while "Building" is the sum of "Pipes, Traps, and Bends" and "Sheet Lead". The trend of lead use in "Construction" can be most clearly analyzed by examining the three separate series, "Pipes, Traps, and Bends", "Sheet Lead", and "Calking".

Pipes, Traps, and Bends.

The term "pipe" is self-explanatory. Lead pipe was used to carry water even in Roman times and is still found preserved in ancient ruins, and this permanence, together with freedom from corrosion products which clog the pipe, a flexibility which allows settling of the building without disruption of the pipe, and ease of joining has made lead pipe popular in domestic and industrial plumbing for years. Until recently its only major competitor was galvanized steel pipe, which is cheaper but corrodes badly and often stains the water. Today lead water pipe competes with a

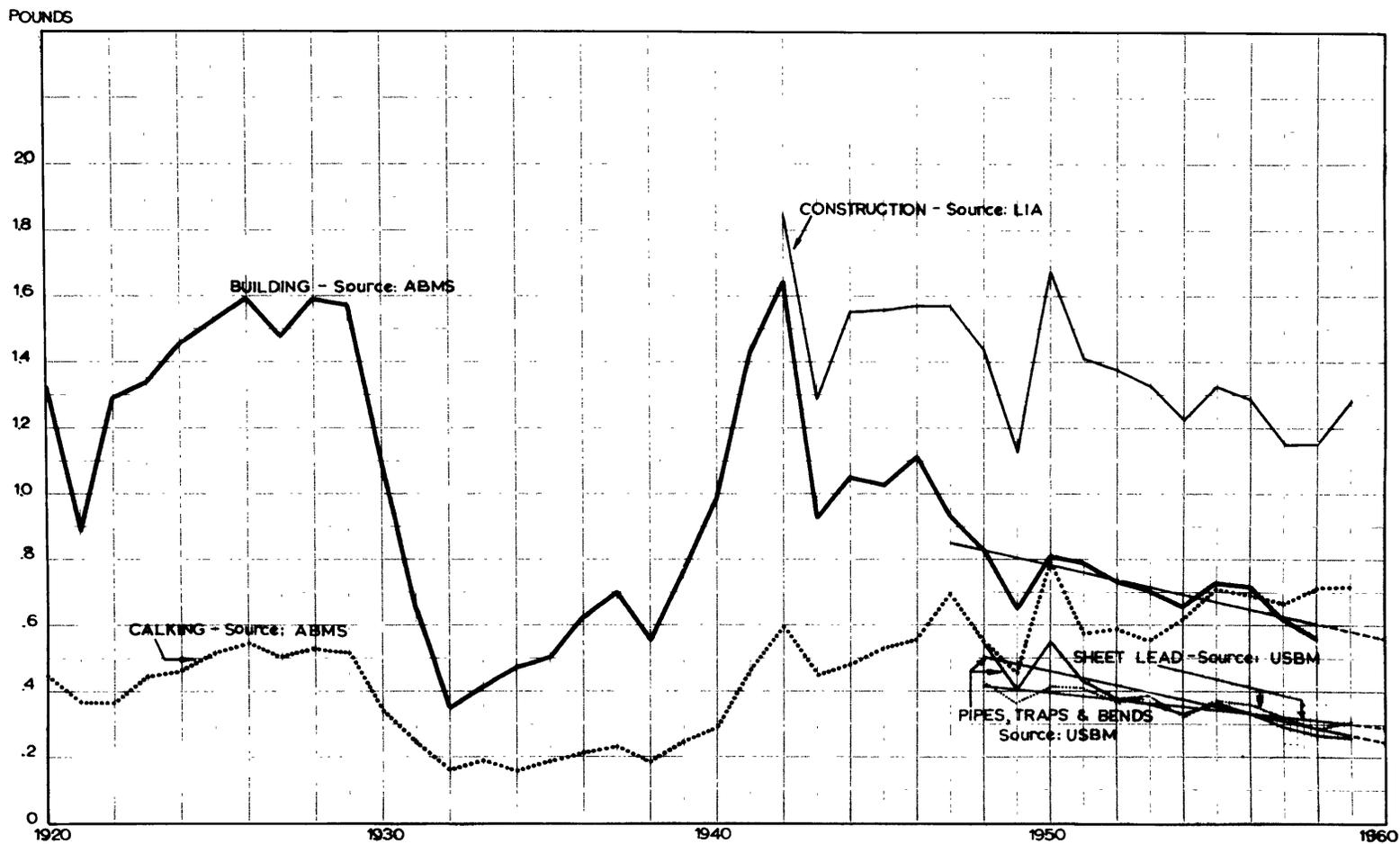


CHART 25. PER CAPITA CONSUMPTION OF LEAD IN BUILDING

stronger adversary, copper pipe, and is beginning to face another in plastic pipe. Copper has two major advantages over lead pipe: first it is more rigid and lighter weight, hence can be installed with far less support than lead pipe, and second it is almost one-half the price of lead pipe of an equal service weight (on a pressure basis), even if scrap return is considered¹. Furthermore the simple sweated joint now used makes copper pipe even cheaper to install. The postwar decline in this category of consumption is caused mainly by the substitution of copper in residential, industrial, and commercial plumbing, and it is expected that this decline will continue to the point where very little lead is used for pipes, except in some chemical construction.

"Traps" are the S- or otherwise curved pipes designed to hold enough water in the bend of the pipe to close off the drainage so that gases do not escape into the room. Other fittings made of lead are also included here, such as flanged toilet connectors. Competition in this use is mainly from cast iron and plastic, both of which are quite resistant to the gases and acids encountered and are also competitively priced.

"Bends" are preformed bent pipes, and can be used in lead plumbing systems and also in copper ones to avoid right-angle bends.

It is impossible to separate the series of data for pipes, traps, and bends. The former probably dominates the three, and since there do not seem to have been any important new developments in traps, except perhaps plastic, which is still not accepted by many plumbing codes, it is the impact of copper plumbing on the use of pipes and bends that accounts for most of the declining trend. While it is expected that lead pipe use will diminish to only a few thousand tons per year, to be used in special applications and repair work, traps and bends will probably continue in use at about current level or somewhat less, but since the data do not show what this is, it is difficult to quantify the forecast.

¹ Based on delivered Boston prices for small lots.

Sheet Lead.

"Sheet Lead" and "Rolled Products", the latter an LIA classification are essentially the same. The construction demand for this product, which is assumed here to represent the only use, is divided between chemical plant construction and domestic construction, but in an unknown ratio, although it may be about 1:1 since its use in the latter category is restricted mainly to roofing and flashing, with some use for lead shower pans and vent pipe flashings, while it is widely used in plant construction, especially where sulphuric acid is made or handled. No successful substitute has been found for lead in the manufacture of sulphuric acid, and in many applications lead is used because it is cheapest in original cost, low in maintenance, very adaptable and easily modified, and has a good scrap value. Its disadvantages are poor abrasion resistance (hard lead has lower corrosion resistance), and a tendency to creep and sag, eventually cracking, so that special construction methods are necessary. Furthermore its low strength may require lead lining in steel or bronze etc. pipes, pumps, and fittings, and it always requires some kind of backing, e.g. wood or steel tanks may be lined with lead sheeting. A new high speed lead cladding device which cuts costs on simple, repetitive jobs from \$18-25 per square foot to \$3.50-5.00 and requires only unskilled labor makes lead more competitive with the many substitutes which are taking advantage of the problems encountered with it¹. Rubber, ceramics, plastics, glass, stainless steel, and monel metal all compete. Usually the material cost for these competing products is as high or higher than lead, and the equipment less adaptable, but it is easier to construct and lasts well. This widespread competition and substitution accounts for some part of the decline in sheet lead use.

In roofing, the higher total cost of lead has caused inroads by galvanized steel, aluminum, and copper, the first

¹ Chem. Eng., June 30, 1958, no. 65, p. 122.

mainly on a cost basis, the latter two because of strong advertising campaigns and architectural trends as well. However, monumental work and large public buildings continue to use sheet lead roofing and flashing. It is possible that recent studies showing that sheet of about half the weight formerly used gives adequate service may make lead more popular for domestic roofing construction but the downtrend shown in chart 25 has not shown any effect so far. It is possible that the decline in plant construction use might offset any such increase.

Forecast.

To forecast building use of lead, a multiple correlation was tried with the variables millions of square feet of building, and new construction of producers' plants related to the ABMS "Building" series, but as explained in the section on correlation problems although the cyclical movements show fair correlation the trends are not parallel so that some trend line must be used. There is then no advantage to using correlation methods since the interest is not to forecast cycles but the trend, and the complications of calculation and interpretation introduced by correlation do not help either in estimating the trend or understanding the demand pattern.

A second method of prediction, by analysis of the overall technology of the application, does not entirely solve the problem, as can be seen by the above discussion, since the technology is closely tied with the cost of using lead, and its multipurpose use in building makes each application a separate consideration with respect to possible substitution.

Therefore, it appears that for forecasting this major use of lead simple trend projection must be used, although knowledge of the technology involved can be helpful in evaluating the results. As previously mentioned, the data since 1946 are the most applicable, and although a rather short base for a 15 year projection, have been used here.

(USBM data are only available since 1948, hence this was the first year used). "Pipes, Traps, and Bends" and "Sheet Lead" were each forecasted separately, and "Building" was forecasted as a check.

Three trends were fitted, with the following equations resulting.

Lbs. of Lead Used Per Capita For Pipes, Traps, and Bends =
= 0.50543 - 0.023533 (Year - 1948)

Lbs. of Lead Used Per Capita For Sheet Lead =
= 0.41779 - 0.010202 (Year - 1948)

Lbs. of Lead Used Per Capita For Building =
= 0.85004 - 0.022349 (Year - 1947)

Projected to 1975, the "Pipes, Traps and Bends" trend gives a negative value, due in part to using a small number of years, in which the first is on a high cyclical movement and the last is on a low, causing the trend to tilt downward more than justified. Reducing the number of data points is not desirable since they are already too few, and there is no clear reason for using a higher order curve as trend. The forecast is therefore a simple estimate that this use will flatten off at 0.15 pounds per capita. The "Sheet Lead" data fluctuate less and the straight line passes through the cycles, and gives a forecast of 0.14 pounds per capita. For 1975 the predicted values are as follows, based on a range of population forecasts of 205,907,000 to 242,880,000.

Pipes, Traps, and Bends.....	15,400 - 18,200 tons
Sheet Lead	14,400 - 17,000 "
Total	29,800 - 35,200 "
Building	25,400 - 30,000 "

It should be noted that the annual sums of the "Pipes, Traps, and Bends" and "Sheet Lead" series are higher than the "Building" series, until about 1954, essentially coinciding thereafter, so that their combined trend should show a steeper downtrend and a lower forecast than the "Building" series and if the trend were used for "Pipes, Traps, and Bends" this would be the case.

Because the data are so meager, and the applications and possible substitutions so complex in the construction sector, this is one of the less dependable predictions. The forecast is taken as 25,000-35,000 tons or $30,000 \pm 5,000$ tons per year by 1975.

Calking Lead.

Chart 25 also shows the ABMS series of per capita data on calking (or caulking) lead, which has not a downtrend but an increasing trend ever since 1920; hence it has been treated separately from other uses of lead in construction.

It is surprising to learn that currently over 60,000 tons of lead is used each year as a calking material for flanged cast iron pipe in water mains, sewers and soil pipe lines, and downpipes or stacks in houses. A single joint in soil pipe requires 12 ounces of lead per inch of pipe diameter, and for pressure joints the amount is doubled. Oakum (a fibrous material) is packed into the joint and lead is poured over it and calked into place. If the use of molten lead is not practical, lead wool is used, and similarly calked. While this seems to be a use of lead which might easily be displaced either by a new type of joint or a new calking material, the flexibility of the joint has not so far been matched, and it can be easily repaired without shutting off the water. It is also a root-proof joint. Because of the excellent resistance of cast iron pipe to the type of conditions encountered, it will no doubt continue to be used, but it requires a flexible joint to avoid cracking and leaking upon settling. According to the Cast Iron Soil Pipe Institute¹, many years of effort by different interests have failed to turn up a successful substitute for the lead-oakum flanged joint because of the easy working qualities and high performance of lead. The latest substitutes include cement, plastics and aluminum wool, but none have gained much acceptance, although changes in plumbing codes could easily accelerate a

¹ Personal communication

change-over. It is assumed here that no substitute will become important over the next 15 years at least.

Forecast.

Lead demand for calking can be related to the volume of new construction, and since it is used in all types of building, an overall index or measure of construction volume is best. However, of the many indices and other series of data available, only one specific long-term forecast could be found. The Econometric Institute (Roos, 1957) forecasts the Construction Material Index to 1975, and the ABMS "Calking" data and this index are shown in chart 26, as well as the tons of calking lead used per index point (upper part). The latter shows a fairly constant relationship at about 4,500 tons per point, i.e. the trends are essentially parallel, and the cycles show reasonably close coincidence, the lead cycles being greater in amplitude and somewhat upset by the Korean War. On the basis of a forecasted index level of 23.0 for 1975 (based on a predicted population of 220 million), the forecast for calking lead consumption is 103,500 tons.

A straight line trend fitted to per capita consumption from 1920 to 1957 inclusive, for a population of 220 million, yields a prediction of 81,660 tons, but as with total lead consumption, it is not a simple matter to choose either the most suitable time span or the type of trend to use. Using only post-war data the result falls much closer to the forecast of 103,500 tons and the trend passes through the cycles of recent years, rather than being so strongly influenced by the highs of the 1920's and the lows of the 1930's. On the other hand, if the 1920's are considered relevant but the 1930's are excluded, a logarithmic trend is more suitable.

Since, in the relationship to the Index of Construction Materials, there is a logical basis on which to forecast, and the value of trend projection in this case is not clear, the figure of 103,500 tons is accepted here as the best prediction of lead consumption for calking in 1975.

2000 1500 1000 500 0
1920 1930 1940 1950 1960 1970 1980

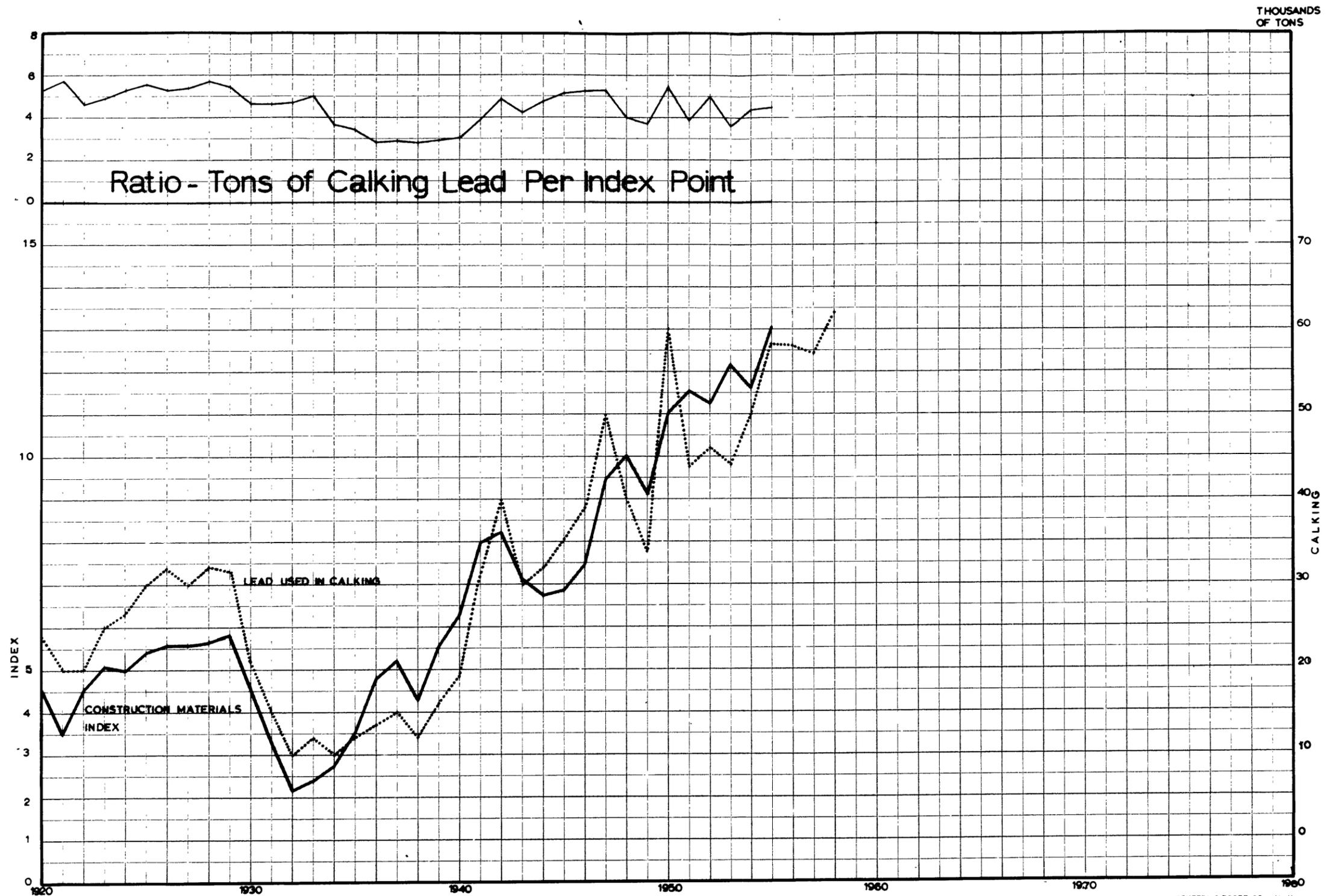


CHART 26. CONSUMPTION OF LEAD FOR CALKING

KEUFFEL & ESSER CO. N. Y.
SOURCES: CALKING DATA - ABMS
CONSTRUCTION MATERIALS INDEX -
ROOS (1957)

Total Forecast.

There is no logical reason to try to forecast this demand sector in toto, i.e. using the LIA series, since over one half is used in calking, which shows continuing growth, while the remainder is declining. Combining the forecasts for "Pipes, Traps, and Bends", "Sheet Lead", and "Calking", the forecast for lead use in construction in 1975 is 133,000 tons, or an increase of 21 percent over 1958 consumption.

Oil Refining and Gasoline

By far the most important completely dissipative use of lead is in tetraethyl fluid ($\text{Pb}(\text{C}_2\text{H}_5)_4$), often referred to as TEL, which is used in high test gasolines to increase their octane ratings and antiknock properties. From this use there is of course no scrap return. Two companies manufacture TEL in the United States -Ethyl Corporation and Du Pont.

Nearly all of the lead included in this sector is used in tetraethyl, and is consumed for the most part by automobiles, trucks, and buses with possibly 15 to 20% being used in aircraft, small boats and stationary plants. Thus, aside from the effects of changes in inventory, there should be good correlation between the number of vehicles miles travelled and lead consumption for TEL. This relationship is complicated by many factors, which result in an increasing apparent consumption of lead per vehicle mile travelled, as shown in chart 27. The term "apparent" is used because it is impossible to separate the amounts of TEL not used for motor vehicles from the total. It should be remembered that small boat use and aircraft travel have increased rapidly since the war years, so that some of the postwar increase in lead used per mile travelled is only "apparent". In what follows the word is not used, but is implied. Forecasts of vehicle miles travelled are available¹, but in forecasting TEL use of lead it is important to assess the factor of lead use per vehicle mile travelled with care since it is vulnerable to many possible technological changes.

Causes of Increased Use of TEL per Mile.

Factors which since 1920 have tended to increase lead consumption per mile travelled include the very innovation of TEL; highway development which has led to higher rates of

¹ In particular, the forecasts used in this study are those of the "Third Progress Report of the Highway Cost Allocation Study", 86th Congress, First Session, House Document No. 91.

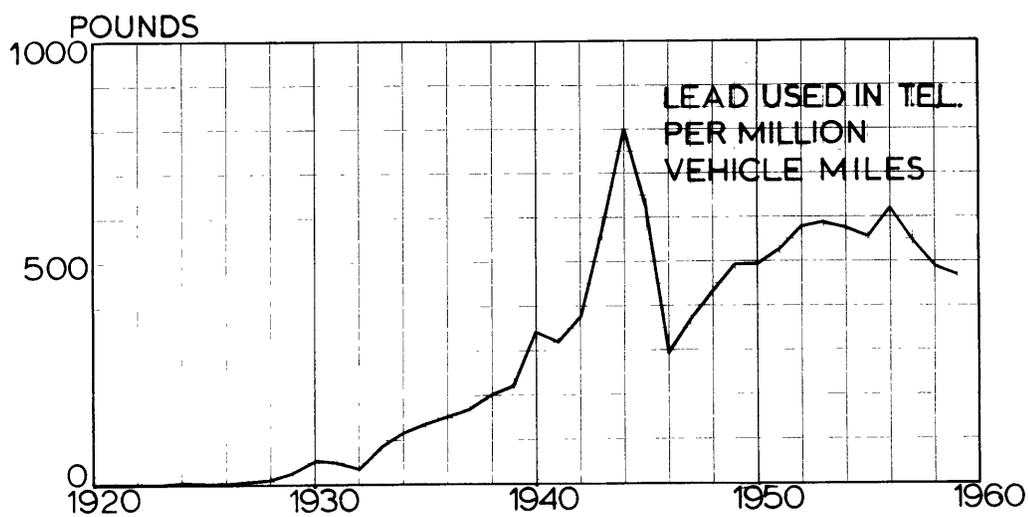
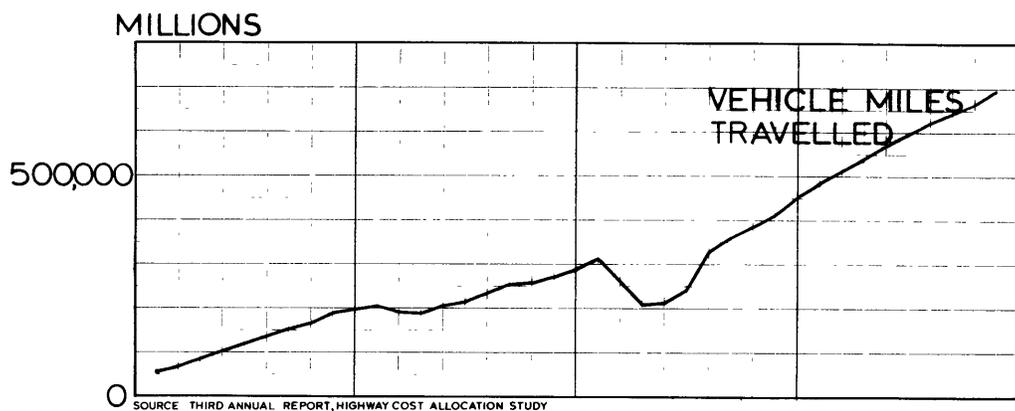
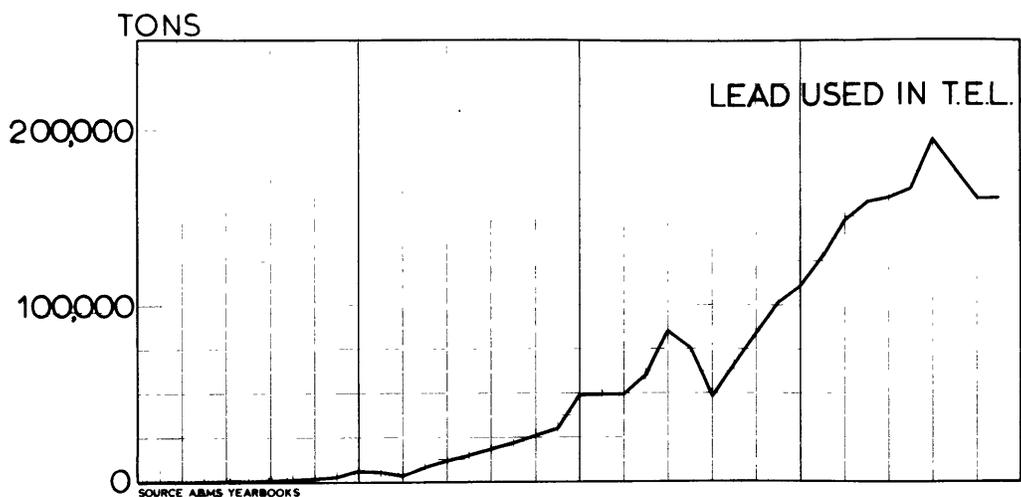


CHART 27. CONSUMPTION OF LEAD IN TETRAETHYL

travel, requiring better performing engines; an increase in size and weight of cars, requiring larger engines, usually with higher compression ratios (these went from 6.55:1 in 1946 to 8.8:1 in 1956); widespread winter use of cars, with consequent need for premium fuels; the introduction of premium and even third grade fuels which contain more TEL per gallon than regular.

New Motive Power Developments.

Almost all of the factors which may reduce the use of lead in gasoline are of currently unknown importance, making this forecast difficult. Some of these can be eliminated on the basis of opinions of experts in the fields involved.

A prospect of long standing is the development of a new type of engine which will operate efficiently on low cost, low grade fuel, and will be suitable for powering an automobile. The nearest approach which has been made is the diesel engine, but lack of rapid acceleration characteristics has limited its use for the most part to trucks, buses and stationary power plants.

Other challengers for the position of the standard gasoline engine include: the turbine, the free piston engine, nuclear-powered engines, electric motors. Turbines have been installed in prototype vehicles, and electric motor vehicles have actually been produced. The turbine, while simple in principle, requires extremely close tolerances in manufacture, working with difficult materials to fabricate, such as niobium, and the cost of production has been impractical. Also, if such motors found wide application there would be a great demand for high temperature metals, which are comparatively rare.

The free piston engine is essentially a diesel which produces gas rather than mechanical motion, the gas then being allowed to expand in a turbine and the power extracted. Automotive experts¹ see little chance that either this engine or the turbine will find wide application in automobiles over

¹ D.M.Borcina, Lead Industries Assoc., verbal communication, April, 1960.

the next decade at least. It is expected that the application of the turbine will be in uses now served by diesel engines, hence will not affect TEL use. Free piston engines are already in use in merchant ships.

Nuclear power plants for automobiles appear to be out of the question for a long time to come, due primarily to the problems of shielding and disposal of radioactive waste. Radiation release caused by traffic accidents would also be a problem. It is concluded that, only the electric motor may be of importance in the period of forecast.

While electric automobiles were manufactured at one time and have lost their popularity, there is today considerable effort being bent toward production of a popular electric, and kits are to be offered for converting from gas engines to electric motors¹. The firm producing this kit is "confident that by the year 1970, fully 50% of the automobiles used in urban areas will be electrically propelled". Such a change-over would appear to significantly decrease TEL requirements, although such cars would presumably be family second cars or for commercial deliveries and calls, due to their limited range (50 to 100 miles), and gas powered cars exclusively would be used for highway travel.

How far the electric car market will develop in the next 15 years is a point of speculation, but electric vehicles may be needed to combat air pollution by exhaust gases in at least some major urban areas. Therefore, the net effects of a change to electric cars are considered in the prediction of TEL use and battery use of lead. Assumptions are necessary. The first is that the kit producer's forecast is optimistic, and that 50% of the automobiles in urban use will be electrics, but only by 1975.

Table 5 shows total United States motor vehicle mileage travelled, urban and non-urban. In recent years the urban

¹ Taylor, Evans, (April, 1959), Appendix, being a resume of articles appearing in Electrical World Magazine.

portion has been slightly less than the non-urban mileage. If this relation persists, and 50% of the urban travel will be done by electric autos by 1975, this would reduce the demand for TEL by about 25%. Below, a forecast of lead demand for gasoline is made which depends upon a forecasted number

Table 5. United States Motor Vehicle Mileage Driven.

Year	Urban	Rural	Av. Miles per Gallon (Passenger Vehicles)
1940	149993	152195	13.79
1945	130161	120012	13.05
1950	218248	239998	12.87
1953	236058	308375	12.77
1954	243639	317218	12.69
1955	267281	336153	12.67
1956	275464	352379	12.54
1957	296699	350305	12.47

Source: Auto Facts and Figures, 1959.

of vehicle miles and a forecasted lead use in TEL per mile driven. The possible 25% reduction in demand outlined above can therefore be subtracted from the demand so estimated without consideration of electric autos.

Historical Trend.

Turning now to the actual forecasting of United States demand for lead in TEL in 1975, the historical trends of lead use in gasoline are examined next. Chart 27 shows total consumption, vehicle miles driven, and consumption per million vehicle miles. Clearly, none of these trends appear to be linear over their entire length, and the downtrend in consumption per vehicle mile since 1953 would not have been foreseen by the forecaster analysing trends even shortly before that time. The reason for the flattening-off in actual use can be seen in the panel "Lead Used in TEL per Million Vehicle Miles". This use of lead illustrates two problems -one the selection of the long-term trend, the second the appraisal of the meaning of the post-1953 decline in lead used per mile driven.

Selection of a trend line for lead used in TEL per mile, given data to 1953 only, would probably result in a straight line being used, and the result would be incorrect by 1959. On the other hand, a trend fitted from 1953 to 1959 shows a rapid decline which will not continue over a much longer period, as will be shown in the next paragraph. In trying to select a trend for the entire period from first production to 1959 the choice of curves is quite limited, mainly to the logistic and Gompertz curves; it is still quite necessary to evaluate the meaning of the recent down-trend, as follows.

The Post-war Trend in TEL Use.

From data published in trade journals, conversations with those in the industry, and correspondence with Ethyl Corporation, the following picture emerges. Octane numbers can be raised by adding TEL, but there are two limiting factors, one the health law that only 3 c.c. per gallon can be used (this was recently increased to 4 c.c.) and the other the economic factor that each additional cc. added increases the octane number by a smaller amount. At current levels, adding one further c.c. of TEL to an average blend would increase the motor octane number by 1 to 1.25 (Oil and Gas Journal, v. 57, no. 40, Sept. 28, 1959, p. 46). Because of pressure from auto manufacturers for higher octane gasolines, producers have invested large amounts in processing units, mainly for catalytic cracking and reforming, alkylation, and isomerization, to increase octanes. Thus the base fuel being used requires less lead to bring it up to a given octane, and although the allowable limit for TEL has recently been increased from 3 c.c. to 4 c.c. per gallon, the lead content of regular grade gasoline shows a marked decrease from about 2.28 c.c. in 1956 to as low as 1.48 c.c. in early 1959. Premium grade gasoline has shown a fairly steady increase in lead content, but standard grade accounts for by far the largest share of sales. However, due to the general recession and a

high rate of gasoline processing construction in previous years, there has been an overcapacity of processing equipment and because of the high capital cost thereof, this has still been run at close to capacity, and the higher octane product used in place of part of the lead. Hence the current lead content is far below the average of the past 5 years, which is reflected in the downtrend of chart 27.

It is therefore concluded that the post-1953 downtrend is a deviation from the long-term trend, due to coincidence of several factors and that the combination of a decrease in construction of new capacity, a return to more normal demand levels, and a continuing demand for higher octane regular grade gasolines to satisfy the "thrifty" engines now advertised to run on regular gasoline will lead to a reversal of this trend. However, there is a difficulty in foreseeing what the long-term growth rate will be.

Projected TEL Use.

Fitting a series of logistic curves by the three-point method to the lead use per million vehicle miles and selecting those based on the most logical years (e.g. excluding the war and 1958 and 1959) gives a range of upper asymptotes from 572 to 590 pounds per million miles, or in other words a flattening-off to an almost horizontal trend is indicated. This trend agrees in general with comments made by Ethyl Corporation¹, and is used here.

Several changes are currently taking place which will tend to keep TEL consumption per mile from increasing at as high a rate as previously. One of these is the widespread use of jet aircraft, using diesel fuel, which will tend to decrease demand for TEL in aviation gasoline, the amount of which has been 12-13% of total domestic use and has been included in the lead statistics used in the above forecast.

¹ M.E.Collins, Manager, Market Analysis and Planning, written communication to the author, dated April 22, 1960.

A decrease in this market, which had previously been a growing one, will tend to flatten the above curve.

The popularity of sports cars, small, and compact cars is contributing toward a higher average mileage per gallon of gasoline, and, assuming that the trend toward such cars continues, the demand for gasoline may decrease from what it may have been for all big cars. This may not be the case. Smaller cars are easier to drive and park, and evidently cheaper to operate, and these advantages lead to more use of the car, hence extra miles per year. Furthermore, the lower prices and operating costs will probably make cars available to more people, and increase the number of cars per family. To 1957 at least, the average mileage per gallon of gasoline for passenger vehicles showed a steady decline, from 13.79 in 1940 to 12.47 in 1957 (Statistical Abstract, 1959). This trend has increased the lead consumed per mile driven, but the use of smaller cars, should it develop further, will tend to flatten off the trend.

At least one company now markets unleaded premium gasoline. This is done by using in the unleaded gas more of the high octane products produced by refining and processing, but requires either more such capacity or higher lead content in the standard grade gas, which is leaded. The second method of course involves no capital investment and is thus the method used. Furthermore, such unleaded gasoline represents only about 2% of total sales in the United States and this percentage is not expected to increase¹. This therefore adds little to the flattening-off.

Thus, to forecast a flattening-off of the trend in lead used per million vehicle miles at a level about that of 1953, or to forecast an uptrend much flatter than pre-1953 appears to be reasonable in view of what is known about the industry.

¹ M.E. Collins, *ibid.*

Substitution.

One other possibility of great interest in the TEL picture has not been discussed -that is the question of substitution. Two problems preclude its discussion in quantitative terms. The first is that the technology of the industry is exceedingly complex and it is impossible to know how good the competing products really are, especially in view of the great power of the Ethyl Corporation in the industry, through its size and especially through long-term contracts. The price per unit of any additive is really of little meaning without careful technical study of its total effects. The second problem is that several new additives have recently been developed, but it is likewise impossible, even knowing their overall qualities and effects, to estimate how much they might affect the TEL industry. It should be noted however that many hundreds of substitutes have been carefully tested since and before TEL was discovered, and none has been successful. Furthermore, most of the new additives are to be used in conjunction with TEL. Shell has already marketed TCP (tri-cresyl phosphate), while Ethyl Corporation has AK-33X, a manganese additive, reportedly close to marketing, and Texaco has TLA (tertiary butyl acetate)¹. How these will affect TEL use is unknown, but they are used in conjunction with it to obtain greater octane increase per unit of TEL than is possible with lead alone, i.e. they could either increase or somewhat decrease TEL use.

Iron carbonyl ($\text{Fe}(\text{C})_5$) was developed as a substitute for TEL during wartime when lead was in short supply, but deposition of iron oxides causes spark plug fouling and severe engine wear, according to several sources. No market could be found for the product when TEL again became available.

The long-term effects of the new additives, although important to future lead consumption, must be ignored (but not forgotten) in making the prediction derived below.

¹ Oil and Gas Journal, September 1959, p. 47.

Forecast.

Estimates of vehicle miles driven are available for 1976 from the Third Annual Report of the Highway Cost Allocation Study. On the basis of the projected growth rate from 1966 to 1976, the mileage for 1975 can be estimated at 1,081,100 million miles. This figure can then be combined with pounds of lead per million vehicle miles travelled, or 572 to 590 pounds, to predict lead consumption in TEL in 1975 at 309,200 to 313,950 tons.

From the estimated levels of 1975 demand, the 25% previously considered to be vulnerable to replacement by electric power can now be subtracted. This gives future demand for 1975 on this assumption as 231,900 to 235,500 tons per year. Because of the major effect on lead consumption in TEL and in batteries that the change to electric automobiles would have, the ultimate forecast presented shows predictions both including and excluding this development.

One further comment might be added. In 1952, it was reported that one large TEL manufacturer did not anticipate that the industry would be using 300,000 tons of lead per year by 1975, as was estimated by the Paley Commission (Andrew Fletcher, American Metal Market, September 23, 1952). No decrease in TEL use per mile driven is indicated over the long run, at least so far as can be determined, so that unless the forecasts of vehicle mileage are in grave error it would appear that the Paley Report figure may be a close estimate.

Cable Covering

For many years, the cable covering market has held the position of second, sometimes third largest user of lead. Its low working temperature, ease of joining, softness and pliability, and the resistance to corrosion made lead the best material available in the past. However, post-war development of various plastics, notably polyethylene, has yielded new covering materials in which these properties are as good as or better than those of lead, and which have high tensile strength, good fatigue resistance, and are lighter and easier to handle.

Three types of multiple cable sheathings are common: 1) lead-plastic sheath, (called by Western Electric "Lepeth"), using slightly less lead than an all-lead sheath; 2) aluminum-polyethylene sheath, called "Alpeth"; 3) steel-aluminum-polyethylene sheath, called "Stalpeth".

In the manufacture of lead sheathed cable, the lead is extruded over the wires (usually insulated with manila) in a continuous process, hence must work at temperatures and pressures low enough to avoid damaging the wires. No problem exists for the plastic coverings since they are easily workable at low temperatures. The metal sheath in Alpeth and Stalpeth is quite thin, and is corrugated to increase strength and flexibility. In Stalpeth cable the thin steel covering provides a hermetic seal over the initial metal sheath and gives the maximum protection against moisture and other physical damage. In Alpeth and Stalpeth, polyethylene is extruded over the metal sheaths, while in Lepeth the polyethylene is inside the lead sheath.

The short supply of lead during the war and in the ensuing few years accelerated the development of these new sheaths, and in 1947 commercial amounts of Alpeth became available. Although as late as 1948 the lead industry considered the cable covering market safe, by 1953 the rate of substitution was intense, and can be expected to become even

more intense. Many important advantages result from using polyethylene-sheathed cables: it eliminates the use of a strategic material (the amount of aluminum used is inconsequential); weight is reduced by 50% with consequent savings in handling effort in installation, and shipping costs; the lighter weight permits longer duct runs and aerial runs, eliminating many costly splices, and, in the latter, allowing the use of smaller supporting wires; greater corrosion resistance, with a considerable reduction in cable failures, improves service and lowers costs¹. There would have to be a strong price differential in favor of lead in order to assure it a competitive position. Perhaps the only advantages lead sheathed cables enjoy over multi-sheathed are good scrap return and relative ease of splicing. The extra number of splices required with lead sheathed cable, and the shorter runs necessitated by its weight are, however, sufficient to outweigh these advantages. There is thus no technological reason to believe that lead will find a major use as a cable sheathing material in the future.

A large percentage of the sheathed cable produced is currently used in communications, power transmission using lesser amounts. Thus, in chart 28 there is seen to be an excellent correlation from 1920 to 1941 between the annual changes of wire mileage in all telephone systems of the United States, and the number of tons of lead used for cable covering. (In 1958, 97.6% of total wire mileage was in cable.)² War demands disrupted the relation from 1941 to 1945, but in the phenomenal growth of telephone systems since the war, lead has played a rapidly decreasing role as multi-sheath cables took over the market, and by 1958 cable covering had fallen from a position of second largest use of lead in pre-war years to fourth largest.

¹ Western Electric Co. Inc., October 1958, Cable sheath -a review from iron pipe to Stalpath: Staff Report, The Western Electric Engineer, v. 11, no. 4, pp. 2-9.

² Statistical Abstract, 1959.

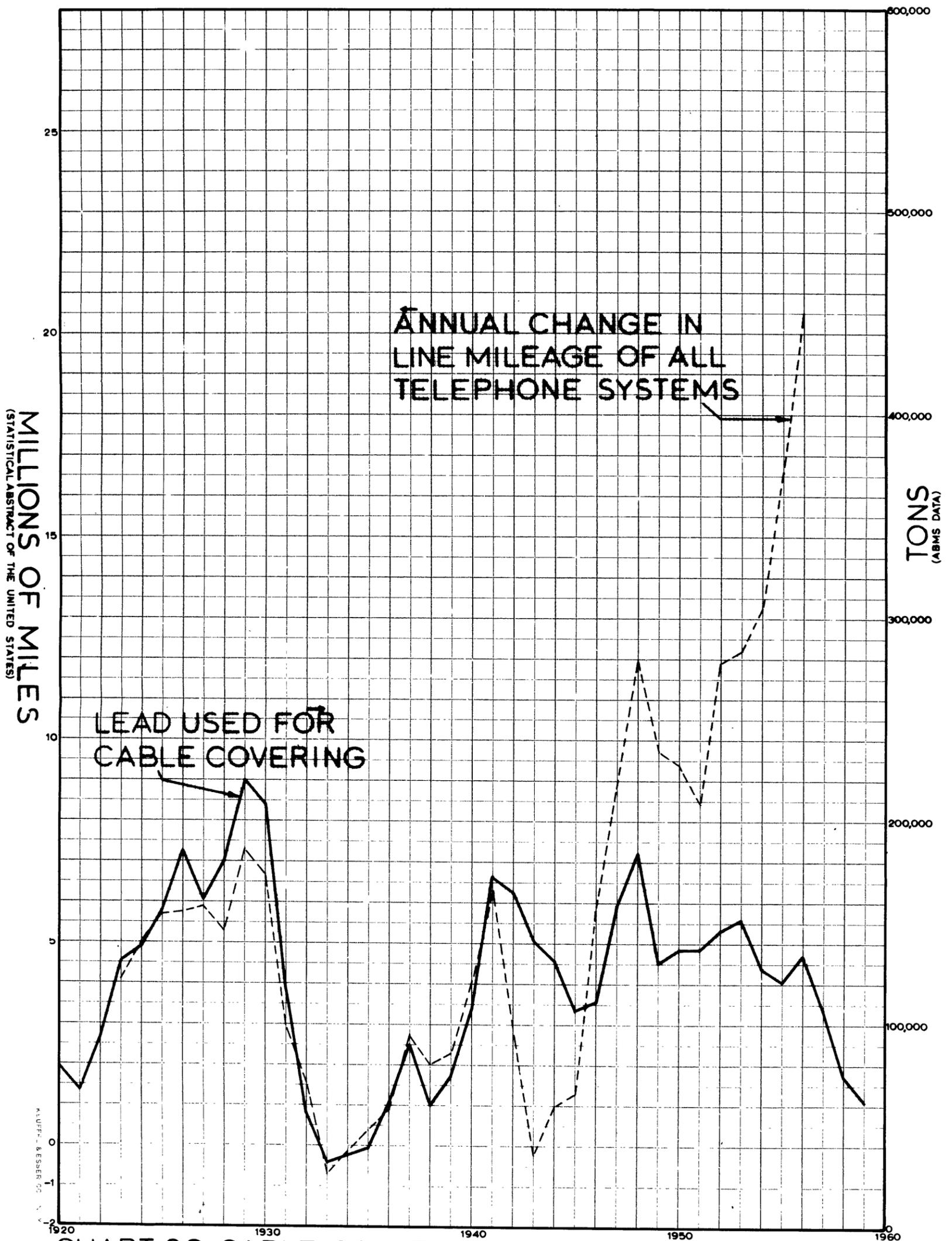


CHART 28. CABLE COVERING AND COMMUNICATIONS.

Table 6 presents some recent statistics on the cable production of Western Electric Company, which produces a large amount of this country's sheathed cable, to indicate how rapid is the transition. The name "Lepeth" refers to lead-plastic sheathed cable. Western Electric, at one time one of the nation's largest lead buyers, now obtains almost all its lead from cable junked by the Nassau Smelting and Refining Company¹,

Table 6. Relation of Lead-sheathed Cable to Total Cable Produced by Western Electric Company.

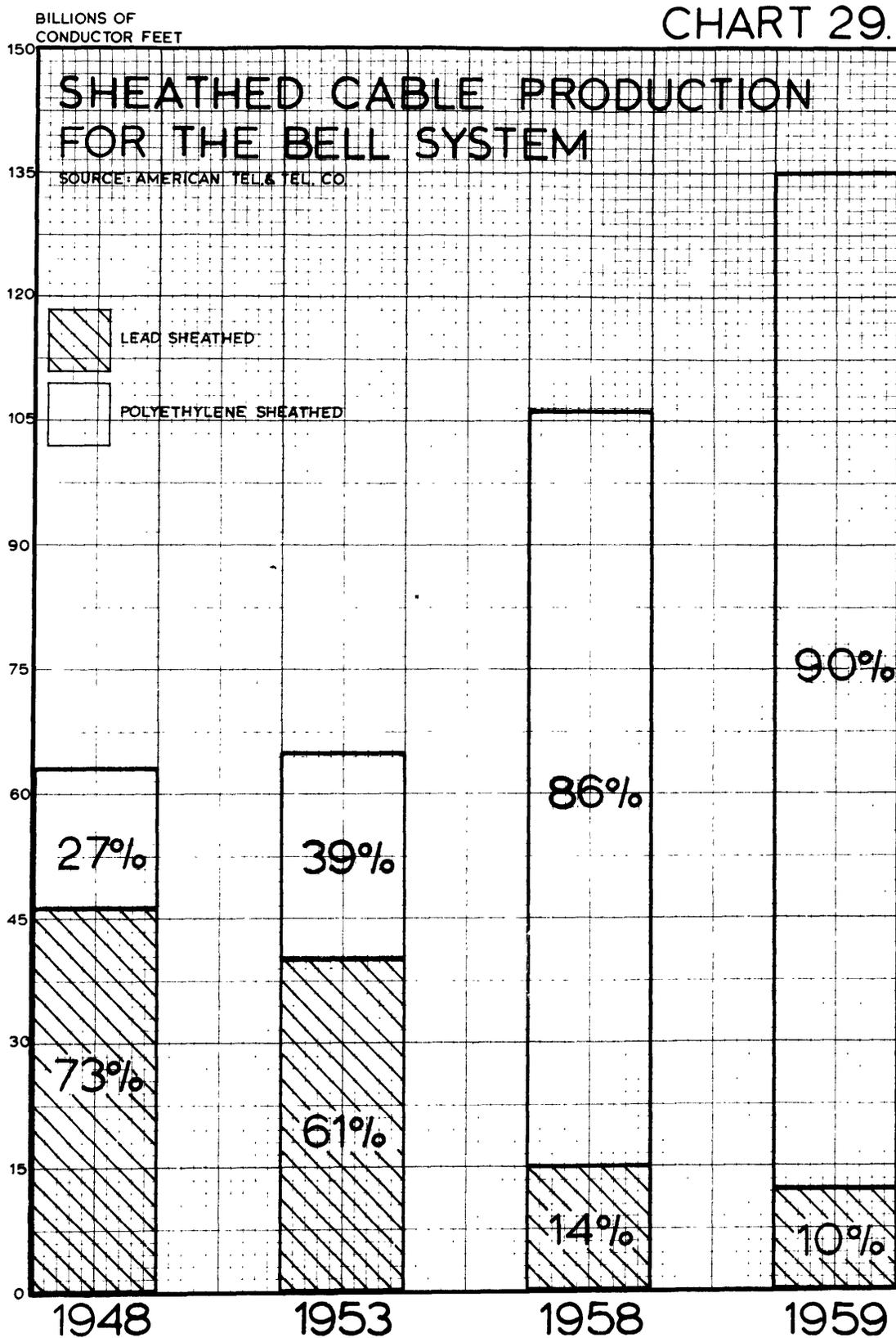
Billions of Conductor Feet of Cable				
Year	Total Produced by Western	Lepeth Produced by Western	Lead-sheathed Cable Purchased by Western	Lead-sheathed Produced as a Percentage of Total Produced
1957	105.933	22.366	27.280	21.11
1958	104.155	12.181	2.139	11.69
1959	129.053	6.359	2.628	4.92

SOURCE: F.G. Zimmer, Western Electric Co. Inc., letter to the author, March 18, 1960.

Chart 29 shows the production of exchange type cable for the Bell System for the years 1948, 1953, 1958 and 1959, divided into polyethylene and lead sheathed types. The decline in lead sheathing over the eleven years is even more dramatic than over the three years shown in the table.

In addition to the declining demand for lead-sheathed cable, there is a superimposed factor of substitution for all sheathed cables of the types mentioned. New systems of communication which do not require sheathed cable, or much less of it are being developed or are already in use. These include microwave and coaxial cable systems, and the experimental systems which reflect signals from the moon or from artificial

¹ Western Electric Co. Inc., October 1958, Cable sheath - a review from iron pipe to Stalpeth: Staff Report, The Western Electric Engineer, v. 11, no. 4, pp. 2-9.



satellites. (The latter is already in use by the United States Navy on the west coast.) In Canada, the microwave system, which reflects signals from the troposphere, is in use for much of the long range communication ordinarily serviced by cables. Coaxial conductors are still exclusively lead-sheathed, but these and other special cables which are lead-sheathed represent only a small part of the total cable production. (In 1958, only 90,442 miles of coaxial tube were in use.)¹.

In view of the combined effect of these two dynamic changes taking place in the communications industry, the outlook for lead for 1975 in this field is dark indeed. But the lead industry is spending money on research and looking for ways to hold the market. The Lepeth cable is an improvement over fully lead-sheathed cable to only a small extent, for its weight and the attendant problems remain. Using 2-inch diameter core as an example the following comparison can be made for aerial-type cable².

	<u>Fully Lead Sheathed</u>	<u>Lepeth Sheathed</u>
Lead Sheathing: thickness	0.115"	0.105"
wt. in lbs/100'	3,741	3,690
Polyethylene: thickness	-	0.075"
wt. in lbs/100'	-	195
Paper Liner: thickness	-	0.005"
wt. in lbs/100'	-	22.69

A further improvement in lead sheathing comes as a result of Lead Industries Association research, which developed a press for extruding three types of lead alloy sheathing, previously impossible due to segregation of the alloy metals in the extruding device. This and other alloys make the lead sheath more resistant to mechanical damage, and, it

¹ Statistical Abstract, 1959.

² F.G. Zimmer, Western Electric Co. Inc.; personal letter to author, dated April 14, 1960.

is reported, to corrosion. However, the lead sheathed cable still suffers from the difficulties of stiffness and weight.

It is here concluded that on the basis of their service to date the Alpeth and Stalpeth cables will not be found to fail in service, and that therefore their superior characteristics of weight, flexibility, and corrosion resistance will lead to essentially complete domination of the cable covering field by 1975. The rate of replacement may be retarded for perhaps another ten years until the new materials have been adequately use-tested under a variety of conditions, but barring failure in general service, replacement should be virtually complete, except for a small volume of coaxial and other special cables, over the next 15 years, and this large use of lead will have declined to perhaps two thousand tons per year at most.

Paint and Varnish¹

Several compounds of lead are used in the paint industry, either as base pigments or colors, the latter being discussed here under a separate heading.

White Lead.

The three major components of a paint are: the "raw material" or white pigment; the "vehicle", some kind of oil; and the color. Lead, zinc and titanium compounds are the usual white pigments, and historically white lead was used, 64,000 tons of lead being employed for this purpose in 1934. Several factors have caused zinc and titanium to cut deeply into this application however, and in 1958 only 13,700 tons of lead was so employed, or less than 20% of the 1934 figure. Of these factors some are inherent faults of the lead compounds, others are superior characteristics of the type found important in paints, others are circumstance.

The theoretical composition of white lead, ($2\text{PbCO}_3 \cdot \text{Pb}(\text{OH})_2$), or basic carbonate of lead is 68.90% lead carbonate, 31.10% lead hydrate but the lead carbonate may vary from 68 to 75 percent. Under weathering tests, ready-mixed pure white lead paint chalks badly, yellows, especially in sulphurous air and sulphuretted hydrogen, and holds dust and dirt very firmly. Chalking can be controlled by adding inert fillers, and white lead does mix perfectly with most pigments. It forms compounds with the vegetable oil vehicles called lead soaps, which results in a tough, flexible film. However, the use of synthetic resin vehicles and most recently water for exterior paints nullifies this useful property, and has hastened the decrease in use of white lead.

The major fault of white lead is its toxicity in manufacture, application and use, the lead poisoning effect being cumulative.

¹ General Reference: Remington and Francis (1954)

on the application. This pigment spreads well, is very opaque, and covers or hides nearly as well as pure white lead. In 1958 the lead content of leaded zinc oxide made up one percent of all lead used in pigments, or 3,402 tons (Minerals Yearbook, 1958).

Lithopone, a mixture of barium sulphate and zinc sulphide, is a pigment of fairly good performance and low cost, and is therefore being used in many enamels, although suffering much competition from titanium oxide.

Titanium oxide, or titanium white, is the second major competitor for white lead. It may be used in its pure form, or blended with barium sulphate. This pigment is the whitest and most permanent used in paints, and also the most opaque. It is non-poisonous, unaffected by acids or acid fumes, sea water, chemical salts, or sulphur compounds. The brightness value is very high, while at the same time it has excellent hiding power. Paints made with titanium oxide have excellent flow and levelling properties and are chemically non-reactive. Permanence, whiteness, non-toxicity and lower cost make titanium oxide preferable to white lead.

Lithopone and lead titanate are sometimes blended with titanium oxide to give improved gloss retention, and a 1:1 blend with zinc oxide gives the best chalking control.

Barite, whiting, gypsum, china clay and silica are used in paints as extenders of the main pigment, and have been used to replace some white lead.

The following figures show the relative position of white lead with respect to the coefficient of reflection, magnesium carbonate taken as 98:

Titanium oxide	86
High grade lithopone	83
High grade French Process zinc oxide	83
White lead	79
5% leaded zinc oxide	77

The hiding powers are:

Titanium oxide	91	sq.	cm.	per	gram	for	complete	hiding
Lithopone	50	"	"	"	"	"	"	"
Zinc oxide	49	"	"	"	"	"	"	"
White lead	47	"	"	"	"	"	"	"

The combination of the excellent qualities of the zinc and titanium pigments, notably the latter, and the failings of white lead, notably its tendency to yellow and its cumulative toxic nature have accentuated the effect of the higher price of white lead, and account for the declining use of lead as a white pigment.

Other Lead Compounds.

Chart 12 shows per capita consumption of lead in paint and varnish. But it does not show the entire picture since white lead, as stated above, is only one of several lead compounds used in paints. In 1959, red lead accounted for as much lead in paints as did white lead, litharge almost half as much, and miscellaneous compounds more than half as much (LIA accounting). Thus the trend is a composite of several presumably rather divergent trends. The decline of white lead consumption to approximately one-tenth of its 1934 level, with only a slight uptrend in red lead and litharge consumption accounts for most of the decline. No LIA series of data are available for the use of miscellaneous compounds, part of which is accounted for by the lead content of leaded zinc oxide, or for red lead, white lead, and litharge separately, as used in the paint industry. However, the U.S. Bureau of Mines accounting¹ does separate white lead, red lead and litharge used in paints and pigments. These are shown in chart 30, and although the accounting procedure gives different totals from those of LIA, it is the trends which are important.

¹ See the annual Minerals Yearbooks, under Lead Pigments in the chapter on Lead.

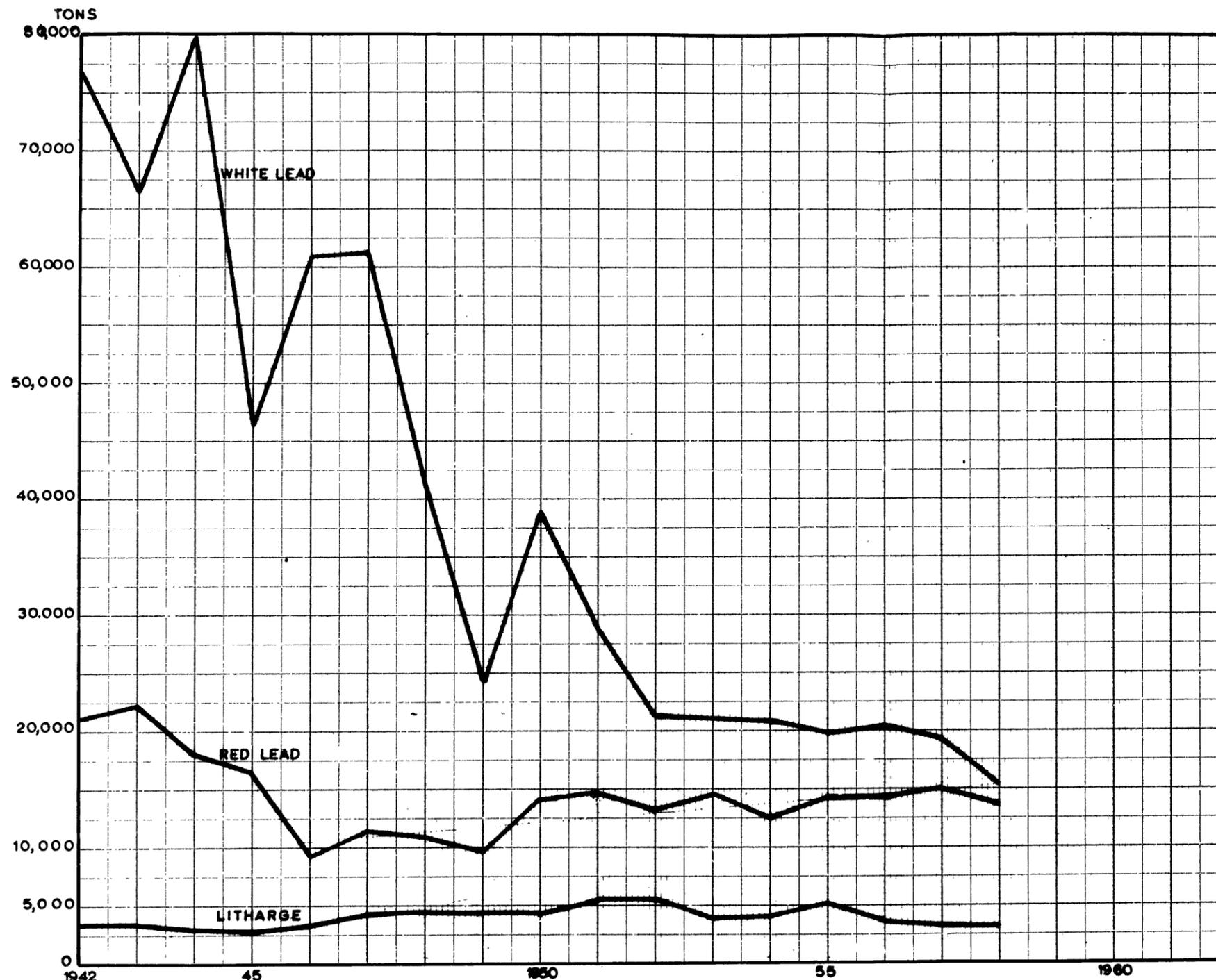


CHART 30. CONSUMPTION OF LEAD IN PAINTS & VARNISH

SOURCE: MINERALS YEARBOOKS, USBM

Litharge is used to make chrome pigments, 3,731 tons being so employed in 1958 (Minerals Yearbook, 1958), and as a drier in varnishes and oils, a small amount being used as a metal protective paint. The amount used in chrome pigments is included in the accounting for Colors. In 1958, the USBM reported 3,223 tons used in varnishes and oils. This application is expected to continue at about the current level.

For many years red lead has been the accepted primer for protecting iron and steel against rust in construction work, bridges, highway signs, posts, and railings, railroad structures, ships, etc. Extensive testing has shown it to be superior to all other types of primer for protecting steel¹. However, several other primers are currently finding considerable application, especially aluminum paints, iron oxide-base paint, and the alkaline, cement-like paints, (e.g. Tnemec) all of which are cheaper than red lead paints. Price and drying time required apparently account for red lead being supplanted as an auto body primer. Another trend which diminishes the use of red lead is increasing use of aluminum. Thus, the Federal Highway Aid Program should increase the use of red lead for structural steel, signs, etc. but there is more and more aluminum, both natural and enamelled, being used for highway signs and markers, and several aluminum bridges have already been built, as well as numerous bridge railings. The result is that the trend of red lead consumption (chart 30) shows only a slight uptrend, in spite of increased construction and an accumulating stock of objects to be painted.

Forecast.

The decline of the large-tonnage item, white lead, dominates the trend of lead consumption in paints since 1920 and is not offset by the weak uptrend in red lead consumption due to increased construction activity and the continued

1

Lead in Modern Industry, p. 161.

recognition of red lead as an excellent anticorrosive paint. The diminishing in the use of white lead appears to be easing somewhat, but continues through 1959.

The development of a new type of white lead pigment by National Lead Co., marketed in 1958, will probably add to the decline, since it consists of a fusion of basic lead chromate and silica on a fine-grained silica core, requiring much less lead per gallon of paint¹. Any trend line fitted to the white lead consumption figures shows a vanishing of this market even before 1975, but it is more reasonable to assume that some at least will continue to be used. What amount this might be is speculative; it is clear that the important position previously held has been lost to zinc and titanium oxides, and a consumption of 5,000 to 10,000 tons per year by 1975 is probably a generous estimate, and assumes a high quality product at a minimum price.

Litharge demand for paint and varnish has been projected on the basis of a straight line trend fitted from 1946 to 1958 inclusive, yielding a forecast of 3,186 tons per year by 1975².

It is difficult to quantify the possible effect of widespread use of aluminum on red lead consumption. The priming of structural steel and steel used in water will continue to demand considerable tonnages, and chances of replacement of these uses of steel by aluminum seem slight. Increased competition from cheaper primers is slight; compared to that of the structures being protected, the price of the primer is of little importance. However, some of these cheaper primers also claim to outperform red lead and there does appear to be increasing use of aluminum paint directly on steel, and of basic, cement-like primers. It is assumed here that since

¹ Minerals Yearbook, 1958.

² The trend line is: $y = 4237 - 45.7 x$, origin 1952, x units half-years.

there is already a stock of objects which require painting, to which additions continue to be made, the trend established since World War II is the best estimate available of future demand. Should aluminum replace large amounts of painted steel in the next 15 years, the forecast will be too high. Projecting a straight line trend fitted to USBM data from 1946 to 1958 yields a forecast of consumption of 22,358 tons per year by 1975¹.

Total lead consumption in paints is thus predicted as the sum of the three uses, or 30,500 to 35,500 tons per year for 1975.

¹ The trend line is : $y = 12974 + 408 x$, origin 1952, x units half-years.

Ammunition

Little can be said about the technology of the use of lead in ammunition. Because of its high specific gravity it imparts greater momentum to the projectile than would be accomplished with lighter slugs. However, with high-powered, high muzzle velocity firearms, the bullet, if made of lead, becomes deformed and the trajectory is poor. Steel-jacketed lead bullets are one answer. But most lead is used today as shot by sportsmen, (about 5 tons for every ton of lead used for bullets in peace time)¹, and although during the second World War lead consumption in ammunition rose sharply, this was due to its use only in shortarms, and some gas seals on large shells, since other military arms do not use lead bullets.

Chart 31 shows the trend of the ammunition sector of lead demand from 1920, and chart 12 from 1942, on a per capita basis. The long-term trend from 1920 is well established as a slow downtrend, but from 1953 there appears to have been a rejuvenation, according to the long-term data (ABMS). Note the sharp discrepancy with the LIA data from 1954 on. This is due to differences in accounting, the LIA coverage being less complete. Since the longer series of ABMS data is available for the Ammunition sector, it has been used in making the forecast. Lead shot is used in ammunition and also in coating steel sheet, and the latter lead is included in the ammunition data, but since it amounts to perhaps 2,000 tons of lead per year², it does not seriously distort the trend.

A straight line trend appears to adequately describe this sector on a per capita basis. Four trends have been fitted, the first from 1920 to 1959, including the war years, the second with the war years reduced to the level of 1939,

¹ Lead in Modern Industry, p. 126.

² Estimated by D.M. Borcina, Lead Industries Association in letter to the author.

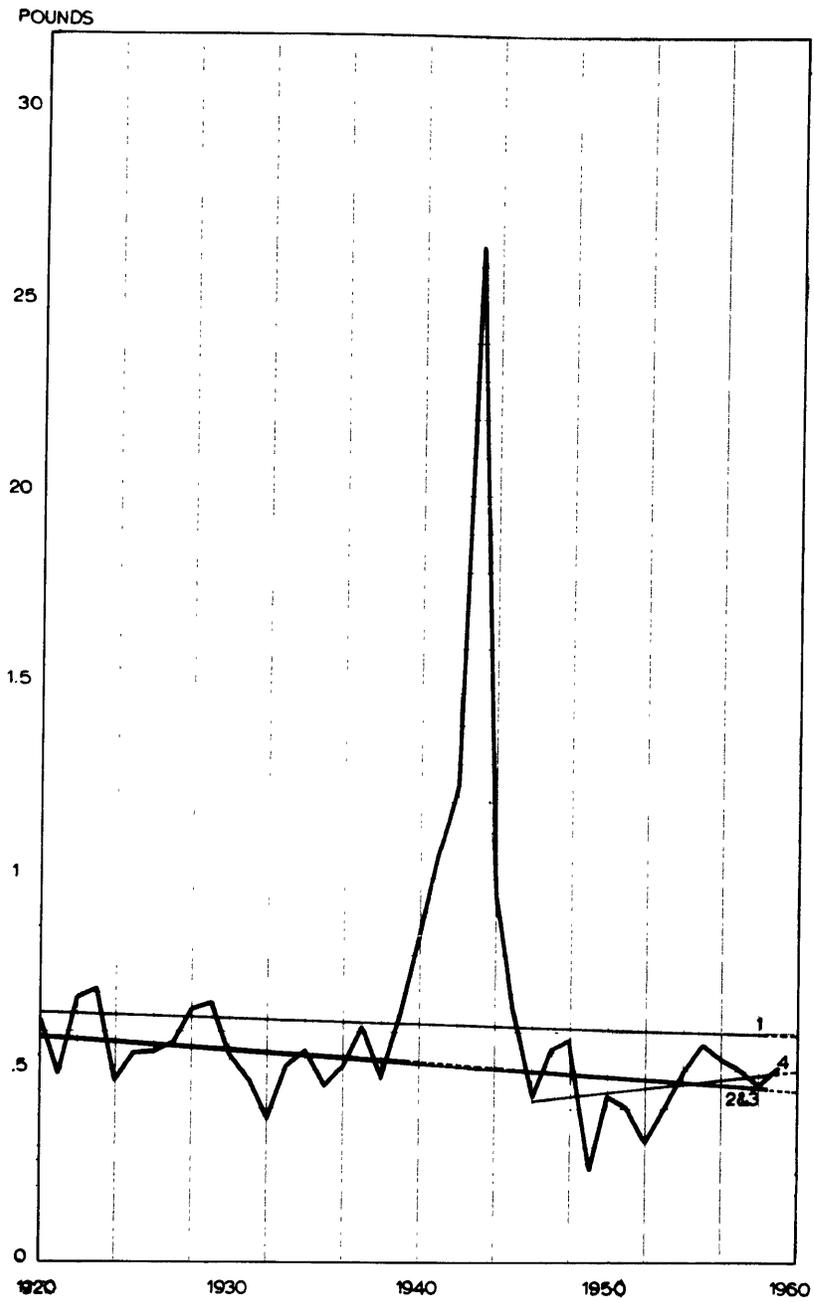


CHART 31. PER CAPITA CONSUMPTION OF LEAD IN AMMUNITION

the third with the war years removed, the fourth to postwar data only. Results are tabulated in table 7 and the trends are shown in chart 31. Combined with population forecasts, the range for 1975 consumption is 35,800 to 70,200 tons. Trend line 1 is obviously biased upward by the high consumption of the war period, and does not pass through or near the cycles of normal years. It can therefore be rejected. Trend 2, for which the years 1940 to 1945 were reduced to the 1939 level, and trend 3, appear to represent the trend rather well over

Table 7. Trend of Lead Consumption for Ammunition.

No:	Trend Equation	Years Fitted	Forecast for 1975 (Tons x 10 ³)
1	$Y=0.65165 - 0.0013585X$	1920-1959	57.1-67.4
2	$Y=0.58600 - 0.0033507X$	1920-1959	35.8-42.2
3	$Y=0.59041 - 0.0038442X$	1920-'39, '46-'59	39.0-46.0
4	$Y=0.4599 + 0.002622X$	1946-1959	59.5-70.2

the entire period (except the war), although it may be argued that the possible uptrend since the war is not well represented. It is necessary therefore to consider the significance of this uptrend. The first possibility is that it is due to the inclusion in the data of the small but growing amount of lead shot used in coating steel, but this has only a small effect. The second, and more important, is that the Korean War has distorted the use pattern. While it may have had some effect, it would be expected to increase consumption, while in fact the years 1950-1953 are the period farthest below the rather even trend, and recovery to the normal level thus appears as an uptrend. Considering that recent years have seen an increase in leisure time, and prosperity for most people, the sport of hunting should attract more interest. But with increased population the area available per hunter becomes decreased, and, since the population is becoming concentrated in urban areas, it becomes more of an effort to get out to hunt. Therefore it seems likely that the slow decline observed in per capita use of lead in ammunition will

continue, and that the uptrend shown by trend 4 is based on too short a period to show the actual trend. Technological change seems to have little to do with the trend, and the problem of trend is a much a psychological one as technological. Therefore, there is no alternative but to use trend projection, on a per capita basis, numbers 2 and 3 being apparently the most representative. The forecast for 1975 is therefore 36,000 to 46,000 tons, or 41,000 tons \pm 5,000. Note that this is about the current level of consumption, so no growth is forecast for the ammunition sector.

Since most ammunition lead is used for sporting shot, the author cannot agree with the Paley Commission's forecast of a doubling of 1950 consumption, based on a doubling of military defense material needs.

Colors¹

Lead chromates are used for primrose, yellow, orange, scarlet, and red paints and are the most important yellow pigments in the paint and allied industries, and are essential in process color printing. Relatively cheap, they have been improved with time so that this market for lead has little to fear from widespread substitution, and it is not surprising to see that the per capita use of lead in colors shows a positive increase over the long term (chart 32).

Generally, light shades contain 58-60% lead chromate, the remainder being lead sulphate or phosphate. Orange and red chromes contain about 85% lead chromate and 15% lead oxide.

Chrome yellows are much less fast to light than zinc yellow, but have good body and brightness of tone. The remarkable color permanence of zinc yellow makes it a strong competitor however.

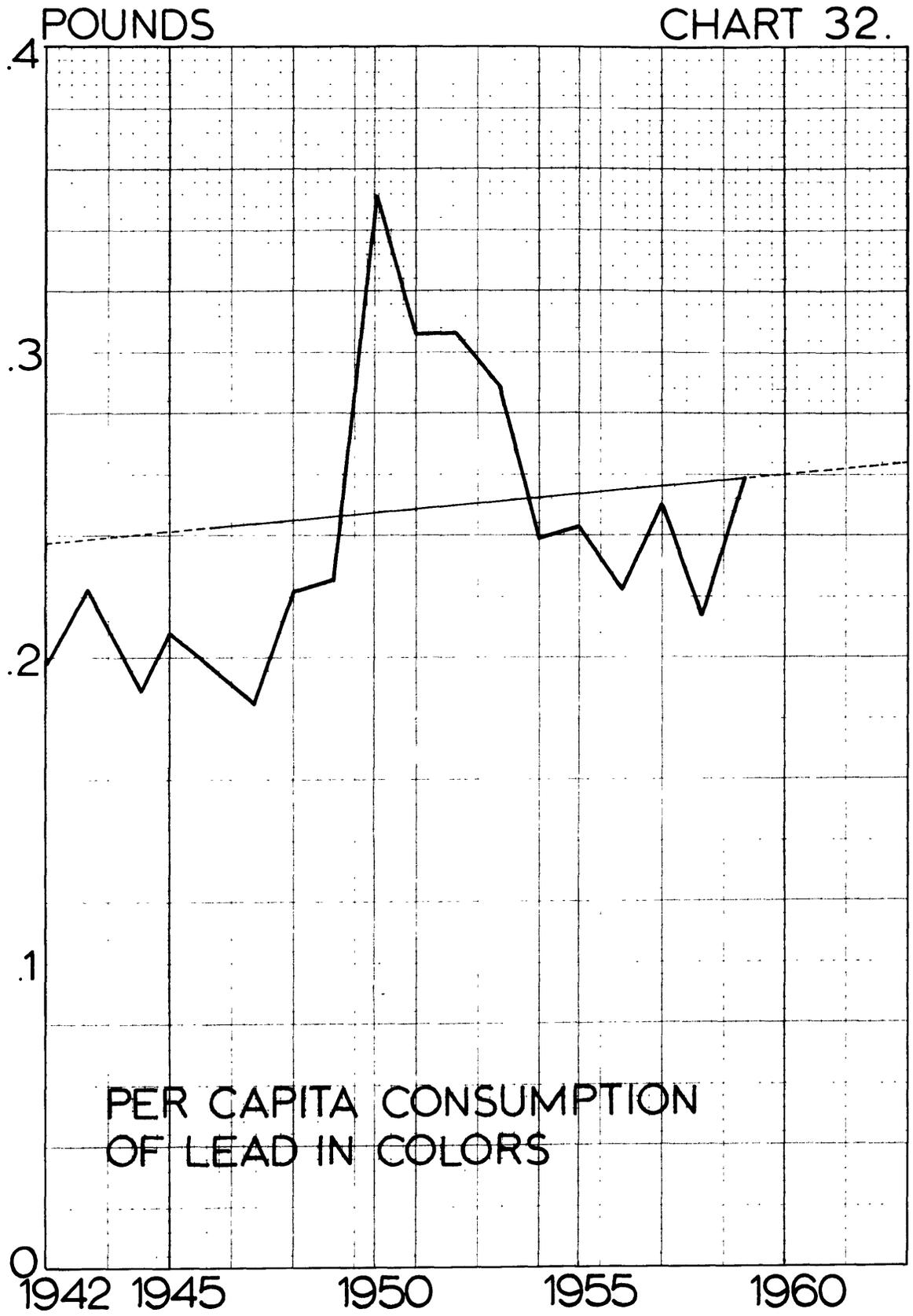
Only small amounts of red lead are used in manufacturing colors, the main lead compound used being litharge, but other forms of lead are often purchased and converted by the color manufacturer.

There is no foreseeable reason why this well-established use for lead should not continue to grow at least as fast as population, and in fact chart 32 shows that the use per capita is increasing slowly but steadily since the war. This is considered to be due to the wider use of paints in decorating, the popularity of bright shades, and the growing per capita amount of housing, automobiles, and other consumer goods, many of which require paint.

A straight line trend has therefore been fitted to per capita consumption of lead in colors. The expression is:
 (Per capita consumption of lead in colors, in pounds) =

$$0.242354 + 0.00128070 \cdot (\text{Year minus } 1946)$$

¹ General reference: Remington and Francis (1954)



which for 1975 gives a predicted demand of 0.2795 pounds per capita. Combined with high and low population forecasts (205,907,000 and 242,880,000, respectively), this gives a net demand for 1975 of 28,775 to 33,942 tons.

The data for color use exemplify several of the problems typical of this forecasting procedure. First, the number of years of available data is small. Second, the atypical wartime demand precludes the use of data from 1942 to 1945. Third, the hoarding and eventual use of manufacturers' stockpiles in 1950, 1951 and 1952 distorts the consumption pattern and changes the trend line (fitted by least squares). Nonetheless, the period 1946 to 1959 used to determine the trend is the most suitable on the basis of available statistics, and although the trend line may appear to be slightly too high, it must be remembered that the large amount of lead purchased by color manufacturers at the start of the Korean war was eventually consumed over the following few years and thus represents true demand, although somewhat distorted over time.

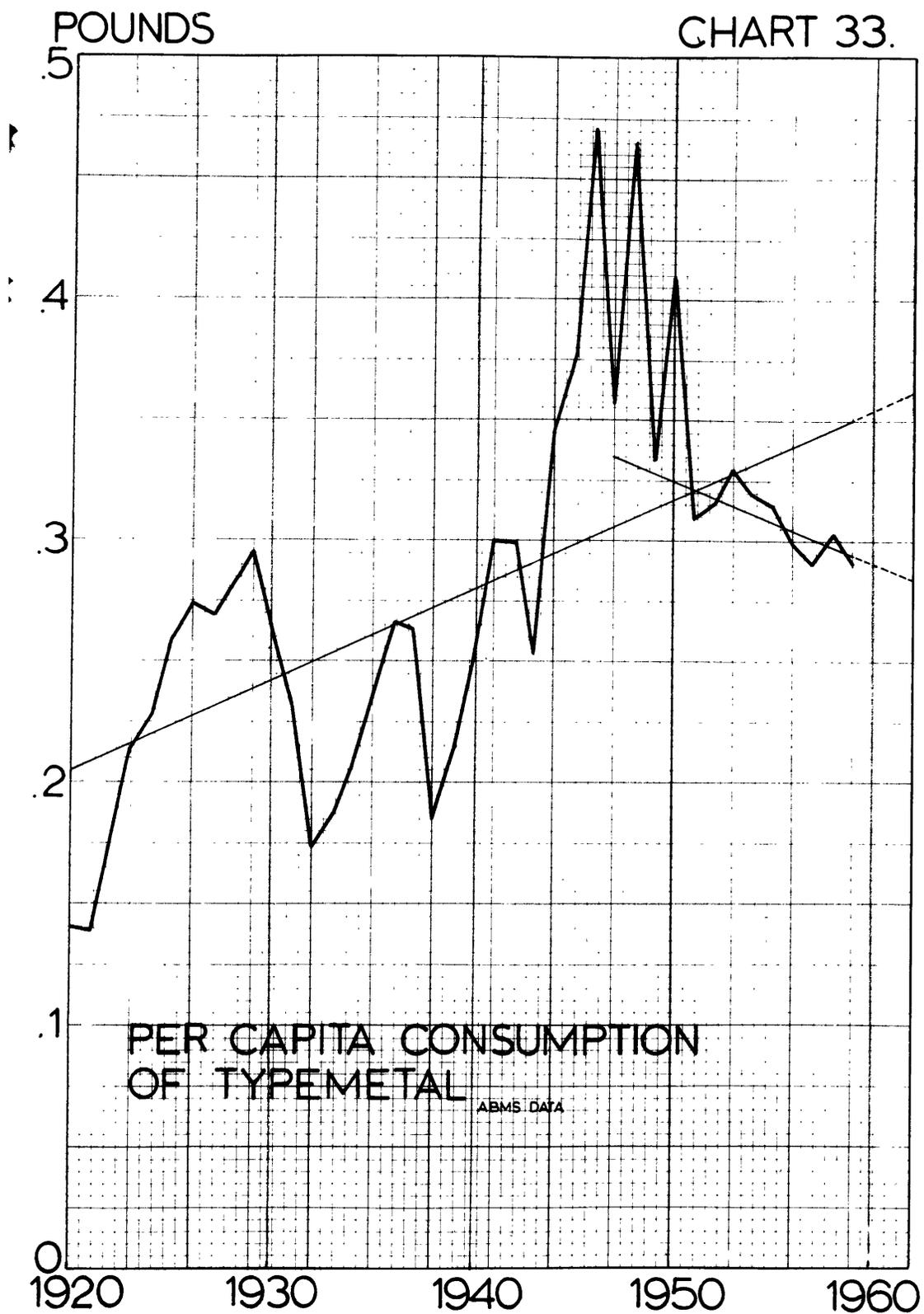
If instead of the least squares line, a straight line is fitted by eye to pass through the middles of cycles from 1954 on, and to take approximate account of the exaggerated demand of the years from 1950 to 1953, there appears to be less than 2,000 tons difference in the forecast for 1975, so the least squares trend has been used here.

Printing¹

Type metal is a combination of lead, tin and antimony which may contain from 64 to 94% lead, depending on its purpose, and is useful because of its relatively low melting point and the fact that it does not clog modern typesetting machines. Many years ago, hand-set type was made from lead, and today the most commonly used typographic machines, linotype and monotype, still use lead by keeping it molten in small pots in the machine and casting it around the various type pieces, either a line or a letter at a time. The resulting type is used directly for small jobs, but for most long runs, such as newspapers, stereotype plates are made by impressing the form of the composed type sheet in a fiber composition material, drying and hardening it, and casting stereotype metal, a lead alloy, into this die to form a much thinner sheet, which may be flat, or curved to fit high speed rotary printing presses. Electrotypes are made similarly with an impression of the sheet in wax or sheet lead, onto which copper is electroplated to form a thin sheet which is then backed up with electrotype metal, another high lead alloy.

Chart 33 shows that per capita typemetal consumption, having grown steadily with the printing industry until World War II, has experienced a continuing decline in the postwar period although the industry continues to expand. The small tonnage of metal reported used represents only a minor fraction of the amount employed each year in printing, since most large printing shops remelt their own type in electric pots and send out only the dross to be resmelted. One Boston daily newspaper estimates that it alone handles more than 4,500 tons of typemetal per year. Therefore, the accounting

¹ N.B. : Data given by LIA departs widely from ABMS data since 1956, and the data used in this study are the ABMS series, which includes the antimony content of type metal in the figures given for lead.



used here shows only typemetal dross, new metal added to the circulating load, (if any), and the small amount of typemetal sent out of the printing shop for remelting, mainly from small shops, the latter comprising by far the largest percentage of typemetal consumption reported, (70 to 90%).

The decline may therefore be interpreted as due to a reduction in the number of small printing shops, or an increase in the number which remelt their own typemetal, in which case it is only apparent and not real. But the technology of printing has seen innovations in recent years which indicates that the decline is due to the supplanting of hot-metal methods and machines. The first of these developments, offset, (or offset-lithography), came into use about 1900 and although faster and more economical than typographic methods using the monotype or linotype machines, the poor quality impression and inability to withstand long runs restricted its use until after 1945, when the method was greatly improved, although numerous units were in use in the 1930's and 1940's.

In this method no lead type or engraved plates are used, but the printing is done from a rubber roller which has on it an inked image from a metal plate, or lithograph. The lithograph is a flat plate on which a greasy substance to which ink adheres has been reproduced in the form of the copy. In modern machines this is done by photography. The problem lies in getting the original copy to be photographed, and it previously had to be set up in type and one clear copy run off. Since World War II photographic composition machines have been developed which do not use lead at all, and which compose the plate directly. The advantages over typographic methods seem great, and there is the possibility of using electronic processes, undoubtedly the basis of the future printing industry. For example, the Photon machine using automatic tape input can handle 8 to 10 characters per second compared to the ordinary typist on a monotype or lino-

type, and the spacing of the letters is done electronically. Probably the most important money saving is due to the ease of changing type size (by magnification) and style. A matrix disc for a Photon machine has in effect 192 characters, and can be so easily and rapidly removed and replaced by another that over 17,000 characters are available on a single machine. The equivalent variety of one Photon matrix disc, which costs about \$2,000 made to order, in a hot-metal machine costs about \$76,000.

Because of expense and delay in producing photo-engravings (i.e. of pictures, etc.), photo-offset methods have not been adopted wholeheartedly by newspaper printers, but recently a new process has been developed that produces a photo-etched magnesium plate in minutes, and a photo-sensitive plastic is also being developed that will save both time and money. Five or more important newspaper are therefore using the Photon machine now, and others can be expected to follow. In printing catalogues, business forms, and advertising, photocomposition has already been widely accepted.

It seems reasonable then that the old typographic printing techniques requiring the use of molten metal will be steadily superseded by faster, simpler, safer, and more flexible methods which have already been developed, and used on a production basis. The rate of replacement is estimated by Mr. Durham Miller of Photon Company¹ to be such that on a conservative basis, about one-half of the lead used today will not be needed by 1975, and it may not be at all unrealistic to consider that closer to one hundred percent will be replaced, small amounts being used for specialty purposes. It is likely that small printers will not change to these new methods for some time since the replacement costs will often not be justified for the advantages gained, and used typographic equipment will probably be available at low

¹ Personal communication

prices. Since it is the small printer whose typemetal accounts for much of the apparent consumption of lead, the decline in this use will probably not become any steeper than it currently appears but will continue as obsolescence and wear remove typographic machinery from service and it is replaced by offset equipment. Accordingly, the extrapolation of a straight line trend fitted to per capita consumption is expected to be representative of what may be expected. The forecast from a line fitted from 1947 to 1959 is for an apparent consumption of 13,300 to 15,700 tons in 1975. Note however that since 1951 there has been a flattening-off in the decline -a trend fitted to the years 1951-1959 gives a forecast of 24,750 to 29,200 tons, or almost double the other trend forecast¹. Although the number of data used is very small this trend is considered the most likely, since a large amount of lithographic equipment was put into use from 1945 to 1950, and 25,000 tons, less 10% for estimated average antimony content, or 22,500 tons, is the forecast figure. It must be remembered that because only a small sample of the actual tonnage of typemetal used enters into the accounting, this forecast is very difficult to quantify on a rational basis, even if one were an expert in the printing field, and the forecast figure must be regarded as provisional.

¹ Trend equations are: a) Fitted to 1947-1959:
$$y = 0.389869 - 0.0093171 x$$
b) Fitted to 1951-1959:
$$y = 0.33538 - 0.0033922 x$$
Origin 1947, x units years.

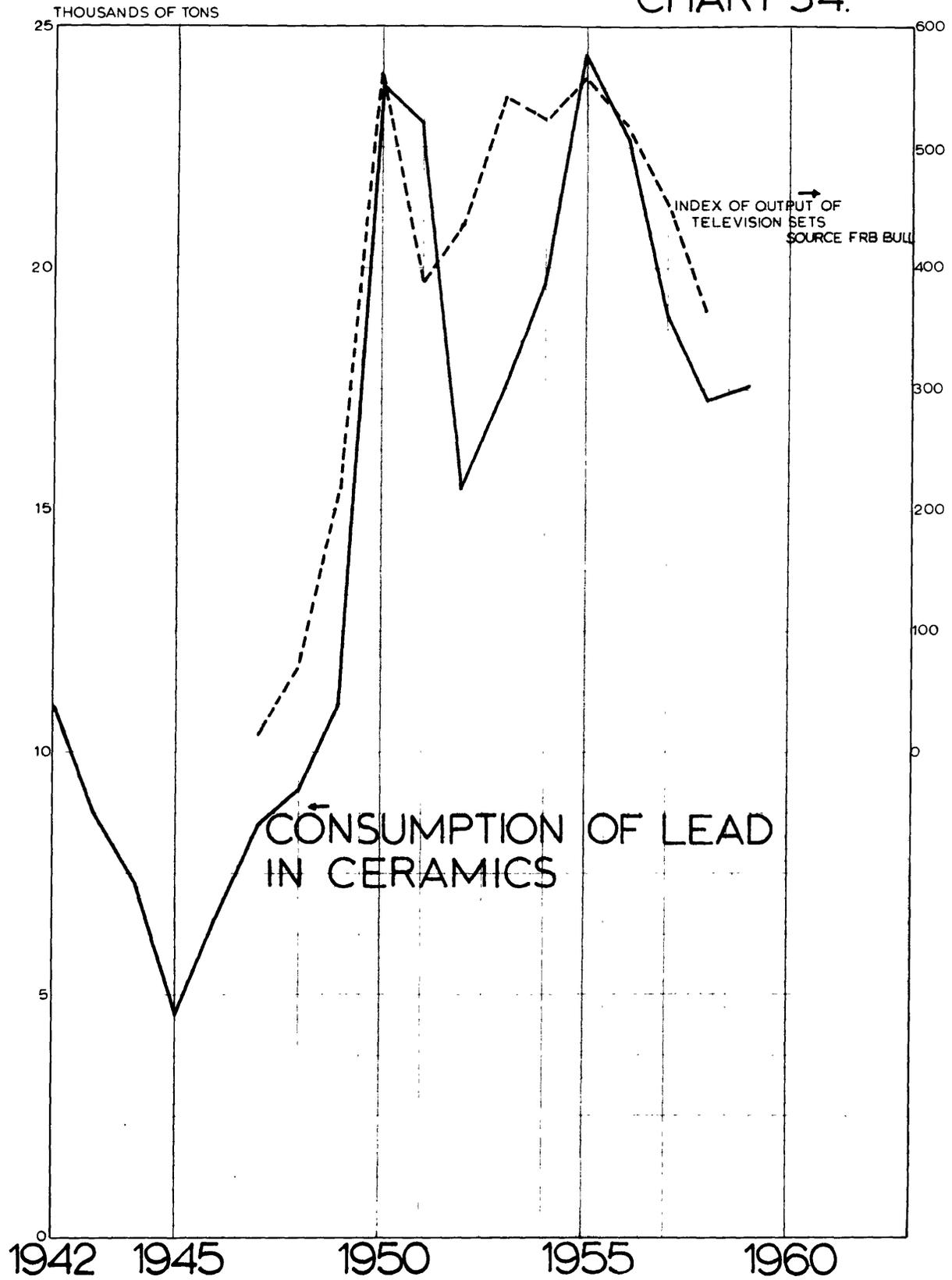
Ceramics

Litharge is the form of lead most commonly used in ceramics, which includes glassmaking as well as pottery making and enameling. The beautiful sparkle and pleasing resonance of fine crystal is due in large measure to its lead content, which may be as high as 80% by weight. Television picture tubes contain as much as 30% lead, and have accounted for a large part of the considerable postwar increase in the use of lead in ceramics to be noted in chart 34. Also shown is an index of output of television sets, although this does not take into account replacement tubes. None of this growth can be attributed directly to increasing use of flat or plate glass in housing, since this type of glass does not ordinarily contain lead, although some double-pane windows use a lead seal.

Many kinds of ceramics are coated with leaded glazes -artware, plumbing fixtures, tableware, wall tiles, etc. Lead has long been used in ceramic glazes because of the low melting points and good flow characteristics of lead glazes, and today fine china still often carries a highly transparent lead glaze. A use which is becoming important is in porcelain enamel; at one time this was a large use of lead, to the extent that twenty five years ago the majority of enamellers used lead based "frits" -the granular material from which porcelain enamel is formed. But here as in paints the toxic nature of lead caused handling difficulties and it lost markets. The low melting point of lead base frits may still provide lead with one of its largest markets in the future.

Since 1955 lead has become the usual base for enamels on aluminum, for the simple reason that standard enamels must be fired at 1450-1550°F, a temperature at which aluminum warps badly and may even melt. Lead base frits (30-40% lead) can be rapidly applied at 975°F. Other low temperature enamels recently developed do not perform as well as lead-based enamels, and result in more rejects. When porcelain

CHART 34.



enamels are applied to aluminum they actually bond to an oxide coating, and can be punched, sheared, sawed or drilled with little spalling. The toxic qualities of lead are not important, since the porcelainized surfaces are not usually in contact with food, and technology can avoid poisoning in manufacturing, mainly through the process of "fritting" the glaze, which gives a granular material rather than a powder.

One of the newest and most rapidly growing uses for enameled aluminum is in building sheathing, due to the spreading use of curtain wall construction. Other potentially large uses in construction are for colored aluminum sidings, primary and storm windows and doors, eaves troughs and down-pipes, and interior wall panels. The penetration of aluminum into the field of windows and doors alone increased 1400 percent from 1946 to 1954. Rosenzweig (1957) forecasted a further 300 percent increase from 1954 to 1965 for this use.

Miscellaneous products, such as fences, gates, railings, awnings, venetian blinds, lawn furniture, and so on, provide further markets.

Highway signs and railway equipment are two current uses of porcelainized aluminum in transportation.

The possibility of extending the lead-base frits to steel and aluminized steel is even more exciting, because of the many uses of enameled steel -in buildings, appliances, cabinets, etc. The selling point is that the lower firing temperatures would allow the use of thinner sheet metal and cheaper grades of steel, and that lead is particularly useful in colored enamels, which are now popular.

In the new field of aluminized steel, lead composition frits will probably be used for all porcelainizing, again because of the low working temperatures, and the market potential for porcelainized, aluminized steel appears large indeed, since it combines advantages of both steel and aluminum. It consists of a layer of aluminum on a steel base. The aluminum provides light weight, a good bonding and protective layer, more consistent production results in

enamelling, and improved workability. The steel base gives structural strength, allows thinner gages than all aluminum, resists warping better, and is cheaper than aluminum¹.

Although porcelainizing is currently a far smaller use of lead than glass manufacturing (less than 500 tons per year), the potential is easily appreciated. Recognizing this, the LIA has begun to promote the use of lead base frits, and hopes to make ceramics a major consumer industry.

Forecast.

As mentioned previously, ceramics is a composite demand sector for which no detailed data have ever been collected. Trends for ceramic glazes, glass manufacturing, and porcelainizing are different and should be treated separately, but without the data it is impossible to know even which is the most important use. Because of the high lead content of television picture tubes, and the large number manufactured, this probably dominates at present, while porcelainizing is known to be small. Several new and important developments may also be placed under this heading; namely lead-base colored glazes on building blocks, which are proving quite popular, and high-lead glass for transparent shielding in "hot labs" (see "New Uses"). In checking the forecast in the future it will be necessary to determine how these uses are classified.

It appears that the meager available data reflect only the superposition of an increasing consumption of litharge in television tubes on the basic level of consumption in pottery glazes and in other kinds of glass. Demand for lead in porcelain is too small to be noticed at present, but the potential seems tremendous. Unless lead-base frits for aluminum are supplanted by some other type, and it appears at present that their only problem is for surfaces with which food will be in contact, lead consumption should grow as

¹ Iron Age, August 8, 1959, p.46.

rapidly as uses for porcelainized aluminum¹. Added to this potential is the possibility of a huge market for glazing steel and aluminized steel.

To make a forecast based on data available to date would tend to ignore these possibilities, although the data show an uptrend, due probably to glass manufacture taking an increasing amount of lead for television tubes.

Growth beyond the present level is confidently expected. As long ago as 1948, the ceramics industry forecasted more than a tripling from 1948 to 1973 for architectural porcelain, and a doubling for major home appliances². Rosenzweig (1957) estimates a growth for total aluminum use in construction (not including possible structural beam use) of 13.1% per year from 1954 to 1965, or a 288 percent increase in 11 years. If lead frits were also applied widely to steel, the market could easily show a 5-fold increase by 1975, which would make this one of the most important demand sectors, using perhaps as much as 100,000 tons of non-recoverable lead annually.

In the other demand sectors where there is an indication of great potential markets, e.g. batteries, the forecast used here is essentially based on the projection of historical data, and the existence of the great potential can merely be noted. This procedure will bias the forecast downward, assuming that some at least of the potential is realized, but the degree of speculation in most cases is such that to include strong forecasts for these factors would alter the entire nature of the attempted prediction. Thus, the forecast for this sector might be taken as the projection of a linear trend fitted to per capita or total consumption from 1947 to 1959. Not only is the data series short, but because the first and last years lie below the general level the trend will be

¹ This is essentially a psychological problem, since once glazed into porcelain the lead is no longer free.

² Ceramics Industry, June, 1948.

biased downward. Furthermore, the trend is due at least in part to picture tube lead use, while the future level is concluded to depend on consumption for porcelainizing aluminum. Therefore, for this demand sector it is more reasonable simply to estimate future consumption by relating it to expected expansion of porcelain enamelled aluminum. However, no forecast of this market could be obtained from the industry.

Noting that use has grown from 3.5 million square feet in 1956 to an estimated 23 million in 1961, a 7.5-fold increase of which building accounted for about 65 percent¹, it is very conservatively estimated here that by 1975 lead consumption for porcelainizing will increase 3-fold to about 50,000 tons per year.

1

Modern Metals, March 1958, v. 14, no. 2, p. 72-3, from a study by the Porcelain Enamel Institute.

Railroads

Storage batteries find considerable application in railroad equipment, being used mainly in diesel locomotives and railway carlighting, but these uses are included in the storage battery classification in this study. The other major use of lead in the railroad industry is in journal bearings¹, in the form of babbitt, or bearing metal, which use makes up most of the consumption reported in the railroads classification. The discussion which follows thus centers about the use of lead in railway car truck bearings.

Railway Journal Bearings.

While some 19,486 (61%) of the total of 32,006 passenger cars owned by all railroads plus the Pullman Company were equipped with roller bearings by 1958, the total number of passenger cars is only 1.56% of the total number of freight cars in the United States, (2,051,580 in 1958)² and the more rapid change to roller bearings in passenger cars is thus not significant in the study of lead use. Discussion is therefore limited to freight cars.

Each freight car has eight bearings, known as journal bearings, which may be solid bearing, cartridge, or roller bearing type, only the first two using lead, and the cartridge type using only small amounts. The solid bearings consist of a shell, usually bronze, lined with lead-base babbitt, or white metal, which is the actual bearing surface, and weigh about 30 pounds each, being about 82% bronze and 18% babbitt when new. For every hundred pounds of bearings used, some four pounds of babbitt is worn away and hence lost. Average life of a solid bearing is about 3.3 years, and since the

¹ The part of the axle in contact with the bearing surfaces is termed the journal, and the solid bearing assembly box is often called the "hot box", since if poorly lubricated it overheats.

² Railroad Car Facts, 1958, published by the American Railway Car Institute, for this and other freight car statistics.

approximately 2 million freight cars have a total of some 16 million solid bearings, about 5 million are currently replaced each year¹. This accounts for the apparent consumption of about 10,000 to 15,000 tons of lead per year, much of which is of course returned for scrap. For the present, there are about 2,000 freight cars equipped with the sleeve bearing cartridge type journal bearing, and 35,163 freight cars, as of 1958, equipped with roller bearings.

It would appear that the future demand for lead in railroad use (excluding batteries) could be predicted directly on the basis of the future growth of ton-miles of freight handled, for which forecasts are available. However, there are complications. Chart 35, the first and second panels, shows consumption of lead by railways for bearings and the number of ton-miles of freight carried respectively. The decline in consumption of lead is obviously sharper than the decline in freight carried, and the third panel shows the number of pounds of lead consumed per ton-mile of freight carried. Part of this decline is due to a small increase in the number of tons carried per carload.

Preliminary Forecast.

Thus at least two factors enter the forecast for lead demand -the growth of ton-miles of freight carried, and the declining consumption of lead per ton-mile. Each of these can be estimated, and the estimates combined to predict lead demand for 1975. Plotting on a semilogarithmic scale the trend of lead use per million ton-miles of freight hauled (chart 36) yields a series of fluctuations about a straight line, discounting the wartime years, and the trend fitted to this series yields a projected lead consumption of 10.117 pounds per million ton-miles.

¹ G.T. Butcher, Product Manager-Bearings, American Brake Shoe Co., Railroad Products Division; letter to author dated March 25, 1960.

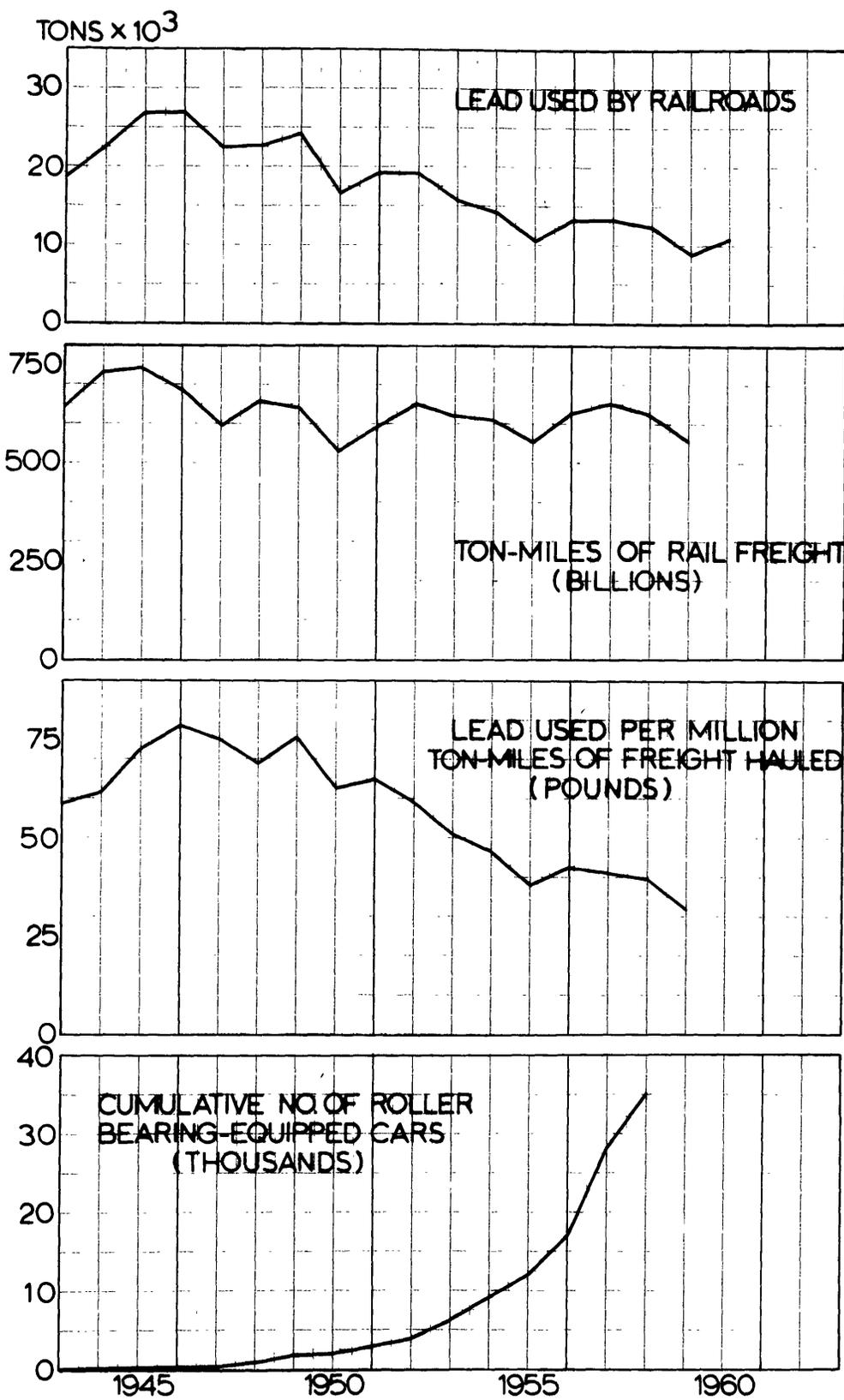


CHART 35. CONSUMPTION OF LEAD BY RAILWAYS

CHART 36.

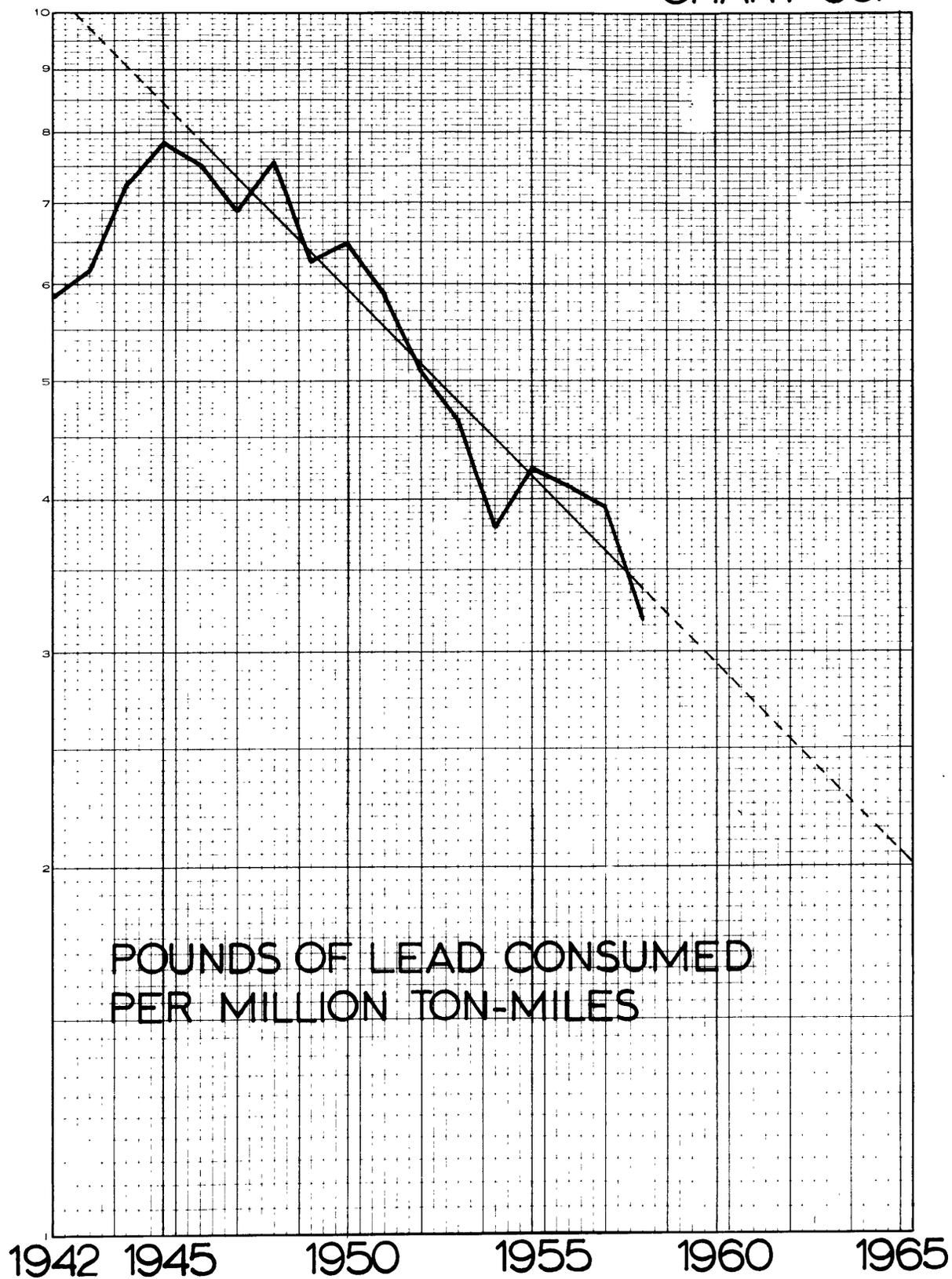


Table 8 shows a forecast of ton-miles prepared by the Bureau of Railway Economics of the American Association of Railroads¹. For 1975 the forecast is for 1,019,000 million ton-miles. Chart 37 shows the long-term trend determined by the author, which gives a forecast to 1975 of 634,400 million ton-miles, based on a straight line fitted to the data for 1946 through 1957. A trend projection of a straight line fitted to ton-miles of freight carried per capita, (chart 38) yields 2,487 for 1975 and combined with the population figure for 1975 used by the American Association of Railroads (235,246,000) gives 585,125 million ton-miles. The American Association of Railroads forecast was described by Mr. Monroe as "highly speculative and probably unduly optimistic from a railroad standpoint". Also, it apparently includes passenger traffic as well as freight, but this is a very small fraction of the difference in predictions of about 385,000 million ton-miles. If, instead of 45% of total intercity ton-miles, the railroads obtain only 30%, they would haul some 679,260 million ton-miles. Table 8, column 7, shows the historical trend of the railroads' share of intercity freight. The growth of the trucking industry since the war accounts for much of the decline. Even assuming that piggyback service will increase railroad traffic, it would appear optimistic to forecast retention of 45% of the traffic. Thus a compromise of 800,000 million ton-miles for 1975 is used here, or about 35% retention of traffic.

Combining this figure with lead consumption per million ton-miles (10.117 pounds) gives a forecasted demand of 4,047 tons or about 4,000 tons of lead per year by 1975. If the "optimistic" figure of 1,019,000 million ton-miles is used, the forecast is still only 5,150 tons.

¹ J.E. Monroe, Vice-President; letter to the author dated March 11, 1960.

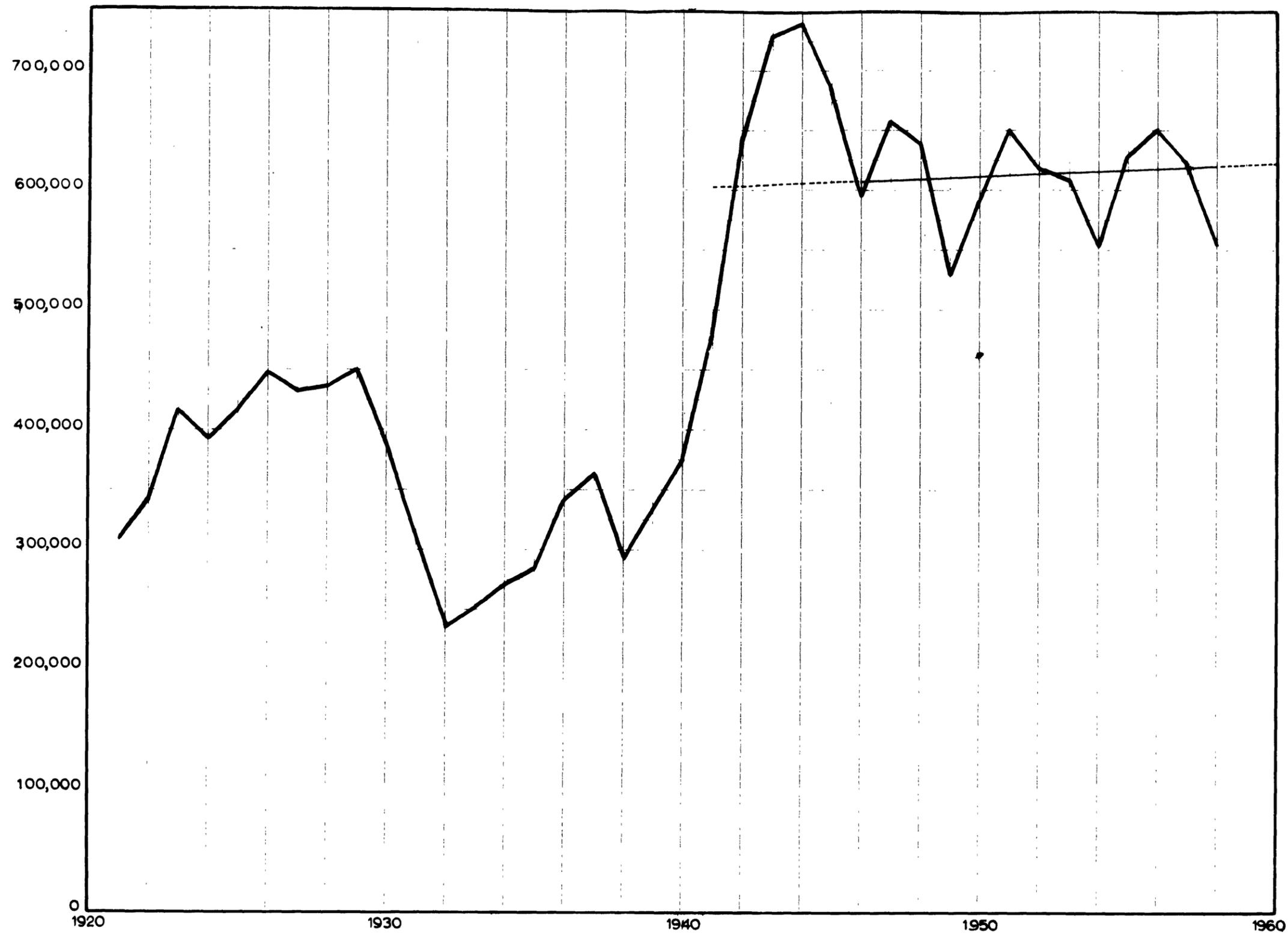
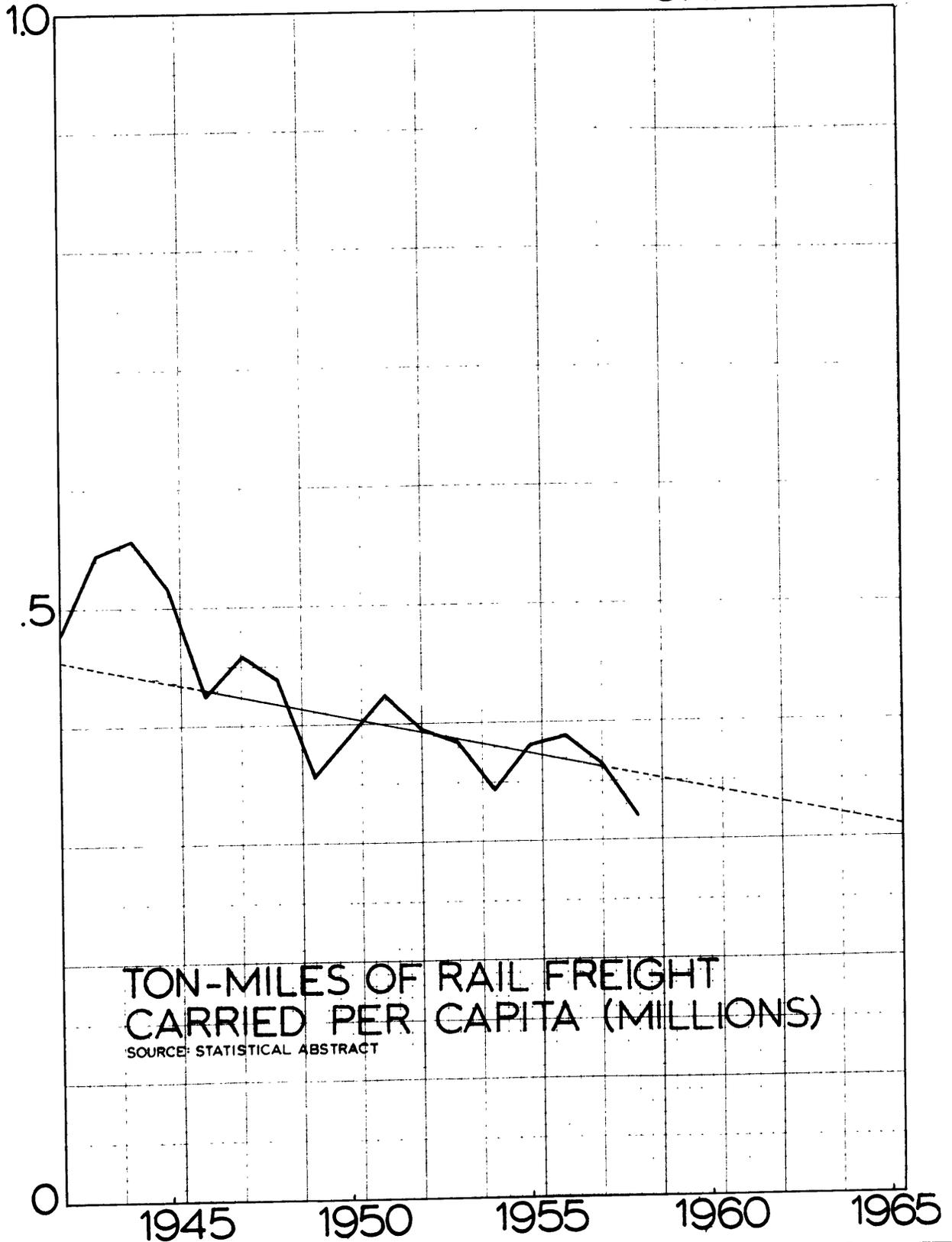


CHART 37. MILLIONS OF TON-MILES OF RAILWAY FREIGHT CARRIED

SOURCE: STATISTICAL ABSTRACT OF THE UNITED STATES

CHART 38.



TON-MILES OF RAIL FREIGHT
CARRIED PER CAPITA (MILLIONS)
SOURCE: STATISTICAL ABSTRACT

Table 8. Population, Gross National Product and Freight Traffic in the United States.

Year	1	2	3	4	5	6	7
1929	121770	181800	607375	454800	1493	3.34	74.9
1939	131028	189300	543534	338850	1445	2.87	62.3
1947	144126	282300	1018651	664523	1959	3.61	65.2
1953	159636	369000	1204098	614199	2312	3.26	51.0
1956	168176	402200	1360142	655891	2392	3.38	48.2
Projections							
1965	195747	515200	1710500	770000	2632	3.32	45.0
1970	213810	591200	1962800	883000	2765	3.32	45.0
1975	235246	682000	2264200	1019000	2899	3.32	45.0
1980	259981	788300	2617200	1178000	3032	3.32	45.0

Col.

1. Population (thousands).
2. Gross National Product (millions of 1954 dollars).
3. Total intercity ton-miles of freight traffic (millions).
4. Railroad ton-miles (presumably including passenger traffic) (millions).
5. Gross National Product per capita (millions of 1954 dollars).
6. Intercity ton-miles per dollar of Gross National Product.
7. Railroad share of total intercity ton-miles (percent).

Basis of projections:

1. Bureau of Census projections based on fertility level of 1955-57.
2. Column 1 x column 5.
3. Column 2 x column 6.
4. Column 3 x column 7.
5. Based on 1953-56 trend.
6. Based on average of 1953 and 1956.
7. Assumes rails will hold approximate 1959 share of traffic.

Source: Bureau of Railway Economics, Association of American Railroads, from letter to the author from J.E. Monroe, Vice-President, dated March 10, 1960.

Another method of projection used was to fit a straight line to the logarithms of annual lead use (chart 39). For 1975 this gives a demand of 3,200 tons, which is in good agreement with the above.

Roller Bearings.

The fourth panel of chart 35 explains a small part of the declining lead consumption, since it shows the cumulative number of freight cars in service which use roller bearings in place of lead-base babbitt solid bearings. The 35,000 cars so equipped would otherwise be consuming approximately 130 tons of lead per year to service the solid bearings. This is an important trend, although quantitatively insignificant for the moment, and may indicate that the economics of solid bearings are such that all freight cars will eventually use roller bearings, eliminating this market for lead. Therefore roller bearing application is discussed below in some detail.

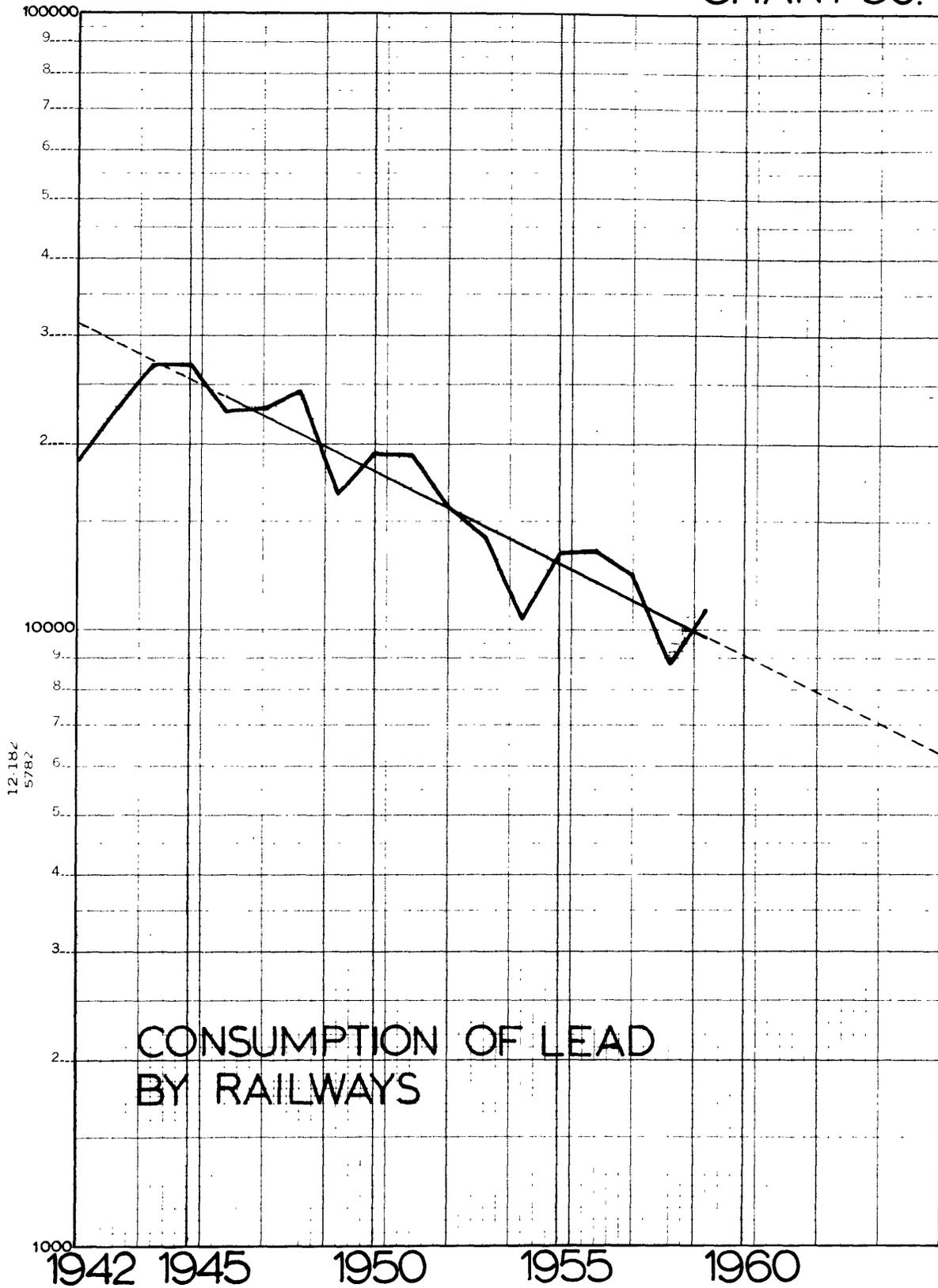
The use of roller bearings actually increases the dead weight of the cars, the adaptor alone for the roller bearings weighing almost as much as the solid journal bearing. But starting friction is much reduced, allowing longer strings of cars to be started by a given locomotive, and rolling friction is also reported by some agencies to be less, yielding savings in motive costs per mile. Also, overheating of solid bearings is a fire hazard, the lubricants being inflammable, and this hazard is reduced with roller bearings.

On the other hand, solid bearings purchased under toll agreements cost approximately \$11.00 per hundred pounds. After a bearing life of 3.3 years they are returned as scrap, and the net cost is less than \$25.00 per carset (8 bearings). A set of roller bearings, installed, costs over \$1,000.00, but a longer life means lower cost per mile and lower labor expenses for replacement work¹.

¹ Butcher, *ibid.*

TONS

CHART 39.



There appears to be a long-term advantage to the use of roller bearings, otherwise continuing substitution would not be taking place, and the important point for forecasting becomes the rate at which the substitution will occur. The change can be made in two ways -in new cars being produced, and in replacement on used rolling stock. In the opinion of G.T. Butcher¹ the high initial cost of roller bearings precludes any widespread replacement of solid bearings in old cars which have already consumed part of their life since the total savings on the miles of life left would be too small. He suggests that somewhat less than 5% of the old cars may eventually be converted. Since 2.2% of the cars in service now are less than one year old and 7.1% are one to two years old this would seem to be a reasonable estimate and this premise is followed here.

Table 9 shows the number of cars equipped with roller bearings each year, the number of cars built, and the ratio of one to the other (as a percentage). Since a few of the cars equipped with roller bearings are probably converted old cars, the percentage may be slightly high. To 1943, a total of 14 cars (exclusive of special U.S. Army cars) had been so equipped. The rapid increase since 1950 in the percentage of new cars with roller bearings would indicate their successful use-testing and acceptance by the industry (they have been in use on passenger cars since 1921) and it is logical to expect that their use will continue to grow, at the expense of the market for lead-base solid bearings. There were 60,000 new cars ordered in 1959 and as of October 1, 1959, 13,423 of these were to be equipped with roller bearings, or about 30% of those on order at October 1².

1 Butcher, *ibid.*

2 J.E.Monroe, Vice-President, Association of American Railroads, Bureau of Railway Economics; letter to the author, dated March 11, 1960.

Table 9. Conversion of Railway Freight Cars from Solid Bearings to Roller Bearings.

Year	No. of Cars Eq. w. Roller Brg. per Yr.	No. of Cars Built per Year	Roller Bearing Eq. Cars as a % of Cars Built	Total Roller Cars Owned	Brg. Eq. Cars as a % of Total Cars Owned
	(1)	(2)	$[(1)+(2)] \times 100$	(3)	$[(1)+(3)] \times 100$
1943	0	31836	0.00	2082179	.0000
1944	15	43006	0.03	2093343	.0009
1945	32	43864	0.07	2080380	.0015
1946	6	41955	0.01	2055383	.0003
1947	57	68522	0.08	2049766	.003
1948	498	112640	0.44	2076458	.02
1949	748	92562	0.81	2069656	.04
1950	344	43991	0.78	2032966	.02
1951	932	95993	0.97	2070821	.04
1952	1111	77833	1.43	2081885	.05
1953	2286	81021	2.82	2099335	.11
1954	2870	35696	8.04	2058550	.14
1955	2923	37545	7.78	2018398	.14
1956	4985	62080	7.43	2031203	.25
1957	11215	99290	11.30	2075219	.54
1958	6953	42760	16.26	2051580	.34

Source: Railroad Car Facts, 1958.

The same source estimates that:

"It would seem likely that for the next several years the rate of transition to roller bearings will approximate one percent of ownership each year, and this rate may increase in the future".

Two main factors govern the rate of transition. One is car building capacity, the other the capacity to produce roller bearings. Current domestic capacity is approximately 50,000 carsets or 400,000 roller bearing assemblies per year. "It is doubtful if more than 100,000 cars per year will be ordered, due to car building capacity"¹. Thus if all cars being built were on roller bearings, and 100,000 were built per year, 1.5 million roller bearing cars would be on hand by 1975, and lead consumption would be quite small.

¹ Butcher, *ibid.*

Cartridge Bearings.

The application of lead in journal bearing metal affords an excellent example of how a forecast can be logically formulated, yet be incorrect in a short time due to a technological innovation unknown or of unknown potential at the time of making the prediction. Since this demand sector is one for which the manufacturers and associations have been most helpful, it also is an example of the manner in which detail becomes important in the forecast. The world's largest independent manufacturer of sleeve-type bearings is currently developing a sleeve-type sealed cartridge bearing which will have a long life (9 years or over, compared to 3.3 years for solid bearings), and freedom from the so-called "hot box" problem, i.e. overheating and burning of the bearings¹. Since a service period is established at 9 years intervals for car wheels, this means little special servicing. Two most important points are the ease of installation, with no major capital expenditures for equipment, and a selling price of 60% to 70% of roller bearing price. Furthermore, (the manufacturer claims), this bearing allows 3/8 to 7/8 inch lateral movement of the journal, and has high impact resistance, which are apparently the two main areas of disadvantage of roller bearings.

Because of the considerably lower cost of this type of bearing it is quite possible that much more than 5% of the cars already in use will be converted and it is the conversion market for which the manufacturer is aiming, claiming a strong competitive position against solid bearings on a cost-per-mile basis. The effect of such a change on lead demand is almost impossible to predict quantitatively since no historical data are available, and any forecast made by a producer must at such an early stage be essentially a guess, and quite possibly biased upward. But the new bearing has been installed on some

¹ A.F.Kaminskas, Engineering Supervisor, Cleveland Graphite Bronze; letter to the author, dated March 30, 1960.

2,000 cars and its acceptance is gaining momentum.

A further complication exists in that the cartridge bearing uses a lead-tin-base babbitt on the bearing surface. The plain bearing in common use has about a $\frac{1}{2}$ inch layer of babbitt, while these use only about 1/16 inch. However, only some 4 pounds of babbitt is worn away for every hundred pounds consumed in the plain bearing and since the accounting of lead use by railroads used here does not include a deduction for scrap return, the amount of lead used on this basis would show a decline by a factor of about 8 on the basis of lead used per bearing alone, (assuming 100% replacement with this type and ignoring the tin content -about 10%- of the babbitt). Furthermore some .01 inch of the .06 inch thick babbitt is expected to be worn away over 8 to 10 years, and no scrap return is expected from the sleeve-type bearings. A life three times longer would further reduce apparent use by a factor of three, the combined reductions making this classification a minor use of lead.

Summary.

Assuming that the solid lead babbitt bearing manufacturers will not evolve a competitive bearing which requires as much lead as those currently in use, the combined effect of roller bearing use on a rapidly growing percentage of new cars being built and replacement of solid bearings of the standard type with the thin-walled cartridge bearings would undoubtedly be a markedly declining demand for lead by the railroads. Furthermore, there has recently been developed a new bearing which does not use lead -an aluminum journal bearing which it is also claimed has a longer life than lead base bearings¹.

Over the next 15 years, the average car will have had 4 to 5 bearing set replacements. As labor costs increase, the tendency is to favor installation of equipment requiring less

¹ Light Metal Age, No. 17, p. 22, August, 1959.

servicing and replacement. Assuming that the cartridge bearing will show a saving in operating costs, it is likely that replacement will be made in a high percentage of all cars not roller bearing-equipped. The aluminum bearing may also be highly successful. Of the new cars manufactured, it is estimated that all or nearly all will be built with roller or cartridge bearings in future years. By means of modern car inventory control and car handling, mainly increasing average speed of travel, the volume freight carried by approximately 2 million cars was almost doubled between 1939 and 1956 (chart 40). Further traffic increases might add another few hundred thousand cars by 1975, but these will probably all be on roller-bearings, since at a one percent changeover per year over 15 years, about 300,000 cars might be so equipped. Since it is assumed that the sleeve-type bearings will be generally used, the demand for lead will have decreased to about 1/24 of what it would be for solid bearings. At the current rate of use of some 35 pounds of lead per million ton-miles, this would mean about 1.46 pounds, or with a predicted 800,000 million ton-miles, about 580 tons of lead per year by 1975. In other words, this sector is expected to be inconsequential by 1975.

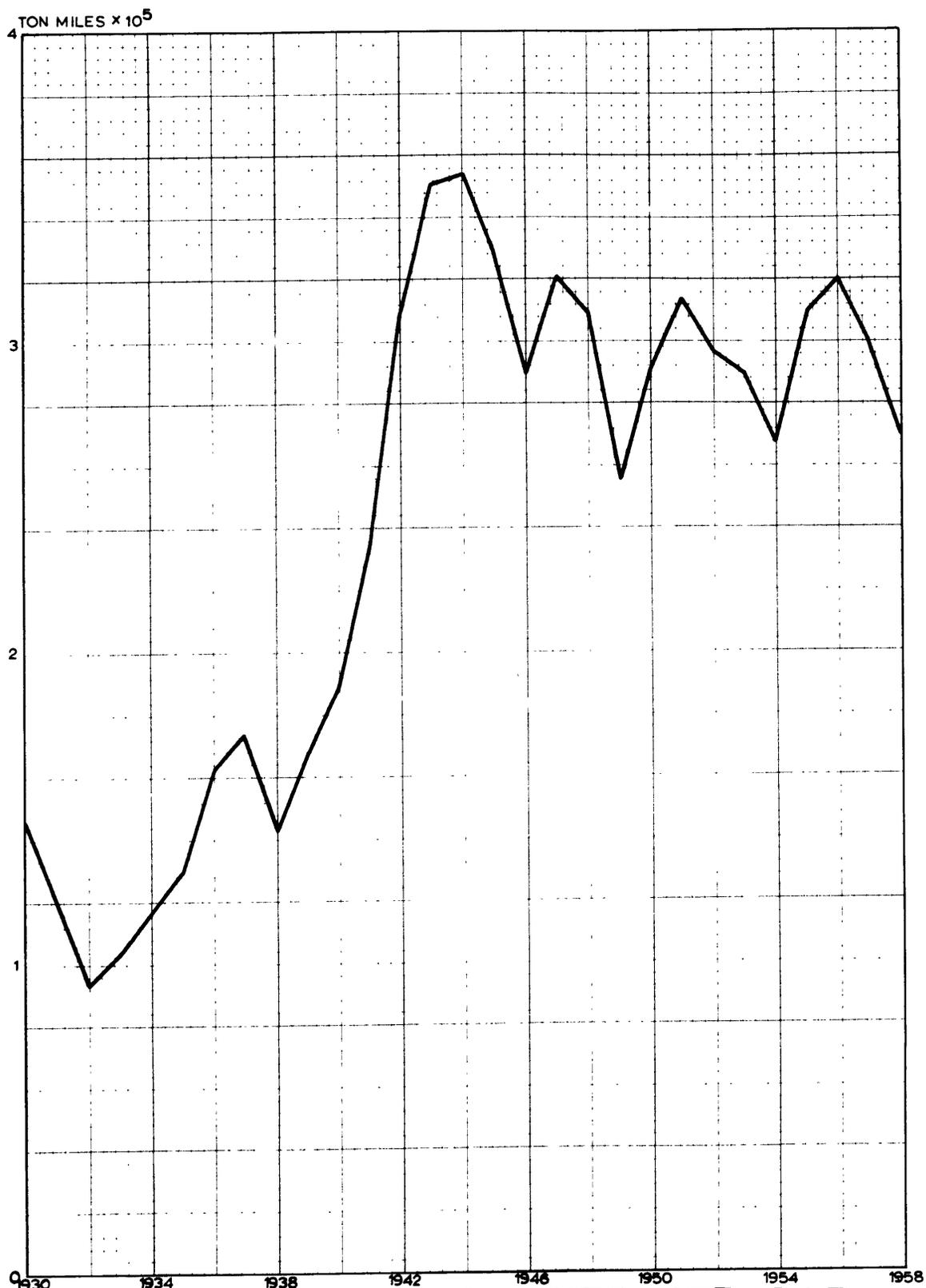


CHART 40. FREIGHT CARRIED PER CAR

Automobile Manufacturing

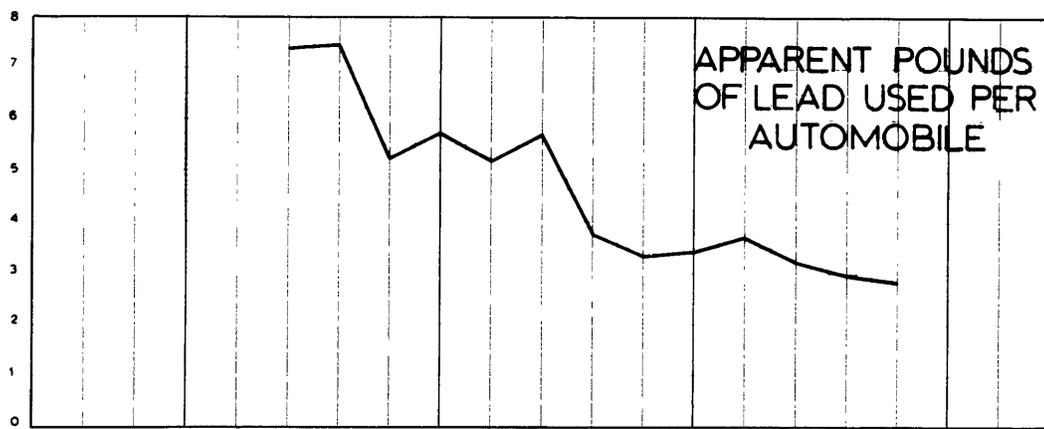
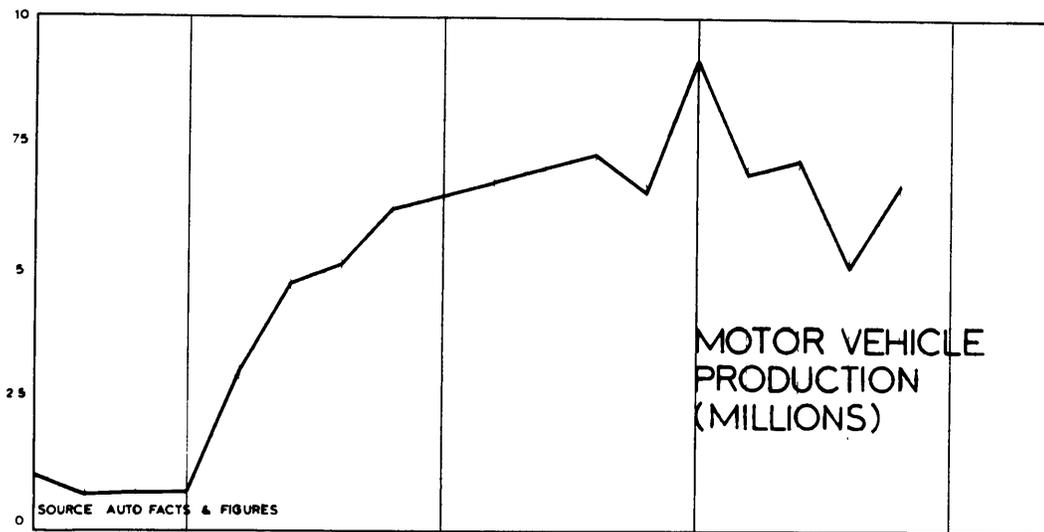
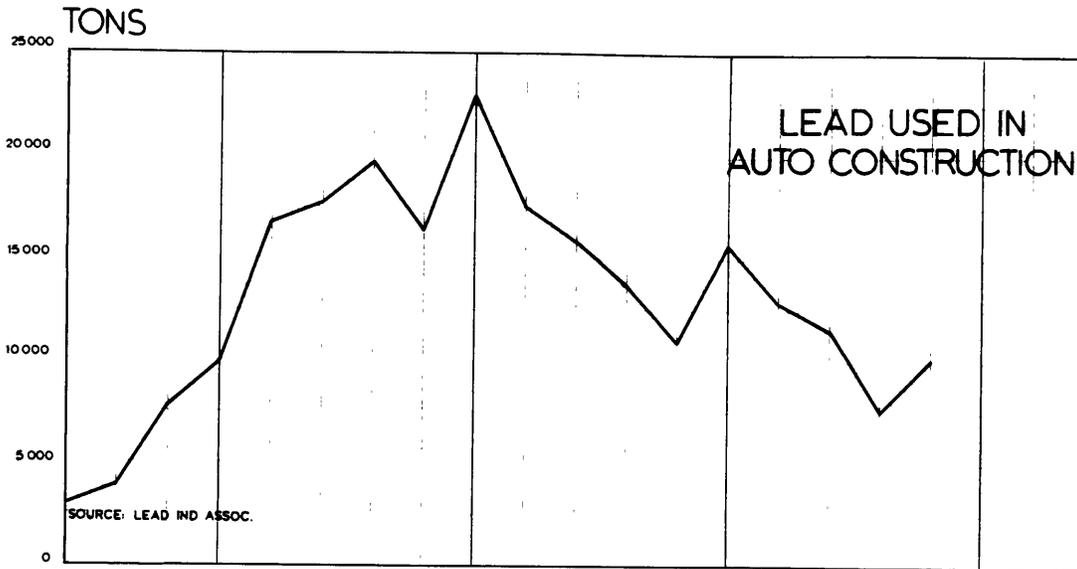
Chart 41 shows the postwar trend of lead use in the automobile manufacturing industry, motor vehicle production, and the apparent number of pounds of lead used per automobile, not including the battery. The latter set of figures is probably low, since the LIA accounting does not give full coverage, the main item reported being solder. This is, however, the most consistently used item, as well as the largest in tonnage consumed. Body solder is used to fill outside welded seams, called "lead loading", to give a smooth seam. The engine and heater radiators are soldered, as well as parts of the electrical system. In 1940, 6,000 tons of solder were used, and in 1955 some 21,000 tons, according to J.M. Campbell of the General Motors Research Staff (LIA annual meeting, 1956). The latter figure is far in excess of the total use figure of 15,675 tons reported for 1955 by LIA, but since LIA reports mainly on the basis of solder purchases, the 5,325 tons could possibly be due mostly to reduction of auto manufacturers' inventories. In fact, in many of the data referring to lead use in autos there are apparent discrepancies, and the available data are tabulated below.

Table . Lead Used in Automobiles, in Pounds per unit.

<u>Source</u>	<u>Year</u>	<u>Total Lead Used</u>		<u>Solder</u>	<u>Other</u>
		Incl. Battery	Less Battery		
Campbell(G.M.)	1955	30.4	11	4.6	6.4
Chrysler	1955	24 (not incl. body solder)	4.6 (not incl. body solder)	?	4.6
Chrysler	1957	25.5	5	2-4	1-3
Ford	1959	29	8 +	?	?
LIA	1955-59	—	3.5	?	?

Source: Chrysler and Ford, letters to author; J.M.Campbell, (General Motors), loc cit.

The exact weight of lead in the battery is difficult to discover, but the above allowances should be close to the



1942 1945 1950 1955 1960

CHART 41. CONSUMPTION OF LEAD IN AUTO PRODUCTION

truth. (See "Storage Batteries"). While the amounts of body solder vary from model to model, depending mainly on the styling, it is difficult to accept such a wide variation in lead use per car as from 3.5 to 11 pounds. In the calculation of 3.5 pounds, no additional weight was given trucks or buses. The Ford and Chrysler studies are by far the best documented, so lead use is probably between 5 and 8 pounds, of which some 3 to 4 pounds is body solder. Unfortunately no series of data but the LIA figures are available, hence these were used in the forecast. In earlier years, the ABMS reported on a classification called "Automobiles" but the tonnage reported each year grew less and less, until by 1946 it was only 1,000 tons, and no further accounting appeared, which would lead one to believe that lead is no longer used in automobiles. It is hoped that the LIA series is somewhat better. It can be pointed out where some of the deficit in accounting occurs by studying the other uses of lead in automobiles.

Terne plate is used in some gas tanks since it makes forming easier and reduces corrosion, but welding difficulties and better gasoline have caused some manufacturers to use uncoated metal. It is used on Ford cars at least, and is also used thereon for the radiator supports, amounting in total to 20 pounds of long terneplate, which would carry about $\frac{1}{4}$ pound of lead. Battery cables are also covered with lead in some automobiles. In the engine, copper-lead metal, 20 to 40% lead, has replaced the tin-base babbitt previously used for the main bearings and the camshaft bearings. This bearing metal is probably the major cause of discrepancy in the consumption data, since as much as 10,000 tons per year of bearing metal is unclassified by use by LIA. The new "compact" cars use less of all raw materials, probably including lead, but no figures for lead use are available, and it would be a sheer guess to try to estimate compact car production in 1975. At any rate, the decrease in lead used per unit would probably be slight.

All industry sources consulted agree that unless some new method of joining body panels is developed or the solder composition is radically changed (and cheaper substitutes are being sought), lead use per automobile should continue at the current rate. This seems feasible in the light of the uses outlined, since they are all small amounts per unit, hence not under intense price competition. The postwar trend (chart 41) shows a strong downtrend to about 1953, with a levelling-off since then at about 3-3.5 pounds per automobile produced. The downtrend is interpreted here as due mainly to style changes, which have resulted in fewer body seams. It may also be only apparent in part, due to the collection of purchase statistics by LIA and the effect of inventory changes in the years of the Korean crisis. With a levelling-off over some 7 years, the level of 3-3.5 pounds of lead per car is reasonably well established on the basis of LIA accounting, and is used here in combination with a forecast of automobile production to 1975.

New developments which could lessen this figure are the use of air-cooled engines, as on the Chevrolet Corvair for 1960, and aluminum radiators, considered by the industry as early as 1956 and first applied on a production model on the 1960 Chevrolet Corvette. Should these developments become widespread, auto use of lead might decrease by one third, or half of the current level reported by LIA.

The forecasts of production assembled by the Automobile Manufacturers Association¹ are from 7.5 million cars per year by 1965 to 11.5 million by 1975. Projecting forecasts short of 1975 at the same rates as used from 1959 gives a range of forecasts for 1975 from 7.7 to 11.5 million cars per year. Fitting a straight line to new car, truck and bus production from 1949 to 1959 inclusive gives a projection of only 6.3

¹ From many sources, compiled in a letter to the author dated January 25, 1960.

million units for 1975, since it shows a declining trend over these years. However, 1958 and 1959 sales were both below the normal level that manufacturers expect on the basis of population and number of households. Evidently household purchase of an automobile is often held off for one or two years in periods of economic slowdown, and later made up. The Paley Commission estimated a 1975 demand for 11.4 million car-equivalents, assuming each truck the equivalent of 1.5 automobiles. Using the same equivalence, the forecasts of truck demand of about 1.7 million per year have a car-equivalence of 2.5 million. Most of the forecasts noted for car sales imply that the figures are for total sales, including imports, and that some 10% of sales might be imports. The forecasts mentioned would then be equivalent to 8.4 to 12.8 million car equivalents of domestic manufacture for 1975. The upper figure appears definitely too high, and only one forecast was for this level. A 1975 production of 10 million car-equivalents is therefore used here as a compromise figure.

Combined with a unit use of 3-3.5 pounds per car equivalent, 1975 demand for lead for automobile production is predicted as 15,000 to 17,500 tons, on the assumption that neither aluminum radiators nor air-cooled engines will be widely used in United States automobiles by that time.

Cans

While a small amount of terne plate (lead-tin coated sheet) is used for manufacturing cans, data reported by the LIA since 1942 is based on sales of solder to the canning industry. Because of the many applications of solder it is quite possible that the accounting contains some error, but the amount cannot be closely estimated. The statistics are less than those of the USBM, reporting soft solder use, by about 5,000-6,000 tons each year, which is in reasonable accord since the soft solder data include tin content. (See Metal Statistics, 1958, p. 473). Chart 12 shows that per capita use of solder for cans has been declining steadily since 1953, after a rather steady climb from wartime levels.

Use of Cans.

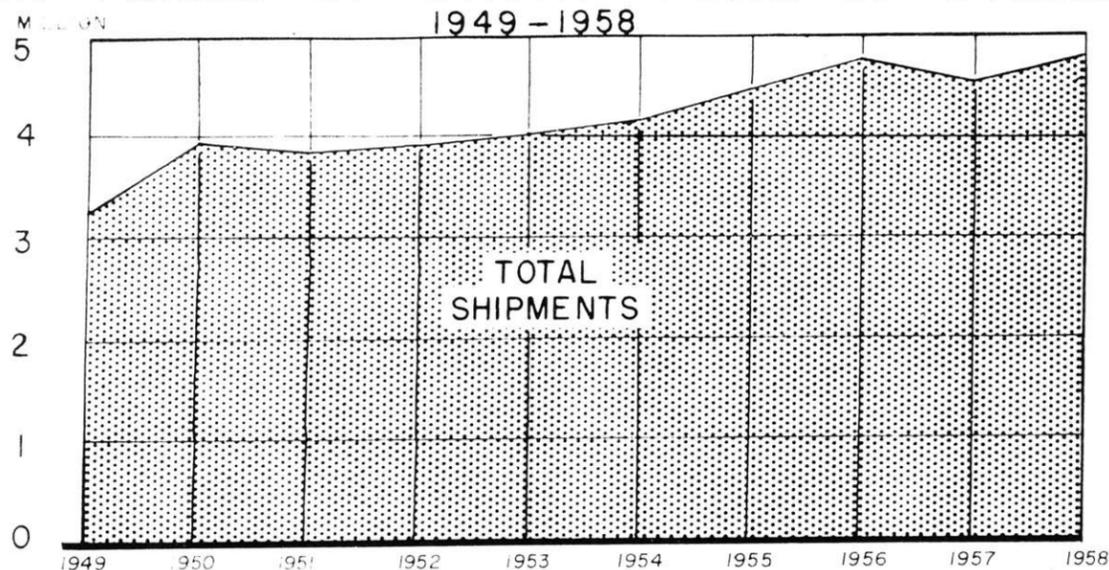
The most obvious possible cause of the decline in lead used is a decreasing use in cans, with substitution by frozen food packages and jars, but neither of these has achieved an important percentage of the market according to tables in the Statistical Abstract of the United States, and chart 42 is taken from the Annual Report of Steel and Tin Consumed in Metal Cans, 1958, of the Can Manufacturers Institute Inc., showing increasing consumption of cans each year. Furthermore, the production of cans for aerosols, beer and soft drinks is expected to increase more rapidly in the future. Thus, the decline in lead used in cans is not due to a decrease in the number of cans used.

Substitution.

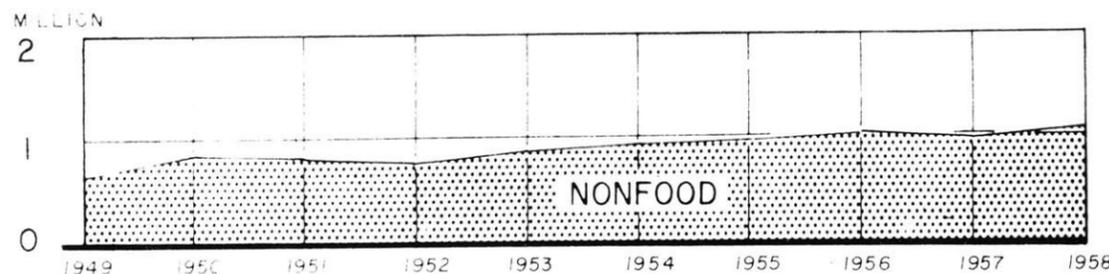
Rather than substitution for cans, part of the decline is due to replacement of can solder by cement or adhesives, developed during World War II under pressure of restrictions on lead use. This material cannot be used for most food products, since they are processed in the can, or for cans under pressure or vacuum, but since 1953 it has been the standard seal on motor oil cans and since 1954 or 1955

CHART 42.

SHIPMENTS OF METAL CANS
IN TERMS OF SHORT TONS OF STEEL

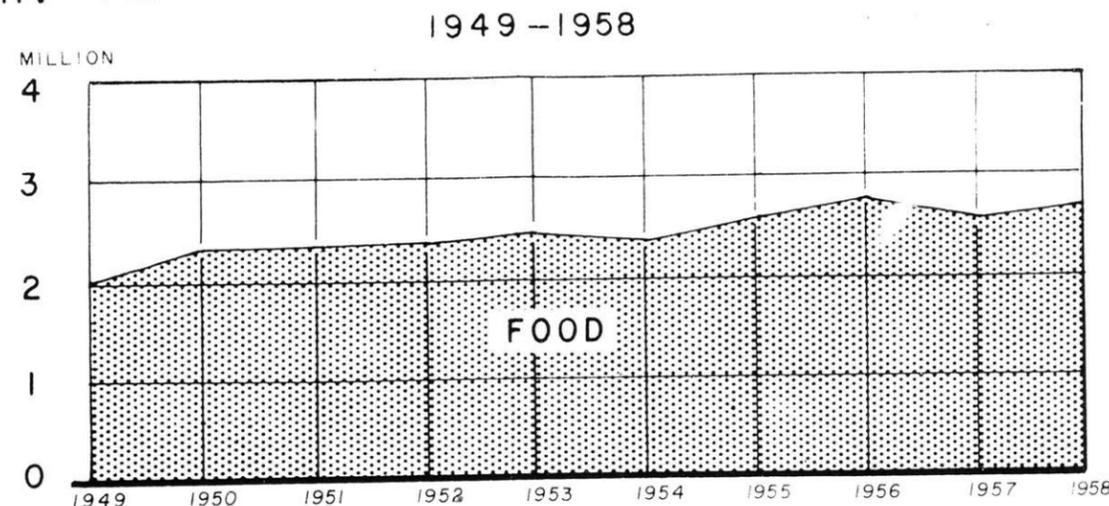


YEAR	TONS OF STEEL	YEAR	TONS OF STEEL
1949	3,276,918	1954	4,143,329
1950	3,893,384	1955	4,483,999
1951	3,804,551	1956	4,785,666
1952	3,842,170	1957	4,594,968
1953	4,082,254	1958	4,760,704

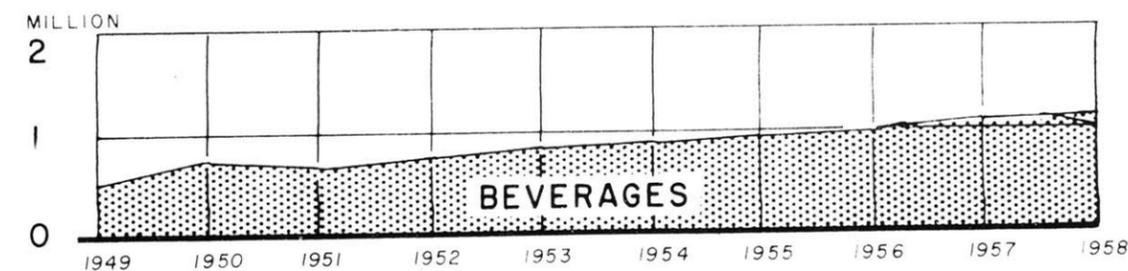


YEAR	TONS OF STEEL	YEAR	TONS OF STEEL
1949	655,754	1954	891,687
1950	850,386	1955	958,232
1951	814,262	1956	1,006,372
1952	795,220	1957	987,168
1953	851,788	1958	1,028,703

SHIPMENTS OF METAL CANS
IN TERMS OF SHORT TONS OF STEEL



YEAR	TONS OF STEEL	YEAR	TONS OF STEEL
1949	2,089,470	1954	2,382,790
1950	2,378,949	1955	2,583,008
1951	2,386,111	1956	2,785,785
1952	2,374,582	1957	2,581,336
1953	2,411,844	1958	2,665,857



YEAR	TONS OF STEEL	YEAR	TONS OF STEEL
1949	531,694	1954	868,852
1950	664,049	1955	942,759
1951	604,178	1956	993,509
1952	672,368	1957	1,026,464
1953	818,622	1958	1,066,144

SOURCE: Annual Report of Steel and Tin Consumed in Metal Cans, 1958: Can Manufacturers Institute Inc.

for citrus concentrate cans¹. Thus the currently developing application of aluminum containers for motor oil will have no direct effect on lead use. There are few other cans on which cement can be used (see table 10). Lard, shortening, wax, detergents, antifreeze, and paint cans are possible uses.

Aluminum is confidently expected to become an important can raw material -a survey made by A.D.Little Inc. in 1958 reported that aluminum might have 20% of the can market in the next 10 years. H.S.Allnutt of Kaiser Aluminum and Chemical Sales² states that in the aluminum industry "an often-heard 'shoot-for' figure is 10% of the can or container industry in the next two years". Aluminum cans are more versatile than tinsplate cans since they can be produced by three methods: 1) three-piece cans, of the conventional type with a cemented side seam (ultimately it would probably be a welded seam); 2) Two-piece drawn and ironed cans, with no side or bottom seams; 3) two-piece impact-extruded cans. Unusual shapes, such as for aerosol cans, can easily be made by the last two methods.

Aluminum has other advantages. Lighter weight, higher heat conductivity, non-rusting quality, attractive appearance of the bare metal, salvage value (both for in-plant scrap which returns 60% of its initial value compared with 10% on tinsplate, and for used cans)³, and fewer leaking cans (because the side seam can be eliminated) all save money for the user. The greatest drawback is that for the same strength as steel the wall sections must be thicker, so that although for the average gauge used in cans tinsplate costs about 10.5 cents per pound and aluminum costs 9.5-12.3 cents for an equivalent

¹ R.R. Hartwell, Chairman, Can Manufacturers Research Committee; letter to author dated June 21, 1960.

² Letter to the author dated June 15, 1960.

³ Modern Packaging, January 1960.

TABLE 10. STEEL CONSUMED IN METAL CANS

1958

TOTAL CONSUMPTION 4,760,704 Tons 100.0 %

<u>COMMODITY</u>	<u>Short Tons</u>	<u>% of Total</u>
<u>FOOD CANS:</u>		
Fruit & Vegetable	1,559,758	32.8 %
Miscellaneous Foods	494,060	10.4 %
Evaporated & Condensed Milk	206,987	4.3 %
Meat (Including Poultry)	135,961	2.9 %
Fish & Seafood	123,602	2.6 %
Lard & Shortening	109,811	2.3 %
Other Dairy Products	<u>35,678</u>	<u>0.7 %</u>
Total Food Cans	2,665,857	56.0 %
<u>BEVERAGE CANS:</u>		
Beer	820,480	17.2 %
Coffee	209,004	4.4 %
Soft Drink	<u>36,660</u>	<u>0.8 %</u>
Total Beverage Cans	1,066,144	22.4 %
<u>NON-FOOD CANS:</u>		
Miscellaneous Non-Foods	583,608	12.3 %
Oil, open top (1 qt. & 5 qt.)	271,960	5.7 %
Pet Food	<u>173,135</u>	<u>3.6 %</u>
Total Non-Food Cans	1,028,703	21.6 %

SOURCE: Annual Report of Steel and Tin Consumed in Metal Cans, 1958 :
Can Manufacturers Institute Inc.

area in the same gauge¹, it is only in the smaller cans that it is currently competitive. On the other hand Esso Oil and other major producers have been using the aluminum quart oil cans for two years economically, and 7 oz beer cans are being sold by American Can at the same price as equivalent steel cans².

While this is an extremely abbreviated treatment of a detailed subject, it has been concluded on this and other evidence that the "20% of the market" estimate of A.D. Little Inc. is indeed well founded and that some 30% may be taken by 1975. Part of this will be citrus concentrate cans and oil cans, which do not contain lead, but aerosol, beer, and soft drink cans, which contain more lead than other cans will probably be mostly made in aluminum. Therefore, the forecast for lead use calculated on the basis of tinplate cans has been reduced by 25%.

Another important innovation is an economical technique for welding the side seams of the light tinplate used in most cans, as has been done on heavy gauge pails and drums for some time. Active development by the industry indicates a good chance of success, the usual number of years being needed to introduce the process, once developed³, and it would be expected that soldered cans would meet considerable competition from this source also.

Reduced Solder per Can.

Aside from the use of cements, none of these developments has added to the downtrend of solder use in cans shown in the chart. The second factor of "substitution", which accounts for most of the decline, is due to a reduction in the amount of metal used in the side seam of a given can. The following is quoted from Heinen (loc. cit.).

1 Modern Packaging, January 1960.

2 Modern Packaging, October, 1959, p. 116.

3 J.M. Heinen, Manager, Packaging Engineering, Continental Can Co.,; letter to the author dated June 6, 1960.

"The amount of solder used for a specific can has been reduced as a result of a reduction of the amount of metal used for the side seam. Side Seam Allowance (SSA) is the term used for the amount of metal in the can body in addition to the π d quantity. Before World War II most cans had a 0.365" SSA. The introduction of low tin-high lead solders (2% tin - 98% lead) increased the creep resistance of the solder bond and this along with improvements in manufacturing made it possible to reduce the SSA. Most soldered side seams now have a 0.260" SSA, but further reduction to 0.200 for many cans is in progress. Such a reduction in SSA (0.260 to 0.200) probably will reduce the solder used from about 0.9# to about 0.5# per 1000 cans 4-13/16 inches high (approx. 0.4# reduction). Cans for pressure products such as beer, carbonated soft drinks and aerosols are made with the side seams completely filled with solders and, in addition, a small fillet of solder is maintained along the seams on the inside of the can. For cans requiring less side seam strength, all of the seam is not completely filled with solder and an inside solder fillet is not obtained except for a small area near the ends of the can. With 0.260 SSA, one thousand beer cans each 4-13/16 inches tall require about 1.4# of solder. One thousand fruit or vegetable cans of the same height require about 0.9# of solder."

The change from soldered end seams to crimped seams was made too long ago to be reflected in the statistics. Evaporated milk cans and 5 gallon square cans still use them, but even these are being changed. Also, some nozzles and other fittings are attached with solder, but the amount used is unimportant.

The introduction of high lead solders has not offset the decreased SSA, and no further increase in lead content is possible.

Forecast.

Projecting a straight line trend fitted to per capita use of solder indicates almost zero consumption by 1975. Should welded seams and cemented aluminum cans be widely used, as appears highly possible, this may not be far from the truth. If a per capita consumption of 0.090 pounds (the 1959 level) is used to forecast, the 1975 consumption of lead would be predicted as 9,300 to 10,900 tons, or if the average of 1957-59 is used, 0.116 pounds, as 11,900 to 14,000 tons. Table 10 shows that about 22% of the steel used in cans in 1958 was for beverage cans, in which the reduced SSA means a decrease to about $\frac{1}{1.4}$ the amount of solder per can, while in the other 78% the reduction is to $\frac{5}{9}$. Hence, at the average 1957-59 level of per capita lead use, 22% of the demand, or .0255, will be reduced to $\frac{1}{1.4}$, or .0182, and 78%, or .0905, will be reduced to $\frac{5}{9}$, or .0503, for a total per capita demand of .0685 pounds. This yields a predicted 1975 consumption of 7,000-8,300 tons.

Although aerosol and beverage cans are expected to be increasing markets, use of aluminum will hold down the demand for solder in this field. The application of welded seams will decrease solder demand in many fields of can manufacture, and the figure of 7,000-8,300 tons is a maximum consumption of lead to be expected on the basis of a tinplate material for most cans, while the discussion above shows that 25% will probably be aluminum, so that the final forecast is reduced to 5,200-6,200 tons.

Miscellaneous Uses

The following is a brief summary of the uses of lead reported by LIA which consume such a small percentage of the total that individual detailed forecasts are not warranted. The important point is to note whether any of these have potential which might make them important uses in the next 15 years. Generally speaking, none of them has grown importantly over the period of record, while some have diminished sharply. The discussion does not include new small-tonnage uses, but only those reported by the LIA, as shown in chart 12.

Insecticides.

As increased insect control became recognized as economically sound, lead arsenates were employed in insecticides and increasing amounts were used, until in 1947, 32,670 tons were consumed. However, the development of DDT and other non-lead insecticides has given stiff competition, and in 1958 only 3,535 tons were reported consumed. Expansion of this use sector is not expected, although it will probably continue to be used as it has certain special applications. On the basis of population growth at a constant per capita use, this market may consume 4,000-5,000 tons in 1975.

Foil.

The sudden loss of a market which consumed as much as 45,000 tons of lead per year may be attributed to competition from aluminum foil, which is cheaper, has been reported as stronger, has no toxic effects so can be used directly on foods, is lighter, and can have colors anodized onto it. Lead does have the advantage of soldering easily to give an airtight seal. The continuing use of lead foil will be in specialized uses such as electronic equipment, packaging X-ray film, the Polaroid camera, tinsel, etc. and assuming a linear growth with population, the consumption could reach 5,500 to 6,000 tons per year by 1975. This, far from a firm figure, could well be less, in spite of strong growth in the general field of packaging.

Steel and Wire.

The soft, ductile nature of lead makes it a good lubricant when applied as a coating on materials to be formed and drawn, and this accounts for a small part of its use in steel and wire fabrication. Most is used in heat treating and annealing the products. Almost 7,000 tons of lead were so used in 1956, and this use can be expected to increase with the volume of manufacturing. Using the FRB Index of Total Manufactures as a guide, since consumption per index point is quite constant, 1975 consumption is expected to be 9,500 to 11,000 tons, which will make it a relatively important use as some of the currently larger ones, such as cable covering, decline.

Collapsible Tubes.

This is another market which suffered badly from substitution of aluminum and which continues to decline as plastics too compete. Some applications such as for artists' oil colors, are probably immune to this substitution, due to the type of product contained, but in general the possibilities of using unlined aluminum tubes for many applications will probably continue to diminish this market, and only 1,000 to 2,000 tons might be used by 1975, although accounting procedures may by then group this in "Other Uses".

Rubber and Hose.

Litharge is the main product used here, in the manufacture of rubber, especially synthetics. It acts as an accelerator and toughener, and while it lends to some products a rapid curing rate, less expensive materials have been developed with equally good properties. Potential growth in this field is considered small, and it is projected at the current level of about 2,000 tons per year by 1975. Some metallic lead is used for casts, but the total used is unimportant, and it is mostly runaround.

Coatings.

Terne metal is the main lead product used in coating other metals, the lead being alloyed with tin (10-25% tin)

and applied by dipping. LIA research is currently aimed at developing better coatings and expanding sales (See New Uses). Terne plate is used for gasoline tanks and radiator supports in Ford Company automobiles. No major expansion of this use can be foreseen, other coatings than terne plate being discussed under New Uses, and this sector is forecasted at current level, 1,000-2,000 tons per year.

Brass Manufacturing.

It should be mentioned that although LIA accounting shows only small amounts of lead used in making brass (4,085 tons in 1959), the USBM reports some 24,500 tons in 1957 for example. No explanation for the discrepancy could be obtained.

Lead is used in making leaded brass, which contains one to 4 per cent lead. Without extensive research into the use of this product, a definitive forecast is difficult to make. Assuming a continuation of the postwar trend shown by the LIA data, lead consumption should be 3,000 to 4,000 tons per year by 1975.

Lead Headed Nails.

These are used where a tight seal is required, as when laying a roof or applying lead sheet for shielding. The latter may cause a slight increase in this use, but consumption will probably remain at less than 2,000 tons per year.

Seals.

Lead seals are familiar in many uses where tamper-proof seals are needed, such as on meters, cash boxes, box cars, etc. The consumption is very small, and will not increase by an important amount. Perhaps 500 tons per year will be used by 1975.

Abrasives.

This use consumes some 200-400 tons per year and has little current importance. Lead is used mainly as arbors on abrasive wheels, and the current level of use is forecasted as continuing.

Total Forecast for Miscellaneous Uses.

The predictions for minor lead uses can now be totalled, as follows.

Insecticides	4,000 - 5,000 tons
Foil	5,000 - 6,000 "
Steel and Wire	9,500,-11,000 "
Collapsible Tubes	1,000 - 2,000 "
Rubber and Hose	2,000 "
Coatings	1,000 - 2,000 "
Brass Manufacturing	3,000 - 4,000 "
Lead Headed Nails	2,000 "
Abrasives	200 - 400 "
Total	28,200 - 34,900 tons

New Uses for Lead

A discussion of the long established markets for lead and their future course is in many ways discouraging, since it is clear that in the aggregate lead consumption will not grow apace with the industrial expansion which is foreseen for the next 15 years. It would seem that the unique physical properties and actually quite low price per pound of lead should guarantee it a special place among the non-ferrous metals, and while this is generally the case, it is from non-metallic materials and new efficiencies in lead use that growth of consumption will suffer most. Lead Industries Association has begun an expanded research program to find new uses for lead that will take advantage of its unique properties¹. Some of those developed thus far are interesting and novel, but cannot possibly consume large tonnages, while other uses show excellent potential and are being stressed. These are discussed below in whatever detail could be assembled, but of course the problem is one of evaluating long term potential for a newly-marketed or unmarketed product. This means that either a "guess" must be made at the growth over the next 15 years, or the forecast must simply be termed conservative, having noted the new developments on the horizon. Here, a combination of the two has been used, but it seems appropriate at this point to state that the existence of great potential for growth in several fields in which lead has no history of participation is of great concern to the forecaster, and for all the statistical methods which may be used and the calculations of probable errors and standard deviations, the source of error suspected of being by far the greatest is the force of research applied to finding new applications for an inexpensive metal with unique physical properties, and compounds which have proven useful in many

¹ Radtke (1960), director of research for LIA, gives a general description of some of the lead research programs currently in progress.

fields of industry.

Radiation Shielding.

In shielding for gamma radiation, two factors enter the calculation of how much shielding is required to reduce radiation of a given energy level to a desired level-space (or thickness of the shield), and density of the shielding material. For a given energy level of radiation, the more dense the material the less thickness is needed, hence the use of lead for shielding against gamma radiation. However, in shielding nuclear reactors concrete, water and air can usually be used much more cheaply than lead, and the concrete also plays a structural role. Thus, most reactors being built do not rely heavily on lead for shielding, except where space is important, as for example on the top of some types of reactors, in laboratory models, or on ships or other vehicles, and specially mixed concrete is most widely used.

On the other hand, lead is by far the most practical material for isotope shipping containers and safes, and for walls through which technicians must work by remote control. It has been used for years as a shielding against X-rays used in hospitals, etc. by lining the X-ray rooms with sheet lead, and supplying lead-impregnated garments to workers. Sinks and table tops, and other "hot lab" equipment are often made of lead, as well as parts of various instruments, such as beam attenuators.

Estimated Requirements, Radiation Shielding.

It is impossible to determine how much lead is used or will be used in conjunction with each reactor built. It was reported that currently the approximately 100 fixed reactors in the United States take from 10 to 1000 tons of lead for shielding (Steel, May 4, 1959), and that by 1965 between 5,000 and 10,000 tons will be used per year for reactor shielding. (Dr. C.E. Crompton, associated technical director, National Lead Co., in Steel, April 29, 1957). The same source estimates that 1,000 tons will be used for isotopes and 2,000 tons for reprocessing the nuclear fuel

elements¹. No data are available on current consumption, but it might be noted that lead containers for isotopes at Brookhaven National Laboratories weigh over 1,000 tons (Mineral Facts and Problems). This is, however, an especially large laboratory and certainly not typical, because of the number and volume of isotopes shipped.

A few simple calculations show roughly the tonnages that might be used in fixed reactors, although admittedly based on numerous assumptions. First, nuclear power plants will be quantitatively unimportant over the next 15 years - plants now organized in the United States will produce 1.4 million kilowatts of power by 1964, which is a very small fraction of consumption. Nuclear power is expected to be economic by 1968, but only in high load areas where conventional power costs are high². The number of nuclear power plants planned is not great, and concrete will probably be the most-used shielding.

Considering reactors in existence, if they use between 10 and 1,000 tons of lead each, 500 tons average is a generous estimate which would mean that 10,000 tons per year of lead is sufficient to construct 20 reactors annually. Although some years may see this rate of construction, it will not persist as an average over any long period of time. Nor is there any need to renew the lead if high purity material is used originally. The estimate of 10,000 tons per year for reactor shielding alone seems overly high. If one includes the shielding and containers etc. needed for the "hot" laboratories which are being built, the figure becomes more reasonable, but again once installed no renewal of the lead is necessary, and this will not be a market which will

¹ The National Lead Co. was unwilling to divulge any details of the basis on which these estimates were made.

² Staekler, U.M., Senior Assistant Director, Reactor Dev. Div., U.S. Atomic Energy Comm., Northern Miner, April 7, 1960.

be continuing to consume large tonnages each year, except for expansion. Including construction materials used for all types of shielding, containers, and special uses such as isotopes, 10,000 tons per year is probably a reasonable estimate for lead consumption in the field of fixed reactor and laboratory shielding by 1975.

Probably as important as fixed reactors with respect to lead consumption is the building of a nuclear-powered navy. The Nautilus used about 225 tons for shielding¹, the Savannah, which is the first nuclear-powered merchant ship, some 500 tons (Steel, May 4, 1959). As of May, 1959, 33 nuclear-powered submarines and 3 surface ships were either approved, begun, or launched by the United States. Aircraft carriers and other large ships with multiple reactors will use even more per ship, and since 20 or 30 ships might be built each year for a considerable number of years, 10,000 to 15,000 tons could be used each year for shielding. Since the life of a ship is normally long, scrap return will not be operative for many years to come.

Other applications of nuclear energy for motive power may well be developed in the next 15 years. It is improbable that nuclear-powered automobiles will appear (see Oil Refining and Gasoline), but the first "A-plane" is reportedly planned for the 1960's (Steel, June 3, 1957, p. 55). It is assumed here that these will not be quantitatively important by 1975.

It is impossible to evaluate what the demand for lead might be if people become worried enough about fall-out to install glass with a high lead content in their windows. Up to 80% lead is used in fine crystalware, and this content in a $\frac{1}{4}$ inch window pane would diminish low energy gamma rays. It would of course make the glass expensive. In actual fact, it would not be worth the cost of installing such windows, since an atomic blast would surely destroy them, and the greatest danger from fall-out, at least at current levels, is not gamma radia-

¹ D.M. Borcina, LIA, personal communication.

tion, but the possibility of ingesting sources of alpha particles, which ionize the material they encounter, including body tissues. These could be ingested for example in milk from cows which fed on contaminated grass, or by eating their flesh. Thus, if installation of window glass containing lead became highly touted, it is probable that government officials would clarify the situation to the public, and the market would essentially disappear.

The same principle of leaded glass is now being used in glass walls in "hot labs", but since lead brick has previously been used for this there is no increase in the use of lead.

Another large and incalculable market might develop if the general public began constructing atomic bomb shelters. However, the use of concrete seems logical for the structural support, and would also serve as shielding.

Assuming that neither of these two developments occur, radiation shielding, other than that used for hospital shielding for X-rays which is included in the building sector, is concluded to represent a possible market for 20,000 to 25,000 tons of lead per year by 1975, isotopes, etc. another 5,000.

Sound Attenuation.

The LIA research program for new lead uses has evolved several lead-impregnated self-adhesive rubber or plastics in sheeting which will reduce noise transmittal through walls, floors, partitions around machinery, etc. The surface can be left in its natural color, which can be offered in various shades, or painted. Lead content varies from 70% to 90% (by weight?). One of its recent applications was in soundproofing the DC7 and DC8 aircraft cabins (the amount used per aircraft could not be determined). The potential for its use in factories, public buildings, multiple dwellings, automobiles, and numerous other applications are great, since it has little competition at present from any material that is so compact. It is essentially unmarketed, and there is no way to estimate the future demand or the limit of applications it may find. Therefore no demand figure can be included in the fore-

cast but this current development should be kept in mind, and it is a potentially large market for lead: "We believe that lead in the form of these (lead-plastic and lead-rubber) combinations, as well as sheet lead laminations and other forms, has a potential running into the tens of thousands of tons per year"¹.

Antivibration Pads.

In buildings where vibration from traffic, air-conditioning and other machinery may be bothersome, a combination pad of lead and asbestos is being used in the foundations as a damper². The pad is usually quite thin (a few inches at most), and of an area slightly larger than the steel column feet or other base, so that even though this installation may be standard practice, the tonnage of lead used is small, and will not be more important in the future.

Electronic Equipment.

Solder is of course a major use of lead in many industries, of which electronics is one of the largest. But lead zirconate-titanate has recently been recognized to have useful piezoelectric properties and is being used in high-fidelity phonograph equipment (American Metal Market, Sept. 3, 1958, p. 6). The volume which could be consumed in this use is exceedingly small. It is also employed in high speed computer electronic components, again an application of limited tonnage potential.

Alloys.

Hard lead, which contains antimony, and solder, which contains tin, are the two most widely used lead alloys. Bearing metals are usually alloys of lead and tin, or lead, tin and copper, although many variations exist. LIA is spending considerable effort to develop new alloys which could open up whole new areas of applications. The results

¹ R.L. Ziegfeld, Secretary, LIA, letter to the author dated June 8, 1960.

² Lead, (1959), vol. 23, no. 4, published by LIA.

are of course impossible to predict, since neither the types of alloys which might result nor their applications can be known at this time. It is also possible that nothing of very practical value will result. The primary objective is to learn the fundamentals of lead powder metallurgy and investigate dispersion hardening techniques for producing stronger and more creep-resistant wrought lead products.

While not true alloys, fiber-reinforced lead may be described here. The use of a lead-impregnated phosphor-bronze fiber structure increases tensile strength from the 2230 psi for 99.9 per cent lead to 3140-6400 psi. What effect this development will have on lead markets, especially construction is unknown.

Ceramics.

The possible expansion of lead-based frit use in porcelainizing aluminum has been discussed under the Ceramics demand heading above. Further research is being done in this field, as well as work to determine the properties of lead used for ceramic whiteware body glaze, its behavior in glass, and the nature of lead ferrites. Having this data at hand, ceramic engineers will probably be encouraged to use lead more often in their designing.

Lead Greases.

Differential gear oil for automobiles, and greases for heavy machinery contain over 20% lead by weight as an effective aid in lubrication under pressure, where the lead fills and smooths any scores or pits on the bearing surfaces and actually coalesces to become the bearing surface itself, giving quieter running and absorbing grit and dirt which would otherwise pit the steel. This is not really a very new use of lead. No data are available for the amount used, so it is probably rather small.

Plastic Lead.

A recent development is a putty-like mixture of 94% lead powder and 6% epoxy resin which can be formed to any desired shape, then cured in three hours to a rigid, tough,

strong mass, without the use of heat or pressure. It bonds well to most common structural materials and after setting can be machined, hammered, etc. almost as easily as lead. Using a special curing agent, a flexible material can be made¹.

The possible applications are many and varied -for repairs to lead-lined equipment and cable, lining tanks with lead, molds and castings, weights, etc. It is probable however that the amount of lead used will not be great, possibly one or two thousand tons per year at most.

Plastics.

Basic lead sulphate, an intermediate product in the manufacture of leaded zinc oxide, is also used as a stabilizer in vinyl plastics, along with many other lead compounds which are mostly made from litharge. They are economical, efficient, and in some cases the only effective stabilizers, although there are of course applications for which they are unsuited. No accurate figures exist on the amount used, but R.L. Ziegfeld² estimates it at around five or six thousand tons per year. Future consumption depends upon the use of vinyl plastics, assuming no substitution for lead, and the technology of this application is so involved that it is impossible to evaluate the possibilities of substitution.

The vinyl plastic industry grew a hundred-fold between 1940 and 1945 due to war demands. In 1947, 150 million pounds of vinyl plastics were used by the consumer market, 215 million pounds in 1948, and for 1960 700 million to one billion pounds is forecasted. From 1945 to 1955 consumption showed a four-fold increase³. It could easily double again by 1975, and a market for 10,000 to 15,000 tons of lead be created.

¹ Reference: Lead, (1960), vol. 24, no. 1, published by LIA.

² Letter to the author dated June 8, 1960.

³ Dutch Boy Handbook on Stabilizers for Vinyl Resins:
National Lead Company, 1956.

Lead Coatings.

A program of evaluation of the methods of coating steel with lead is under way and may lead to wider application of lead-coated sheet metal. An immediate objective is to find the most economical way of producing lead-coated stock for non-food cans.

About 16 percent of the total number of cans is used for non-food items, including oil cans, for which aluminum is being substituted. In 1958, some 23,000 tons of tin was used in all types of cans in tin and terne plate, and if all of the non-food 16 percent were replaced by pure lead-coated stock, it would amount to about 3,700 tons of lead per year currently, assuming about an equal coating weight per unit of area. Since complete substitution is improbable, the potential growth of this use is seen to be rather small, in spite of increasing consumption of cans. Of course other lead coatings may show better growth, but the potential for them cannot be analysed at present.

Unclassified

Because in any accounting of metal use, some applications consume very small quantities, or some form of the metal is distributed in such a manner that it cannot be reported on an end-use basis, there is always an "Unclassified" amount of consumption. Under LIA accounting this has varied from 15.3% of total consumption in 1957 to 9.6% in 1949. In 1959, it totalled 136,925 tons or 14.2%.

This percentage varies on the basis of the accounting reports, and no way was found to relate it to any general variable which can be forecasted, e.g. the FRB Index. In 1959, solder represented 36,000 tons, bearing metal 10,000, anti-monial (hard) lead products 20,000, litharge 5,000, miscellaneous lead products 59,000 (LIA information). All of these materials are used in widely scattered applications and can not be singly evaluated.

The variation in the unclassified category, as a percentage of the total classified consumption is given in table 11.

Table 11. Unclassified Consumption as a Percentage of Total Classified Consumption.

Year	Percentage
1942	14.6
1943	14.7
1944	13.8
1945	13.0
1946	15.9
1947	13.0
1948	11.0
1949	11.0
1950	13.3
1951	14.0
1952	13.8
1953	12.8
1954	15.8
1955	18.8
1956	18.3
1957	20.2
1958	20.1
1959	19.2
Average	15.2

Note that the "Unclassified" category is independent of the "Understatement of Consumption" and that it is the latter which has been adjusted when ABMS data were used. Since the ABMS data in several instances show uptrends where the LIA data show downtrends, the projections based on ABMS data will also be higher than if LIA data had been used. Therefore it would not be correct to fit a trend to the above percentage unclassified and project it to get the percentage for the forecast year, the percentage changing as the ABMS trend departs from the LIA trend. The problem of unclassified consumption is one for which no really satisfactory solution could be found. The average of the above percentages has been used here, and for 1975, 15.2% of 1,269,500 tons or 192,964 tons has been added to the total forecast of classified consumption. If the level of recent years had been used, about 20% would have to be added, or 253,900 tons, a considerable difference.

Understatement of Consumption

Comparing the total consumption indicated by LIA to that of the USBM for each year shows a discrepancy, which the LIA balances out by adding an estimate of understatement of consumption. In recent years this amounts to close to 12% of total reported consumption. Since ABMS and LIA data were used in this study, (and their totals are the same) it is against their accountings that the forecast should ultimately be checked. Two exceptions are the categories "Paints" and "Construction" for which some USBM data were used. However, the classifications which characteristically account for most of the understatement are "Batteries", "Brass Manufacturing", "Ammunition", and "Cable Covering". For the first and third of these, ABMS data were used and the understatement has already been distributed across ABMS accounting. For "Cable Covering" the forecast is so small, on the basis of technology not trend, that the difference is inconsequential. For "Brass Manufacturing" the LIA data were used. Thus, in checking the forecast, the LIA totals, including the understatement but less the difference between USBM brass use of lead and the forecast for this use, should be used for comparison.

The effect of using USBM data for "Paints" and "Construction" is somewhat more difficult to quantify since the trends are different than those which would result using the insufficiently-divided LIA data. However, the disparities between the series in the various accounts in recent years have been very small, and it is believed that since in both classifications some assumptions have been made about the future level of demand, the effect should be negligible.

TOTAL CONSUMPTION FORECAST ON AN END-USE BASIS

The total forecast of domestic lead consumption for 1975 can now be assembled from the forecasts of the various demand sectors, as listed below. A "D" indicates a dissipative use, an "S" one from which considerable secondary lead will probably be recovered.

Storage Batteries	547,000 ± 100,000	Tons	S
Construction: Pipes, Traps, & Bends	16,800 ± 1,400		S
Sheet Lead	15,700 ± 1,300		S
Calking	103,500		S
Oil Refining and Gasoline	311,600 ± 2,400		D
Cable Covering	2,000		S
Paint and Varnish: White Lead	7,500 ± 2,500		D
Litharge	3,200		D
Red Lead	22,400		D
Ammunition	41,000 ± 5,000		D
Colors	31,400 ± 2,500		D
Printing	22,500		S
Ceramics	50,000		D
Railroads	0		S
Automobiles	16,200 ± 1,200		D
Cans	5,700 ± 500		D
Miscellaneous Uses	31,500 ± 3,500		-
New Uses: Radiation Shielding	27,500 ± 2,500		S
Plastic Lead	1,500 ± 500		D
Plastic Stabilizers	12,500 ± 2,500		D
Others	?		-
Unclassified (15.2% of total of above)	<u>192,964 ± 19,122</u>		-
Total Consumption Forecast	1,462,464 ± 144,922	Tons	
or about	1,460,000 ± 145,000	Tons	
or	1,315,000 to 1,605,000	Tons	

Comparing this forecast to the result of total consumption forecasting, 1,475,000 ± 125,000 tons, the agreement is surprisingly good, in fact excellent, and both forecasts were derived entirely independently, with no counter-checking,

and no addition of the various demand sector forecasts until all were completed. Neither was any final adjustment made.

This agreement is taken to indicate that the projection of overall demand is, in this case, a more efficient method than end-use analysis, and gives acceptable results. Rosenzweig (1957) found a similar agreement in using both methods for forecasting aluminum demand to 1965. Adams (1951) likewise found that mechanical forecasts for employment worked better than forecasts based on models. It must be remembered that these are only three investigations, and that there are many cases where such methods have failed.

The case for electric automobiles is one which deserves special mention, because the potential lead demand is so great. Should these autos actually become popular, the forecast must certainly be revised, at which time both battery consumption and tetraethyl use will be reevaluated. Since sufficient information is already available, the approximate effect on tetraethyl use has already been calculated. If, under the gross assumptions made about battery consumption, the forecast were altered, it would have to be increased by some 1.5 million tons!

DISCUSSION OF THE END-USE ANALYSIS

There can be no doubt that some of the many assumptions made in the above forecast will prove to be incorrect. Furthermore it is quite possible that there are important technological points of which the author is unaware which could change the future consumption pattern considerably, and there are certainly current developments, such as in sound attenuation, that do not permit forecasting at this time. Some of these sources of error could be reduced by spending considerably more time in studying each application of lead, and it is therefore suggested that the Lead Industries Association or other trade association pursue an organized policy of studying developments in lead-consuming industries for the purpose of estimating future trends.

Many of the companies and associations consulted had no clear idea of how much lead is used for various applications in their industry, nor of how its use might change. Other companies and associations no doubt have considerable information from technical and marketing studies which is not easily obtained by an individual. The data assembled here might be regarded as a base on which to collect and analyse further information and better statistics.

The problems of available statistics were discussed above. Perhaps the study herein attempted will illustrate that there are further divisions of classifications needed to be able to follow developments in both the new and the old uses of lead. Only one new classification -tetraethyl- has been added to ABMS statistics since 1919, while a number of old ones have been combined under miscellaneous uses, and several potentially important new uses have sprung up, without any accounting being made of them.

An effort should be made, in conjunction with the above, to reduce the "Unclassified" and "Understatement" classifications to a small percentage of total use. Furthermore there should be available for each source of statistics a detailed and current description of how the data are collected, including the number of companies reporting, the percentage of the industry they represent, the method of extrapolating to get 100 percent coverage, possible sources of error, and estimates of the amount of error.

Perhaps if such improved data were available it might be worthwhile to try some of the more refined forecasting techniques, e.g. econometric methods. It is true however that the reason for abandoning even correlation methods in this study was not so much because of the nature of the statistics as because of methodological failings. It is an easy matter to allow statistical analysis of the data to become an end in itself rather than a tool to aid in the forecast. In many cases in this forecast, trend lines have been extrapolated as

the sole means of forecasting, but it is believed that rational analysis of the particular field of lead consumption allows the forecaster to evaluate his result sufficiently well to justify this method. The trend projections are considered only a mechanical guide.

THE USEFULNESS OF END-USE ANALYSIS FOR FORECASTING

Throughout the entire end-use analysis and forecast there are many points of doubt, and many assumptions that must be made. That so many assumptions do not have to be made for the total demand forecast seems a decided advantage, but the fact is that as many assumptions are implicitly made in this type of forecast as well. The end-use forecast has the great advantage of acquainting both the forecaster and the users of the forecast with the actual assumptions made, the sector in which they were made, and the possibilities of error. Similarly one becomes aware of possible changes in consumption which could occur, given some technological change, and can tell if the forecast is conservative in nature, such as the present one, or if it includes possible consumption in new and untried areas of use.

To give an example of these advantages, if after several years electric autos become popular, the forecast can be revised accordingly. The total forecast may not change much in these few years since the use would only be beginning to show in the statistics of total consumption, hence a recalculated trend would fall short of the potential growth of this very large market.

Finally, the most important asset of the end-use forecast for the mineral industry is that it allows one to evaluate the future importance of dissipative uses versus those which return scrap, hence indicates the possible amount of increase or decrease for demand for new metal, the market available to the domestic producer. How he will share this with the foreign producer is a field for considerable discussion and study.

The end-use forecasting method has one serious drawback -it takes a considerable amount of time. For the mineral industry the author believes that the extra time is justified because it allows further analysis of secondary lead futures, and because for the planning of corporate development, more faith can be placed in a logically developed forecast supported by technical knowledge than in a simple total prediction.

FORECASTS OF LEAD CONSUMPTION FROM OTHER SOURCES

It will be of interest to compare the above total forecast of lead consumption in the United States in 1975 to predictions made by others.

Perhaps the best-known forecasts of metal consumption are those of the President's Materials Policy Commission (or Paley Commission)(1952). The forecast for lead was presented on an end-use basis for 1975 and is included in the table below. It should be remembered that data were available only to 1950 when the forecast was made and that the length of the forecast period was nine years longer. The year 1950 was one of abnormally high lead consumption and it is the author's opinion that the Commission used the final year to represent the then-current demand when it should have examined historical trends more extensively. Hence the forecast is considerably higher than the one obtained here.

New developments account for some of the discrepancies. For example the very rapid advancement of multi-sheath cables, introduced only a few years previous to 1950, was not foreseen, nor were changes in the technology of printing, railroad bearings, and automobile building. The figure given by Paley for ammunition appears to be an error, as mentioned in the forecast above, under "Ammunition".

Another forecast, by the United States Bureau of Mines¹ is noted in the table in total. The details of the prediction

¹ From preprints of Bulletin 585, Mineral Facts and Problems, 1960 edition.

are not given. Battery use is expected to show a low growth rate due to longer battery life (cf "Storage Batteries" above).

	Thousands of Short Tons (Rounded)			
	White (1960)	Paley (1950)	USBM (1959)	Arbiter (1959)
Storage Batteries	547.0	707	400?	
Construction	136.0	141	125?	
Oil Refining & Gasoline (TEL)	311.6	300	241	
Cable Cover	2.0	67		
Paint & Varnish	33.1	77	23?	
Ammunition	41.0	66		
Colors	31.4	39		
Printing (Typemetal)	22.5	54		
Ceramics	50.0	48		
Railroads	0.0	38		
Automobiles	16.2	31		
Cans	5.7	-		
Insecticides	4.0	36		
Foil	6.0	3		
Solder	-	27		
Other	263.5	243		
Total	1,460	1,950	1,350	1290-1430

Tetraethyl use of lead is expected to grow at $1\frac{1}{2}$ percent per year to 1961 and 3 percent annually thereafter, equivalent to a forecast of about 241,000 tons for 1975.

Building use is projected as increasing slightly.

Pigments and coatings are expected to show little or no growth.

Apparently the major disagreements between the two forecasts are battery lead and TEL consumption.

In Bulletin 556, Mineral Facts and Problems, 1956, the forecast was for 1.45 million tons by 1975, evidently revised since.

Arbiter (1959) has forecasted total United States lead consumption by relating its trend to that of per capita steel consumption, to which he fitted a series of linear trends. The range of population estimates used coincides closely with that used here -207 to 229 million. His fore-

casts are as follows:

1965 - 1.24 - 1.28 million tons
1970 - 1.26 - 1.35
1975 - 1.29 - 1.43

In 1952, the National Industrial Conference Board published a study of growth patterns in many domestic industries, based largely on the Gompertz curve. The Gompertz curve fitted to the USBM total lead consumption statistics from 1870 to 1948 has the equation

$$y = (752.1717)(.072278)^{.968675^x}$$

For 1975, the curve gives a value of 685,420 tons. Like the logistic, it is not a successful forecasting device, at least for this series. It is not logically clear why lead consumption should be expected to approach an asymptote (in this case 752,172) in the near future, since both population and per capita consumption can increase. Eventually of course both will reach a limit, but in this case the limit is set too low.

The forecast derived in this study is seen to be slightly higher than those of the United States Bureau of Mines and Arbiter, although both are included in the range of the forecast. Since most opinions heard on the Paley Report consider its estimates too high, the present forecast is concluded to be essentially in agreement with other published predictions.

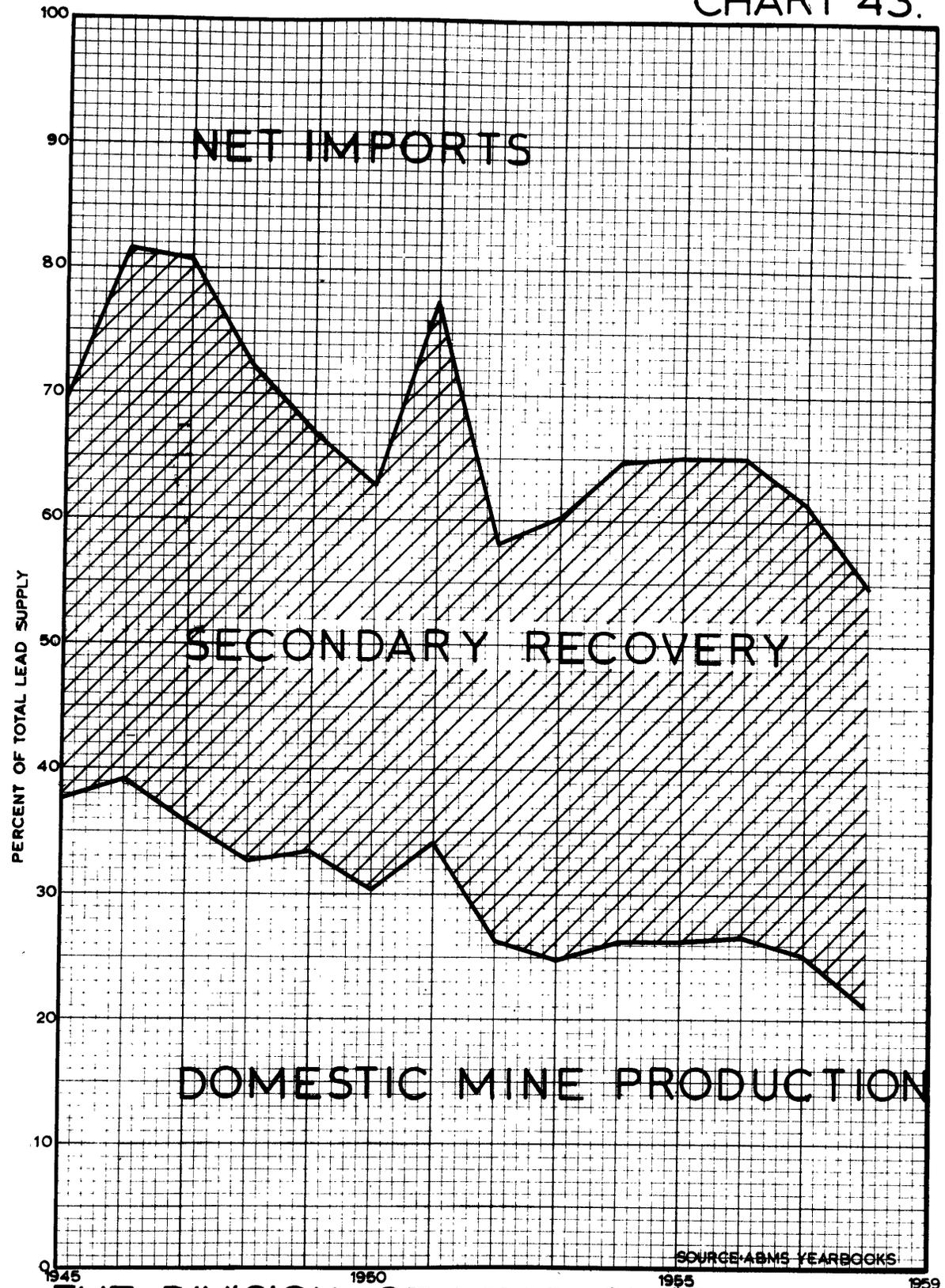
STATISTICS OF SECONDARY LEAD CONSUMPTION

As can be seen from chart 43, secondary lead is currently a more important source of supply than domestic production of new metal. There is no series of statistics available which reports scrap return of lead on a basis parallel to the divisions of consumption. The series of data which is most detailed is that of the USBM, and it covers only the consumption sectors of battery lead, cable lead, babbitt (bearing metals), and type metal, other scrap being reported as hard lead, soft lead, etc. Therefore it is not possible to know exactly how much scrap is returned from each use of lead, but in 1958 the four sectors mentioned accounted for 371,089 tons of a total scrap lead consumption of 442,591 tons, or 84%, and batteries alone accounted for 69% of the total. These four sectors have been taken as representative of the trend of scrap return as a whole, and their trends are shown in chart 43.

It should be remembered that there are usually slight discrepancies among the total consumption figures reported by ABMS, LIA and USBM and that the former two series have been used for the most part in the demand forecast. This is not sufficiently important to affect the relation of scrap to total consumption within the limits of error of the following treatment of the problem.

The data used in this study will be found in the Appendix. It is not clear from the manner of reporting of the USBM what figures should be used for total recovery from scrap. Following Merrill (1950), the figure used here is the consumption of old scrap, (which in effect includes nearly all scrap included in the accounting except that in drosses and residues) in the table entitled "Stocks and consumption of new and old lead scrap in the United States", table 9 in the Minerals Yearbook, 1958, and the equivalent in preceding years.

CHART 43.



THE DIVISION OF LEAD SUPPLY.

SOURCE: ABMS YEARBOOKS

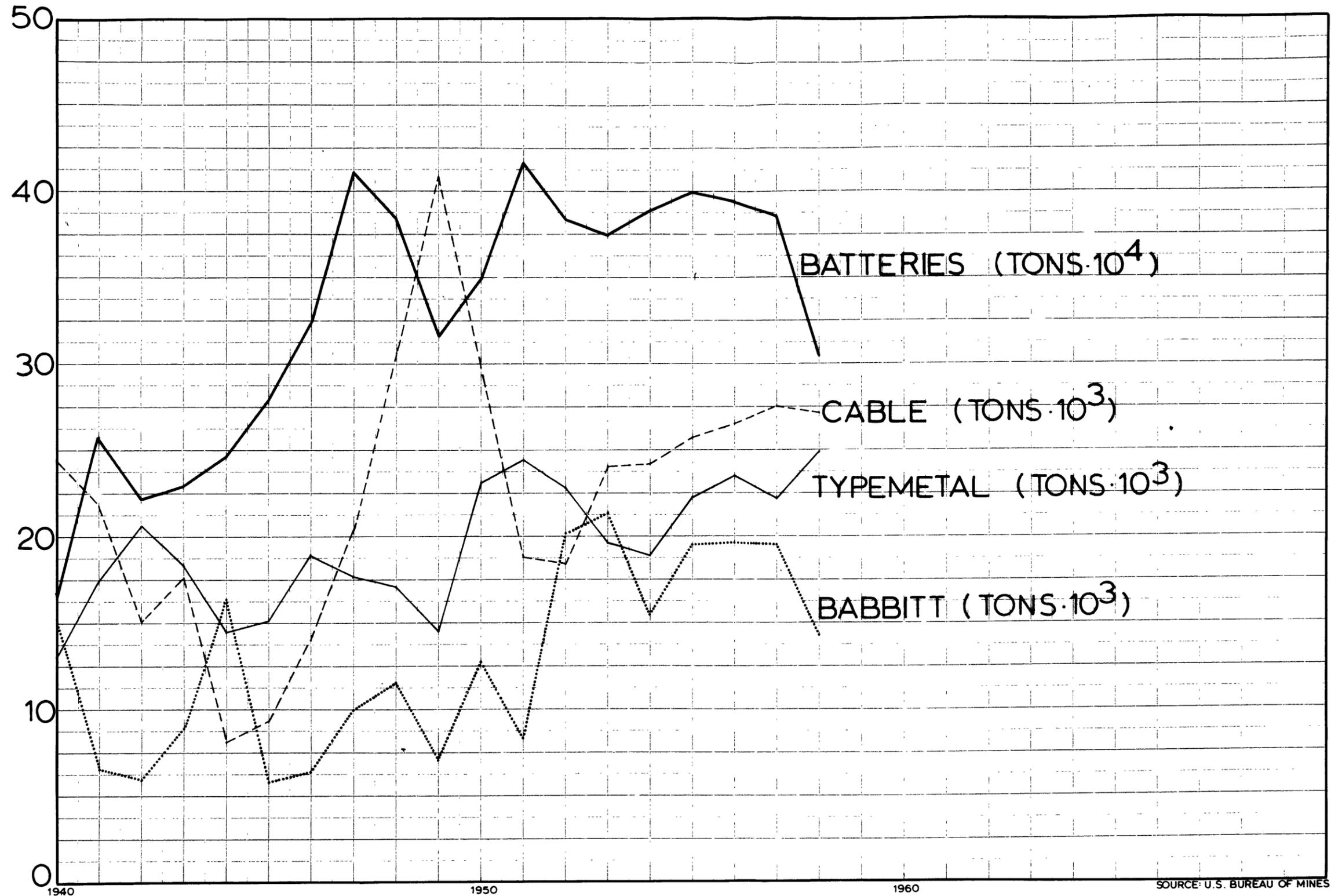


CHART 44. TRENDS OF SECONDARY LEAD RECOVERY.

SOURCE: U.S. BUREAU OF MINES

General Considerations¹

To predict future secondary metal production for any metal, the first estimate of interest is that of the pool of metal in use. The United States Bureau of Mines has recently estimated this for lead at 5 million tons of recoverable lead in use, distributed as follows².

Batteries	-one million tons
Cable Cover	-three million tons
Bearings, Pipe, Sheet and Type metal	-one million tons

Metal in the strategic stockpile was apparently not included.

The United States Bureau of Mines has also apparently estimated lead-in-use at 3,000,000 tons as of the end of 1959, a considerable discrepancy³.

This estimate can be kept up to date by calculating the amount of lead put into use each year which is potentially recoverable, from which is subtracted the amount of metal recovered from scrap in that year, the balance being added to the pool of metal in use⁴.

A second factor, necessary to calculate potentially recoverable lead put in service, is the percentage which is not dissipated. Also, each use which is essentially non-dissipative has a certain waste involved and a "recoverable lead factor" must be derived for each. Merrill (1950) has estimated the following factors.

Storage batteries	85%
Cable cover	90%
Building	15%
Bearing metal	60%
Type metal	90%
Other	32%

¹ A general reference on the topics covered is Merrill (1950)

² Preprint, Bulletin 585, Mineral Facts and Problems, 1960 ed.

³ G.R.Gwinn, Commodity Specialist, U.S. Bureau of Mines; letter to the author dated June 27, 1960.

⁴ See for example "Economics of the Minerals Industries", A.I.M.E.

Presumably the figure for "Building" includes calking lead used, otherwise the percentage would be higher. Tetra-ethyl, paint and varnish, insecticides, ceramics, colors, and many of the minor uses return no scrap. Ammunition, automobiles, and cans are possible sources of scrap but are not shown in any available accountings. It may be noted that Paley (1952) used 90% recoverability for battery lead.

It should be remembered that a considerable amount of the scrap being used never enters the accounting since it is what is termed runaround or home scrap, being consumed in the plant of generation. Most type metal falls in this class, as does some bearing metal, battery lead, ammunition, and calking lead. This is not to be confused with prompt scrap or new scrap, which is produced in the process of manufacturing, e.g.terne plate pieces remaining after a stamping or cutting operation, but is sent out to be reclaimed. However, some prompt scrap is also runaround, and vice versa. Neither prompt scrap nor runaround scrap are considered here, since the first is a small fraction of reported scrap (aside from drosses and residues) and the second is not reported.

By far the largest amount of scrap in the USBM accounting is old scrap, salvaged from worn-out, obsolete, or damaged articles. In each use, the recycling time is different, so that the amount of scrap generated at any particular time is a function of the amount of lead each sector used at various times in the past. This is one of the greatest problems in attempting to forecast scrap return. For example, the following recycling intervals have been estimated for various lead uses¹.

Storage batteries	2-3 years
Cable cover	25 years (minimum)
Building	30 years

¹ R.H.Mote, U.S. Bureau of Mines, letter to Professor R.D. Parks dated May 26, 1950.

Bearings	8-10 years
Type metal	3-5 years
Solder	?
Calking	?

Thus the 1975 level of consumption of secondary metal will depend on the 1950 level of cable covering use, the 1945 level of use in construction, the 1972 or 1973 level of battery use, etc.

Problems

By combining recycling time with lead consumption and applying the factor for recoverability, the amount of lead recoverable in 1975 can be estimated. This is not the amount that will be recovered, the difference being the amount added to the pool of lead in use. To find the amount which will be recovered one must calculate the rate of addition of lead to each demand sector, i.e. if 90% of the lead used in cable covering in 1950 is available for scrap in 1975 what percentage could be expected to be returned on the basis of historical rates? There is no other basis from which to work, although this is not a highly dependable one, since the lag of scrap return from original consumption is only an estimate, the scrap return flow tends to be distorted by price changes, and the war period disrupted the pattern by changing the pool of metal in use from its equilibrium position due to shortages for replacements. This may be a problem which could be investigated by correlation methods, and lead-lag relationships statistically determined. There is a possibility that the many approximations, such as recoverability factors, may prove too much for this approach however, and the first task in such a study would be to evaluate these as carefully as possible.

An apparently insurmountable problem, as in many of the sectors in the demand study, is the lack of early statistics of secondary lead recovery, the earliest available being for 1939. The war period is obviously atypical, and

the pattern for many uses has been grossly distorted for many years following the war. The result is a very short series of useful data. Because early data is not available it is impossible to establish any definite pattern of recovery for products having a long recycling time, such as cable covering and building, and for those with a shorter period, the length of that period must be subtracted from the length of the series of useful data, in order that the pattern be reestablished.

Further distortion of the relation between scrap return and metal use occurs when an important technological change occurs in an industry. For example, adoption of lithographic methods of printing means the retirement of "hot metal" machines, and the inventories of pig lead and runaround scrap associated with them. The recycling time for some of this metal will be much less than the normal 3-5 year period. This may account for the large amount of type metal scrap handled in 1950, 1951 and 1952.

An Example

Consider battery scrap as an example, since the recycling time is apparently least for this use, and it is the most important single source of secondary lead. By plotting secondary lead recovered from this source on a transparent overlay and comparing it to lead consumption the recycling time can be approximately found. Using monthly or quarterly data will improve the technique. By correlating the two series in their approximate lead-lag relationship, as well as for shorter and longer leads, the maximum value of the coefficient can be found which indicates the closest relationship.

Using only annual data, chart 45 shows the consumption of battery lead (ABMS data) and the recovery of battery lead scrap (USBM data) with the former leading one, two and three years. The first anomaly to be noted is that scrap recovery has exceeded consumption in almost every year, although only

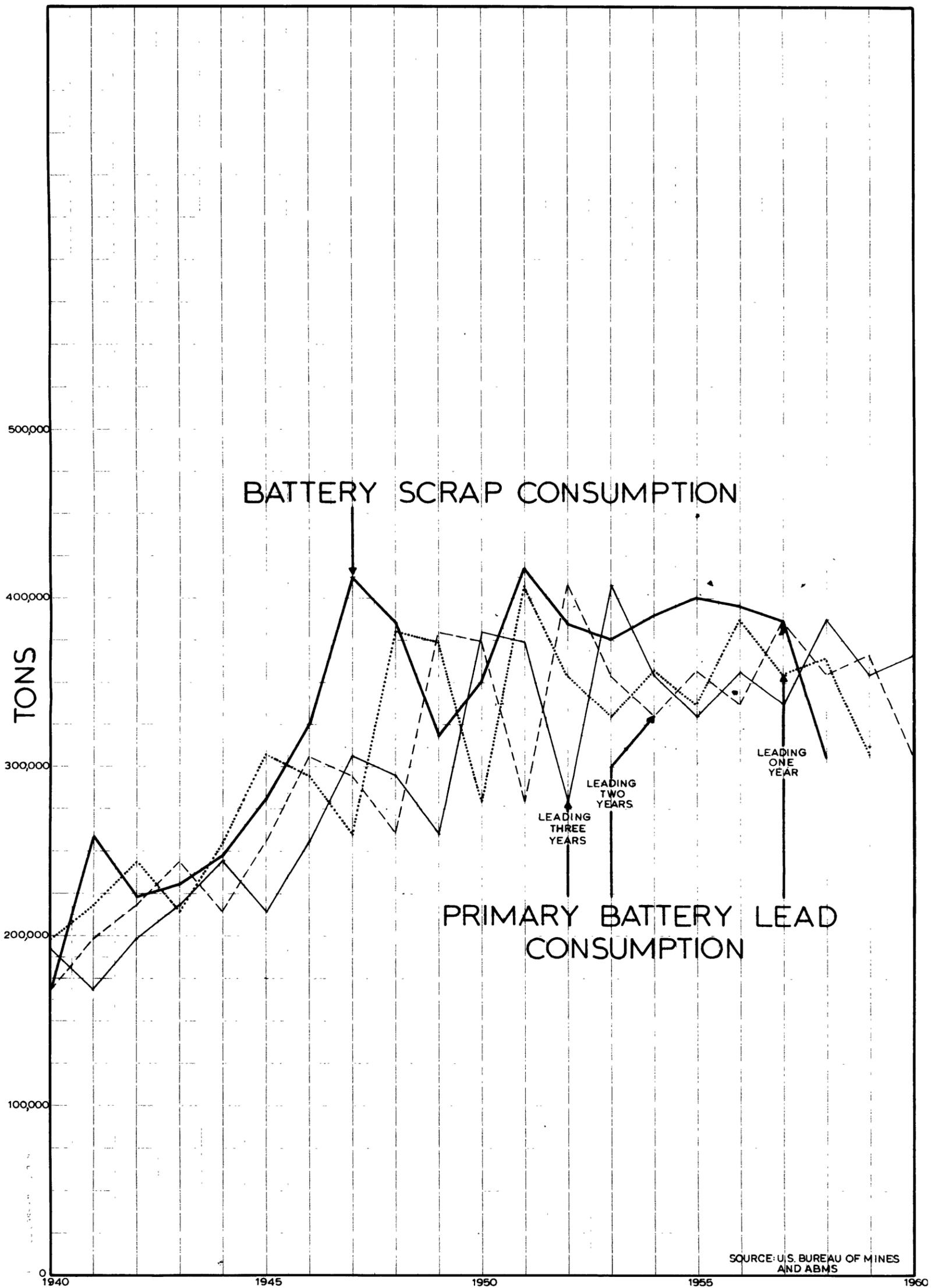


CHART 45. BATTERY LEAD CONSUMPTION & RECOVERY.

SOURCE: U.S. BUREAU OF MINES AND ABMS

85% is considered recoverable. It appears that lead is being removed from the pool of metal in use, but the number of cars in use is increasing, hence so must be the number of batteries in service. Furthermore, scrap battery dealers could not have been holding so much scrap in stock. The cause of this anomaly is undoubtedly statistical.

The second noteworthy point is that the pattern of scrap return is by no means identical to that of consumption. Without calculating correlation coefficients, it appears that the three year lag of scrap shows the best relationship to consumption, but there are many irregularities. This period seems logical since batteries now last slightly over two years on the average, and time is required for collection, etc. The high recovery in 1955 and 1956 may be due to high prices for scrap, which calls out a higher percentage thereof.

Thus, without considerable study, the estimation of secondary metal to be recovered must only be an approximate one. Assuming that the recoverability factors and recycling times given above are reasonably close to fact, secondary lead recovery for 1975 is estimated below. It should be noted that no allowance is made for addition to the pool of metal in use, for in order to do this on other than a total basis, the amount to be expected from each use would have to be estimated, and for most uses this cannot be done with the limited data available.

Estimate of Secondary Metal Recovery in 1975

(Preliminary)

Storage Batteries.

On the same basis as the forecast for 1975, estimated battery lead consumption for 1972 is about 530,000 tons, including antimony. Assuming 85% recovery, no addition to the pool of metal in use, and a recycling time of three years, 1975 scrap recovery from batteries should approximate 450,500 tons, including antimony content, or 432,500 tons of lead. This makes no allowance for possible use of electric cars. Battery lead scrap is reported by the USBM as 304,860 tons in 1958.

Cable Covering.

Assuming a recycling time of 25 years and a recoverability of 90% for lead used in cable covering, the amount used in 1950, an average year for this use, will dictate the scrap return to be expected. Since 136,800 tons of lead were used, 123,000 tons should be recovered from scrapped cable in 1975, but this source should decline markedly shortly after that year. Only 27,136 tons were reported recovered in 1958. In 1933, 31,400 tons were put in service.

Type Metal.

It is not easy to see how one can estimate closely the recycling time of type metal since some comes back as dross and is reported with other drosses, while the rest, as mentioned previously, is only a small fraction of the amount actually in use. The pattern of scrap return shows the closest relation to consumption when lagged about 5½ years, although quarterly data would be more satisfactory to use, and 5 years is used here. On the basis of the trend used for the 1975 forecast, typemetal consumption in 1970 is estimated at 0.2574 pounds per capita or about 26,500 tons. With a recovery factor of 90%, some 24,000 tons of scrap should come from this use in 1975.

Babbitt Metal.

Because LIA data were used for railroads, which use about 50% of the bearing metal reported consumed annually, the scrap return from this use might be doubled to estimate total lead to be recovered from babbitt metals. However, railroad use is projected as declining to essentially zero level by 1975, while other uses of bearing metal are expected to continue. Furthermore, from the discussion of railway car truck bearings above, it appears that they last only 3.3 years.

As a first approximation, if bearing metal use other than for railroads is taken to grow from the 1959 level of about 10,000 tons (LIA data) at about the rate of the FRB Index, 1966 consumption would be about 12,800 to 13,000 tons.

Combined with a recovery factor of 60%, which is probably somewhat incorrect when journal bearings are not included, 1975 scrap recovery might be about 9,500 tons. USBM reports 14,822 tons of lead recovered from babbitt metal in 1958, or about 15,000 tons.

Building.

Although no scrap is reported on the basis of origin from building, estimates of recoverability and recycling time are available. In 1945, lead use in building was reduced somewhat due to wartime restrictions, but the level of use was fairly constant at about 100,000 to 110,000 tons from 1944 to 1949. With a 30-year recycling period and 15% recoverability, some 15,000 to 16,500 tons of secondary lead may originate from this use in 1975.

Radiation Shielding.

Although no significant quantity of scrap has been returned from shielding use, it may be expected that small amounts will be scrapped from ships being decommissioned, laboratory changes, etc. but the amount will not be important.

Other Sources of Secondary Lead.

The amount of lead to be expected from the other usual sources, such as solder, soft lead, hard lead, tinny lead, drosses and residues, is difficult to estimate since it cannot be traced through the uses of the scrapped goods. Therefore, a percentage of the other scrap estimated is used.

The above classifications, not including building, should return about 594,500 tons of secondary lead in 1975, as an approximation. Over the past 19 years, these have accounted for 81.4% of the total scrap recovered, on the average, varying from 77.1% in 1943 to 84.8% in 1957 (USBM data). On this basis a total scrap return of 743,125 tons can be expected in 1975, or about 743,000 tons.

The Market For New Metal

In any year the amount of new metal supplied equals total consumption, less secondary metal recovered. Thus for 1958, the USBM reported 986,387 tons consumed, of which

442,591 tons were secondary metal (excluding inventory changes), so that 543,796 tons of new lead were put into use. For 1975 the predicted supply is as follows.

Total consumption	1,315,000 - 1,605,000
Secondary Lead	743,000
New Lead	572,000 - 862,000

Of course a question of great interest is how much of the new lead will be domestically produced. Perhaps one manner of indicating what may be expected is to say that if world trade conditions and United States tariff policies remain as they were in 1958 for the next 15 years, the same percentage of domestic and foreign metal will be used as in 1958, i.e. 21% domestic production, 45% imports and 34% secondary lead. In 1975, scrap is expected to supply about 50% of consumption, hence imports would supply 33%, and domestic production 17%, or domestic production will be about 122,000 tons.

The obvious weakness in this analysis is that trade conditions have not and will not remain the same. In the last few years, GATT notwithstanding, protectionism appears to be increasing in the United States. Furthermore, without doubt there will be continuing industrialization in many of the countries from which the United States obtains its lead, including Peru, the Union of South Africa, Canada, and Australia, and in other countries competing for the world's lead supply. This means that the wholesale dumping of foreign lead on the United States market which has caused an oversupply in recent years will begin to taper off. At the same time, the price of lead is expected to increase somewhat, since demand will be increasing¹. If the government decides to drop all quotas and tariffs, and encourage foreign sales, these developments may be frustrated.

Thus, although domestic demand for new metal will probably increase little by 1975, it is the opinion of this

¹ cf Shea, (1950).

author that the share supplied by new metal from domestic ores will show a further increase, in spite of the current limitations of ore reserves. This conclusion is preliminary, and should be investigated by a non-biased study in depth on the question of future foreign trade in mineral raw materials, and a concurrent study of domestic supplies. The forecast from the logistic curve (for $K = 33$ million, chart 9) is for about 90,000 tons production in 1975. The author believes that the suggested study, and history, will show this figure to be considerably too low.

STATISTICAL APPENDIX

STATISTICS OF ANNUAL UNITED STATES PRODUCTIONOF COPPER, LEAD, ZINC AND PETROLEUM

<u>YEAR</u>	<u>COPPER</u>	<u>LEAD</u>	<u>ZINC</u>	<u>PETROLEUM</u>
1801-10		est. 10000		
1811-20		est. 15000		
1821		1900		
1822		1900		
1823		2068		
1824		1987		
1825		2232		
1826		2379		
1827		4490		
1828		7452		
1829		8572		
1830		8000		
1831		7500		
1832		10000		
1833		11000		
1834		12000		
1835		13000		
1836		15000		
1837		13500		
1838		15000		
1839		17500		
1840		17000		
1841		20500		
1842		24000		
1843		25000		
1844		26000		
1845	112	30000		
1846	168	28000		
1847	336	28000		
1848	561	25000		
1849	784	23500		
1850	728	22000		
1851	1008	18500		
1852	1232	15700		
1853	2240	16800		
1854	2495	16500		
1855	3360	15800		
1856	4480	16000		
1857	5376	15800		
1858	6160	15300	20	
1859	7056	16400	50	2
1860	8064	15600	800	500
1861	8400	14100	1500	2114
1862	10580	14200	1500	3057
1863	9520	14800	1700	2611
1864	8960	15300	1800	2116
1865	9520	14700	2100	2498
1866	9968	16100	2000	3598
1867	11200	15200	3200	3347
1868	12992	16400	3700	3646
1869	14000	17500	4300	4215
1870	14114	17830	5400	5261

STATISTICS OF ANNUAL UNITED STATES PRODUCTION

OF COPPER, LEAD, ZINC AND PETROLEUM (Cont'd)

<u>YEAR</u>	<u>COPPER</u>	<u>LEAD</u>	<u>ZINC</u>	<u>PETROLEUM</u>
1871	14560	19970	6900	5205
1872	14000	25720	7800	6293
1873	17360	41940	9600	9894
1874	19600	51230	13100	10927
1875	20160	58590	16700	8788
1876	21280	62940	17000	9133
1877	23520	80380	17500	13350
1878	24080	89130	19000	15397
1879	25740	90840	21000	19914
1880	30240	95725	23239	26286
1881	35840	114495	30000	27661
1882	45323	129780	33765	30350
1883	57763	140297	36872	23450
1884	72473	136297	38544	24218
1885	82938	126192	40688	21859
1886	78882	127189	42641	28065
1887	90734	141630	50340	28283
1888	113181	147379	55903	27612
1889	113388	151787	58860	35164
1890	129882	139720	63683	45824
1891	142061	174511	80873	54293
1892	172499	168266	87260	50515
1893	164677	158969	78832	48431
1894	177094	153911	75328	49344
1895	190307	159649	89686	52892
1896	230031	179749	81499	60950
1897	247039	198498	99980	60476
1898	263256	202203	115399	55364
1899	284333	202121	129051	57071
1900	303059	260918	123886	63621
1901	301036	256610	140822	69389
1902	329754	267286	156927	88767
1903	349022	280615	159219	100461
1904	406269	297602	186702	117081
1905	444392	307514	203849	134717
1906	458903	336200	199694	126494
1907	434498	352381	223745	166095
1908	471285	311666	190749	178527
1909	546476	352839	230235	183171
1910	540080	375402	252479	209557
1911	548616	391995	271621	220449
1912	621634	392517	323907	222935
1913	612242	411878	337252	248446
1914	575069	512794	343418	265763
1915	694005	507026	458135	281104
1916	963925	552228	563451	300767
1917	943060	548450	584597	335316
1918	954267	539905	492405	355928
1919	643210	424433	452272	378367
1920	604531	476849	450045	442929

STATISTICS OF ANNUAL UNITED STATES PRODUCTIONOF COPPER, LEAD, ZINC AND PETROLEUM (Cont'd)

<u>YEAR</u>	<u>COPPER</u>	<u>LEAD</u>	<u>ZINC</u>	<u>PETROLEUM</u>
1921	252793	398222	198232	472183
1922	475143	468746	353274	557531
1923	717500	543841	508335	732407
1924	817125	566407	515821	713940
1925	837435	654921	555631	763743
1926	869811	680685	611991	770874
1927	842020	668320	576960	901129
1928	912950	626202	591525	901474
1929	1001432	672498	612136	1007323
1930	697195	573740	489361	898011
1931	521356	390260	291996	851081
1932	272005	255337	207148	785159
1933	225000	259616	306010	905656
1934	244227	299841	355366	908065
1935	381294	310505	412184	996596
1936	611410	387698	491803	1099687
1937	834661	443142	551165	1279160
1938	562328	331964	436007	1214355
1939	712675	420967	491058	1264962
1940	909084	433065	589988	1353214
1941	966072	470517	652599	1402228
1942	1087991	467367	629957	1386645
1943	1092939	406544	594250	1505613
1944	1003379	394443	574453	1677904
1945	782726	356535	467084	1713655
1946	599656	293309	459205	1733939
1947	862872	381109	510058	1856987
1948	842477	339413	537966	2020185
1949	757931	404449	591454	1841940
1950	911352	418809	588291	1973574
1951	930774	342644	621826	2247711
1952	927365	383358	575828	2289836
1953	943391	328012	495436	2357082
1954	834381	322271	380312	2314988
1955	1007311	321132	582913	2488428
1956	1117580	349188	470093	
1957	1081055	347675	539692	
1958	990000prel.	302000prel.	390000est.	

COPPER, Smelter production from domestic ores. (Sh. tons)

Source: 1845-1925, U.S. Department of Commerce, Economic Paper 1.

1926-1956, U.S. Bureau of Mines, as reported by Metal Statistics.

LEAD, Smelter production from domestic ores. (Sh. tons)

Source: 1801-1925, U.S. Department of Commerce, Economic Paper 5.

1926-1956, U.S. Bureau of Mines, as reported by Metal Statistics.

ZINC, Smelter production of slab zinc from domestic ores. (Sh. tons)

Source: 1858-1907, U.S. Department of Commerce, Economic Paper 2.

1908-1955, U.S. Bureau of Mines, as reported by Metal Statistics.

PETROLEUM, Thousands of U.S. barrels (42 U.S. gallons).

Source: 1859-1953, World Oil, August 15, 1955 issue.

1953-1956, Metal Statistics, 1957.

STATISTICS OF CUMULATIVE UNITED STATES PRODUCTIONOF COPPER, LEAD, ZINC AND PETROLEUM

<u>YEAR</u>	<u>COPPER</u>	<u>LEAD</u>	<u>ZINC</u>	<u>PETROLEUM</u>
1801-10		10000		
1811-20		25000		
1821		26900		
1822		28800		
1823		30868		
1824		32855		
1825		35087		
1826		37466		
1827		41956		
1828		49408		
1829		57980		
1830		65980		
1831		73480		
1832		83480		
1833		94480		
1834		106480		
1835		119480		
1836		134480		
1837		147980		
1838		162980		
1839		180480		
1840		197480		
1841		217980		
1842		241980		
1843		266980		
1844		292980		
1845	112	322980		
1846	280	350980		
1847	616	378980		
1848	1177	403980		
1849	1961	427480		
1850	2688	449480		
1851	3696	467980		
1852	4928	483680		
1853	7168	500480		
1854	9663	516980		
1855	13023	532780		
1856	17503	548780		
1857	22879	564580		
1858	29039	579880	20	
1859	36095	596280	70	2
1860	44128	611880	870	502
1861	52528	625980	2370	2616
1862	63108	640180	3870	5673
1863	72628	654980	5570	8284
1864	81588	670280	7370	10400
1865	91108	684980	9470	12898
1866	101076	701080	11470	16496
1867	112276	716280	14670	19843
1868	125268	732870	18370	23489
1869	139268	750180	22670	27704
1870	152880	768010	28070	32965

STATISTICS OF CUMULATIVE UNITED STATES PRODUCTION
OF COPPER, LEAD, ZINC AND PETROLEUM

<u>YEAR</u>	<u>COPPER</u>	<u>LEAD</u>	<u>ZINC</u>	<u>PETROLEUM</u>
1871	167440	787980	34970	38170
1872	181440	813700	42770	44463
1873	198800	855640	52370	54357
1874	218400	906870	65470	65284
1875	238560	965460	82170	74072
1876	259840	1028400	99170	83205
1877	283360	1108780	116670	96555
1878	307440	1197910	135670	111952
1879	333200	1288750	156670	131866
1880	363440	1384475	179909	158152
1881	399280	1498970	209909	185813
1882	444603	1628750	243674	216163
1883	502336	1769047	280546	239613
1884	574839	1905344	319090	263831
1885	657777	2031536	359778	285690
1886	736659	2158725	402419	313755
1887	827393	2300355	452759	342038
1888	940574	2447734	508662	369650
1889	1053962	2599521	567522	404814
1890	1183844	2739241	631205	450638
1891	1325905	2913752	712078	504931
1892	1498404	3082018	799338	555446
1893	1663081	3240987	878170	603877
1894	1840175	3394898	953498	653221
1895	2030482	3554547	1043184	706113
1896	2260513	3734296	1124683	767073
1897	2507552	3932794	1224663	827549
1898	2770808	4134997	1340062	882913
1899	3055141	4337118	1469113	939984
1900	3358200	4598036	1592999	1003605
1901	3659236	4856646	1733821	1072994
1902	3988990	5123932	1890748	1161761
1903	4338012	5404547	2049967	1262222
1904	4744281	5702149	2236669	1379303
1905	5188673	6009663	2440518	1514020
1906	5647576	6345863	2640212	1640514
1907	6082074	6698244	2863957	1806609
1908	6553359	7009910	3054706	1985136
1909	7099835	7362749	3284931	2168307
1910	7639915	7738151	3537410	2377864
1911	8188531	8130146	3809031	2598313
1912	8810165	8522663	4132938	2821248
1913	9422407	8934541	4470190	3069694
1914	9997476	9447335	4813608	3335457
1915	10691481	9954361	5271743	3616561
1916	11655406	10506589	5835194	3917328
1917	12598466	11055039	6419791	5252644
1918	13552733	11594944	6912196	4608572
1919	14195943	12019377	7364468	4986939
1920	14800474	12496226	7814513	5429868

STATISTICS OF CUMULATIVE UNITED STATES PRODUCTION
OF COPPER, LEAD, ZINC AND PETROLEUM

<u>YEAR</u>	<u>COPPER</u>	<u>LEAD</u>	<u>ZINC</u>	<u>PETROLEUM</u>
1921	15053267	12894448	8012745	5902051
1922	15528410	13363194	8366019	6459582
1923	16245910	13907035	8874354	7191989
1924	17063035	14473442	9390185	7905929
1925	17900470	15128363	9945816	8669672
1926	18770281	15809048	10557807	9440546
1927	19612301	16477368	11134767	10341675
1928	20525251	17103570	11726292	11243149
1929	25126683	17776068	12338428	12250472
1930	22223878	18349808	12827789	13148483
1931	22745234	18740068	13119785	13999564
1932	23017239	18995405	13326933	14784723
1933	23242239	19255021	13632943	15690379
1934	23486466	19554862	13988309	16598444
1935	23867760	19865367	14400493	17595040
1936	24479170	20253065	14892296	18694727
1937	15313831	20696207	15443461	19973887
1938	25876159	21028171	15879468	21188242
1939	26588834	21449138	16370526	22453204
1940	27497918	21882203	16960514	23806418
1941	28463990	22352720	17613113	25208646
1942	29551981	22820087	18243070	26595291
1943	30644920	23226631	18837320	28100904
1944	31648299	23621074	19411773	29778808
1945	32431085	23977609	19878857	31492463
1946	33030741	24270918	20338062	33226402
1947	33893613	24652027	20848120	35083389
1948	34736090	24991440	21386086	37103574
1949	35494021	25395889	21977540	38945514
1950	36405373	25814698	22565831	40919088
1951	37336147	26157342	23187657	43166799
1952	38263512	26540700	23763485	45456635
1953	39206903	26868712	24258921	47813717
1954	40041284	27190983	24639233	50128705
1955	41048595	27512115	25222146	52617133
1956	42166175	27861303	25692236	
1957	43247230	28208978	26231928	
1958	44147230	28510978	26621928	

Source: See Annual Data.

Units: Copper, Lead and Zinc -short tons.
Petroleum -thousands of U.S. barrels.

APPENDIX Per Capita Consumption of Lead by Industries
(In pounds)

	1	2	3	4	5	6	7
1942	3.2785	2.3961	1.4805	1.1879	.8103	1.8479	.3039
1943	3.7610	2.1750	1.4057	2.3917	1.0318	1.2899	.2524
1944	4.4715	1.9106	1.4825	.7609	1.4248	1.5538	.3398
1945	4.3954	1.5412	1.1252	.6017	1.3225	1.5568	.3838
1946	3.5993	1.3320	1.1184	.4302	.8607	1.5715	.4848
1947	5.1218	2.1627	1.2778	.5568	1.1794	1.4690	.3574
1948	4.8544	2.3933	.9489	.5805	1.3194	1.4350	.3932
1949	3.5858	1.5300	.6186	.2425	1.4255	1.1276	.3359
1950	5.1107	1.7700	.8523	.4336	1.5930	1.6773	.4088
1951	4.3824	1.6117	.6823	.4060	1.7610	1.4069	.3088
1952	4.0212	1.7577	.5722	.3123	1.9974	1.3782	.3153
1953	4.2791	1.7548	.5641	.4037	2.0881	1.3256	.3296
1954	3.6647	1.5746	.4676	.3657	2.0538	1.2154	.3244
1955	4.2648	1.5027	.4749	.3931	2.0633	1.3246	.3128
1956	3.6337	1.5568	.4481	.3402	2.3604	1.2847	.2988
1957	3.6383	1.2122	.4034	.3175	2.1020	1.1553	.2763
1958	3.0478	.7717	.3729	.2793	1.8728	1.1520	.2615
1959	3.7231	.5688	.3590	.3373	1.8539	1.2796	.2464

	8	9	10	11	12	13	14
1942	.1626	.1977	.2799	.0445	.1310	1.9289	.7967
1943	.1307	.2224	.3340	.0572	.1053	2.1165	.9823
1944	.1109	.1890	.4031	.1165	.1552	1.9640	1.1876
1945	.0701	.2080	.4053	.1499	.1822	1.7126	1.2209
1946	.0942	.1976	.3188	.2387	.1494	1.6954	.8839
1947	.1192	.1851	.3160	.2466	.1702	1.8013	.9293
1948	.1265	.2219	.3312	.2697	.1880	1.4625	.5917
1949	.1481	.2253	.2228	.2196	.1906	1.0901	.3846
1950	.3144	.3515	.2538	.3032	.2368	1.7850	.5251
1951	.2994	.3060	.2498	.2283	.1743	1.5226	.3677
1952	.1983	.3064	.2017	.2025	.1961	1.5137	.3203
1953	.2215	.2889	.1787	.1730	.2061	1.4326	.3424
1954	.2443	.2388	.1299	.1358	.1942	1.5655	.3367
1955	.2968	.2427	.1618	.1908	.1796	2.0610	.3808
1956	.2719	.2226	.1595	.1531	.1676	1.8321	.2627
1957	.2238	.2497	.1438	.1361	.1396	1.8501	.2168
1958	.1997	.2140	.1016	.0879	.1151	1.5198	.2179
1959	.1992	.2588	.1216	.1137	.0943	1.5517	.2378

1. Storage Batteries; 2. Cable Cover; 3. Paint & Varnish;
4. Ammunition; 5. Oil Refining & Gasoline; 6. Construction;
7. Printing; 8. Ceramics; 9. Colors; 10. Railroads; 11. Auto-
mobiles; 12. Cans; 13. Unclassified; 14. Other Uses.

Source: Consumption Data - Lead Industries Association, as
given in American Bureau of Metal Statistics Year-
books. Population Figures - Statistical Abstract of
the United States, 1959. Not including Armed Forces
Overseas.

Year	White Lead Used in Paints	Red Lead Used in Paints	Litharge Used in Varnish	Lead Used in Pipes, Traps & Bends	Sheet Lead
1929	136526	11855	3124		
1930	91563	10906	698		
1931	91832	9256	641		
1932	63399	6389	1360		
1933	68368	7182	610		
1934	75008	8766	414		
1935	91297	8721	564		
1936	113363	11786	2307		
1937	93580	10440	1865		
1938	95018	8698	2449		
1939	92380	11421	2428		
1940	73137	11949	3003		
1941	100665	20130	3165		
1942	76837	21017	3423		
1943	66551	22271	3453		
1944	79948	18074	2988		
1945	46418	16438	2752		
1946	60943	9318	3302		
1947	61265	11362	4258		
1948	40892	10863	4424	39843	31559
1949	24284	9634	4286	29858	27144
1950	38920	14103	4347	41361	30778
1951	28718	14740	5584	33095	31210
1952	21223	13149	5572	29465	28697
1953	21030	14570	3915	28693	30476
1954	20929	12568	4162	26832	26014
1955	19825	14308	5206	29757	30466
1956	20288	14331	3571	28028	30249
1957	19253	15993	3227	24739	27474
1958	15288	13726	3223	23044	25104
1959				22827	27153

SOURCES:

White Lead (column 1), Red Lead (column 2) and Litharge (column 3) U.S. Bureau of Mines, Mineral Yearbooks.

Lead Used in Pipes, etc. (column 4) and Sheet Lead (column 5)

U.S. Bureau of Mines, Mineral Yearbooks. N.B. Data prior to 1948 does not include secondary lead consumed, hence is not comparable to later data.

CONSUMPTION OF OLD LEAD SCRAP IN THE UNITED STATES,
GROSS WEIGHT IN TONS

Year	Total Consumption	<u>T y p e o f S c r a p</u>			
		Type Metal	Cable Lead	Battery Lead	Mixed Common Babbitt
1940	272273	13073	24383	165387	15008
1941	383294	15483	21779	257638	6574
1942	327977	20646	15074	221573	5923
1943	355325	18321	17607	229059	8907
1944	359353	14482	8159	246255	16313
1945	381553	15143	9303	279454	5791
1946	432575	18962	13995	323594	6349
1947	558791	17705	20362	411037	9955
1948	543375	17056	30675	384270	11573
1949	453845	14515	40818	316031	7067
1950	526692	23142	29764	349376	12736
1951	555565	24422	18726	416612	8300
1952	529018	22803	18392	383280	20137
1953	540656	19587	24032	371928	21362
1954	547394	18861	24198	388630	15452
1955	575952	21990	25736	398607	19484
1956	571999	23509	26517	393525	19653
1957	535705	22225	27519	385149	19514
1958	442591	24857	27136	304860	14236

SOURCE: United States Bureau of Mines, Minerals Yearbooks.

Year	Population of U.S., not incl. Armed Forces Overseas (Thousands)	Consumer Price Index 1947-49=100	FRB Index 1947-49=100	Annual Change In FRB Index	Index of Construction Materials 1947-49=100
1920	106446		40	+ 2	4.55
1921	108541	76.4	30	-10	3.49
1922	110055	71.6	39	+ 9	4.54
1923	111950	72.9	45	+ 6	5.08
1924	114113	73.1	43	- 2	4.98
1925	115832	75.0	48	- 5	5.41
1926	117399	75.6	50	+ 2	5.58
1927	119038	74.2	50	0	5.57
1928	120501	73.3	52	+ 2	5.64
1929	121770	73.3	58	+ 6	5.81
1930	123077	71.4	48	-10	4.52
1931	124040	65.0	39	- 9	3.25
1932	124840	58.4	30	- 9	2.14
1933	125579	55.3	36	+ 6	2.39
1934	126374	57.2	39	+ 3	2.73
1935	127250	58.7	46	+ 7	3.52
1936	128053	59.3	55	+ 9	4.78
1937	128825	61.4	60	+ 5	5.21
1938	129825	60.3	46	-14	4.28
1939	130880	59.4	57	+11	5.52
1940	131954	59.9	66	+ 9	6.28
1941	133121	62.9	88	+22	7.97
1942	133920	69.7	110	+22	8.22
1943	134245	74.0	133	+23	7.11
1944	132885	75.2	130	- 3	6.73
1945	132481	76.9	110	-20	6.85
1946	140054	83.4	90	-20	7.46
1947	143446	95.5	100	+10	9.46
1948	146093	102.8	103	+ 3	10.03
1949	148665	101.8	97	- 6	9.10
1950	151234	102.8	113	+16	11.02
1951	153384	111.0	121	+ 8	11.54
1952	155761	113.5	125	+ 4	11.25
1953	158313	114.4	136	+11	12.17
1954	161191	114.8	127	- 9	11.61
1955	164302	114.5	140	+13	13.05
1956	167261	116.2	144	+ 4	
1957	170293	120.2	145	+ 1	
1958	173260	123.5	139	- 6	
1959	176481		158(Prel- iminary)	+19	

SOURCES:

Population (column 1) Statistical Abstract of the U.S., 1959.

Consumer Price Index (column 2) Economic Report of the President, 1959.

SOURCES (Cont'd):

FRB Index (Federal Reserve Board Index of Industrial Production, Total Manufactures) (column 3) Federal Reserve Board Bulletin.

Index of Construction Materials (column 5) Roos (1957).

Year	SLI Battery Shipments Not Incl. Exports	Apparent Pounds of Lead per Battery	Outboard Motor Sales	Outboard Motors in Use
1936	14,940,662			
1937	15,649,219			
1938	14,248,407			
1939	17,004,889			
1940	18,814,286			
1941	20,767,502			
1942	16,232,524	21.6		
1943	17,698,828	29.0		
1944	19,853,134	30.9		
1945	18,285,215	32.3		
1946	20,611,565	25.2		
1947	30,625,621	24.8	584,000	1,857,000
1948	30,360,544	24.6	499,000	2,321,000
1949	25,636,651	21.9	329,000	2,643,000
1950	32,445,056	25.0	367,000	2,811,000
1951	28,984,263	24.4	284,000	3,010,000
1952	27,991,959	23.6	337,000	3,219,000
1953	30,937,214	23.1	464,000	3,419,000
1954	30,372,071	22.2	479,000	3,740,000
1955	34,997,276	21.4	515,000	4,210,000
1956	31,934,590	22.2	642,000	4,740,000
1957	33,163,520	22.0	550,000	5,140,000
1958	30,405,106	20.2	504,000	5,485,000
1959	34,224,000	21.1	540,000	5,845,000

SOURCES:

Battery Shipments and Apparent Pounds of Lead per Battery:

1959 Yearbook of The Association of American Battery
Manufacturers, Inc.

Outboard Motor Data: Boating 1959, A Statistical Report
Jointly Presented by the Outboard Boating Club of America
and the National Association of Engine and Boat Mfrs.

Year	Per Capita Consumption of Lead in Pounds	Average Annual Price of Lead Cents Per Pound	Consumption of Lead in Tetraethyl	Telephones All Systems Total Miles of Wire (Thousands)	Annual Change in Wire Mileage (Miles x 10 ³)
1920	11.364	8.08	000		
1921	9.592	4.55	000		
1922	12.415	5.71	000	37260	
1923	13.720	7.25	000	41400	+ 4140
1924	14.234	8.08	275	46500	+ 5100
1925	14.789	9.02	125	52200	+ 5700
1926	15.348	8.42	225	57960	+ 5760
1927	14.129	6.75	600	63840	+ 5880
1928	15.451	6.31	1150	69130	+ 5290
1929	15.969	6.83	2580	76460	+ 7330
1930	12.490	5.52	5710	83110	+ 6650
1931	9.153	4.24	5170	86100	+ 2990
1932	6.676	3.18	3330	87680	+ 1580
1933	7.159	3.87	8050	87000	- 680
1934	7.723	3.86	11950	86800	- 200
1935	8.470	4.06	14800	87200	+ 400
1936	9.895	4.71	17900	88100	+ 900
1937	10.537	6.01	See	90830	+ 2730
1938	8.411	4.74	ABMS	92850	+ 2020
1939	10.192	5.05	Data	95150	+ 2300
1940	11.853	5.18		99250	+ 4100
1941	15.774	5.79		105550	+ 6300
1942	14.934	6.48		108300	+ 2750
1943	16.388	6.50		108000	- 300
1944	16.405	6.50		109000	+ 1000
1945	15.096	6.50		110700	+ 1300
1946	13.209	8.11		116600	+ 5900
1947	16.341	14.67		125500	+ 8900
1948	15.456	18.04		137600	+12100
1949	11.677	15.36		147300	+ 9700
1950	16.015	13.30		156700	+ 9400
1951	15.008	17.49		165100	+ 8400
1952	14.137	16.47		176000	+11900
1953	14.541	13.48		188200	+12200
1954	13.585	14.05		201500	+13300
1955	14.677	15.14		218100	+16600
1956	14.134	16.01		238600	+20500
1957	13.366	14.66			
1958	11.386				

SOURCES:Lead Consumption (column 1) ABMS.Lead Price (column 2) Metal Statistics, 1958, p. 505.Tetraethyl (column 3) From M.E. Collins, Ethyl Corp. records.Wire Mileage (column 4 & 5) Statistical Abstract of the U.S. 1957.

Year	Factory Sales of Cars, Trucks, Buses	Motor Vehicles Registered (Thousands)	Vehicle Miles Travelled (Millions)
1921	1616119	10494	56681
1922	2544176	12274	68340
1923	4034012	15102	84045
1924	3602540	17613	102423
1925	4265830	20069	119057
1926	4300934	22200	135905
1927	3401326	23303	150533
1928	4358759	24689	167317
1929	5337087	26705	188617
1930	3362820	26750	199263
1931	2380426	26094	203777
1932	1331860	24391	190728
1933	1889817	24159	188784
1934	2737070	25262	204070
1935	3971241	26546	215428
1936	4461462	28507	235205
1937	4820219	30059	253818
1938	2508407	29814	257087
1939	3588889	31010	271379
1940	4472286	32453	288155
1941	4840502	34894	312307
1942	1041524	33004	259990
1943	699828	30888	207887
1944	738134	30479	213066
1945	725215	31035	245145
1946	3089565	34373	328431
1947	4797621	37704	360689
1948	5285544	40960	387209
1949	6253651	44448	413597
1950	8003056	48945	451771
1951	6765263	51643	484582
1952	5538959	52966	512689
1953	7323214	55939	539199
1954	6601071	58219	558801
1955	9169276	62343	595856
1956	6920590	65119	622932
1957	7220520	67159	647004
1958	5135106	68328	664653
1959	6728644	70446	696000(Prel- iminary)

SOURCES :

Sales (column 1) Auto Facts and Figures. Includes military vehicles.

Registrations (column 2) Third Progress Report, Highway Cost Allocation Study. Includes publicly owned vehicles . After 1956, Auto Facts and Figures.

Miles Travelled (column 3) As for Registrations.

FORECASTS FOR 1975Population (not including armed forces overseas)

U.S. Bureau of the Census (Stat. Abst.)	205,907,000
	to 227,463,000
United Nations (1958)	215,790,000
	to 243,880,000
Roos (1957)	220,000,000
Brown and Hansen (1957)	218,064,000
Highway Cost Allocation Study (for 1976)	229,758,000
RANGE USED: 205,907,000 - 242,880,000	

Federal Reserve Board Index of Total Manufactures

Roos (1957) (1947-1949 = 100)	255
Brown and Hansen (1957) (1947-1949 = 100)	293

Construction Materials Index

Roos (1957) (1947-1949 = 100)	23.0
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Automobile Registrations

Third Progress Report of the Highway Cost Allocation Study, 86th Congress, First Session, House Document No. 91	111,162,000
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Automobile Production

See text.

USE OF LEAD IN THE UNITED STATES (a)
(In tons of 2,000 lb.)

Purpose	1919	1920	1921	1922	1923	1924	1925	1926	1927	1928	1929	1930	1931	1932	1933	1934	1935	1936	1937	1938	1939	1940	1941	1942	1943	1944	1945	1946	1947	1948	1949	1950	1951	1952	1953	1954	1955	1956	1957	1958			
White lead (b)	111,000	116,500	136,000	156,000	130,000	150,000	131,000	120,000	126,000	123,000	119,700	83,900	77,500	54,500	59,100	64,500	80,000	85,500	86,000	71,000	75,000	65,500	85,000	75,000	50,000	60,000	41,000	48,000	55,000	34,400	20,600	34,000	25,700	21,100	19,200	18,800	19,300	17,400	15,800	13,700			
Red lead and litharge (c)	32,000	45,000	23,500	30,000	46,000	34,000	42,000	36,000	38,000	31,000	30,000	32,000	18,000	15,800	19,000	42,000	47,500	54,000	57,000	43,000	57,200	59,400	89,100	68,000	79,000	80,000	68,200	61,000	65,600	58,900	48,600	75,900	78,700	65,000	66,000	76,500	87,500	78,000	75,000	60,000			
Storage batteries (d)	100,000	108,000	87,000	130,000	143,000	170,000	180,000	190,000	175,000	220,000	210,000	163,000	157,000	138,000	147,000	163,000	175,000	191,000	192,000	167,000	198,000	220,200	245,000	215,600	257,000	307,000	295,000	260,000	380,000	373,300	280,600	406,800	353,800	330,000	356,500	337,200	375,000	355,000	365,000	307,000			
Cable covering (d)	53,000	79,000	67,000	93,000	131,000	138,000	156,000	185,000	161,000	180,000	220,000	208,000	117,000	55,000	31,000	35,200	38,900	61,400	90,000	60,000	74,400	107,400	173,000	165,000	141,000	131,000	107,000	111,000	158,700	184,300	130,100	136,800	136,900	146,200	151,500	127,900	121,100	133,900	107,000	74,500			
Building (e)	68,000	71,000	48,000	71,000	75,000	83,100	88,400	93,700	88,000	96,000	96,000	67,000	40,000	22,000	26,000	30,000	32,000	40,000	45,000	36,000	50,000	65,000	95,000	110,000	62,000	70,000	68,000	78,000	66,600	60,800	48,200	61,500	60,700	57,100	55,600	52,800	59,700	60,100	52,500	48,400			
Automobiles (e)	4,000	4,750	4,600	7,200	10,400	10,700	12,800	16,700	12,000	17,000	18,000	11,000	6,000	3,500	5,000	7,300	10,000	11,100	12,000	6,000	8,900	11,000	12,000	3,000																			
Tetraethyl																																											
Locomotives (e)	1,700	2,000	1,000	850	2,100	1,100																																					
Railway cars (e)	5,600	2,800	1,800	2,500	6,000	4,500	5,200	4,800	3,350	2,500	5,700	5,200	1,000	300	200	1,100	500	2,400	3,800	1,200																							
Shipbuilding (e)	2,250	1,550	850	100	100	200	100	100	200	100	300	500	400	200	100																												
Ammunition (d)	32,000	33,600	26,500	36,500	39,000	26,500	31,500	32,000	34,000	39,600	41,100	33,300	29,700	23,300	32,300	34,800	29,200	32,500	39,500	31,200	42,300	56,000	71,500	83,000	178,000	63,000	43,000	30,000	40,000	42,400	18,000	32,800	31,100	24,300	32,000	40,200	46,800	41,200	43,000	40,200			
Terne plate (d)	3,400	3,100	3,000	4,700	4,600	4,050	4,500	4,100	3,900	4,400	4,200	2,200	1,400	2,500	2,600	4,700	6,200	6,400	4,300	6,000	6,000	8,700	4,900	5,000	6,000	5,000	3,100	(f)	(f)	(f)													
Foil (d)	21,000	21,000	23,800	26,800	33,300	35,000	32,500	35,000	30,000	35,000	39,800	26,000	20,000	14,000	22,500	16,200	15,900	28,500	21,700	22,000	21,800	23,500	45,000	7,700	13,000	16,000	15,600	4,000	3,600	3,100	2,700	3,300	2,800	2,900	5,000	4,400	5,200	4,600	5,000	4,600			
Bearing metal (g)	27,000	26,000	21,000	26,000	28,000	32,000	34,000	36,000	31,000	32,000	30,000	12,000	10,000	15,000	12,100	13,000	16,500	15,000	9,000	12,800	14,000	20,000	20,000	20,000	35,000	41,000	42,000	41,000	40,100	41,500	26,600	32,800	33,700	29,400	29,800	27,200	33,100	27,800	26,500	18,500			
Solder (f)	15,000	15,000	12,000	20,000	30,000	30,000	35,000	37,000	35,000	37,000	37,000	27,000	20,500	14,000	16,000	16,000	20,000	22,000	22,000	15,000	20,000	24,000	36,000	38,000	38,000	41,000	47,000	53,000	59,000	67,500	58,300	88,100	65,400	64,900	65,900	71,100	88,300	72,000	70,000	57,200			
Typemetal (f)	7,500	7,500	7,500	9,500	12,000	13,000	15,000	16,000	16,000	17,000	18,000	16,000	14,400	10,800	11,000	13,000	15,000	17,000	17,000	12,000	14,000	16,800	20,000	20,000	17,000	23,000	25,000	33,000	25,600	28,700	25,000	30,900	23,700	24,600	26,100	25,700	25,000	24,800	26,300				
Calking (f)	24,000	24,000	20,000	20,000	25,000	26,500	30,000	32,000	30,000	32,000	31,500	21,000	15,000	10,000	12,000	10,000	12,000	13,500	15,000	12,000	16,000	19,200	31,000	40,000	30,000	32,000	35,200	39,000	49,900	40,200	33,800	60,200	43,800	46,000	43,900	49,900	58,300	58,000	57,000	62,000			
Castings (f)	15,000	15,000	12,000	15,000	15,000	15,000	18,000	18,000	17,000	18,000	18,000	12,000	7,000	5,000	5,000	5,000	5,000	5,750	6,000	6,000	7,500	9,000	14,000	(g)																			
Other uses (h)	26,000	29,000	25,000	32,500	36,500	38,000	40,500	44,500	40,500	46,000	50,000	40,000	30,000	22,200	30,000	36,300	40,700	48,600	54,100	51,500	63,100	85,000	99,700	149,800	134,700	134,800	132,100	115,900	161,300	110,100	73,800	136,800	167,200	141,500	141,200	102,500	120,400	112,700	100,200	103,000			
Totals (i)	548,450	604,800	520,550	681,850	767,000	811,850	856,500	900,900	840,950	930,600	972,300	768,600	567,700	400,000	433,700	488,000	538,900	633,550	678,700	546,000	667,000	782,000	1,050,000	1,000,000	1,100,000	1,090,000	1,000,000	925,000	1,172,000	1,129,000	868,000	1,211,000	1,151,000	1,101,000	1,151,000	1,094,900	1,205,700	1,182,000	1,118,000	974,800			

ESTIMATED DOMESTIC LEAD CONSUMPTION

As compiled by LEAD INDUSTRIES ASSOCIATION
(Lead content, in tons of 2000 lb.; not including pig lead exported or delivered to U.S. permanent stockpile)

CONSUMPTION BY INDUSTRIES

	1942(e)	1943	1944(e)	1945	1946	1947	1948	1949	1950	1951	1952	1953	1954	1955	1956	1957	1958
Storage batteries	219,530	252,445	297,095	291,155	252,050	367,350	354,595	266,540	386,455	336,095	313,470	338,715	295,355	350,360	303,890	309,785	264,030
Cable	160,440	145,995	128,945	102,090	93,275	155,115	174,820	113,725	133,840	123,605	136,890	138,905	128,905	123,450	130,290	103,215	66,850
Paint and varnish	99,135	94,355	98,500	74,535	78,320	91,645	69,315	45,985	64,445	52,325	44,560	44,655	37,690	39,010	37,475	34,350	32,305
Ammunition	79,540	180,540	50,555	39,855	30,125	39,935	42,400	18,025	32,790	31,135	24,329	31,955	29,475	32,295	28,455	27,030	24,195
Oil refining and gasoline	54,255	89,260	94,670	87,605	60,270	84,590	96,375	103,960	120,455	135,058	155,555	165,285	165,530	169,500	197,405	178,980	162,245
Construction	123,735	86,580	103,240	105,125	110,945	112,535	104,825	83,820	120,830	107,895	107,335	104,930	97,955	108,820	107,440	98,370	99,800
Insecticides	17,325	20,220	24,185	25,420	33,945	25,635	28,725	24,965	30,915	23,685	24,555	26,090	26,145	25,695	24,985	23,530	22,650
Foil	5,840	13,230	16,070	15,645	3,640	3,305	3,075	2,670	3,295	2,760	2,930	5,990	4,010	5,555	4,010	5,555	4,010
Ceramics	10,890	8,770	7,370	4,645	6,595	8,550	9,240	11,010	23,775	22,965	15,445	17,530	19,690	24,385	22,740	19,060	17,300
Colors	13,235	14,930	12,560	13,780	13,840	13,275	19,210	16,745	26,580	23,465	23,865	22,870	19,250	19,940	18,615	21,265	18,540
Railroads	18,740	22,420	26,785	26,850	22,325	22,665	24,195	16,565	19,190	19,160	15,710	14,145	10,470	13,295	13,340	12,245	8,800
Automobiles	2,980	3,840	7,740	9,930	16,715	17,690	19,700	16,320	22,925	17,510	15,775	13,790	10,945	15,675	12,800	11,585	7,615
Cans	8,770	7,065	10,310	12,070	10,460	12,205	13,735	14,170	17,905	13,370	15,270	16,315	15,655	14,755	14,020	11,890	9,975
Steel and wire	6,910	5,010	4,575	4,135	3,425	4,085	2,960	2,980	4,845	3,545	4,660	4,130	5,580	5,915	6,780	5,305	5,750
Collapsible tubes	9,865	15,920	16,255	15,630	13,135	11,125	6,750	2,140	4,965	3,980	2,575	3,375	3,030	2,280	1,685	1,485	1,330
Rubber and hose	4,035	3,225	4,135	3,580	4,000	4,195	3,640	6,565	4,500	3,235	3,260	3,635	2,605	3,020	3,245	2,855	2,370
Coatings	2,730	2,835	5,500	4,960	3,615	4,515	4,710	2,645	4,065	2,910	1,620	1,855	2,225	2,145	2,240	1,890	1,600

COMPANIES AND ASSOCIATIONS
WHICH HAVE CONTRIBUTED INFORMATION TO
THIS STUDY

American Brake Shoe Company
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American Railway Car Institute
American Society for Metals
American Telephone and Telegraph Company
American Typefounders Company
Association of American Battery Manufacturers
Association of American Railways
Automobile Manufacturers' Association
Bell Telephone Laboratories
Building Research Institute
Can Manufacturers Institute
Cast Iron Soil Pipe Institute
Ceramic Industry
Chrysler Corporation
Clay Sewer Pipe Association
Cleveland Graphite Bronze Company
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Kaiser Aluminum and Chemical Sales Incorporated
Lead Industries Association
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National Association of Engine and Boat Manufacturers
National Cannery Association
National Lead Company

COMPANIES (Cont'd)

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Printers' Ink Publishing Company
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Remington Arms Company Incorporated
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Society of Plastics Engineers
Society of the Plastics Industry
Twentieth Century Fund
Typefounders Sales Company
United States Bureau of Mines
United States Department of Commerce
Vinyl Fabrics Institute
Western Electric Company

IBM 704 COMPUTER PROGRAMS

Three programs have been written for fitting logistic curves. Program number one fits a skew logistic, third order, by Pearl's method, and a block diagram and description of the make-up of the deck as assembled are included here with the program.

Program number two is the same as number one except that in this case K is also reevaluated with the other parameters. Although all programs have been debugged, this method failed to give convergence for the mineral production series, presumably because they have not yet approached the asymptote sufficiently closely.

Program number three fits a symmetrical logistic, first order, by first estimating the parameters by the three point method, then improving the estimates by least squares.

(Choose a value)
(of K in the range)

(Subtract each Y from K)

Estimate Parameters (Divide K-Y by Y)

(Look up $\ln \frac{K-Y}{Y}$)

(Fit a third order
(polynomial to logs of $\frac{K-Y}{Y}$)) LSQPF1

Improve Estimate (Substitute the parameters of the polynomial for
(a_0, a_1, a_2, a_3 , in the logistic expression and ex-
pand in a Taylor's Series, solving for correc-
tion terms) LAS872

(Iterate a selected number of times)

Calculate a point on the curve (Use the new values & compute $a_1x+a_2x^2+a_3x^3$)

(Compute $1+e^{a_0+a_1x+a_2x^2+a_3x^3}$)

(Compute $\frac{K}{1+e^{a_0+a_1x+a_2x^2+a_3x^3}}$)

(Compute $Y_{obs} - Y_{calc}$)

(Square)

RMS Deviation (Sum the squares)

(Divide by number of points)

(Square Root)

(Display curve on cathode ray tube) SCOPE

Results (Print out K, a_0, a_1, a_2, a_3 , curve values, original
(values, deviations, annual differences, and RMS)
(deviation)

(Repeat for a new K)

(Repeat for a new commodity (i.e. Copper, Lead, etc.)

MAKE-UP FOR DATA DECK FOR FITTING
A THIRD ORDER LOGISTIC

1. RUN M361-292-WHITE
2. BIN M361-WHITE-SUBRT Binary deck with CLLSQ1,
LAS872, SCOPE
3. BIN M361-FITTING Binary deck for main program,
including octal corrections
(before the END card)
4. PMD M361-FITTING Error post-mortem deck -used
only if an error stop occurs.
5. RIP M361-WHITE-SUBRT
6. RIP M361-FITTING
7. DPR PLEASE SWITCH TO SINGLE SPACE
8. BGN M361-FITTING
9. DATA

7660	DEC	FLONY	Number of Y values (in floating point, i.e. with a decimal)
7661	DEC	ITEM	Number of commodities being fitted (in fixed point, i.e. with no decimal)
7662	DEC	SIZE	Maximum value to go on scope (float)
7663	DEC	ITER	Number of iterations desired (fixed)
7664	DEC	NX	Number of X values (fixed)
7665	DEC	NY	Number of Y values (fixed)
7666	DEC	NK	Number of K values (fixed)
7667	DEC	K's	Actual K values (float)
7692	DEC	Y's	Actual Y values (float)
7942	DEC	X's	Actual X values (float)
10. TRA 2,4
11. DPR PLEASE RETURN TO PROGRAM CONTROL
12. TER M361-292-WHITE

* * *

To reassemble the main program, substitute for 3 the following:

CST M361-WHITE-SUBRT
SAP M361-FITTING

followed by the symbolic deck (the SAP deck) for the main program.

PROGRAM NUMBER ONE

THIS PROGRAM FITS A THIRD ORDER LOGISTIC CURVE TO UP TO 250 DISCRETE DA
POINTS ,BY MAKING A FIRST APPROXIMATION THEN IMPROVING THIS BY APPROXIM
THE FUNCTION BY A TAYLORS SERIES TRUNCATED AT THE FIRST ORDER TERM
THE VALUE OF K, THE UPPER ASYMPTOTE IS SPECIFIED BY THE USER AND IS NOT
CHANGED BY THE PROGRAM

CST M361-WHITE-SUBRT

SAP M361-FITTING

PRG OFF

SMT OFF

REM LOGISTIC CURVE M361 JAMES A.L. WHITE,MAY,1958

REM THIS PROGRAM HAS BEEN DEBUGGED AND RUNS

ORG 320

START TSX LOAD,4

PZE K

CAL NY ADDRESS MODIFICATION

COM

ADD ONE

ANA MASK

ALS 18

STD BACK

CLA NY

STA LOCNY

ALS 18

STD GO+4

CAL NK

COM

ADD ONE

ANA MASK

ALS 18

STD CYCLE

CLA NX

ADD FIFTY

COM

ADD ONE

ANA MASK

ALS 18

STD PRAM

CLA LX

ADD NX

STA FWA5+1

STA FWA5+5

STA FWA5+9

STA FWA5+29

STA FWA5+32

STA FWA5+35

CLA SIZE

STO NUMB+3

CLA LY

ADD NY

STA FWA5+40

TSX WOT,4

PZE NAME,,NAME+5

CLA NY

SET

ADD FIFTY

UP

LRS 35

FINAL

MPY FIVE

ADDRESS

LLS 35

ADD PRSET

OF

```

ALS 18
STD PRSET
CFE
CHANGE FILM FRAME
PUT ZERO IN INDEX REGISTER ONE
PUT ZERO IN INDEX REGISTER TWO
LOOP CLA K,1
STO K
FSB Y,2
FDH Y,2
CLM
LTS 35
TSX 4347,4
HPR 0
STO TEMP,2
TXI *+1,2,-1
DO THIS FOR EACH Y VALUE
BACK TXH LOOP,2,0
TSX L50PF,4
FIT A THIRD ORDER POLYNOMIAL
PZE X,0,TEMP
LOCNY MZE 0,3,3POL
HTR
SXD KEEP,1
LXA COCO,1
COUNT CLA 3POL+11,1
STO FWAI+4,1
TX COUNT,1,1
LXA ITER,1
GO TSX LAS872+0,4
HPR
PZE FWAI,0,FWA2
PZE FWA3,0,FWA4
PZE FWA5,0,0
DECR WILL BE MODIFIED TO NUMBER OF YS
PZE 4,0,20
TX GO,1,1
REM PARAMETERS ARE COMPUTED. FOLLOWING COMPUTES VALUES OF POINTS
REM ON THE CURVE PLUS FIFTY FORECAST YEARS
STZ DIF50
PUT ZERO IN DIF50
CLA NX
ADD FIFTY
STA NUMB
MODIFY ADDRESS FOR PLOTTING ON THE SCOPE
PAX 0,1
LXA IRO,2
CURVE LDO FWAI+3
FMP X,2
FAD FWAI+2
STO DENOM
FAD FWAI+1
TIMES X
FMP X,2
LDO DENOM
FAD FWAI+1
TIMES X
FMP X,2
LDO DENOM
FAD FWAI+2
STO DENOM
FAD FWAI+2
TIMES A3
FMP X,2
LDO FWAI+3
43
PUT ZERO IN INDEX REGISTER TWO
PUT ZERO IN INDEX REGISTER ONE
EXPONENTIAL SUBROUTINE
TSX S816EX,4
HPR
ERROR RETURN FOR SUBROUTINE

```


	FAD FLONE	ADD ONE TO THE EXPONENTIAL
	STQ DENOM	THIS IS THE FRACTION DENOMINATOR
	CLA K	
	FDH DENOM	
	STQ POINT,2	A POINT ON THE CURVE
	REM A ROOT MEAN SQUARE DEVIATION WILL BE COMPUTED NEXT.	
	TXL STUP,1,50	WHEN IR ONE HAS 50 IN IT ,NO MORE Y OBS
	CLA Y,2	
	FSB POINT,2	Y OBS MINUS Y CALC
	STO DEV,2	IS STORED IN DEV
	LDQ DEV,2	
	FMP DEV,2	DEVIATION SQUARED
	FAD DIFSQ	ADD TO CONTENTS OF DIFSQ
	STO DIFSQ	STORE TOTAL IN DIFSQ
	TXL **4,2,0	COMPUTE
	CLA POINT,2	YEARLY
	FSB POINT-1,2	PRODUCTION
	STO YANN,2	
STUP	TXI **1,2,-1	COMPUTE CURVE VALUES FOR ALL VALUES OF X
	TIX CURVE,1,1	
	CLA DIFSQ	
	FDH FLONY	DIVIDE BY THE NUMBER OF POINTS
	CLM	
	LLS 35	
	TSX SQRT,4	TAKE SQUARE ROOT OF DEVIATION FUNCTION
	TRA **3	SUBROUTINE ERROR RETURN
	STO RMSD	ROOT MEAN SQUARE DEVIATION IS STORED HERE
	TRA **2	READY TO PRINT OUT RESULTS
	HTR	
	REM EQUATION,VALUES,AND RMS DEV STORED IN FWA4 TO FWA4+5, OINT,	
	REM TO POINT + NY + 50 AND RMSD.CURVE WILL NOW BE PLOTTE	
	TSX SCOPE,4	
	PZE X,,POINT	LOCATIONS OF FIRST X AND FIRST Y
NUMB	PZE	NUMBER OF POINTS
	DEC 250.,0.	MAXIMUM AND MINIMUM VALUES OF X
	DEC 0.,0.	MAXIMUM AND MINIMUM VALUES OF Y
	LXA IRO,3	
MOVE	CLA X,1	
	STO PRBLK,2	
	CLA Y,1	
	STO PRBLK+1,2	
	CLA POINT,1	
	STO PRBLK+2,2	
	CLA DEV,1	
	STO PRBLK+3,2	
	CLA YANN,1	
	STO PRBLK+4,2	
	TXI **1,2,-5	
	TXI **1,1,-1	
PRAM	TXH MOVE,1,0	
	TSX WOT,4	
	PZE HEAD,0,HEAD+14	
	TSX FORM,4	
PRSET	PZE PRBLK,0,**	
	TSX PRK,4	
	PZE K,0,K	

```

    TSX Q,4
    PZE RMSD,0,RMSD
    TSX R,4
PRINC PZE FWA1,0,FWA1+3
    LXD KEEP1,1
    TXI *+1,1,-1
CYCLE TXH BEGIN,1,0
    CFF
    CLA CTIT
    ADD ONE
    STO CTIT
    SUB ITEM
    TMI START
    HLT
    REM THIS IS A FUNCTION ROUTINE TO EVALUATE THE PARTIALS
FWA5 LDQ FWA1+3          A3
    FMP 0,2             X TIMES A3
    FAD FWA1+2         PLUS A2
    STO E
    LDQ E
    FMP 0,2             TIMES X
    FAD FWA1+1         PLUS A1
    STO E
    LDQ E
    FMP 0,2             TIMES X
    FAD FWA1           PLUS A0
    STO E               A0+A1X+A2XSQUARE+A3XCUBE
    SXD KEEP4,4
    TSX S816EX,4
    HTR 0
    LXD KEEP4,4
    STO E
    FAD FLONE          PLUS ONE
    STO G              ONE PLUS THE EXPONENTIAL
    CLA FLONE
    FDH G
    STQ 10VG
    CLS K
    LRS 35
    FMP E
    FDH G
    FMP 10VG
    STO FWA2           PARTIAL WRT A0
    LDQ FWA2
    FMP 0,2             TIMES X
    STO FWA2+1         PARTIAL WRT A1
    LDQ FWA2+1
    FMP 0,2             TIMES X
    STO FWA2+2         PARTIAL WRT A2
    LDQ FWA2+2
    FMP 0,2             TIMES X
    STO FWA2+3         PARTIAL WRT A3
    CLA K
    FDH G              CALCULATE AN APPROXIMATE Y FOR THE SUBROUTINE
    STQ YCALC
    CLA 0,2

```

	FSB	YCALC	Y OBS MINUS Y CALC			
	LRS	35				
FINISH	TRA	1.4				
G	BSS	1				
YCALC	BSS	1				
E	BSS	1				
10VG	BSS	1				
TEMP	BSS	250				
3POL	BSS	11				
KEEP1	BSS	1				
KEEP2	BSS	1				
KEEP4	BSS	1				
DENOM	BSS	1				
FWA1	BSS	5				
FWA2	BSS	5				
FWA3	BSS	30				
FWA4	BSS	5				
POINT	BSS	300				
RMSD	BSS	1				
DEV	BSS	200				
CTIT	BSS	1				
DIFSQ	BSS	1				
YANN	BSS	300				
PRBLK	BSS	1300				
LX	PZE	X				
LY	PZE	Y				
Y	SYN	7692	ORIGIN OF Y VALUES IN STORAGE			
X	SYN	7942	ORIGIN OF X VALUES IN STORAGE			
K	SYN	7667	ORIGIN OF K VALUES IN STORAGE			
NK	SYN	7666	NUMBER OF KS			
NY	SYN	7665	NUMBER OF YS			
NX	SYN	7664	NUMBER OF XS			
ITER	SYN	7663	NUMBER OF ITERATIONS DESIRED			
SIZE	SYN	7662	MAXIMUM POINT FOR SCOPE PLOTTING			
ITEM	SYN	7661	NUMBER OF DATA DECKS BEING RUN AT ONE TIME			
FLONY	SYN	7660				
IRO	DEC	0				
COCO	DEC	4				
FIVE	DEC	5				
FIFTY	DEC	50				
FLONE	DEC	1.0				
MASK	OCT	000000077777				
NAME	BCD	6JAMESA.L.WHITE,292,PROBLEMM361				
HEAD	BCD	9	X	Y OBS	YCALC	
	BCD	6	DEV	YANN		
FORM	TRA	BLOCK				
	BCD	5F10.0,F20.0,F20.0,F20.4,F20.0				
PRK	TRA	BLOCK				
	BCD	22HK=12.0				
Q	TRA	BLOCK				
	BCD	35HRMSD=F20.0				
R	TRA	BLOCK				
	BCD	93HA0=E13.5,6H	A1=E13.5,6H	A2=E13.5,6H	A3=13.5	
	END	START				

PROGRAM NUMBER TWO

```

REM JAMES A.L.WHITE,292,PROBLEM M361 1959 LOGISTIC CURVE (SKEW)
ORG 256
START TSX LOAD,4
PZE K
CAL NY ADDRESS MODIFICATION
COM
ADD ONE
ANA MASK
ALS 18
STD BACK
CLA NY
STA LOCNY
ALS 18
STD GO+4
CAL NK
COM
ADD ONE
ANA MASK
ALS 18
STD CYCLE
CLA NX
ADD FIFTY
COM
ADD ONE
ANA MASK
ALS 18
STD PRAM
CLA LX
ADD NX
STA FWA5+1
STA FWA5+5
STA FWA5+9
STA FWA5+30
STA FWA5+33
STA FWA5+36
CLA SIZE
STO NUMB+3
CLA LY
ADD NY
STA FWA5+41
TSX WOT,4
PZE NAME,,NAME+5
CLA NY SET
ADD FIFTY UP
LRS 35 FINAL
MPY FIVE ADDRESS
LLS 35
ADD PRSET OF
ALS 18 PRINT
STD PRSET BLOCK
CFF CHANGE FILM FRAME
BEGIN LX A IRO,1 PUT ZERO IN IR ONE
LOOP LX A IRO,2 PUT ZERO IN IR TWO
CLA K,1 K TO AC
STO K
FSB Y,2 K-Y
FDH Y,2 K-Y/Y
CLM CLEAR AC

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LLS 35      MQ TO AC
TSX 4347,4      LN LOOKUP
HPR 0      RETURN X MINUS
STO TEMP,2      STORE LOGS
TXI *+1,2,-1
BACK TXH LOOP,2,0      DECR WILL HAVE 2S COMP OF NO OF YS
TSX LSQPF,4      FIT A THIRD ORDER POLYNOMIAL
PZE X,0,TEMP      LOCATION OF X1 AND Y1
LOCNY MZE 0,3,3POL      ADDRESS WILL BE NUMBER OF YS
HTR      ERROR RETURN
SXD KEEP1,1      STORE IR ONE IN KEEP1
LXA COCO,1      PUT 4 IN COCO
COUNT CLA 3POL+11,1      PUT COEFFS INTO FWA1 BLOCK
STO FWA1+5,1
TIX COUNT,1,1
CLA K
STO FWA1
LXA ITER,1      COUNT NUMBER OF ITERATIONS
GO TXS LAS872+0,4
HPR      RETURN SINGULAR SET
PZE FWA1,0,FWA2
PZE FWA3,0,FWA4
PZE FWA5,0,0      DECR WILL CONTAIN NUMBER OF YS
PZE 5,0,30      NO. OF PARAMETERS EQUALS 5
TIX GO,1,1
REM PARAMETERS ARE COMPUTED. FOLLOWING COMPUTES VALUES OF POINTS
REM ON THE CURVE PLUS FIFTY FORECAST YEARS
STZ DIFSQ      PUT ZERO IN DIFSQ
CLA NX      COUNT NUMBER OF XS
ADD FIFTY      FOR FORECAST
STA NUMB      MODIFY ADDRESS FOR SCOPE
PAX 0,1      PUT NX PLUS FIFTY IN IR ONE
LXA IRO,2      PUT ZERO IN IR TWO
CURVE LDQ FWA1+4      A3
FMP X,2      X TIMES A3
FAD FWA1+3      PLUS A2
STO DENOM
LDQ DENOM
FMP X,2      TIMES X
FAD FWA1+2      PLUS A1
STO DENOM
LDQ DENOM
FMP X,2      TIMES X
FAD FWA1+1      PLUS A0
STO DENOM      A1X + A2XSQUARE + A3XCUBE + A0
CLA DENOM
TSX S816EX,4      EXPONENTIAL SUBROUTINE
HPR      ERROR RETURN
FAD FLONE
STO DENOM      THIS THE DENOMINATOR OF THE FRACTION
CLA FWA1      K
FDH DENOM
STQ POINT,2      A POINT ON THE CURVE
REM A ROOT MEAN SQUARE DEVIATION WILL BE COMPUTEDNEXT.
TXL STUP,1,50      WHEN IR 1 CONTAINS 50 NO MORE YOBSE
CLA Y,2

```

```

FSB POINT,2      YOBS MINUS YCALC
STO DEV,2        DEVIATION
LDQ DEV,2
FMP DEV,2        DEVIATION SQUARED
FAD DIFSQ        ADD TO CONTENTS OF DIFSQ
STO DIFSQ        STORE TOTAL IN DIFSQ
TXL *+4,2,0      COMPUTE
CLA POINT,2      YEARLY
FSB POINT-1,2    PRODUCTION
STO YANN,2
STUP TXI *+1,2,-1 COMPUTE CURVE VALUES FOR ALLVALUES OF X
TIX CURVE,1,1
CLA DIFSQ
FDH FLONY        DIVIDE BY NUMBER OF POINTS
CLM
LLS 35
TSX SQRT,4       TAKE SQUARE ROOT
TRA *+3          ERROR RETURN
STO RMSD         THE RMSD IS STORED HERE
TRA *+2          READY TO PRINT OUT
HTR
REM PARAMETERS,VALUES,AND RMSD STORED IN FWA4 TO FWA4+5,POINT,
REM TO POINT+50+NY,AND RMSD. CURVE WILL NOW BE PLOTTED.
TSX SCOPE,4
PZE X,,POINT     LOCATIONS OF FIRST X AND FIRST Y
NUMB PZE         NUMBER OF POINTS
DEC 250.,0.      MAXIMUM AND MINIMUM VALUES OF X
DEC 0.,0.        MAXIMUM AND MINIMUM VALUES OF Y
LXA IRO,3
MOVE CLA X,1
STO PRBLK,2
CLA Y,1
STO PRBLK+1,2
CLA POINT,1
STO PRBLK+2,2
CLA DEV,1
STO PRBLK+3,2
CLA YANN,1
STO PRBLK+4,2
TXI *+1,2,-5
TXI *+1,1,-1
PRAM TXH MOVE,1,0
TSX WOT,4
PZE HEAD,0,HEAD+14
TSX FORM,4
PRSET PZE PRBLK,0,**
TSX PRK,4
PZE K,0,K
TSX NUK,4
PZE FWA1,0,FWA1
TSX Q,4
PZE RMSD,0,RMSD
TSX R,4
PRINC PZE FWA1+1,0,FWA1+4
LXD KEEP1,1
TXI *+1,1,-1

```

```

CYCLE TXH BEGIN,1,0
  CFF
  CLA CTIT
  ADD ONE
  STO CTIT
  SUB ITEM
  TMI START
  HLT
  REM THIS IS A FUNCTION ROUTINE TO EVALUATE THE PARTIALS
  FWA5 LDQ FWA1+4      A3
      FMP 0,2          X TIMES A3
      FAD FWA1+3      PLUS A2
      STO E
      LDQ E
      FMP 0,2          TIMES X
      FAD FWA1+2      PLUS A1
      STO E
      LDQ E
      FMP 0,2          TIMES X
      FAD FWA1+1      PLUS A0
      STO E          A1X + A2XSQUARE + A3XCUBE + A0
      SXD KEEP4,4
      TSX S816EX,4
      HTR 0
      LXD KEEP4,4
      STO E
      FAD FLONE          PLUS 1
      STO G          1+EXPONENTIAL
      CLA FLONE
      FDH G
      STQ 10VG
      STQ FWA2          PARTIAL WRT K
      CLS FWA1
      LRS 35
      FMP E
      FDH G
      FMP 10VG
      STO FWA2+1        PARTIAL WRT A0
      LDQ FWA2+1
      FMP 0,2          MULT BY X
      STO FWA2+2        PARTIAL WRT A1
      LDQ FWA2+2
      FMP 0,2          MULT BY X
      STO FWA2+3        PARTIAL WRT A2
      LDQ FWA2+3
      FMP 0,2          MULT BY X
      STO FWA2+4        PARTIAL WRT A3
      CLA FWA1
      FDH G          CALCULATE AN APPROXIMATE VALUE OF Y
      STQ YCALC
      CLA 0,2
      FSB YCALC          YOBS MINUS YCALC
      LRS 35
  FINISH TRA 1,4
  G      BSS 1
  YCALC BSS 1

```



```

E      BSS 1
10VG  BSS 1
TEMP  BSS 250
3POL  BSS 11
KEEP1 BSS 1
KEEP2 BSS 1
KEEP4 BSS 1
DENOM BSS 1
  FWA1 BSS 6
  FWA2 BSS 5
  FWA3 BSS 30
  FWA4 BSS 5
POINT BSS 300
RMSD  BSS 1
DEV   BSS 200
CTIT  BSS 1
DIFSQ BSS 1
YANN  BSS 300
PRBLK BSS 1300
LX    PZE X
LY    PZE Y
Y     SYN 7692
X     SYN 7942
K     SYN 7667
NK    SYN 7666      NUMBER OF KS
NY    SYN 7665      NUMBER OF YS
NX    SYN 7664      NUMBER OF XS
ITER  SYN 7663      NUMBER OF ITERATIONS DESIRED
  SIZE SYN 7662
  ITEM SYN 7661
FLONY SYN 7660
IRO   DEC 0
COCO  DEC 4
FIVE  DEC 5
FIFTY DEC 50
FLONE DEC 1.0
MASK  OCT 000000077777
NAME  BCD 6JAMESA.L.WHITE,292,PROBLEMM361
HEAD  BCD 9      X      YOBS      YCALC
      BCD 6      DEV      YANN
FORM  TRA BLOCK
      BCD 5F10.0,F20.0,F20.0,F20.4,F20.0
PRK   TRA BLOCK
      BCD 22HK=12.0
Q     TRA BLOCK
      BCD 35HRMSD=F20.0
R     TRA BLOCK
      BCD 93HA0=E13.5,6H  A1=E13.5,6H  A2=E13.5,6H  A3=13.5
NUK   TRA BLOCK
      BCD 22HK=12.0
      END START

```

PROGRAM NUMBER THREE

```

REM LOGISTIC CURVE M361 JAMES A.L. WHITE, MAY, 1958
ORG 320
START TSX LOAD,4
PZE K
CLA NY
ALS 18
STD GO+4
CLA NX
ADD FIFTY
COM
ADD ONE
ANA MASK
ALS 18
STD PRAM
CLA LX
ADD NX
STA FWA5+1
STA FWA5+22
CLA LY
ADD NY
STA FWA5+27
TSX WOT,4
PZE NAME,,NAME+5
CLA NY          SET
ADD FIFTY       UP
LRS 35          FINAL
MPY FIVE        ADDRESS
LLS 35
ADD PRSET       OF
ALS 18          PRINT
STD PRSET       BLOCK
BEGIN LXA IRO,2  PUT ZERO IN INDEX REGISTER TWO
LDQ Y0
FMP Y1
STO Y01
LDQ Y01
FMP Y2
STO Y012        YOY1Y2
LDQ Y01
FMP Y1
STO Y011
LDQ Y1
FMP Y1
STO Y11         Y1Y1
LDQ Y11
FMP Y2
STO Y112        Y1 SQUARE Y2
LDQ Y0
FMP Y2
STO Y02         YOY2
FSB Y11
STO BELOW
CLA Y012
FAD Y012
FSB Y011
FSB Y112
STO NUMER       NUMERATOR OF K EXPRESSION
FDH BELOW

```

```

STQ K
STQ FWA1
CLA K
FSB Y0
STO KMY0
CLA K
FSB Y1          K MINUS YU
FDH Y1          K-Y1/Y1
FMP Y0
FDH KMY0        Y0(K-Y1)/Y1(K-Y0)
CLM
LLS 35
TSX 4347,4      NATURAL LOG LOOKUP
HPR 0           ARGUMENT NEGATIVE
FDH SPACE
STQ A1
STQ FWA1+2
CLA K
FSB Y0
FDH Y0
CLM
LLS 35
TSX 4347,4
HPR 0
STO A0
STO FWA1+1
LXA ITER,1
GO  TSX LAS872+0,4
HPR 0
PZE FWA1,0,FWA2
PZE FWA3,0,FWA4
PZE FWA5,0,0    DECR WILL BE MODIFIED TO NUMBER OF YS
PZE 3,0,12      3 PARAMETERS-K,A0,A1
TIX GO,1,1
REM PARAMETERS ARE COMPUTED. FOLLOWING COMPUTES VALUES OF POINTS
REM ON THE CURVE PLUS FIFTY FORECAST YEARS
STZ DIFSQ       PUT ZERO IN DIFSQ
CLA NX          COUNT THE NUMBER OF XS
ADD FIFTY       FOR A FIFTY YEAR FORECAST
PAX 0,1         PUT NX PLUS FIFTY IN INDEX REGISTER ONE
LXA IRO,2       PUT ZERO IN INDEX REGISTER TWO
CURVE LDQ FWA1+2  A1
FMP X,2
FAD FWA1+1      A0+A1X
STO DENOM
TSX S816EX,4
HPR 0
FAD FLONE
STO DENOM
CLA FWA1
FDH DENOM
STQ POINT,2
REM A ROOT MEAN SQUARE DEVIATION WILL BE COMPUTEDNEXT.
TXL STUP,1,50  WHEN IR ONE HAS 50 IN IT ,NO MORE Y OBS
CLA Y,2
FSB POINT,2    Y OBS MINUS Y CALC

```

	STO DEV,2	IS STORED IN DEV
	LDQ DEV,2	
	FMP DEV,2	DEVIATION SQUARED
	FAD DIFSQ	ADD TO CONTENTS OF DIFSQ
	STO DIFSQ	STORE TOTAL IN DIFSQ
	TXL *+4,2,0	COMPUTE
	CLA POINT,2	YEARLY
	FSB POINT-1,2	PRODUCTION
	STO YANN,2	
STUP	TXI *+1,2,-1	COMPUTE CURVE VALUES FOR ALL VALUES OF X
	TIX CURVE,1,1	
	CLA DIFSQ	
	FDH FLONY	DIVIDE BY THE NUMBER OF POINTS
	CLM	
	LLS 35	
	TSX SQRT,4	TAKE SQUARE ROOT OF DEVIATION FUNCTION
	TRA *+3	SUBROUTINE ERROR RETURN
	STO RMSD	ROOT MEAN SQUARE DEVIATION IS STORED HERE
	TRA *+2	READY TO PRINT OUT RESULTS
	HTR	
	LXA IRO,3	
MOVE	CLA X,1	
	STO PRBLK,2	
	CLA Y,1	
	STO PRBLK+1,2	
	CLA POINT,1	
	STO PRBLK+2,2	
	CLA DEV,1	
	STO PRBLK+3,2	
	CLA YANN,1	
	STO PRBLK+4,2	
	TXI *+1,2,-5	
	TXI *+1,1,-1	
PRAM	TXH MOVE,1,0	
	TSX WOT,4	
	PZE HEAD,0,HEAD+14	
	TSX FORM,4	
PRSET	PZE PRBLK,0,**	
	TSX PRK,4	
	PZE K,0,K	
	TSX Q,4	
	PZE RMSD,0,RMSD	
	TSX R,4	
PRINC	PZE FWA1,0,FWA1+3	
	CLA CTIT	
	ADD ONE	
	STO CTIT	
	SUB ITEM	
	TMI START	
	HLT	
	REM THIS IS A FUNCTION ROUTINE TO EVALUATE THE PARTIALS	
FWA5	LDQ FWA1+2	A1
	FMP 0,2	A1 TIMES X
	FAD FWA1+1	A0
	STO E	A0+A1X
	SXD KEEP4,4	

TSX	S816EX,4	
HTR	0	
LXD	KEEP4,4	
STO	E	
FAD	FLONE	
STO	G	
CLA	FLONE	
FDH	G	
STQ	10VG	
STQ	FWA2	PARTIAL WRT K
CLS	K	
LRS	35	
FMP	E	
FDH	G	
FMP	10VG	
STO	FWA2+1	PARTIAL WRT A0
LDQ	FWA2+1	
FMP	0,2	
STO	FWA2+2	PARTIAL WRT A1
CLA	FWA1	
FDH	G	
STQ	YCALC	
CLA	0,2	
FSB	YCALC	
LRS	35	
FINISH	TRA 1,4	
G	BSS 1	
YCALC	BSS 1	
E	BSS 1	
10VG	BSS 1	
TEMP	BSS 250	
3POL	BSS 11	
KEEP1	BSS 1	
KEEP2	BSS 1	
KEEP4	BSS 1	
DENOM	BSS 1	
FWA1	BSS 5	
FWA2	BSS 5	
FWA3	BSS 30	
FWA4	BSS 5	
POINT	BSS 300	
RMSD	BSS 1	
DEV	BSS 200	
CTIT	BSS 1	
DIFSQ	BSS 1	
YANN	BSS 300	
PRBLK	BSS 1300	
LX	PZE X	
LY	PZE Y	
Y	SYN 7692	ORIGIN OF Y VALUES IN STORAGE
X	SYN 7942	ORIGIN OF X VALUES IN STORAGE
K	SYN 7667	ORIGIN OF K VALUES IN STORAGE
NK	SYN 7666	NUMBER OF KS
NY	SYN 7665	NUMBER OF YS
NX	SYN 7664	NUMBER OF XS
ITER	SYN 7663	NUMBER OF ITERATIONS DESIRED

```

SIZE SYN 7662          MAXIMUM POINT FOR SCOPE PLOTTING
ITEM SYN 7661          NUMBER OF DATA DECKS BEING RUN AT ONE TIME
FLONY SYN 7660
Y0 SYN 7657
Y1 SYN 7656
Y2 SYN 7655
SPACE SYN 7654
Y01 BSS 1
Y02 BSS 1
Y011 BSS 1
Y012 BSS 1
Y11 BSS 1
Y112 BSS 1
BELOW BSS 1
NUMER BSS 1
KMYO BSS 1
A1 BSS 1
A0 BSS 1
IRO DEC 0
COCO DEC 4
FIVE DEC 5
FIFTY DEC 50
FLONE DEC 1.0
MASK OCT 000000077777
NAME BCD 6JAMESA.L.WHITE,292,PROBLEMM361
HEAD BCD 9          X          YOBS          YCALC
      BCD 6          DEV          YANN
FORM TRA BLOCK
      BCD 5F10.0,F20.0,F20.0,F20.4,F20.0
PRK  TRA BLOCK
      BCD 22HK=12.0
Q    TRA BLOCK
      BCD 35HRMSD=F20.0
R    TRA BLOCK
      BCD 93HA0=E13.5,6H  A1=E13.5,6H  A2=E13.5,6H  A3=13.5
      END START

```

SUBROUTINES

LSQPF

LAS872

SCOPE

PRG OFF
 SMT OFF
 ORG 3000
 LSOFF XSD LSOFF+0551,4

WORD IN N+1 TO ACC

CLA 1,4
 STA LSOFF+0538

STORE A

ARS 18
 STO LSOFF+0539

STORE B

SXD LSOFF+0584,2
 CLA 2,4

STORE N

STA LSOFF+0554
 PAX 0,2

N IN IR 2

LRS 18
 STA LSOFF+0540

STORE C

CLM
 LLS 3

STA LSOFF+0541

STORE P

ADD LSOFF+0554

A+N

STA LSOFF+0027

STA LSOFF+0031

STA LSOFF+0103

STA LSOFF+0394

SXD LSOFF+0585,1

B

CLA LSOFF+0539

B+N

ADD LSOFF+0554

STA LSOFF+0153

STA LSOFF+0400

DCT
 TRA LSOFF+0027

A+I

CLA 0,2
 SSP

TIX LSOFF+0031,2,1

TRA LSOFF+0035

A+I+1

LDO 0,2
 LRS 0

TLO LSOFF+0029

STORE X MAX

STO COMMON
 IZE LSOFF+0579

CLA LSOFF+0541

P

ADD LSOFF+0542

STO COMMON+1
 ALS 18

STO COMMON+8
 P+1 IN DECR, COMMON+8

ARS 18
 ADD LSOFF+0543

P+4

STO COMMON+6
 LDO COMMON+6

MPY COMMON+1
 LRS 1

STO COMMON+2
 CLA COMMON+2

(P+1)(P+4)/2

ALS 18
 STO COMMON+7

(P+1)(P+4)/2 IN DECR

SUB COMMON+8
 STO COMMON+9

(P+1)(P+4)/2-(P+1) IN DECR

CLA	COMMON+2	
ADD	LSQPF+0544	
STA	LSQPF+0110	COMMON+A+(P+1)(P+4)/2
STA	LSQPF+0160	
STA	LSQPF+0161	
STA	LSQPF+0171	
STA	LSQPF+0177	
STA	LSQPF+0181	
STA	LSQPF+0218	
ADD	LSQPF+0554	COMMON+P+1*P+4/2+N
STA	LSQPF+0105	
STA	LSQPF+0143	
STA	LSQPF+0145	
CLA	LSQPF+0544	
ADD	LSQPF+0541	COMMON+A+P+2
ADD	LSQPF+0545	
STA	LSQPF+0190	
CLA	LSQPF+0541	
ALS	1	2P
ADD	LSQPF+0544	COMMON+A+2P
ADD	LSQPF+0543	COMMON+A+2P+3
STA	LSQPF+0192	
STA	LSQPF+0193	
CLA	LSQPF+0541	P
ADD	LSQPF+0553	5
STO	LSQPF+0556	
LDQ	LSQPF+0556	
MPY	LSQPF+0541	P(P+5)
LRS	1	P(P+5)/2
STQ	LSQPF+0556	
CLA	LSQPF+0556	
ADD	LSQPF+0540	C+P(P+5)/2
STA	LSQPF+0219	
STA	LSQPF+0250	
STA	LSQPF+0252	
STA	LSQPF+0253	
STA	LSQPF+0322	
STA	LSQPF+0378	
STA	LSQPF+0471	
STA	LSQPF+0476	
STA	LSQPF+0485	
STA	LSQPF+0494	
STA	LSQPF+0504	
STA	LSQPF+0514	
STA	LSQPF+0525	
STA	LSQPF+0533	
STA	LSQPF+0417	
LXA	LSQPF+0554,2	N IN IR 2
TQO	LSQPF+0103	
CLA	0,2	X
FDP	COMMON	
STQ	0,2	
TIX	LSQPF+0103,2,1	
TQO	LSQPF+0579	
LXA	COMMON+2,2	(P+1)(P+4)/2
CLA	LSQPF+0552	

STO 0,2	SET STORAGES TO ZERO
TIX LSQPF+0110,2,1	
CLA LSQPF+0541	
SUB LSQPF+0542	P-1
STO COMMON+3	
ADD LSQPF+0543	
STO COMMON+4	P+2
SUB LSQPF+0544	
STA LSQPF+0251	COMMON+A-N-2
CLA LSQPF+0546	COMMON+10
ADD LSQPF+0541	
STA LSQPF+0191	COMMON+10+P
STA LSQPF+0148	
SUB LSQPF+0542	
STA LSQPF+0150	COMMON+9+P
ADD LSQPF+0545	
STA LSQPF+0151	COMMON+11+P
STA LSQPF+0323	
STA LSQPF+0379	
STA LSQPF+0178	
ADD LSQPF+0542	
STA LSQPF+0154	COMMON+12+P
STA LSQPF+0158	
STA LSQPF+0159	COMMON+12+P
CLA LSQPF+0280	
ADD LSQPF+0541	COMMON+A+15+P
STA LSQPF+0287	
SUB LSQPF+0542	
STA LSQPF+0286	COMMON+A+14+P
CLA LSQPF+0571	
STO COMMON+10	
LXA LSQPF+0554,2	N IN IR 2
TOV LSQPF+0143	
CLA 0,2	A+N
STO COMMON+11	XI
FAD 0,2	A+N
STO COMMON+5	2XI
LXA COMMON+3,1	P-1 IN IR 1
LDQ 0,1	COMMON+10+P
FMP COMMON+5	
FSB 0,1	COMMON+9+P
STO 0,1	COMMON+11+P
TIX LSQPF+0148,1,1	
CLA 0,2	B+N
STO 0,1	
LXA COMMON+4,5	P+2 IN IR 1 AND IR 4
SXD LSQPF+0538,2	
LXA COMMON+2,2	(P+1)(P+4)/2 IN I.R. 2
LDQ 0,1	COMMON+12+P
FMP 0,4	
FAD 0,2	COMMON+20+(P+1)(P+4)/2
STO 0,2	
TIX LSQPF+0164,2,1	
TRA LSQPF+0167	
TIX LSQPF+0158,4,1	
LXA LSQPF+0552,4	

TIX LSQPF+0158,5,1
LXD LSQPF+0538,2
TIX LSQPF+0143,2,1
LXD COMMON+7,1
LXA COMMON+1,6
CLA 0,1
TZE LSQPF+0581
STO LSQPF+0577
TOV LSQPF+0579
TQO LSQPF+0176
TNX LSQPF+0212,1,1
CLA 0,1
STO 0,2
CHS
FDH LSQPF+0577
STQ 0,1
TIX LSQPF+0176,2,1
TXL LSQPF+0212,1,1
TQO LSQPF+0579
TXI LSQPF+0186,4,-1
TOV LSQPF+0187
CAL COMMON+1
STO LSQPF+0578
LXA LSQPF+0578,2
LDQ 0,2
FMP 0,4
FAD 0,2
STO 0,2
TIX LSQPF+0190,2,1
CAL LSQPF+0578
SUB LSQPF+0542
STA LSQPF+0578
ADM LSQPF+0192
STA LSQPF+0192
STA LSQPF+0193
TIX LSQPF+0189,4,1
CAL LSQPF+0190
ADD COMMON+1
STA LSQPF+0190
CAL COMMON+1
SUB LSQPF+0542
STO COMMON+1
ADM LSQPF+0190
STA LSQPF+0192
STA LSQPF+0193
TXI LSQPF+0170,1,-1
LXA LSQPF+0556,2
TXI LSQPF+0214,1,1
SXD COMMON+1,1
LXD COMMON+9,4
CAL COMMON+8
STD LSQPF+0220
CLS 0,4
STO 0,2
TNX LSQPF+0225,4,0
CAL LSQPF+0220

SUB LSQPF+0544
STD LSQPF+0220
TNX LSQPF+0228,2,1
TIX LSQPF+0218,1,1
LXD COMMON+1,1
TIX LSQPF+0213,2,1
CAL LSQPF+0251
STA COMMON+7
LXA LSQPF+0556,3
LXA LSQPF+0543,4
SXD LSQPF+0276,4
TIX LSQPF+0234,2,1
CLM
STD LSQPF+0268
CAL LSQPF+0251
STO LSQPF+0578
CAL COMMON+6
SUB LSQPF+0542
STO COMMON+6
ADM LSQPF+0578
STA LSQPF+0251
STA LSQPF+0578
CAL COMMON+6
SUB LSQPF+0545
STO COMMON+1
CLM
STD LSQPF+0256
LXA LSQPF+0552,4
LDQ 0,2
FMP 0,4
FAD 0,1
STO 0,1
TXI LSQPF+0255,2,1
TXI LSQPF+0256,4,1
TXL LSQPF+0250,4,0
SXD LSQPF+0262,4
CAL LSQPF+0256
ADD LSQPF+0544
STD LSQPF+0256
TXI LSQPF+0262,1,1
TNX LSQPF+0263,2,0
CAL COMMON+1
ADD LSQPF+0542
STO COMMON+1
SBM LSQPF+0251
STA LSQPF+0251
TXL LSQPF+0249,4,0
CAL LSQPF+0268
ADD LSQPF+0544
STD LSQPF+0268
CAL LSQPF+0276
ADD LSQPF+0544
STD LSQPF+0276
LXA LSQPF+0552,1
TNX LSQPF+0278,3,0
TXI LSQPF+0238,1,1

LXA LSQPF+0572,1	15 IN IR 1
CLA LSQPF+0552	
STO COMMON+35,1	
TIX LSQPF+0280,1,1	
LXA COMMON+3,1	P-1 IN I.R.1
CLA COMMON	X MAX
STO COMMON+35	
LDQ COMMON	X MAX
FMP 0,1	COMMON+20+14+P
STO 0,1	COMMON+2045+P
TIX LSQPF+0285,1,1	
CLA LSQPF+0551	D45
ADD LSQPF+0541	D45+P
STA LSQPF+0333	
TOV LSQPF+0579	
LXD LSQPF+0551,4	
CLA 2,4	
TMI LSQPF+0306	
LDQ LSQPF+0554	N
MPY LSQPF+0541	NP
LLS 53	
STD COMMON+2	NP IN DECR. OF DX
ARS 18	
ADD LSQPF+0322	C+P(P+5)/2-NP
STA LSQPF+0573	
CLA LSQPF+0573	
STO LSQPF+0401	
TRA LSQPF+0308	
CLA LSQPF+0574	
STO LSQPF+0401	
CLA LSQPF+0554	
ADD LSQPF+0575	
FAD LSQPF+0552	
TZE LSQPF+0579	
STO COMMON+9	NORMALIZED N
LXA LSQPF+0555,1	8 IN IR 1
CLA LSQPF+0552	
STO COMMON+18,1	
TIX LSQPF+0315,1,1	
TOV LSQPF+0318	
LXA LSQPF+0545,4	2 IN IR 4
LXA LSQPF+0556,1	P(P+5)/2 IN IR 1
LXD COMMON+8,2	P+1 IN IR 2
SXD LSQPF+0538,4	
CLA 0,1	C+P(P+5)/2
STO 0,2	COMMON+10+P+1
TNX LSQPF+0331,4,1	
TNX LSQPF+0326,2,1	
TIX LSQPF+0322,1,1	
LXD LSQPF+0539,1	
TNX LSQPF+0370,1,2	
LXD LSQPF+0538,4	
TXI LSQPF+0320,4,1	
SXD LSQPF+0539,1	
LXA LSQPF+0556,4	D45+P
TRA 0,2	LINEAR

TRA LSQPF+0367	QUADRATIC
TRA LSQPF+0341	CUBIC
TRA LSQPF+0345	QUARTIC
TRA LSQPF+0349	QUINTIC
TRA LSQPF+0353	6TH
TRA LSQPF+0357	7TH
TRA LSQPF+0362	
TSX LSQPF+0465,1	
TXI LSQPF+0343,4,-5	
TSX LSQPF+0508,1	
TRA LSQPF+0327	
TSX LSQPF+0459,1	
TXI LSQPF+0347,4,-10	
TSX LSQPF+0498,1	
TRA LSQPF+0327	
TSX LSQPF+0456,1	
TXI LSQPF+0351,4,-16	
TSX LSQPF+0489,1	
TRA LSQPF+0327	
TSX LSQPF+0447,1	
TXI LSQPF+0355,4,-23	
TSX LSQPF+0480,1	
TRA LSQPF+0327	
TSX LSQPF+0438,1	
TQO LSQPF+0359	
TXI LSQPF+0360,4,-31	
TSX LSQPF+0474,1	
TRA LSQPF+0327	
TSX LSQPF+0426,1	
TQO LSQPF+0364	
TXI LSQPF+0365,4,-40	
TSX LSQPF+0469,1	
TRA LSQPF+0327	
TXI LSQPF+0368,4,-1	
TSX LSQPF+0518,1	
TRA LSQPF+0327	
LXA LSQPF+0555,1	PUT ZERO IN
CLA LSQPF+0552	COMMON+10 THRU
STO COMMON+18,1	COMMON+17
TIX LSQPF+0372,1,1	
LXA LSQPF+0545,4	2 IN IR 4
LXA LSQPF+0556,1	$P(P+5)/2$
LXD COMMON+8,2	$P+1$ IN I.R. 2
SXD LSQPF+0539,4	
CLA 0,1	$C+P(P+5)/2$
STO 0,2	COMMON+10+P+1
TNX LSQPF+0387,4,1	
TNX LSQPF+0382,2,1	
TXI LSQPF+0378,1,-1	
LXD LSQPF+0540,1	
TNX LSQPF+0422,1,2	
LXD LSQPF+0539,4	
TXI LSQPF+0376,4,1	
SXD LSQPF+0540,1	
LXA LSQPF+0554,1	N IN IR 1
LXD COMMON+2,4	NP

CLA COMMON+17	
STO LSQPF+0547	
LXA LSQPF+0552,2	0 IN IR 2
LDQ LSQPF+0547	
FMP 0,1	
FAD COMMON+16,2	
STO LSQPF+0547	
TXI LSQPF+0398,2,1	
TXL LSQPF+0393,2,6	
CLA LSQPF+0547	
FSB 0,1	
STO 0,4	$C+P(P+5)/2+NP$
SSP	
FAD LSQPF+0548	
STO LSQPF+0548	SUM RESIDUES
TIX LSQPF+0406,4,1	
TIX LSQPF+0390,1,1	
SXD COMMON+2,4	
TOV LSQPF+0579	
TQO LSQPF+0410	
CLA LSQPF+0548	
FDP COMMON+9	
TQO LSQPF+0579	
TOV LSQPF+0414	
FMP LSQPF+0576	STANDARD DEVIATION
LXD LSQPF+0540,1	
TIX LSQPF+0417,1,1	
STO 0,1	
TOV LSQPF+0579	
CLA LSQPF+0552	
STO LSQPF+0548	
TRA LSQPF+0383	
LXD LSQPF+0551,4	
LXD LSQPF+0584,2	
LXD LSQPF+0585,1	
TRA 4,4	
LDQ COMMON+17	T7
FMP LSQPF+0557	64
STO COMMON+20	64 T7
LDQ COMMON+17	
FMP LSQPF+0558	112
STO COMMON+21	112 T7
LDQ COMMON+17	
FMP LSQPF+0559	56
STO COMMON+22	56 T7
LDQ COMMON+17	
FMP LSQPF+0560	7
STO COMMON+23	7 T7
LDQ COMMON+16	T6
FMP LSQPF+0561	32
STO COMMON+24	32 T6
LDQ COMMON+16	
FMP LSQPF+0562	
STO COMMON+25	48 T6
LDQ COMMON+16	
FMP LSQPF+0563	18

STO COMMON+26	18 T6
LDQ COMMON+15	
FMP LSQPF+0564	
STO COMMON+27	16 T5
LDQ COMMON+15	
FMP LSQPF+0565	20
STO COMMON+28	20 T5
LDQ COMMON+15	
FMP LSQPF+0566	5
STO COMMON+29	5 T5
LDQ COMMON+14	T4
FMP LSQPF+0567	8
STO COMMON+30	8 T4
LDQ COMMON+13	T3
FMP LSQPF+0568	4
STO COMMON+31	4 T3
LDQ COMMON+13	
FMP LSQPF+0569	3
STO COMMON+32	3 T3
LDQ COMMON+12	T2
FMP LSQPF+0570	2
STO COMMON+33	2 T2
TRA 1,1	
CLA COMMON+20	64 T7
FDP COMMON+41	X7
STQ 0,4	C+ P(P+S)/2 A7
TQO LSQPF+0579	
TXI LSQPF+0474,4,1	
CLA COMMON+24	32 T6
FDP COMMON+40	X6
STQ 0,4	A6
TQO LSQPF+0579	
TXI LSQPF+0479,4,1	
TOV LSQPF+0480	
CLA COMMON+27	16 T5
FSB COMMON+21	112 T7
TOV LSQPF+0579	
TQO LSQPF+0484	
FDP COMMON+39	X5
STQ 0,4	A5
TQO LSQPF+0579	
TXI LSQPF+0488,4,1	
TOV LSQPF+0489	
CLA COMMON+30	8 T4
FSB COMMON+25	48 T6
TOV LSQPF+0579	
TQO LSQPF+0493	
FDP COMMON+38	X4
STQ 0,4	A4
TQO LSQPF+0579	
TXI LSQPF+0497,4,1	
TOV LSQPF+0498	
CLA COMMON+31	4 T3
FSB COMMON+28	20 T5
FAD COMMON+22	56 T7
TOV LSQPF+0579	

TQO LSQPF+0503	
FDP COMMON+37	X3
STQ 0,4	A3
TQO LSQPF+0579	
TXI LSQPF+0507,4,1	
TOV LSQPF+0508	
CLA COMMON+33	2 T2
FSB COMMON+30	
FAD COMMON+26	8 T4
TOV LSQPF+0579	18 T6
TQO LSQPF+0513	
FDP COMMON+36	X2
STQ 0,4	A2
TQO LSQPF+0579	
TXI LSQPF+0517,4,1	
TOV LSQPF+0518	
CLA COMMON+11	T1
FSB COMMON+32	3 T3
FAD COMMON+29	5 T5
FSB COMMON+23	7 T7
TOV LSQPF+0579	
TQO LSQPF+0524	
FDP COMMON+35	X
STQ 0,4	
TQO LSQPF+0579	
TXI LSQPF+0528,4,1	
TOV LSQPF+0529	
CLA COMMON+10	T0
FSB COMMON+12	T2
FAD COMMON+14	T4
FSB COMMON+16	T6
STO 0,4	A0
TOV LSQPF+0579	
DCT	
TRA LSQPF+0579	
TRA 1,1	
PZE	A
PZE	B
PZE	C
PZE	P
PZE 1	
PZE 3	
PZE COMMON+20,0,1	
PZE 2	
COMMON+10	
PZE	YJ
PZE	SUM RESIDUES
PZE 6	
LSQPF+0334	2S COMP N IN DECR.
PZE	0
PZE 5	
	N
PZE 8	
	P(P+5)/2
DEC 64.0	

DEC 112.0
DEC 56.0
DEC 7.0
DEC 32.0
DEC 48.0
DEC 18.0
DEC 16.0
DEC 20.0
DEC 5.0
DEC 8.0
DEC 4.0
DEC 3.0
DEC 2.0
DEC 1.0
PZE 15
STO 0,4
NOP 0
OCT 233000000000
DEC 1.253314137

AII
BULK SOLUTION

CLA LSQPF+0571
TRA LSQPF+0582
CLS LSQPF+0571
LXD LSQPF+0551,4
TXI LSQPF+0423,4,1
PZE
PZE

LAS872	SXD	LAS872+00108,1,00000	SAVE C(XA)
	SXD	LAS872+00109,2,00000	SAVE C(XB)
	SXD	LAS872+00110,4,00000	SAVE C(XC)
	CLA	00003,4,00000	FWA3,FWA4 TO ACC
	STO	LAS872+00087,0,00000	ST IN CASQ OF NXN
	CLA	00005,4,00000	(K) AND (K)(K+1)
	STA	LAS872+00088,0,00000	ST K AS ADDR IN NXN
	ARS	00018,0,00000	(K)(K+1) TO ADDR
	PAX	00000,2,00000	(K)(K+1) TO XB
	ADD	LAS872+00087,0,00000	ADD FWA3
	STA	LAS872+00012,0,00000	(FWA3 +K(K+1)) ADDR
	PXD	00000,0,00000	CLEAR ACC
	STO	00000,2,00000	CLEAR MATRIX STORAGE
	TIX	LAS872+00012,2,00001	XX
	CLA	00002,4,00000	FWA1,FWA2 TO ACC
	ADD	LAS872+00088,0,00000	ADD K
	STA	LAS872+00103,0,00000	FWA1 + K ADDR
	STA	LAS872+00104,0,00000	XX
	ARS	00018,0,00000	FWA2 TO ACC ADDR
	ADD	LAS872+00088,0,00000	ADD K
	STA	LAS872+00051,0,00000	FWA2 + K AS ADDR
	STA	LAS872+00052,0,00000	XX
	STA	LAS872+00070,0,00000	XX
	STA	LAS872+00072,0,00000	XX
	CLA	00004,4,00000	FWA5, N TO ACC
	STA	LAS872+00030,0,00000	ST FN RT ADDR
	PDX	00000,2,00000	(N) TO XB
	CLA	LAS872+00088,0,00000	(K) TO ACC
	ADD	LAS872+00116,0,00000	LOC ADDR 1
	STA	LAS872+00108,0,00000	ST (K+1) AS ADDR
	TSX	00000,4,00000	TO FN ROUTINE
	STQ	COMMON+00004,0,00000	SAVE DEL Y IN 1.4
	TZE	LAS872+00054,0,00000	NO WEIGHT
	STO	COMMON+00001,0,00000	TAKE SQRT OF WEIGHT
	ANA	LAS872+00114,0,00000	AND MPY EACH
	LRS	00001,0,00000	PARTIAL BY RESULT
	ADD	COMMON+00001,0,00000	XX
	LRS	00001,0,00000	XX
	ADD	LAS872+00115,0,00000	XX
	LXA	LAS872+00003,4,00000	XX 3 TO XC
	STO	COMMON+00002,0,00000	XX
	CLA	COMMON+00001,0,00000	XX
	FDH	COMMON+00002,0,00000	XX
	STQ	COMMON+00003,0,00000	XX
	CLA	COMMON+00003,0,00000	XX
	FAD	COMMON+00002,0,00000	XX
	SUB	LAS872+00114,0,00000	XX
	TIX	LAS872+00040,4,00001	XX
	STO	COMMON+00003,0,00000	SQRT OF WEIGHT TO 1.3
	LXA	LAS872+00088,4,00000	(K) TO XC
	LDQ	COMMON+00003,0,00000	MPY PARTIALS BY SQRT
	FMP	00000,4,00000	OF WEIGHT AND ST
	STO	00000,4,00000	BACK
	TIX	LAS872+00050,4,00001	XX
	CLA	LAS872+00087,0,00000	FWA3
	SUB	LAS872+00116,0,00000	LOC ADDR 1
	STA	LAS872+00073,0,00000	ST FWA3-1 AS ADDR
	LXA	LAS872+00088,1,00000	(K) TO XA

PXD 00000,1,00000	C(XA) TO ACC DECR
ARS 00018,0,00000	C(XA) TO ACC ADDR
PAX 00000,4,00000	C(XA) TO XC
STO COMMON+00003,0,00000	C(XA) TO 1.3
CLA LAS872+00073,0,00000	FWA3-1 INITIAL
ADD LAS872+00108,0,00000	ADD K+1
STA LAS872+00073,0,00000	ST FWA3-1 + (K+1)
STA LAS872+00074,0,00000	INITIAL ADD K+1
STA LAS872+00082,0,00000	EACH TX ON XA
STA LAS872+00083,0,00000	XX
SUB COMMON+00003,0,00000	FORM EFFECTIVE ADDR
STA LAS872+00075,0,00000	ST AS ADDR
LDQ 00000,1,00000	PARTIAL TO MQ REG
STQ COMMON+00002,0,00000	SAVE PARTIAL IN 1.2
FMP 00000,4,00000	PRODUCT OF PARTIALS
FAD 00000,4,00000	FORM SUMS FOR MATRIX
STO 00000,4,00000	ELEMENTS
STO 00000,0,00000	ST SYMMETRIC ELEMENT
CLA LAS872+00075,0,00000	ADD K+1 TO ADDR FOR
ADD LAS872+00108,0,00000	NEXT SYMMETRIC
STA LAS872+00075,0,00000	ELEMENT
TIX LAS872+00070,4,00001	ACROSS FIRST ROW
LDQ COMMON+00004,0,00000	COMPUTE SUMS IN RIGHT
FMP COMMON+00002,0,00000	HAND COLUMN
FAD 00000,0,00000	XX
STO 00000,0,00000	XX
TIX LAS872+00058,1,00001	DELETE 1 COL, 1 ROW
TIX LAS872+00030,2,00001	FN ROUTINE, NEXT PT.
TSX LAS872+00118,4,00000	TO NXN ROUTINE
PZE 00000,0,00000	FWA3, FWA4
PZE 00000,0,00000	(K) IN ADDR
CLA LAS872+00087,0,00000	FWA4
ARS 00018,0,00000	FWA4 TO ACC ADDR
ADD LAS872+00088,0,00000	ADD K
STA LAS872+00102,0,00000	FWA4 + K
LXA LAS872+00014,4,00000	2 TO XC
CLA LAS872+00111,0,00000	FL PT 1
PSE 00115,4,00000	SELECT SCALE FACTOR
TRA LAS872+00098,0,00000	1, 1/2, 1/10 THRU
CLA LAS872+00114,4,00000	SENSE SWITCHES 1
TIX LAS872+00095,4,00001	AND 2
STO COMMON+00001,0,00000	ST SCALE FACTOR 1.1
LXA LAS872+00088,4,00000	(K) TO XC
LDQ COMMON+00001,0,00000	SCALE FACTOR TO MQ
FMP 00000,4,00000	FWAA + K
FAD 00000,4,00000	FWA1 + K
STO 00000,4,00000	ST CORRECTED A,S
TIX LAS872+00101,4,00001	XX
LXD LAS872+00110,4,00000	REPLACE C(XC)
TRA 00006,4,00000	NORMAL RETURN
HTR 00000,0,00000	C(XA) SAVED, K+1 ADDR
HTR 00000,0,00000	C(XB) SAVED
HTR 00000,0,00000	C(XC) SAVED
DEC 1.000000000000E00	
DEC 5.000000000000E-01	
DEC 1.000000000000E-01	

OCT 001000000000	
OCT 100400000000	
HTR 00001,0,00000	LOC ADDR 1
HTR 00002,0,00000	LOC ADDR 2
DCT 00000,0,00000	BEGIN N+N ROUTINE
NOP 00000,0,00000	TURN DIVIDE CK OFF
CLA 00002,4,00000	ST K IN 1.4 = N PRIME
STO COMMON+00004,0,00000	ADD 2
ADD LAS872+00117,0,00000	ST (K+2) IN 1.6
STO COMMON+00006,0,00000	SUB 1
SUB LAS872+00116,0,00000	ST (K+1) IN 1.5
STO COMMON+00005,0,00000	XX
PAX 00000,1,00000	(K+1) TO XA
SXD LAS872+00164,1,00000	C(XA) AS DECR
SXD LAS872+00193,1,00000	XX
SXD LAS872+00199,1,00000	XX
TNX LAS872+00131,1,00001	DECREASE XA BY 1
SXD LAS872+00216,1,00000	(K) AS DECR
SXD LAS872+00233,1,00000	XX
LDQ COMMON+00004,0,00000	(K)(K+1)=E
MPY COMMON+00005,0,00000	XX
LLS 00035,0,00000	SHIFT TO ACC ADDR
STO COMMON+00007,0,00000	NO. OF ELEMENTS 1.7
ADD 00001,4,00000	FWA3 + (K)(5+1)
STA LAS872+00163,0,00000	ST AS ADDR
STA LAS872+00165,0,00000	XX
STA LAS872+00175,0,00000	XX
STA LAS872+00177,0,00000	XX
STA LAS872+00182,0,00000	XX
STA LAS872+00194,0,00000	XX
STA LAS872+00197,0,00000	XX
STA LAS872+00198,0,00000	XX
STA LAS872+00222,0,00000	XX
STA LAS872+00227,0,00000	XX
SUB COMMON+00007,0,00000	SUB E
ADD COMMON+00005,0,00000	FWA3 + (5+1)
STA LAS872+00174,0,00000	ST FWA3 + (K+1) AS
STA LAS872+00176,0,00000	ADDR
STA LAS872+00181,0,00000	XX
STA LAS872+00183,0,00000	XX
STA LAS872+00195,0,00000	XX
ARS 00018,0,00000	FWA4 TO ACC ADDR
ADD COMMON+00004,0,00000	ADD K
STA LAS872+00223,0,00000	FWA4 + K AS ADDR
STA LAS872+00229,0,00000	XX
SXD COMMON+00003,4,00000	SAVE C(XC) IN 1.3
LXA COMMON+00007,1,00000	E TO XA
SXD COMMON+00000,1,00000	C(XA) TO 1.0 DECR
CLM 00000,0,00000	CLEAR ACC
ADM 00000,1,00000	+ABS 1ST ELEMENT
TNX LAS872+00169,1,00000	TEST +DECR XA
LDQ 00000,1,00000	NEXT ELEMENT TO MQ
LRS 00000,0,00000	SET MQ +
TLQ LAS872+00164,0,00000	COMPARE ACC AND MQ
TRA LAS872+00161,0,00000	XX
LXD COMMON+00000,2,00000	SET XB

CLA COMMON+00004,0,00000	N PRIME TO ACC
ADD LAS872+00117,0,00000	ADD 2
PAX 00000,1,00000	(N PRIME +2) TO XA
TNX LAS872+00179,1,00001	TEST, DECREASE XA
CLA 00000,1,00000	ELEMENT FROM 1ST ROW
LDQ 00000,2,00000	ELEMENT FROM 1TH ROW
STQ 00000,1,00000	INTO 1ST ROW
STO 00000,2,00000	INTO 1TH ROW
TIX LAS872+00173,2,00001	TEST, DECREASE XB
LXA COMMON+00004,2,00000	N PRIME TO XB
LXA COMMON+00007,1,00000	E TO XA
CLA 00000,2,00000	A(I,J)
FDP 00000,1,00000	DIV A(11)
STQ 00000,2,00000	INTO A(IJ)) PRIME
DCT 00000,0,00000	TEST DIVIDE CHECK
TRA LAS872+00234,0,00000	RETURN, SINGULAR SET
PXD 00000,2,00000	C(XB) TO ACC DECR
ARS 00018,0,00000	C(XB) TO ACC ADDR
ADD COMMON+00007,0,00000	ADD E
SUB COMMON+00006,0,00000	SUB (K+2)
SUB COMMON+00004,0,00000	SUB N PRIME
TMI LAS872+00215,0,00000	IS LAST ROW FINISHED
PAX 00000,4,00000	C(ACC ADDR) TO XC
TNX LAS872+00194,1,00000	DECREASE XA
LDQ 00000,1,00000	A(11)
FMP 00000,2,00000	TIMES A(IJ) PRIME
CHS 00000,0,00000	CHANGE SIGN
FAD 00000,4,00000	ADD A(IJ)
STO 00000,4,00000	INTO A(IJ) PRIME
TIX LAS872+00193,4,00000	TEST, DECREASE XC
TIX LAS872+00180,2,00001	TEST DECREASE XB
CLA COMMON+00004,0,00000	N PRIME
SUB LAS872+00116,0,00000	SUB 1
STO COMMON+00004,0,00000	(N PRIME -1) TO 1.4
CLA COMMON+00007,0,00000	E TO ACC
SUB COMMON+00006,0,00000	SUB (K+2)
STO COMMON+00007,0,00000	(E)-(K+2) TO 1.7
CLA LAS872+00174,0,00000	L(A11)+(K+1)
ADD COMMON+00005,0,00000	ADD (K+1)
STA LAS872+00174,0,00000	ST AS ADDR
STA LAS872+00176,0,00000	XX
STA LAS872+00181,0,00000	XX
STA LAS872+00183,0,00000	XX
STA LAS872+00195,0,00000	XX
TRA LAS872+00160,0,00000	DO NEXT ROW AND COL
LXA LAS872+00116,5,00000	1 TO XA AND XC
TXH LAS872+00236,4,00000	TEST END OF CALC
CLM 00000,0,00000	CLEAR ACC
STO COMMON+00000,0,00000	CLEAR 1.0
PXD 00000,4,00000	C(XC) TO ACC DECR.
PDX 00000,2,00000	C(XC) TO XB
TNX LAS872+00227,2,00001	TEST, DECREASE XB
LDQ 00000,1,00000	A(IJ)
FMP 00000,2,00000	TIMES BJ
FAD COMMON+00000,0,00000	FORM SUM IN 1.0
STO COMMON+00000,0,00000	XX

TXI LAS872+00221,1,00001	TEST, DECREASE XA
CLA 00000,1,00000	B1
FSB COMMON+00000,0,00000	-SUM A(I) B(J)
STO 00000,4,00000	INTO XI
TXI LAS872+00231,4,00001	INCREASE XC
SXD LAS872+00232,4,00000	C(XC) AS DECR.
TXI LAS872+00233,1,00000	INCREASE XA BY C(XC)
TXI LAS872+00216,1,00000	INCREASE XA BY K
LXD LAS872+00110,4,00000	SINGULAR SET RETURN
TXI LAS872+00237,4,00002	INCREASE XC BY 2
LXD COMMON+00003,4,00000	RECOVER XC NORMAL
LXD LAS872+00108,1,00000	RECOVER XA
LXD LAS872+00109,2,00000	RECOVER XB
TRA 00003,4,00000	

	SCOPE	SXD COMMON,2	
		SXD COMMON+1,4	
		WTV	
		LXA TEN,2	
		CLA 3,4	
		FSB 4,4	
		STO COMMON+2	X RANGE
		CLA 5,4	
		FSB 6,4	
		STO COMMON+3	Y RANGE
		CLA CONST	
		FDP COMMON+2	
		STQ COMMON+4	X INTERVAL
		CLA CONST	
		FDP COMMON+3	
		STQ COMMON+5	Y INTERVAL
		LDQ COMMON+4	
		FMP 4,4	
		CHS	
		STO COMMON+6	Y AXIS POSITION
		TZE **4	
		TMI **5	
		UFA FIX	
		ALS 18	
		STD YAXIS	
		CPY YAXIS	
		LDQ COMMON+5	
		FMP 6,4	
		CHS	
		STO COMMON+7	X AXIS POSITION
		TZE **3	
		TMI **4	
		UFA FIX	
		STA XAXIS	
		CPY XAXIS	
		CLA 1,4	
		ADD 2,4	
		STA XC	
		ARS 18	
		ADD 2,4	
		STA YC	
		CLA 2,4	
		PAX 0,4	
XC		LDQ **,4	X COORDINATE
		FMP COMMON+4	
		FAD COMMON+6	
		TMI CT	
		CAS CONST	
		TRA CT	
		NOP	
		UFS FIX	
		LRS 17	
		SLQ COMMON+3	X POSITION
YC		LDQ **,4	Y COORDINATE
		FMP COMMON+5	
		FAD COMMON+7	
		TMI CT	
		CAS CONST	

```
TRA CT
NOP
UFA FIX
STA COMMON+3          Y POSITION
CPY COMMON+3
CT  TIX *+4,2,1
    CPY XAXIS
    CPY YAXIS
    LXA TEN,2
    TIX XC,4,1
    LXD COMMON,2
    LXD COMMON+1,4
    TRA 7,4
ONE  PZE 1
TEN  PZE 50
CONST DEC 1024.
FIX  OCT 233000000000
    XAXIS MTW
    YAXIS MON
NA34.1 LIB
UABDC1 LIB
UASTH1 LIB
LAS820 LIB
LAS816 LIB
UADBC1 LIB
UACSH2 LIB
COMMON BSS 300
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