# SENSITIVITY OF MOIST AVAILABLE ENERGY TO INCREASE IN TEMPERATURE

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# SENSITIVITY OF MOIST AVAILABLE ENERGY TO INCREASE IN TEMPERATURE

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## ABSTRACT

Moist available energy is defined by Lorenz (1978) as:

"the amount by which potential plus internal (including latent) energy of a given atmospheric mass field exceeds that of a hypothetical field, which can be constructed from the given field by rearranging the atmospheric mass, under reversible dry-adiabatic and moist-adiabatic processes, to minimize the potential plus internal energy."

Lorenz evaluated the moist available energy (MAE), and the efficiency fields of heating and cooling (N), and evaporation and precipitation (N<sup>'</sup>), for a zonally averaged mass field using the graphical procedure he outlined.

In this thesis, the temperature of the mass field used by Lorenz is increased by 2% everywhere, while the relative humidity is held fixed. MAE, N, N' were then evaluated and the results compared to those of Lorenz. MAE was found to be very sensitive to temperature change. There was a 21% increase in MAE that was due largely to the availability of additional latent energy in the tropics. The generation efficiencies also experienced large increases in the tropics.

Thesis Supervisor: Edward N. Lorenz Title: Professor of Meteorology

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### I. INTRODUCTION

I.1 Energetics of the Atmosphere

Studying the various forms of energy in the atmosphere and their interactions is useful in helping us understand how the general circulation of the atmosphere is maintained. The strength of the general circulation can be measured in terms of kinetic energy (KE), the energy of motion. We can think of the atmosphere as being composed of parcels of equal mass, so minute that there is no spatial variation of thermodynamic variables within them. Each parcel possesses potential energy, due to its height above the earth's surface, and internal energy, thermal as well as latent (heat releasable with condensation of water vapor).

Potential and internal energy are the immediate sources of kinetic energy. It is convenient to think of potential energy and the thermal portion of internal energy as a single form of energy, namely the total potential energy (TPE). We would then treat release of latent energy as a diabatic (external) form of heating.

Not all of the TPE is available for conversion into KE. Available potential energy (APE) suggested by Margules (1903) and formulated by Lorenz (1955) gives a better estimate of the amount that can be converted into KE. Lorenz also defined an efficiency factor N which measured the fraction of external heating actually generating APE. APE is a useful tool in explaining how solar heating, the ultimate source of energy, maintains the general circulation.

Lorenz treated the atmosphere as being dry in deriving APE. Evaporation and condensation of water are actually internal processes.

If we include moist processes in defining our available energy, we will get a better understanding of the structure of the energy cycle. Lorenz (1978) defined moist available energy (MAE), the counterpart of APE for a moist atmosphere, and outlined a graphical procedure for evaluating it.

I.2 The Reference Field and Available Energy

The processes that convert internal plus potential energy (this sum integrated over the whole atmosphere is the state function enthalpy) into KE are reversible and adiabatic (there is no transfer of heat or mass outside the system). If we allow the mass to be redistributed under adiabatic flow, there will be a change in the total enthalpy (TPE in a dry atmosphere) of the atmosphere and an equal and opposite change in KE. If the mass is redistributed so that the atmosphere is in stable static equilibrium, total enthalpy has reached its minimum and KE its maximum value. This maximum gain in KE equals the amount of internal plus potential energy available for conversion into KE under the adiabatic and reversible assumptions. The minimum enthalpy value identifies the mass field's reference state.

The reference field must possess the following properties. First, isobaric surfaces must be horizontal. Otherwise we have horizontal pressure differences and acceleration of motion is possible. Second, the reference field must be in hydrostatic equilibrium. Hence, density must be horizontally stratified. If this were not true, vertical accelerations would be possible. Third, the vertical stratification must be stable. Parcels displaced in the vertical would otherwise accelerate and KE would increase.

A dry atmosphere possesses a reference state that satisfies the above properties. The potential temperature ( $\theta$ ), the temperature of a parcel of air if it descended dry adiabatically to a pressure (P) of 1000 (mb), would be conserved.  $\theta$  is a function of the two variables T and P. In the reference state, the properties listed above restrict  $\theta$ to be horizontally stratified and to increase with height.

APE can be defined with respect to this reference state. APE is equal to the TPE of the mass field minus the TPE of the reference state. It can be generated by the following heating (cooling) processes: radiation, conduction, and latent heat of condensation.

Latent heat makes a significant contribution to APE generation. In hurricanes, for example, latent heat release can be two orders of magnitude greater than the amount of energy needed to maintain KE against frictional drag (Charney, 1964). In mid-latitude cyclones, latent heat is important in their intensification process. Danard (1964) found latent heat release to be just as important an effect as dry air processes, in his numerical experiments.

Moist air processes, which include the release of latent heat, are reversible and adiabatic. We could give them the consideration they deserve by treating them as internal. We would, with our new reference state, get a better estimate of the amount of energy available in our moist atmosphere.

The reference state of a moist atmosphere is found by rearranging the mass field graphically under moist adiabatic as well as dry adiabatic processes. We can then use the reference state to graphically determine MAE and the efficiency fields of heating (cooling) and evaporation

(precipitation) in generating MAE. The procedures are discussed in detail in the next section.

Lorenz compared MAE to APE for the zonally averaged hemispheric mass field shown in Figure 1. He found the moist and dry reference fields to be nearly the same near the ground and the tropopause (top of diagram). The moist reference field was nearly 5 K warmer in the mid-troposphere. This is a direct effect of the moist processes. In the mid-troposphere, rearrangement of the mass field causes a large part of the atmosphere to be saturated in the reference state. There is latent heat release and the reference state of the moist atmosphere is warmer in this region.

MAE was greater than APE by approximately 21% in Lorenz's study. Latent heat is included as a form of internal energy in evaluating MAE, and its release makes a significant contribution.

There are two ways to generate MAE. First, as with APE, we have diabatic heating: radiation and conduction. Latent heating, however, is no longer diabatic. Second, evaporation of water vapor increases the internal energy and can generate MAE.

Lorenz evaluated the efficiency fields of heating (cooling) N and evaporation (precipitation) N<sup> $\cdot$ </sup>. The rate of generation can be found by coupling heating fields with N and evaporation fields with N<sup> $\cdot$ </sup>. Lorenz found generation of MAE by evaporation to be just as important as generation by heating.

### I.3 The Problem

The purpose of this work is to look at the sensitivity of MAE to changes in the atmosphere's temperature (T) field. Temperature of the

mass field used by Lorenz (1978) will be increased uniformly by 2%, while the relative humidity (r) will be held fixed. It can be shown that this would likewise increase APE by 2%. However, it is not obvious how MAE will change. We evaluate MAE and efficiency fields for our perturbed mass field and compare our results with Lorenz's. The procedures used in our evaluations are outlined in detail in Chapter II. The results are discussed in Chapter III. The conclusions reached are presented in Chapter IV.

#### II. PROCEDURE

We will now go through the graphical procedure, described by Lorenz, for finding the reference state for a moist atmosphere. We will then show how to use the reference state to evaluate moist available energy (MAE) and the efficiency of heating and cooling (N) and evaporation and precipitation (N<sup>-</sup>), in generating MAE.

II.1 Graphical Determination of Reference Field

The intial mass field used by Lorenz is shown in Figure 1. The ordinate, pressure (P), decreases upward. The abscissa,  $\chi$ , ranging from 0 to 1 represents fractional area of the earth's surface from equator to north pole.  $1 - \chi$  is equal to the sine of latitude.  $\chi = 0$  at the pole and  $\chi = 1.0$  at the equator.

The temperature and humidity fields are zonally averaged values. Figure 1 is an estimate of the normal distribution of temperature (T) and relative humidity r in the northern hemisphere winter. The mass field is bounded by P = 1000 mb, z = 0, everywhere at the surface, and T = 220 K, r = 0.55, everywhere at 200 mb. This portion of the atmosphere is assumed to be self-contained, with no mass flow across the equator or across the 200 mb surface aloft, in passing from the given state to the reference state.

The field is based on averages and is therefore different from any ins antaneous field that has occurred. In particular there are no clouds. It does, however, display features of a real global mass field. There is a poleward and upward temperature decrease. The air is conditionally unstable and has high moisture content at low levels in the



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Figure 1.

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tropics.

Lorenz evaluated MAE and the efficiency fields for Figure 1. We will compare our results, for the uniformly greater T field, with his. We will refer to Lorenz's work as case A and the results of this thesis as case B.

The perturbed mass field is shown in Figure 2. We have added an isothermal layer, a stratosphere, to our initial mass field. This was necessary to keep the mass field self-contained in finding its reference state. The T field was increased everywhere by 2% and therefore retains its original features. The relative humidity field is unchanged. We assumed r = 0.55 everywhere above 200 mb.

Keeping relative humidity constant instead of absolute water content better represents the field of water vapor in a warmer atmosphere. The amount of water vapor air can hold increases with temperature. The water source, namely the oceans, is large and at the surface. Water content is more dependent on temperature than relative humidity. The increased water content has made the air even more unstable near the equator.

If we were to determine the reference field numerically, we would use the set of thermodynamic equations for reversible adiabatic processes. We would minimize a characteristic state function, the total enthalpy of a column. This is the same as the TPE of a column. If we can minimize the enthalpy of the atmosphere, we have found the reference state.

In a graphical procedure we use a thermodynamic diagram. Equations for reversible adiabatic processes are replaced by state curves of parcels. We must define enough characteristic properties of the reference



Figure 2.

field so that by using the thermodynamic diagram, the reference state can be found.

The state curve defines the thermodynamic state of the parcel at any level in the atmosphere. Figure 3 is a portion of a thermodynamic diagram. The ordinate  $P^{K}$  decreases upward;  $\kappa$  is equal to the ratio of the gas constant to the specific heat at constant pressure, taken to be 2/7. The abscissa, T, increases to the right. We have plotted the state curve of a hypothetical parcel on the diagram.

In the lower portion of the curve, the parcel is unsaturated. As the parcel rises and cools, the saturation mixing ratio decreases. The water vapor starts to condense at the point T, P. Latent heat is released as the parcel rises further, and temperature decreases less rapidly. The dashed line is the constant saturation mixing ratio curve. that passes through T, P. There is a unique T, P for each state curve. Each parcel also has a unique condensation potential temperature  $(\theta_{c})$  and equivalent potential temperature  $(\theta_{e})$  as well.  $\theta_{c}$  is the potential temperature of the parcel before water starts to condense, its value is constant below T<sub>c</sub>, P<sub>c</sub>;  $\theta_c = 285.0$  for our parcel.  $\theta_e$  is the potential temperature associated with the moist adiabat after all the water has condensed;  $\theta_{a}$  = 299.3 for the curve shown. We see that  $\theta_{e}$  is always greater than  $\theta_c$ .  $\theta_c$  is dependent upon water vapor. A warm saturated parcel will release a large amount of latent heat, and has a correspondingly large  $\boldsymbol{\theta}_{\underline{\mbox{\scriptsize o}}}$  . Given temperature, pressure, and mixing ratio of a parcel, we can find its  $\theta_c$ ,  $\theta_c$ , and P using the thermodynamic diagram. This fully identifies the state curve of a parcel.



Figure 3.

The adiabatic chart gives us only an approximation of a state curve. Simplifications had to be made in constructing it to make it readable. Secondary effects of water vapor are ignored. Water vapor changes the density and heat capacity of the air. If these effects were included, there would be an infinite number of dry adiabats and moist adiabats through any point.

Lorenz calls two mass fields equivalent if each parcel can pass from its state in one field to its state in the other field by thermodynamically reversible adiabatic processes. The reference field is the equivalent field possessing the minimum TPE. Parcels retain their state curve and hence their values of  $\theta_c$ ,  $\theta_e$  in passing to the reference field. Therefore, if we can identify each parcel's reference pressure ( $P_r$ ), the reference field is completely described.

We found the  $\theta_c$ ,  $\theta_e$ ,  $P_c$  fields of Figure 2. The results are displayed in Figure 4. We might think of this chart as an alternate description of the mass field. If  $\theta_e$  decreases with height, the atmosphere is conditionally unstable, which means that a saturated parcel displaced in the vertical will be buoyant with respect to the surrounding air and its motion will grow. There is strong conditional instability near the equator. The maximum  $\theta_e$  of case A was 345 K at the equator. In case B,  $\theta_e$  equals 383 K at the equator. This is an 11% increase in  $\theta_e$  with a 2% increase in  $\theta_c$ . With this indicated large latent heat content, we could no longer confine air at the equator to below the 200 mb level in the reference state. We had to extend the atmosphere to 154 mb.

We were able to deduce three properties of the reference field in the introduction based on borizontal and vertical stability criteria.



Figure 4.

We were then able to conclude that if the atmosphere was dry  $\theta$  must be horizontally stratified and increase in the vertical. In a moist atmosphere, three variables (instead of two) must be specified to determine a parcel's thermodynamic state. The deduction that P and  $\alpha$  (inverse of density) must be horizontally stratified does not restrict  $\theta_c$  and  $\theta_e$ to be horizontally stratified everywhere. The state of a parcel is now dependent upon water content as well. We can deduce that instead, for stability,  $\theta_c$  must be horizontally stratified in unsaturated air, and  $\theta_e$  is horizontally stratified if the air is saturated. In the vertical, stability requires that  $\theta_c$  increase with height in unsaturated air and  $\theta_e$  increase with height in saturated air.

A reference pressure  $(P_r)$  curve in dry air is a  $\theta$  curve, meaning  $\theta$ is horizontally stratified. In moist air a  $P_r$  curve consists of a  $\theta_c$ curve in the dry portion and a  $\theta_e$  curve in the moist portion. The intersection of the two portions, if it exists, is along the curve  $P_r = P_c$ .

We will now show how  $P_r$ -curves are located graphically. For example, let us locate  $P_r = 800$  mb on Figure 4. We start by choosing a point along the curve  $P_c = 800$  mb and assume  $P_r = P_c$  at this point. Pick the point where  $\chi = 0.51$ . We find that  $\theta_c = 300$  K and  $\theta_e = 323$  K here. Then the remainder of the  $P_r = 800$  mb curve would consist of a portion of the curve  $\theta_c = 300$  K where the air is saturated,  $P_c < 800$ , and a portion of the curve  $\theta_e = 323$  K where the air is saturated,  $P_c > 800$ . That is the  $\theta_c$  curve upward from our point and the  $\theta_e$  curve downward from our point. By the stable stratification criteria,  $\theta_c$  and  $\theta_e$  increase upward, 8/10th of the area of Figure 4 (extended to P = 0) must lie above  $P_r = 800$  mb and 2/10th must lie below.



Figure 5.

Measurement (counting squares on graph paper) reveals the area to be small. We can make the area larger by choosing a point on the  $P_c$ curve farther to the right. By trial and error, we find  $P_r = 800$  mb intersects  $P_c = 800$  at  $\chi = 0.55$ . It consists of a portion of the curves  $\theta_c = 304$  K and  $\theta_c = 328$  K.

It is possible that  $P_r$  will not intersect  $P_c$  within the graph. We can still find the  $\theta_e$  portion of a  $P_r$ -curve by imagining that we have added a strip of negligible width to the right of  $\chi = 1.0$ . We will then add water vapor to this strip raising the  $P_c$  curve. We add as much water vapor as we need so that  $\theta_c$  intersects the proper  $\theta_e$ , such that the required fraction of area is below.

The  $P_r$ -curves for a representative number of levels are plotted in Figure 5. We have also displayed the curves  $P_r = P$  (solid) and  $P_r = P_c$ (dashed). The  $P_r = P$  curve separates air that rises, to the left, from air that sinks, to the right, to its reference state. The  $P_r = P_c$  curve separates unsaturated and saturated air in the same way. Note that the  $P_r = P_c$  curve is parallel to  $\chi = 1.0$  above P = 340 mb. It was necessary to imagine the narrow strip of mass to the right of  $\chi = 1.0$  to find  $P_r$ -curves above 400 mb. Also, just above  $P_r = 300$  all the air that rises is saturated, rising to its reference state from low levels in the tropics. The warm moist air will release a tremendous amount of latent heat in the process.

II.2 Determination of Moist Available Energy

We use our reference field to evaluate moist available energy. The method is illustrated in Figure 6. The ordinate is  $P^{\kappa}$ , the abscissa

 $(P/P_0)^{\kappa}$ . We defined  $\kappa$  earlier.  $P_0$  is the average surface pressure, in our case,  $P_0 = 1000$  mb.

The diagram is an equal area transformation of a P- $\alpha$  diagram. The specific volume,  $\alpha$ , is equal to the inverse of density. Dry adiabats are vertical. The light curve is a sounding through the reference field in case A. The heavy curve is a sounding through the reference field in case B, Figure 5. The two dashed curves are portions of the state curves for two parcels in Figure 2. The state curve to the right of the sounding follows a dry adiabat only up to P = 951 (mb) and rises moist adiabatic-ally from there, intersecting the reference sounding at P = 154 mb. If we extend the state curve of the parcel to the left of the sounding of the reference state, it descends along a dry adiabat and intersects the reference sounding at P = 870 mb.

As a parcel rises through a portion of its state curve, its specific enthalpy decreases by an amount proportional to the area on the chart (extended to  $\theta = 0$ ) to the left of the segment. The factor of proportionality is  $c_p (=1000 \text{ m}^2 \text{sec}^{-2} \text{K}^{-1})$ . The shaded area in the lower right corner of Figure 6 is of unit area in K and therefore has a specific enthalpy of  $10^3 \text{ J kg}^{-1}$  (joules per kilogram). The parcel on the left descends to its reference state and its specific enthalpy increases. The parcel on the right rises and therefore loses specific enthalpy. The total loss of enthalpy, equal to moist available energy, is obtained by summing up the areas for all parcels. This would consist of adding together a lot of estimated areas of equal and opposite signs. The error could be as large as the result.



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Figure 6.

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Lorenz noted that as many parcels pass upward across each pressure level as downward. We would therefore not change the final sum if we subtracted from each area to the left of the state curve the area to the left of the sounding. Since rising parcels are all to the right of the sounding and sinking parcels all to the left, we obtain positive (or zero) net area for each parcel. The net areas for the two selected parcels have been shaded in Figure 6. We find their specific moist available energy (Am) to be 4100 J kg<sup>-1</sup> for the descending parcel and 7700 J kg<sup>-1</sup> for the ascending one.

## **II.3** Determination of Efficiencies

Now we will describe the method used to evaluate efficiency factors. The efficiency of heating and cooling as well as the efficiency of evaporation and precipitation in generating MAE are evaluated. The procedure involves measurement of the state curve of a parcel before and after the addition of internal energy.

Figure 7, taken from Lorenz (1978), illustrates the procedure. It is an enlarged section of an adiabatic chart. The heavy solid curve is the reference sounding. The two points P correspond to the state of a parcel. The parcels were chosen so that  $P_c$  is between the parcel and its reference pressure. The two curves PCR and their extensions are state curves.

We will first examine the efficiency of heating and cooling. We add enough heat to change the initial state of the parcels to P<sup>2</sup>. The state curves are now P<sup>2</sup>C<sup>2</sup>R<sup>2</sup>. The condensation pressure level has been raised so that the old and new curves come close together and then diverge.



Figure 7.

The change in MAE is represented by the area PCRR CP.

We define an efficiency factor N as the ratio of the gain in moist available energy to the gain in enthalpy. The latter is proportional to the area PP' to the top of the atmosphere. This chart would have to be 3.44 times taller to extend to P = 0, since the ordinate does not decrease linearly but by the 2/7 power of P. N is equal to the ratio of the two areas as P' approaches P. It is important to make P' approach P to minimize the error. N is found to be + 0.043 for the right-hand parcel and -0.062 for the left.

For our case no parcels are saturated in their initial state. All desending parcels do so dry adiabatically. Warm moist parcels from the tropics have state curves that extend far in the vertical before reaching their reference state and will experience large changes in MAE with heating.

We can also find the effect of precipitation and evaporation using Figure 7. Let us evaporate water into parcels on the right and precipitate water from parcels on the left. Condensation then occurs at C<sup>-</sup> and the sounding is intersected at R<sup>-</sup>. The changes in MAE are represented by the areas CRR<sup>-C<sup>--</sup>, which is smaller than the area found for heating.</sup>

Lorenz defined a second efficiency factor N' as the ratio of the gain in MAE to the latent energy added. The latter is equal to the gain in enthalpy for the parcel on the right, and is proportional to the shaded area CC'' to the top of the atmosphere. N' for this example is + 0.20.

N' must be defined a little differently for precipitation since there is no enthalpy change. Lorenz defined N' for this case as the ratio of the (negative) gain in MAE to the latent energy which the precipitating

water possessed before it condensed. We can evaluate this graphically by noting that if following the precipitation, the parcel at P should be lowered to C', the desired amount of latent energy could approximately be added by evaporating as much as had precipitated, without changing the temperature. The condensation level would be lowered from C'' back to C', and the accompanying gain in enthalpy would be represented by the shaded area C''C' to the top of the atmosphere. We find N' = - 0.41 for our example.

For our case there are no clouds in the initial state and therefore no precipitation can take place. We will only have an evaporation efficiency field to concern ourselves with.

Some relations between N and N' that can be used in finding our efficiency fields can be deduced. First of all, if a parcel is nearly saturated, the points P and C nearly coincide and therefore N'  $\approx$  N. This turns out to be true near the equator in our field. Secondly, if C is close to R, evaporation is most inefficient. Thirdly, if an ascending parcel remains unsaturated in the reference state, C passes R and N' vanishes (N' = 0 along the curve  $P_r = P_r$ ). We will now discuss the quantitative results of a 2% increase in temperature of the mass field in Figure 1. In Figures 8 through 11, we present fields of moist available energy (MAE), efficiency of heating and cooling (N), and efficiency of evaporation (N<sup>'</sup>). We will discuss the figures and compare our results, case B, to Lorenz's results, case A.

When we compare the distribution of  $\theta_c$  and  $\theta_e$  fields for case B in Figure 3 to the fields of case A, we find that the most important changes are in  $\theta_e$  at low levels, near the equator. The gradient of  $\theta_e$ (change in  $\theta_e$  with distance) has nearly doubled that of case A in this region. The effect that this has on the reference state can be seen in Figure 5. The parcels with high  $\theta_e$  values (large water content) are farther from the reference state. Besides releasing more latent heat than the parcels in case A, they also experience increased ascent, further increasing their MAE.

The reference soundings for cases A and B are plotted in Figure 6. The light curve is the sounding for case A and the heavy curve is the case B sounding. We see that they diverge at the surface. They are then nearly parallel in the lower troposphere. The soundings diverge again in the mid-troposphere. At P = 400, the difference between the soundings is 9 K, a 3% change from case A. The greatest difference takes place above 230 mb. At 200 mb, the soundings are separated by 13.5 K, or a 3.8% change from case A.

The MAE of two parcels for both case A and case B is graphically evaluated in Figure 6. The upper parcel is located at P = 400,  $\chi$  = 0.0

in the mass field. The lower parcel has coordinates  $P = 1000, \chi = 1.0$ . The state curves for case A are dotted and for case B are dashed. The two points are near the center of maximum MAE for the descending and ascending portion of the atmosphere.

The 2% T increase shifted the upper parcel's state curve 6 K to the right. If the sounding had not also changed the MAE released would have decreased at this point for case B. The reference sounding does change, moving to the right, increasing the area and allowing the parcel to descend further. The increase is proportional to the hatched area between the two soundings. The net change is a 370 J kg<sup>-1</sup> increase in MAE for the parcel.

The dotted line to the right of the case A sounding is the initial state curve of the equatorial parcel. We see that for ascending parcels the shift in the reference sounding decreases the MAE released. The 2% increase in  $\theta$  with constant relative humidity causes a striking difference in the state curve. MAE increases by 3600 J kg<sup>-1</sup>, largely due to additional latent heat release. The MAE increase is nearly 10 times that of the parcel that descended to its reference state.

The MAE distribution for case B is plotted in Figure 8. To the left of the O contour line parcels release MAE by descending to their reference state. The contours are very similar in shape and value to the MAE fields in case A. In descent, water does not condense. Without the effect of latent heat release, changes in MAE are not very pronounced There is a big difference to the right of the O contour line, where parcels ascend to reach their reference state. The greatest change occurs near the equator. The warm moist parcels that ascend far into the

![](_page_29_Figure_0.jpeg)

Figure 8.

troposphere, and even to the stratosphere, are located here. Nearly all water is condensed in reaching the reference state, releasing tremendous amounts of latent heat. Water content of the parcel at the equator increased from 18.5 g/kg to 26 g/kg; a 41% increase with a 2% increase in T. The MAE values have nearly doubled at low latitudes.

We produced the solid curves in Figure 9 by counting squares in Figure 8. The ordinate  $A_m$  is a label on a curve in Figure 8. The abscissa represents fraction of total area to the left of a curve. Above 154 mb we assume that  $A_m$  vanishes. The dashed curve is the result for case A.

We can see that the curves are nearly the same on the left side of the diagram. The big difference in the distribution of MAE near the equator is well illustrated on the right side of the diagram. Any change in MAE will be due to the large increase in a small area near the equator.

The average heights of the solid and dashed curves are proportional to total MAE. We find the average height by counting squares beneath the curve, then we divide by the length of the abscissa, and multiply our results by a proportionality factor (25 J kg<sup>-1</sup> sq<sup>-1</sup>). The total MAE for case B is 496 J kg<sup>-1</sup> and for case A is 408 J kg<sup>-1</sup>; a 21% increase occurs in MAE.

The 21% increase in MAE does not guarantee a large increase in KE. A look at the new efficiency fields would give us more insight into changes we might expect in KE.

The distribution of the heating efficiency N for case B is presented in Figure 10. There is little change in N compared to case A to the left side of the O contour line, just as we found little change in MAE in this region. The maximum negative N is located, as in case A, at the pole

![](_page_31_Figure_0.jpeg)

Figure 9.

![](_page_32_Figure_0.jpeg)

Figure 10.

![](_page_33_Figure_0.jpeg)

Figure 11.

between 400 mb and 300 mb. The maximum value, -0.27, is also nearly the same. Heating at low latitudes is much more efficient, in case B, at generating MAE. The maximum value, at the equatorial surface, + 0.52, is more than twice the value of N here for case A.

The efficiency field of evaporation N<sup> $\circ$ </sup> for case B is plotted in Figure 11. The initial field had no clouds and therefore there is no efficiency of precipitation. We find the same large values of N<sup> $\circ$ </sup> as we found for N at low latitudes. The values of N<sup> $\circ$ </sup> approach N as we near the equator. The maximum N is + 0.47. The N<sup> $\circ$ </sup> field for case B has more than double the values for case A.

#### IV. CONCLUSION

We find a 21% increase in MAE accompanies a 2% increase of a zonally averaged northern hemisphere temperature field. APE of this mass field would have increased by approximately 2%. We do not consider latent heat as a form of internal energy in evaluating APE.

In defining the new mass field, case B, we assumed that the relative humidity field remained the same. The warmer atmosphere of case B had a greater absolute water content. The additional latent heat released by parcels at low latitudes in going to the reference state was responsible for the increase in MAE.

MAE is greater than APE for both case A and case B. The conversion rate of available energy, whether APE or MAE, must remain the same. The atmosphere is therefore less efficient at converting MAE to KE. In case B, the top of the mass involved in adiabatic redistribution processes increased from 200 mb to the 154 mb level. Increased mass was not responsible for the increased MAE. In both cases we are evaluating MAE of the whole atmosphere. In case A, MAE = 0 above 200 mb, while for case B, there is no MAE above 154 mb.

We also evaluated the efficiency of heating and cooling, evaporation and precipitation, in generating MAE. We found a large increase (more than double) in the atmospheric ability to generate MAE from heat and evaporation for case B in comparison to case A. If we assume that the atmosphere in case B is in equilibrium, generation of MAE must equal conversion of MAE to KE, which equals dissipation of KE by friction. The heat and evaporation fields of case B must be greater than those of case A for the mean temperature of the mass field to be greater. The

atmosphere in case B is as efficient, or more efficient (sometimes more than double) than the atmosphere of case A. The circulation of the mass field in case B is therefore likely to be more intense than that of case A, as indicated by the big increase in efficiency, and accompanying increase in heating and evaporation.

We would like to mention that an attempt was made to devise a numerical scheme to find the reference state of a moist atmosphere. Given the thermodynamic state of an array of parcels in the mass field, we assumed that they possessed a unique set of reference levels. We would in essence stack the parcels in one column and try to rearrange the parcels so that the enthalpy of the column was absolutely miminized. Assuming we had a representative number of points in the mass field, we would get a reasonable estimate of the reference sounding. However, we were not able to place enough restrictions on the parcels to eliminate a reasonable portion of the large number of possible arrangements.

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