A Study of the Magneto-Telluric Method of Prospecting

by

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Submitted in partial fulfillment of the requirements for the degree of Bachelor of Science and Master of Science in Geology and Geophysics at the

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ABSTRACT

The Magneto-Telluric Method of Prospecting

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Submitted to the Department of Geology and Geophysics on May 26, 1958 in partial fulfillment of the requirements for the degree of Bachelor of Science and Master of Science.

The purpose of this thesis was to devise a scheme of detecting and recording the horizontal electric and magnetic components of the earth's magneto-telluric field. This was accomplished using porous pot electrodes and an inductance coil as detection instruments. An electrical system then transferred the signal to Sanborn recording tape. A spectrum analysis over the region of investigation, which was .02-2 cycles, revealed that for both E and H the major contribution to the signal was due to the longer period variations. The outline of an interpretation scheme based on Cagniard's plane wave solution for the MT field is included together with tape recordings indicating possible correlations.

Thesis Supervisor: Theodore R. Madden

Title: Lecturer in Geophysics

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Statement of Problem

The problem to be dealt with in this thesis is the laboratory instrumentation and field test of a system of equipment designed to accurately record the variations of the earth's magneto-telluric field. This work is largely a continuation of that initiated by Cunningham in 1957. The major problem encountered was that of detecting the magnetic field variations, and a detailed description of the solution to this problem will occupy the major portion of this paper. In addition field records will be submitted in conjunction with a discussion of their significance.

The purpose behind the development of this system is to afford a means of investigating the electrical and structural properties of the region in the earth penetrated by frequencies in the range .02 - 2 c.p.s. Cagniard (1953) has given the mathematical solution to the problem for the two and three layered cases in one-dimensional geometry, and Neves (1957) has formulated a finite difference approach to the problem in arbitrary two-dimensional geometries.

The MT system devised here has the advantage over such electrical methods as applied field techniques of eliminating a current source, flexibility in the range of data which can be simultaneously recorded, and greater possibilities of interpretation, both quantitatively and qualitatively.

Theory

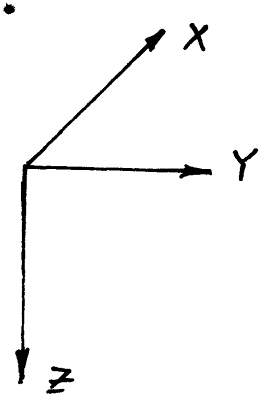
The basic theory involves measuring the horizontal components of the earth's electric and magnetic fields at the surface and comparing them. The ratio of the two vectors determines the conductivity of the earth for a particular frequency, assuming we can make a good estimate of the magnetic permeability. These horizontal field components are damped out exponentially with depth at a rate which will be shown to be proportional to the $(\text{frequency})^{\frac{1}{2}}$, so that the longer period variations will give us information at greater depths. The field procedure will involve simultaneous recording of both electric and magnetic fields over a wide frequency band which can be subsequently filtered in the laboratory to compare narrow band or individual frequency components.

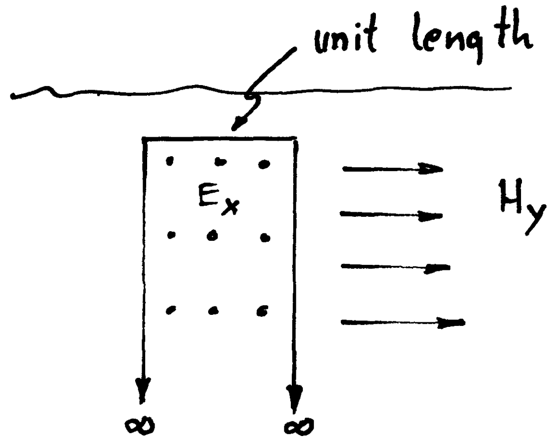
The MT method takes advantage of extensive current sheets which propagate through the conducting layers of the earth. The source electrodes for such phenomena are effectively at infinity, and so this allows us to overcome the problem of large finite electrode separations necessary for deep penetration in applied field work. These currents are caused by the rotation of the earth through the magnetic field of ionospheric currents, magnetic storms, etc. Currents due to local sources are superimposed on these large scale effects, but, according to Cagniard, this has no distorting effect on MT interpretation.

Due to the large scale nature of the source, the

MT field, which is, in fact, an electromagnetic phenomena, may be treated mathematically according to the theory of plane waves. Neves (1957) has pointed out that the refracted waves originating at the earth's surface, due to the large difference in the propagation constants of the air and ground, all travel vertically downwards regardless of the angle of incidence.

Following the basic method of Cagniard, but keeping to MKS units, we consider a unit cross section of earth extending to essentially infinite depth with a coordinate system defined such that positive "Z" is in the downward direction, positive "X" is into the paper, and positive "Y" is to the right.





Since what we plan to measure at the surface are perpendicular tangential components of E and H, these can be designated as E_x and H_y. The plane wave representation for E is given by: $\vec{E} = \vec{a}_x E_0 e^{ik\cdot r} e^{-i\omega t}$

As has already been pointed out, the direction of propagation r is confined to Z, and the propagation constant $k = (\epsilon\mu\omega^2 + i\mu\sigma\omega)^{1/2}$. We may neglect the real part of K, since typical values of the constants are $\sigma = 1/200$, $\mu = 1.26 \times 10^{-6}$, and $\epsilon = 4 \times 10^{-11}$

(all in MKS), bearing in mind that we shall be dealing with frequencies below 10 cycles.

Using the Ampere Circuital Law for the closed loop above:

$$\oint H \cdot ds = \int J_n \, d\vec{S} = i$$

where J = σE, and H damps out with depth, we have:

$$\oint H \cdot ds = H_y = \int_0^\infty J_x \, dz = \int_0^\infty \sigma E_x \, dz$$

Now: $E_x = \bar{a}_x E_o \, e^{i\sqrt{i\mu\sigma\omega}\, z} \, e^{-i\omega t}$

$$i\sqrt{i} = i^{3/2} = e^{3\pi/4} = \cos 3\pi/4 + i \sin 3\pi/4$$

$$= -\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}$$

Hence: $E_x = \bar{a}_x E_o \, e^{-\sqrt{\mu\sigma\omega/2}\, z} \, e^{i(-\omega t + \sqrt{\frac{\mu\sigma\omega}{2}}\, z)}$

Then: $H_y = \int_0^\infty \sigma E_x \, dz$

$$= \sigma \int_0^\infty E_o \, e^{-i\omega t} \, e^{(-1+i)\sqrt{\frac{\mu\sigma\omega}{2}}\, z} \, dz$$

$$= \bar{a}_y \frac{\sigma E_o e^{-i\omega t}}{(i-1)} \sqrt{\frac{2}{\mu\sigma\omega}} \; e^{(i-1)\sqrt{\frac{\mu\sigma\omega}{2}}\, z}\Bigg]_0^\infty$$

$$\left.\frac{E_x}{H_y}\right|_{z=0} = -\frac{E_o e^{-i\omega t}(i-1)}{\sigma E_o e^{-i\omega t}} \sqrt{\frac{\mu\sigma\omega}{2}} \; \frac{e^{i(\alpha+\pi/2)}}{e^{i\alpha}}$$

where α = any angle.

$$\left.\frac{E_x}{H_y}\right|_{z=0} = (i-1)\sqrt{\frac{\mu\omega}{2\sigma}} \, e^{i\pi/2} = \sqrt{\frac{\mu\omega}{\sigma}} \, e^{i\pi/4}$$

measuring positive angles counterclockwise.

The "skin depth", or "depth of penetration" is expressed by the exponent of the vertical damping term, namely, $p = \sqrt{2/\mu\sigma\omega}$ Thus, when the vertical depth is np the amplitude of the field vector has dropped off by a factor of $1/e^n$ of its initial value at the surface.

This fact is used to estimate the depth of material sampled by a particular frequency.

The above solution for E/H is for two semi-infinite, plane boundary, homogeneous media. The general magneto-telluric method involves measurement of amplitudes and relative phase angles of E and H. It can readily be seen that this ratio is solely a function of the electrical properties of the ground to be sampled and the frequency of the sampling wave. Thus for a given point on the surface of the earth the E/H ratio should be constant with respect to time for any particular frequency, since any magnetic or electrical disturbances in the atmosphere or the interior of the earth would induce such changes in both E and H as to keep their ratio constant. In principle, judging from the average values of the electrical properties of the earth, the range of frequencies of interest in this paper should allow us to examine depths as great as 400 km. in the earth.

A further point to be noted, as proved by Neves (1957), is that for an electric or magnetic polarized plane wave, polarization indicating that either E or H points in the direction of the strike in two dimensional structures, the surface E or H field is constant even over regions of changing conductivity. This is a point to be seriously considered in interpretation.

For the one-dimensional layered problem treated by Cagniard data at a single observation point on the surface is sufficient for interpretation. In the two-dimensional structures treated by Neves, however, a line profile of data, preferably perpendicular to the strike line, is necessary for interpretation. With this information a vertical profile map of apparent resistivities may be constructed in a manner analagous to applied field profile maps, in this case using the skin depth as the depth of sampling.

In the actual field measurements to be carried out it will be important to determine how much of the electric field which is recorded is actually due to telluric currents, and how much is merely instrument pickup induced by the power generating device to be used. The same problem will be encountered as far as the magnetic recording is concerned, and every precaution must be taken in the field work to isolate the E and H field detection devices from sources capable of producing fields in addition to the natural fields we wish to detect. In other words, the motor generator unit, for instance, which shall be used as a power supply, will produce a field which we may consider as non-magneto-telluric, and this will tend to obscure the information of interest. Further, there may be natural phenomena such as streaming potentials which will give a detectable electric field, but which may have no associated magnetic field.

Design of Field Equipment

I. The design of suitable field equipment may be treated in three stages: a detection device to convert the Earth field into an electrical signal; possible amplification of this signal; and the recording of the electrical signal in some manner which lends itself to a meaningful interpretation, whether this be qualitative or quantitative. Detecting the electric field is accomplished by means of measuring an effective voltage induced in the ground due to the passage of an electric field through a media of finite conductivity. Contact with the ground is obtained by means of porous pots containing a copper sulphate solution. Copper electrodes immersed in the sulphate solution connect to a wire strung between two pots. The voltage measured between the two pots is related directly to the E field by the equation $\int E \cdot ds = V$ when the surface field is uniform and the connecting wire lies on the ground so that, considering the ground-wire system as a closed loop, all the potential which occurs on a straight line between the two pots will appear directly across the voltmeter. The amplification problem in the case of the electric field is virtually non-existent due to the magnitude of the field and the nature of the parameter to be measured. Since the voltage is linearly proportional to the pot separation, the output signal can be increased merely by increasing this distance. The electrical signal is eventually transferred to Sanborn tape as a permanent record. Typical values are on the order of millivolts for 1000 meter pot separations.

A more challenging problem presents itself in the accurate detection of the horizontal component of the magnetic field. The variations which occur are of extremely small amplitude, so the problem was to design a rather compact coil system which would accurately detect the field. The standard method of detection is to measure the output voltage of a coil which intercepts the field lines, since time variations of the field induce a voltage in the coil.

Using Faraday's Law: $|V| = NA \frac{dB}{dt}$

where V = voltage,

N = total number of turns

A = cross-sectional area of coil,

and $\vec{B} = C e^{-i\omega t} e^{ik \cdot r}$

then $\frac{dB}{dt} = -i\omega \vec{B}$ where $\vec{B} = \mu \vec{H}$

and H is the measurement we are actually interested in.

Thus: $|V| = NA\left(-i\omega\mu H\right)$

The physical parameters which can be varied in the coil design are N and A. In order to get an order of magnitude to estimate what the product NA should be, we use

$$NA = -V / i\mu\omega H$$

To obtain an order of magnitude for H, we use $|H| = |E| \sqrt{\sigma/\mu\omega}$

Using some sample data for the E Field observed by Cunningham, over a region where $\rho = 2000$ ohm-meters, the electric potential was 3 millivolts for a 1000 meter spread. The above equation then becomes $|H| = \left(3 \times 10^{-3}\right) 10^{-3} \sqrt{\frac{10^{7}}{2000 \times 4\pi \times 2\pi f}}$

$\mu_0 = 4\pi \times 10^{-7}$ f = frequency in cps.

$$|NA| = -\left| \frac{V}{\mu\omega} \times \frac{\sqrt{1.6\times10^5 f}}{3\times10^{-6}\sqrt{10^7}} \right|$$

$$\simeq 4\times10^{10}\, V, \text{ using smallest } f = .02 \text{ cps.}$$

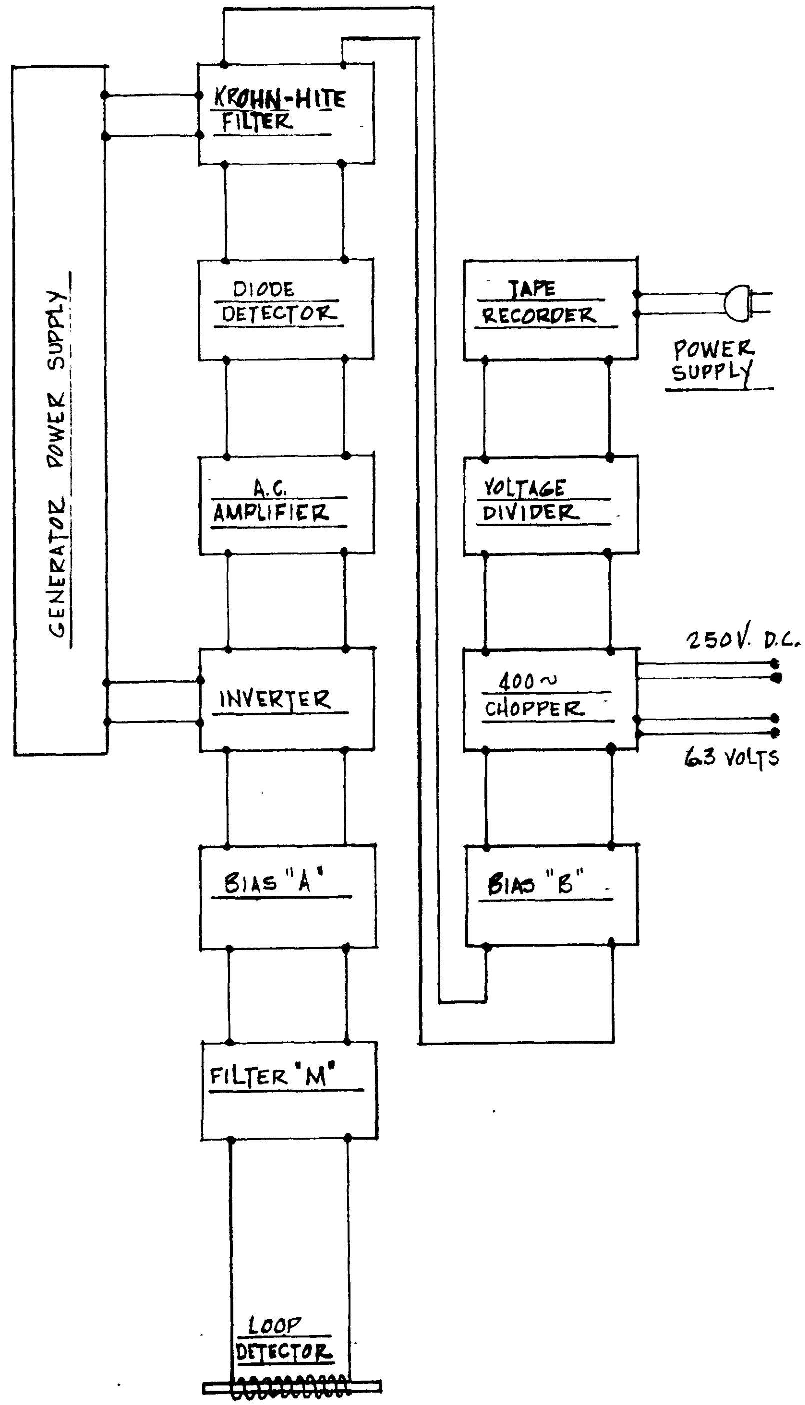
We want to maximize NA, and about the smallest voltage we can deal with is on the order of 2 or 3 x 10^{-4} volts. This means NA must be around 10^7. This calculation was made on the basis of an air coil, since μ_0 was used. The air coil used by Cunningham had an A = 1, and N = 5,000. This only had an NA = 5,000 instead of the necessary order of $\sim 10^7$.

The only parameter which can be varied to increase NA is μ, so this was the next point of investigation. There are various types of high MU-metal cores which can be used to increase the effective B field inside a coil. Investigation of the shape of the core to be used led to the conclusion that maximum efficiency could be attained through use of a long, thin rod. Although the metal processing determines the initial permeability of the rod, the shape factor determines the effective permeability which will increase the flux lines through the core. Sommerfeld has solved the magnetostatic boundary value problem for the relationship between the magnetic field strength inside and outside an ellipsoid of revolution. In the limiting case where the long axis becomes very much greater than the cross-sectional axes, the ellipsoid approximates a thin rod. The solution expresses the following relationship:

$$\mu_e = \mu / 1 + (\mu - 1)\left(\frac{1-\epsilon^2}{\epsilon^3}\right)\left[\frac{1}{2}\log\frac{1+\epsilon}{1-\epsilon} - \epsilon\right]$$

where: μ_e = effective permeability
μ = initial permeability
$\epsilon = c/a = \sqrt{a^2-b^2}/a$
a = length of long axis
b = length of cross-sectional axes.

There are many limitations which influenced the final choice of the type of rod to be used. From Sommerfeld's equation it is apparent that the greater the ratio of length to cross-sectional width of the rod, the greater advantage may be taken of the high initial permeability of the rod. This means that for a given practical upper limit to the rod length, decreasing the cross-sectional area will increase the effective permeability and thus tend to increase the effective coil area. This is counteracted by the fact that we are actually decreasing the coil area to employ this advantage in the first place. Therefore some compromise area had to be arrived at which would give the maximum effective area, and thus the maximum output signal. The rod which was finally used had an effective area of about 1 with an actual area of 3.87×10^{-4} (meters)^2, providing a small area on which to wind the inductance coils.



BLOCK DIAGRAM OF MAGNETIC FIELD RECORDING CIRCUIT

II Coil Design

The final coils were wound on individual plastic spools which were designed to be slipped over the permalloy rod and connected in series to give a manageable set-up readily adaptable to field use. The amount of wire on an individual spool was limited by the resonant characteristics of the coil. Besides the inductance normally associated with a coil of wire there is also a capacitive effect due to the fact that overlapping layers of wire are at slightly different potentials which depend upon the resistive nature of the wire. The coil may be represented equivalently by an RLC circuit which has an associated resonant frequency. The coil should be designed such that the resonant point lies outside the frequency spectrum of interest since the amplitude and phase relationships undergo rapid changes in the frequency range close to resonance.

III Filter Design for Magnetic Recording Circuit

The main function of filter "M" is to attenuate 60 cycle pickup. Under normal field operating conditions the amplitude of the 60 cycle signal is on the order of 5×10^3 greater than the low frequency signal of interest, thus obliterating the effect of the signal. The type of filter to be used is limited by the input and output impedances necessary to comply with the rest of the circuit. The input impedance must be greater than 1 Meg., since the equivalent source circuit has about 1 Meg. in series with it, so that if this faces an equivalent impedance, one half of the generated signal is already lost by the remainder of the circuit. The output impedance of the filter should not greatly exceed 10 Meg, since this is the upper limit to the load which can be hung across the input to the inverter. There are inductive filters which can be designed to have a resistive peak at one particular frequency, e.g. 60 cycles, and pass all other frequencies with negligible attenuation. Unfortunately this type of filter has a very low input impedance due to the inductive effect at low frequencies. An RC filter was finally used, the number of stages being limited by the small impedance range between the input and output.

IV Bias and Inverter

Bias "A", as designed by Cunningham, is used to keep all signal variations above the zero d.c. level before going into the inverter. If this is not done, the demodulated wave at the output of the amplifier will not reproduce the original low frequency wave envelope.

The sensitive inverter acts as a low noise chopper-amplifier which has an output chopped at 60 cycles and amplified by a factor of 100.

V A.C. Amplifier

The 60 cycle modulated output of the inverter is then fed into a battery operated hp a.c. amplifier to increase the signal by a greater amount. A glance at the block diagram for the magnetic recording circuit seems to indicate the rather purposeless steps gone through to amplify a small signal initially on the order of tenths of millivolts only to cut it down in the final stage to the order of millivolts, which is determined by the amount of signal the tape recorder can handle. The reasoning behing this is two-fold. One is that we must have a signal which is much greater than the noise levels associated with the Krohn-Hite filter and the 400 cycle chopper. Also the diodes which are used in the detection circuits do not operate at signals on the order of millivolts, but will, however, function properly when the signal is on the order of volts.

Normal field operating point for this amplifier was -20 db. The amplifier output is then passed through a detection circuit to obtain the original low frequency wave form.

VI Krohn-Hite Filter

The purpose of using this filter in the field recording is so that narrow frequency bands may be recorded individually over the entire range from .02 to 2 cycles providing an electrical method of Fourier analysis in the frequency domain. This analysis is necessary for interpretation since we want to compare amplitude and phase relationships of individual frequency components. Since there was only one Krohn-Hite filter available, it was used in the field recording of the magnetic signal; the electric signal was recorded wide band and then filtered on the playback between the magnetic tape and the Sanborn tape. In this manner both signals pass through the same narrow band, even though this occurs at different stages of the circuit. The disadvantage of recording a wide band signal on the magnetic tape is the fact that some frequency components may have extremely large amplitudes in relation to others, and thus they will completely dominate the total signal. Due to the limited amplitude which may be put onto magnetic tape, the effect of the small amplitude frequencies will be practically undetectable on playback. If, however, a narrow band recording is made so that frequencies of widely differing amplitudes are essentially isolated from each other, then by changing the amplifier setting they can all be recorded on the tape so as to take full advantage of the saturation limit of the tape.

VII Final Stages

After the Krohn-Hite filter the signal, now low frequency with an amplitude adjusted to be on the order of 3 volts RMS is biased and chopped at 400 cycles. This is because the amplifier in the tape recorder playback has very poor response at low frequencies. The voltage divider is designed to cut the signal down to about 4 millivolts RMS, above which the magnetic tape saturates.

Low Frequency Amplifier Design

This amplifier was designed to be a compact, low-cost unit which would replace four stages of the magnetic recording circuit, namely bias "A", the inverter, the a.c. amplifier, and the detector. The basic design problem is that of finding a good coupling circuit between amplifier stages. The practical limitations on the maximum size of the coupling capacitor force the grid resistance to a very high value. Any leakage currents associated with the grid of the tube or else with one of the capacitors flow across this high grid resistance and create a noise voltage which appears at the input to the tube. This will be amplified by all the succeeding stages and the final output will have a signal to noise ratio much less than one. The amplifier in Fig. A had an overall gain of 700; but while this was stable at one cycle, at .1 cycle the drift was on the order of ten times the signal amplitude.

The circuit which appears in Fig. B is essentially the same as A except that the two middle stages employ the trick of applying the tube input voltage between the grid and a tap-in point on the cathode resistor instead of between grid and ground. This has the effect of increasing the grid resistance as seen by the preceding stage and simultaneously decreasing the total gain. This allows us to decrease the actual grid resistance, which will reduce the noise generated by leakage currents, and still maintain a high effective input impedance so that most of the signal won't be lost in the coupling network. The amount by which the effective input impedance increases depends upon the ratio of the cathode resistance above and below the tap-in point. A limit to how much stabilization may be achieved in this manner is set by the lowest value of loop gain which can be tolerated.

The cathode resistor values which appear in Fig. B left the amplifier with a total gain of only 100, but a reasonable degree of stability had been reached. The output signal throughout the spectrum was superposed on a drift variation with a period of about 3 minutes and an amplitude about twice as great as the signal. Since this period is much longer than that of most of the spectrum of interest, it should be possible to separate the signal from the drift by direct observation of the Sanborn tape. Since the average value of signals put out by the coil in the field was $\frac{1}{2}$ mv., this amplifier would boost that to 50 mv., which is well above the noise levels of the Krohn-Hite filter and 400 cycle chopper. This amplifier was not used to make any of the field recordings.

Frequency Spectrum Analysis

In this section the Sanborn tapes will be analyzed to compare relative amplitudes of E or H throughout their separate frequency spectra in order to determine which frequencies dominate the total signal and which frequencies are subordinate. Using these results we may get some idea as to which signals are associated with the magneto-telluric field and which signals are possibly generated within the recording system. Further, this analysis may give some indication as to the nature of the source of the MT field.

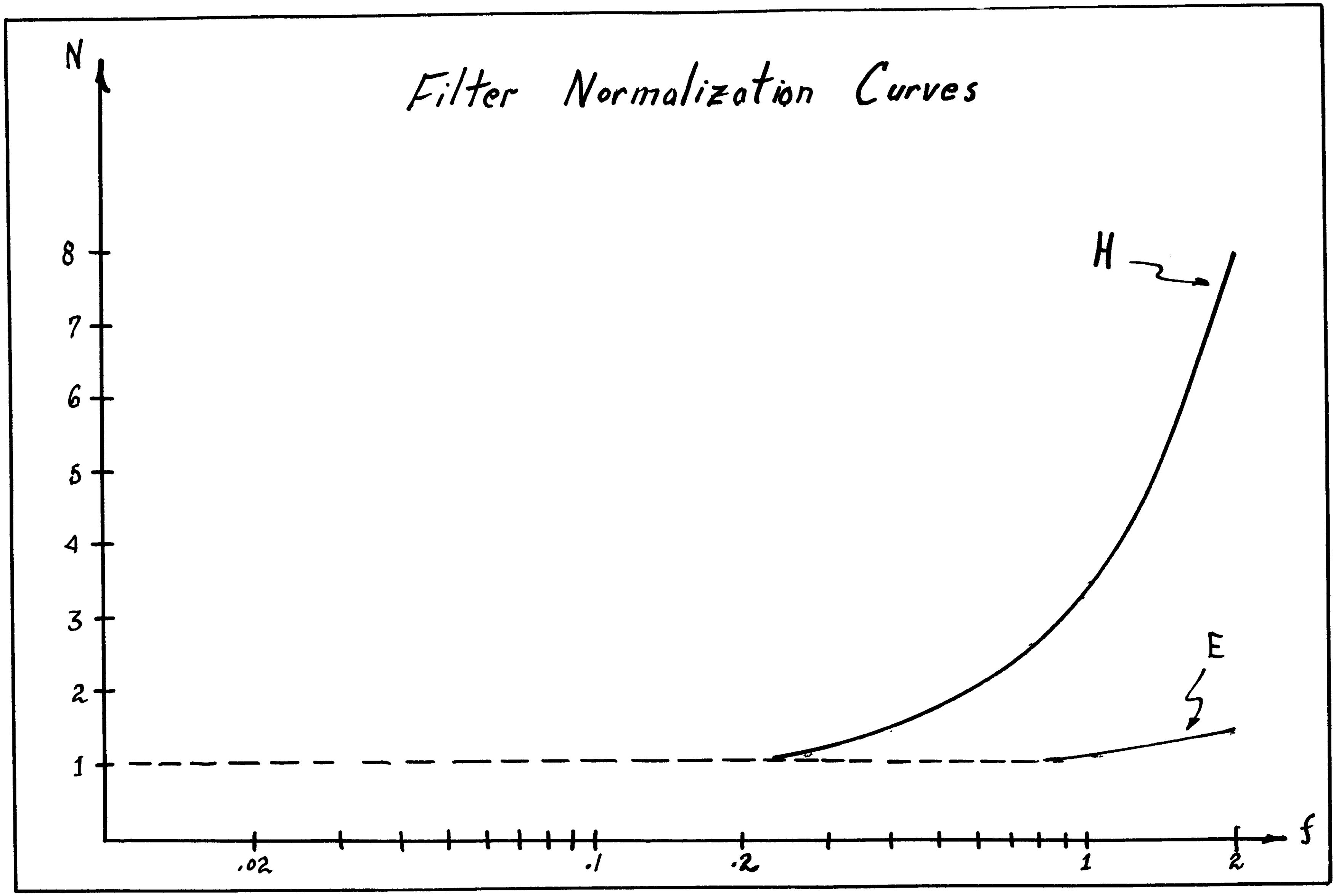
Magnetic Spectrum

Before a spectrum analysis may be made the entire recording system between the coil and the Sanborn tape recorder must be normalized to account for any relative variation of signal amplitude due to instrumentation techniques. For the magnetic system there are three sources of amplitude variation as a function of frequency. The first is due to the nature of the field pick-up device. Recalling the equation $|H| = V/NA\mu\omega$, where NA is a constant due to the coil design, and approximating μ as a constant with depth, it can be seen that H is inversely proportional to the frequency; or, $H \propto VT$, where T is the period of the wave. Thus, since the nature of the transducer is to amplify the magnetic field signal propotional to its frequency, the final system output which appears on the Sanborn tape must be normalized to account for this. This will be designated as the "T" factor.

A second source of amplitude variation due to the recording system lies in the characteristics of filter "M". This filter is designed to attenuate the higher frequencies. If .02 cycles is taken as the reference point, then the filter output at frequencies above .02 must be multiplied by a factor which is a function of frequency to normalize the filter output voltage with respect to input voltage. This shall be designated as the "N" factor, the values of which appear on the Filter Normalization Curve.

The third source of amplitude variation is the amplifier control of the Sanborn. This can be accounted for by reducing all response signals to a common control setting. In the results which follow signal amplitude will be measured in centimeters, as it appears on the Sanborn tape, multiplied by the attenuation factor at which the signal is recorded.

All amplifiers in the recording system remained at the same level during field measurements regardless of what frequency band was being recorded, so there is no calibration necessary due to these components. In this spectrum analysis we are only concerned with relative amplitudes of H, so any amplification or attenuation factors int he system which are not a function of frequency may be neglected. In the results which follow, the predominant frequency in each narrow band was determined by scanning the Sanborn tape.


Filter Normalization Curves
N
8
7
6
5
4
3
2
1
H
E
f
.02
.1
.2
1
2

RESULTS:

May 1 Record

Frequency Range (c.p.s.)	2-.6	.6-.2	.2-.06	.06-.02
Predominant Frequency (c.p.s.)	1	.4	.11	.04
"T" Factor (predominant period)	1	2.5	9	24
Signal Amplitude on Sanborn	1x20	1.2x20	2x20	2x20
"N" Factor	3	1	1	1
H_R (relative H)	3	3	18	48

May 18 Record

Frequency Range (c.p.s.)	2-.6	.6-.2	.2-.06	.06-.02
Predominant Frequency	.67	.3	.14	.04
"T" Factor (predominant period)	1.5	3	7	24
Signal Amplitude on Sanborn	2x5	2x5	3.5x5	2x5
"N" Factor	2	1.3	1	1
H_R (relative H)	6	7.8	24.5	48

where H_R = Sanborn Signal x "T" x "N"

These results indicate that the major part of the signal in the region of investigation is composed of the longer period variations, the 24 second period component contributing most of the total signal with a 7 or 9 second component of about half the amplitude of the 24 second one. This leads one to expect that the longer period variations would be more closely associated with the MT field than those of short period, since there is always the possibility of some random signal appearing at the output and being superimposed on the MT signal. Assuming that this non-magneto-telluric noise has a constant amplitude with respect to frequency, the signal-to-noise ratio at the system output would be greater at low frequencies.

Electric Spectrum

For a spectrum analysis of the electric field we have $E \propto V$, where V is the voltage level appearing on the Sanborn tape. All amplification recording factors remained constant throughout the spectrum, and the filter normalization factor for E is always approximately equal to one between .02 and 2 c.p.s. Only one field recording had an accurate E record on it; the results are the following:

May 18 Record

Frequency Range (c.p.s.)	2-.6	.6-.2	.2-.06	.06-.02
Voltage Amplitude on Sanborn Tape	.8×5	3.4×5	6×20	4×20
Predominant Period (sec.)	1.5	4.5	7.5 and 10	20 and 30
E_R (relative Electric field)	.8	3.4	24	16

From these results it appears that, as in the magnetic case, the major part of the signal is due to the longer period variations. However, it should be noticed that here the relative field value in the .06-.02 range is less than in the .2-.06 range. This may be due solely to the instrumentation limits. An indication that this may be the case is given by the following. The E variations were recorded wide-band in the field, since there was only one Krohn-Hite filter available, while the H field was recorded in the respective narrow bands. When an originally wide-band H recording was band-pass filtered at a point in the system between the magnetic tape and the Sanborn tape, thus performing the various operations in the same order as was actually done in the electric field system, the spectrum analysis results showed one outstanding difference when compared with results filtered before tape recording. The values of H_R for decreasing frequency ranges were 3.5, 5, 17, 6. Thus it seems that the instrumentation has the characteristic of losing the effect of the signal in the .02-.06 cycle range. For this reason it seems necessary that for accurate results to be used for interpretation a band-pass filter is required in the electric system before the chopper.

INTERPRETATION

This part of the paper was not carried as far as was originally expected due to the instrumentation problems which had to be solved first. However, a general scheme of tape interpretation will be outlined and sample tape recordings will be presented to indicate the correlation between the electric and magnetic field signals.

Before any interpretation is possible using the Sanborn tapes, the entire recording system must be calibrated so that the output voltage which appears on the tape may be related directly to the E or H field which it represents. For the magnetic system we have the expression $H = -V/i N A \mu \omega$ where V is the output voltage of the coils. This voltage is further modified by the transfer function of the recording network. This transfer function may be designated as $T_M(f) = \frac{e_o}{e_m}$ where e_o = output voltage and e_m = input voltage. $T_M(f)$ expresses an amplitude and phase relationship between e_o and e_m as a function of frequency. In the present case $V = e_m$, and e_o = voltage recorded on magnetic tape. In addition, there are the scale factors due to amplification by the tape recorder and attenuation by the Sanborn recorder. Denoting A_{1m} as tape recorder amplification and A_{2m} as Sanborn attenuation, we have the relation $V_{SM} = [-i \mu \omega H N A]\, T_M(f)\, A_{1M}\, A_{2M}$ where V_{SM} is the signal voltage recorded on the Sanborn tape. This expresses the relationship between V_{SM} and H in terms of known constants and instrument transfer functions which can be measured for any frequency.

For the electric field measurement the voltage between the two pots may be represented by $e_0 = LE$ where E is the electric field intensity and L is the length of wire. This voltage e_0 is then modified by the transfer function $F_e(s)$ of the recording system between the pots and the tape recorder, the amplification factor A_{1E} of the tape recorder, and the attenuation factor A_{2E} of the Sanborn. Therefore $e_{SE} = LEF_e(s)A_{1E}A_{2E}$ where e_{SE} is the voltage appearing on the Sanborn tape. It should be noted here that A_{2M} and A_{2E} must include a calibration factor relating the Sanborn output voltage to its peak-to-peak amplitude measured in centimeters on the tape.

Once the total transfer function for each system is known, it can be applied to sections of the Sanborn tape which appear to have some correlation. This will determine the amplitude and phase angle of E and H for the particular frequency band, and the results may be used in the equation $E/H = \sqrt{\mu\omega/\sigma}\ e^{i\pi/4}$ to determine σ.

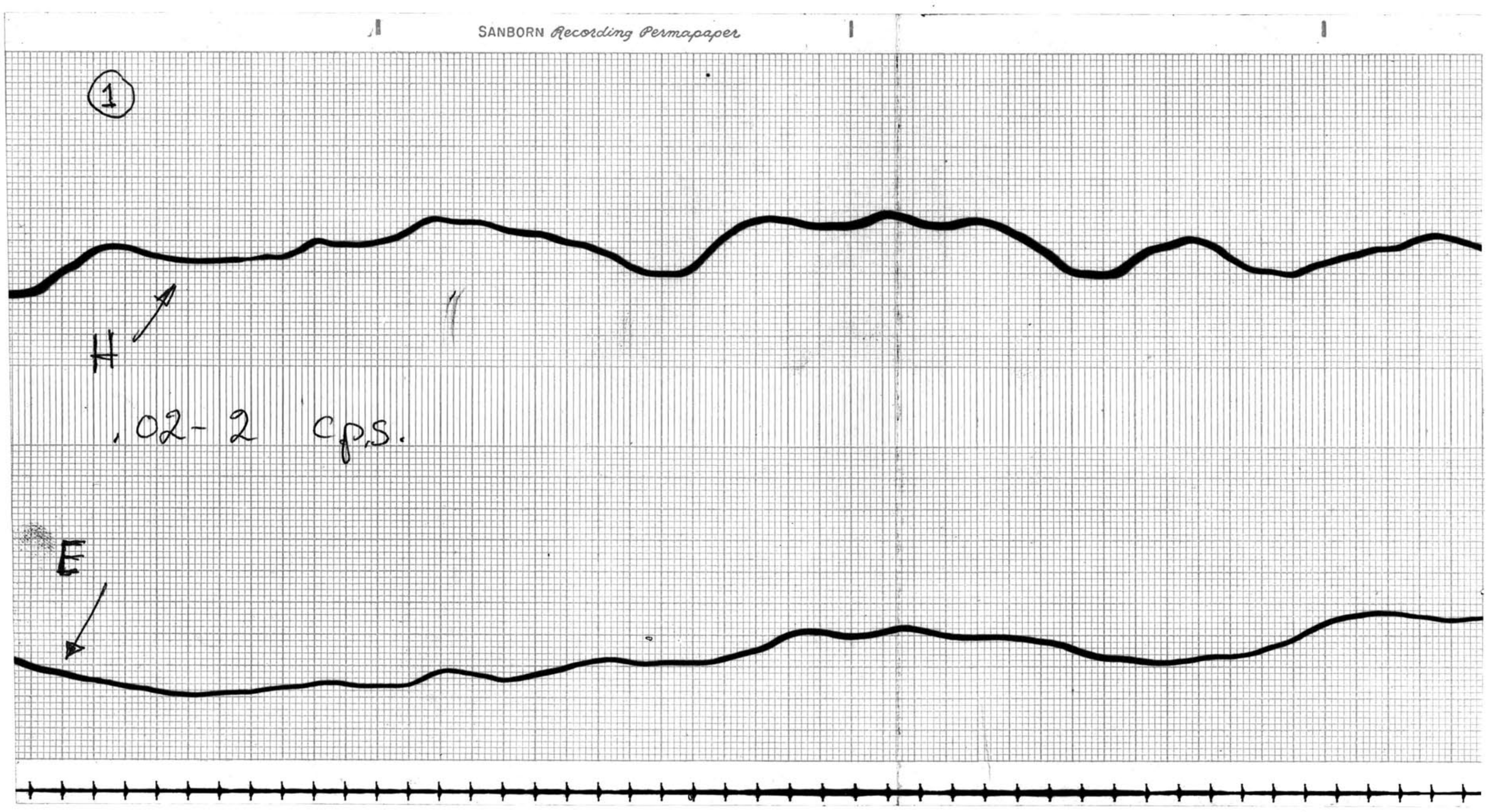
A possible scheme of exploration would be to make field recordings at spaced intervals on a straight line at the surface, and then, for each station, plot the conductivity σ vertically at a depth equal to the skin depth.

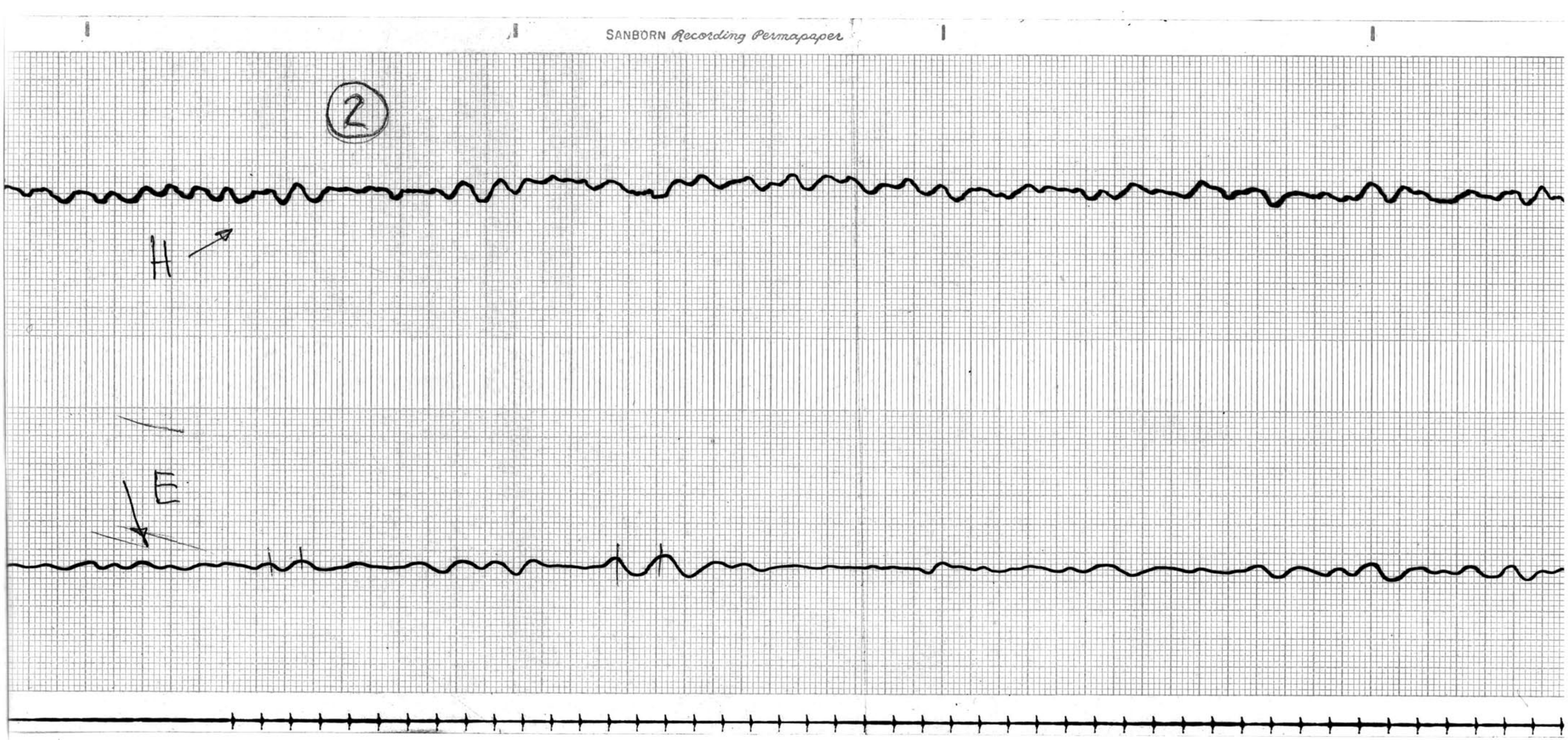
Sanborn Tapes of Field Records

The tapes presented in the folder are to indicate possible areas of correlation between the two signals.

Tape No.	Frequency Range (c.p.s.)	Recording Date
1	.02-2	May 18, 1958
2	2-.6	"
3	.6-.2	"
4	.2-.06	"
5	.06-.02	May 1, 1958

Each timing space at the bottom of the tape represents one second.


SANBORN Recording Permapaper
1
H
.02- 2 cps.
E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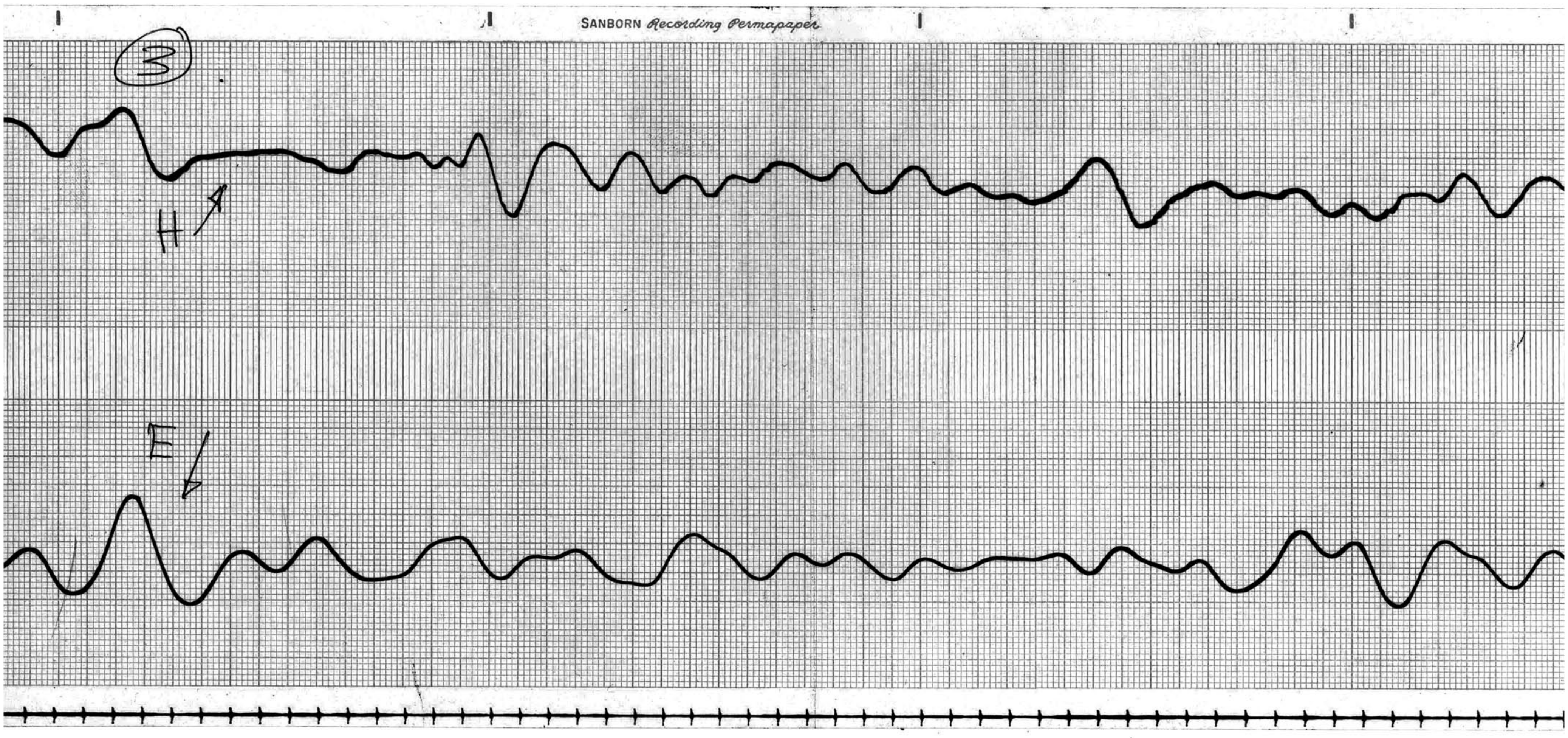
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3
H
E


SANBORN Recording Permapaper
SANBORN Recording Permapaper
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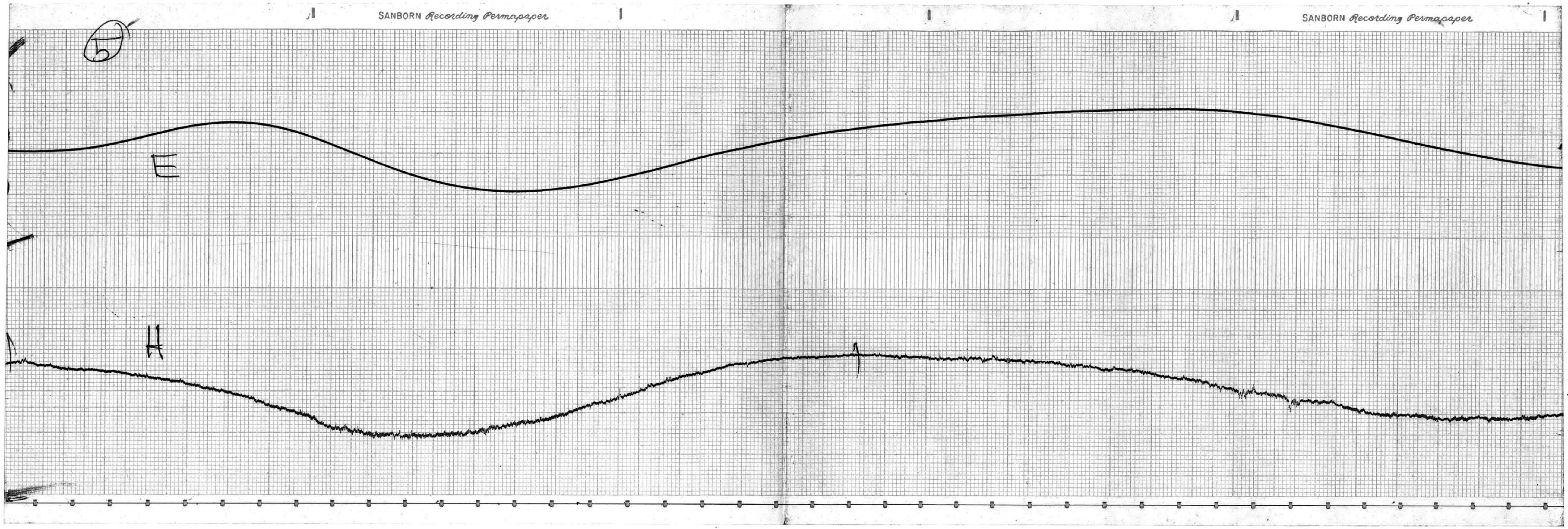
E
2


SANBORN Recording Permapaper
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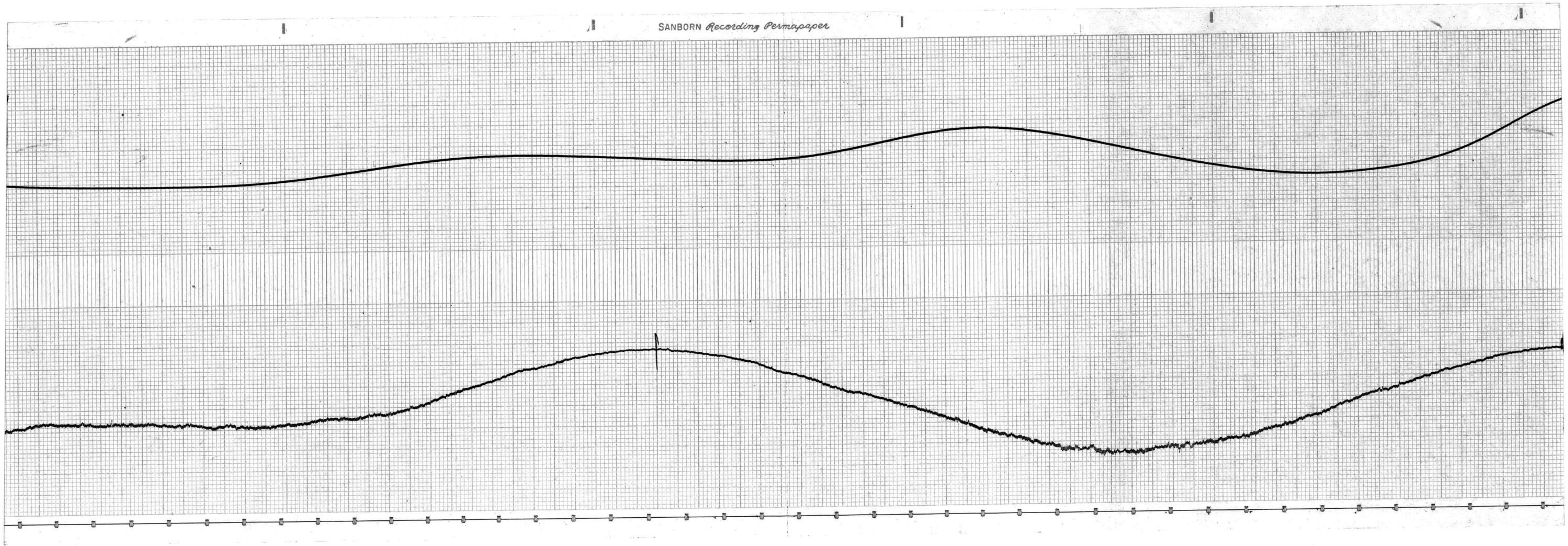

SANBORN Recording Permapaper

28 c

SAMPLE EVALUATION OF FIELD DATA

Using order of magnitude arguments it can be shown that the voltage values measured in the field indicate a plausible value for σ.

The average value of the coil output was about 10^{-4} volts. Using the equation $|H| = V/NA\mu w$ (all calculations done in MKS units), where: $A u$ = effective area x u_o = 1×10^{-6}

$N = 2 \times 10^5$

$w = .6$ (for f equal to 1 c.p.s.)

Thus: $$|H| = \frac{10^{-4}}{2 \times 10^5 \times 10^{-6} \times .6} \simeq 10^{-3}$$ ampere-turns/meter

Typical values for E were on the order of 10^{-1} volts/300 meters.

Then: $E = V/L = 10^{-1}/300 \simeq 3 \times 10^{-4}$. volts/meter

Using the equation $\left|\frac{E}{H}\right| = \sqrt{\frac{\mu w}{\sigma}}$ and taking $u = u_o$.

$$\frac{3 \times 10^{-4}}{10^{-3}} = \sqrt{\frac{10^{-6} \times .6}{\sigma}} \quad ; \quad 3 \times 10^{-1} = 2.4 \sqrt{\frac{10^{-7}}{\sigma}}$$

$$\text{or: } \sigma \simeq 10^{-7}/10^{-2} = 10^{5} \quad (\text{ohm-meters})^{-1}$$

This reasonable value for σ is an indication that the field measurements are of the correct order of magnitude.

APPENDIX I

Equipment Design

1. High-Mu Rod

The final choice of a rod was strongly biased by the availability of materials which could be purchased. The rods selected are 4-79 Mo-Permalloy purchased from the Arnold Engineering Company. The specifications on the rods are $u_i = 20,000$, $2b = 7/8"$, $2a = 90"$. According to the industrial catalog this material is strain sensitive, but they give no order of magnitude for the change of the permeability as a function of strain. It is very doubtful that u_i is actually the specified 20,000, since the rods were delivered with large dents which must have altered u_i somewhat. The actual effective permeability of the rod could be measured by inserting it through a coil of known inductance and applying a known field, then measuring the output voltage of the coil.

The specification curves printed in the technical bulletins published by Arnold indicate that the region of operation which we are concerned with lies in the domain of maximum response for this particular alloy. The flux density should be on the order of 2-4 kilogauss, which lies below the saturation point for Permalloy and in the region of maximum permeability as a function of B.

Using the Sommerfeld expression to calculate the effective permeability of a rod of the dimensions given, we have:

$$\epsilon = \frac{\left(90^2 - \frac{7}{8}^2\right)^{1/2}}{90} = \left[1 - \left(\frac{7}{8 \times 90}\right)^2\right]^{1/2}$$

$$\simeq \left(1 - 10^{-4}\right)^{1/2}$$

$$\epsilon \simeq 1 - 5 \times 10^{-5}$$

or $\epsilon = 1 - \delta$ where $\delta = 5 \times 10^{-5}$

$$\epsilon^2 = (1-\delta)^2 = 1 - 2\delta = 1 - 10^{-4}$$

$$\epsilon^3 = (1-\delta)^3 = 1 - 3\delta = 1 - 1.5 \times 10^{-4}$$

$$\mu_e = \frac{\mu_i}{1 + (\mu_i - 1)\left[\frac{1 - 1 + 10^{-4}}{1 - 1.5 \times 10^{-4}}\right]\left[\frac{1}{2}\ln\frac{2 - 5\times 10^{-5}}{5 \times 10^{-5}} - 1 + 5 \times 10^{-5}\right]}$$

$$\mu_e = \frac{\mu_i}{1 + (\mu_i - 1)(10^{-4})\left[\frac{1}{2}\ln 2 - \frac{1}{2}\ln 5\times10^{-5} - 1 + 5\times 10^{-5}\right]}$$

$$\frac{1}{2}\ln 2 - \frac{1}{2}\ln 5 \times 10^{-5} = \frac{1}{2}\ln 4000 \qquad \text{where } \ln 4000 = 8.294$$

therefore:

$$\mu_e = \frac{\mu_i}{1 + (\mu_i - 1)(10^{-4})(4.147 - 1 + 5 \times 10^{-5})}$$

$$\mu_i = 20,000$$

so:

$$\mu_e = \frac{2 \times 10^4}{1 + 2(10^4)(10^{-4})(3.147)}$$

$$= \frac{20,000}{7.294} = 2,743$$

The minimum NA for an air coil was $\sim 10^7$, but since NA is inversely proportional to u, for the Permalloy core the minimum NA can be $\sim 10^7/3000 \simeq 3.7 \times 10^3$.

The coil area is very small, however, and in MKS units $A = \pi r^2 = \pi \left(2.54 \times 7/16\right)^2 \times 10^{-4} = 3.87 \times 10^{-4}\ m^2$

This means N must be $\simeq 3.7 \times 10^3/3.87 \times 10^{-4} \simeq 10^7$ turns.

This is a rough order of magnitud estimation and experimental results in the field show that for $N = 0(10^6)$ a reliable signal can be recorded. The high mu core has the characteristic of increasing the effective area of the coil. In this case the actual area is 3.87×10^{-4} and the effective u 3×10^3, so the effective area is about one. The effective area of the air coil was also one, but it is going to be much easier to put a large number of turns on a round 7/8" diameter core that it would be on the 1 meter square air core. The final number of turns used in field recording was 256,000, and we might notice that this would put out a signal $256{,}000/5{,}000 \simeq 50$ times as great as the air coil.

2. Coil Design

40 Formex Magnet Wire was used in the winding. This small size wire was used largely to diminish the total weight and volume of the coil for a given number of turns. As it turns out, however, this was not a very good criterion, and it would have been better to have used heavier, lower resistance wire, since the resistance of the coils places a serious limit on the total number of turns possible, as will

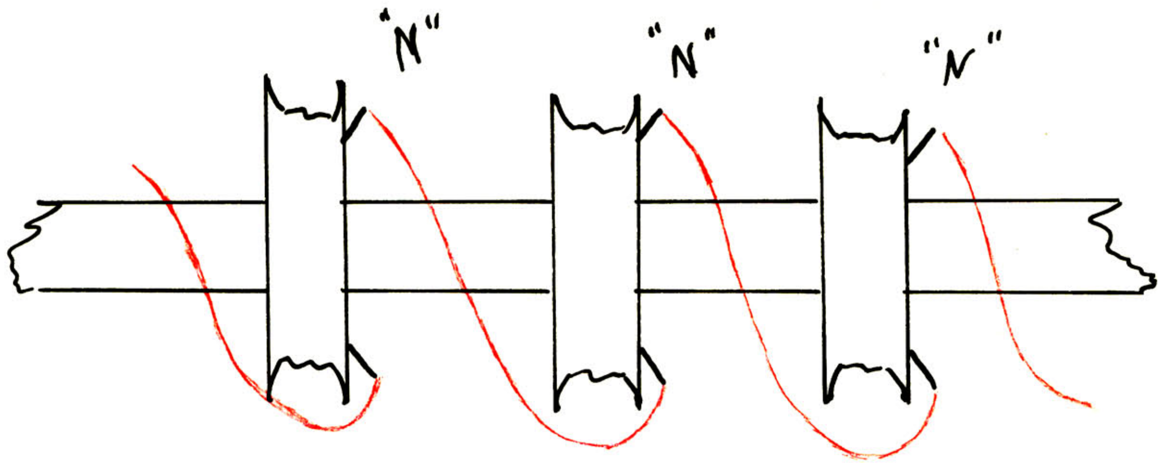
be pointed out later. The individual plastic spools have an inside diameter of 2.3 cm., outside diameter of 7.6 cm., and a width of 2.7 cm.

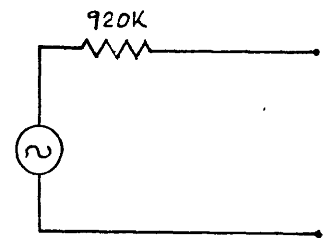
It was found experimentally that the resonant frequency of the coil was inversely proportional to the number of turns. For 50,000 turns the resonant point for a coil with the Permalloy core is about 17 c.p.s. Increasing the number of turns per coil would lower the resonant point into the spectrum of investigation, and this is to be avoided. The overall number of turns may still be increased, however, since several coils with similar resonant characteristics placed in series will have the same resonant point as an individual coil.

The field equipment consists of 4 coils of 50,000 turns each and 1 of 56,000 turns. The spools were wound on a device which consisted of a modified Signal Corps tape winder. This system had a continuously variable adjustment to control the r.p.m.'s which solved the inertia problem of breaking the wire at the first instant on high speed lathe winding. Normal winding time was one hour per 50,000 turns.

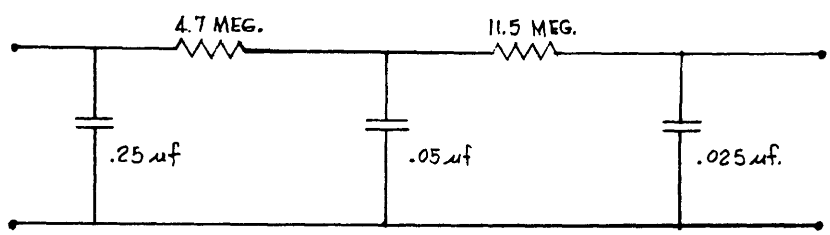
In order that the signals of the respective coils shall add when placed on the rod, the terminal lugs, both of which appear on the same side of the individual coil, should all be on the same side of the observer, whether it be left or right. One lug on each coil is marked "N". These are all like poles, so when the coils are connected in series, each "N" pole should be linked to an unmarked pole.

Series Coil Connection

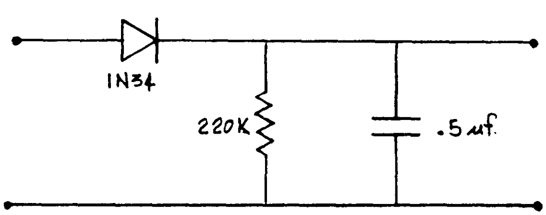




EQUIVALENT LOOP SOURCE

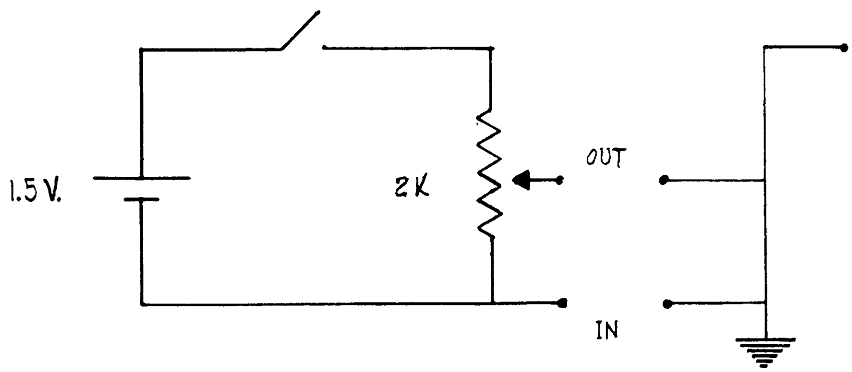


FILTER "M"

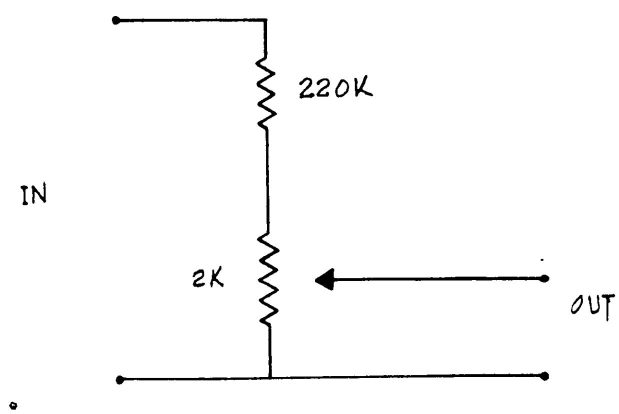


DIODE DETECTOR

INDIVIDUAL UNITS OF MAGNETIC CIRCUIT



BIAS "B"



VOLTAGE DIVIDER

INDIVIDUAL UNITS OF MAGNETIC CIRCUIT

3. Detection Circuit

In order that the detection circuit be able to follow the low frequency envelope, the RC time constant must be adjusted so that $RC \leq \frac{1}{\omega_m} \sqrt{1-m^2}/m$ where ω_m is the envelope frequency in radians/sec. and m is the modulation factor, which can be adjusted in the field to be very close to one. In this approximation it would seem that the capacitance should be zero for 100% modulation. Under these circumstances the output would not follow the wave form of the input voltage. Additional factors such as source impedance modify the detector, however, so that harmonic distortion will not be excessive is $RC \leq \frac{1}{\omega_m m}$. This equation just governs the product of R and C. Individually R should be much greater than the forward resistance of the diode and much less than the back resistance. The capacitor should be on the order of .5 to 10 microfarads, the higher capacitance providing a better smoothing effect.

4. Krohn-Hite Filter

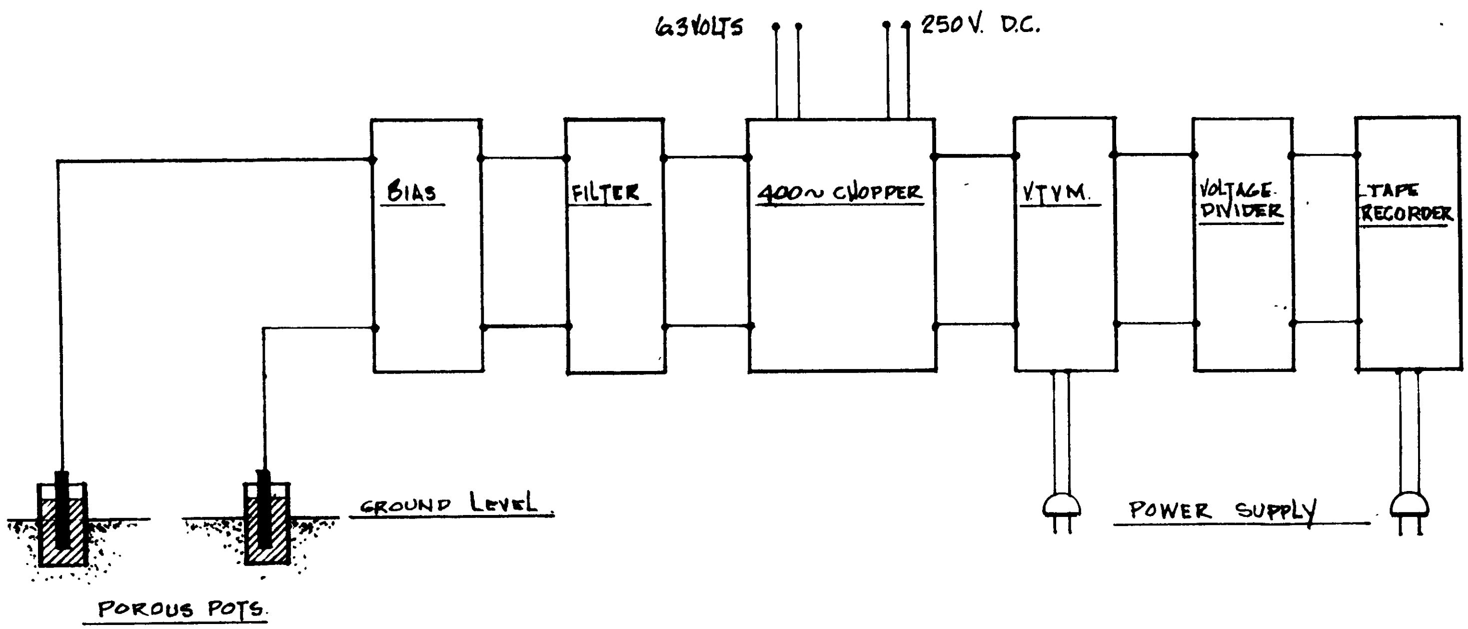
This is an ultra-low frequency band-pass filter with a minimum low frequency cut-off at .02 c.p.s. and a maximum high frequency cut-off at 2,000 cycles. The band-pass gain is 0 db ± 1 db. There is a high input impedance of about 20 Megs. and an output impedance of 5 K. The maximum input amplitude is 10 volts RMS with an associated internal noise generation of one millivolt. Attenuation is 24 db/octave outside the pass band.

NOTE: For components not given here, refer to

Cunningham (1957).

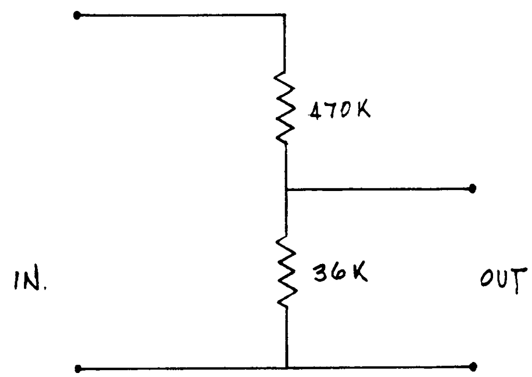
5. Electric Recording Circuit

This is essentially the same as that used by Cunningham, with one modification. The input to the tape is monitored by an hp 400H VTVM. This may be treated as a voltmeter-amplifier with full-scale deflection on the meter being .15 volts RMS regardless of the gain setting. It was found experimentally that when the meter reads half-scale an output sine wave is 10 mv. peak-to-peak, or 3.53 mv. RMS, across the voltage divider. About the best recording voltage for the tape is 4 mv. RMS, which means that the meter should be adjusted to read about 6. If any long period self-potential drift tends to drive the meter above this limit while recording is taking place, the amplification level of the VTVM may be changed to bring the signal level back to the recording level of the tape.

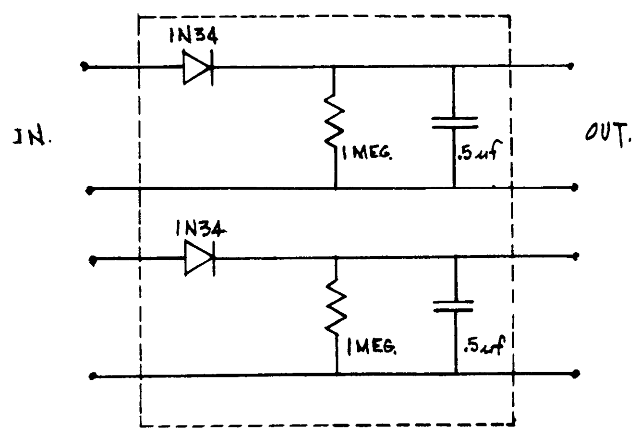


BLOCK DIAGRAM OF ELECTRIC FIELD RECORDING CIRCUIT

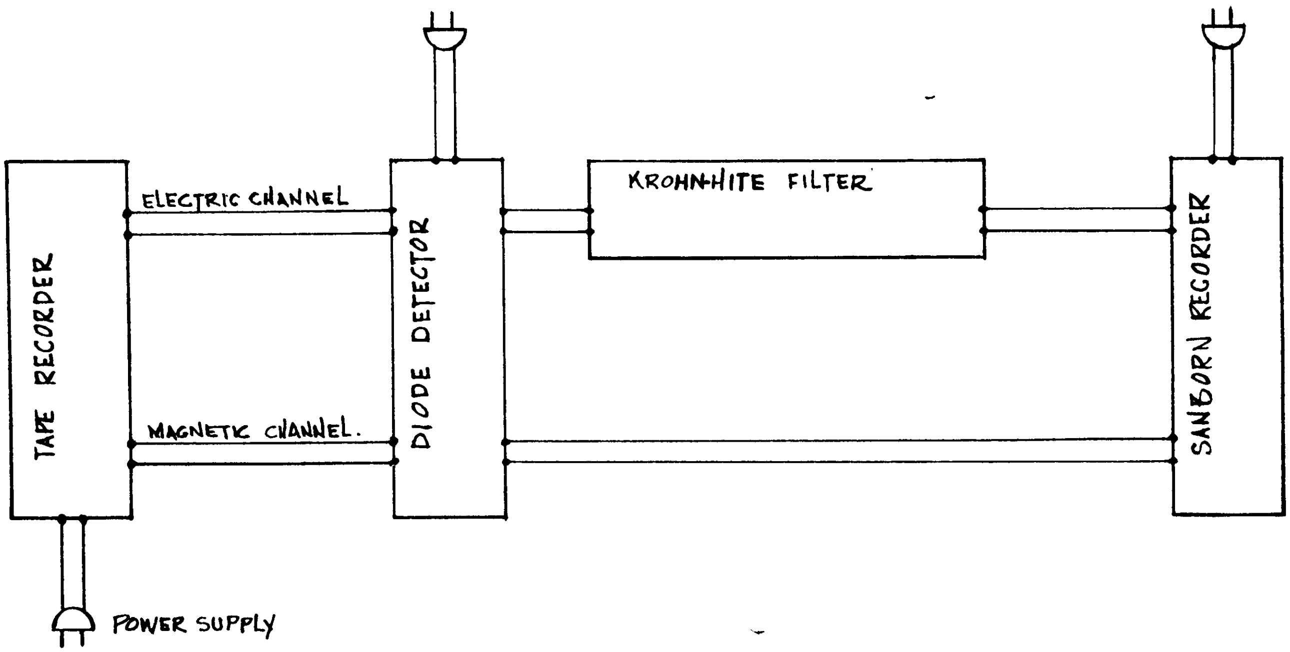
39.



VOLTAGE DIVIDER FOR ELECTRIC CIRCUIT



DIODE DETECTOR FOR TRANSFER CIRCUIT

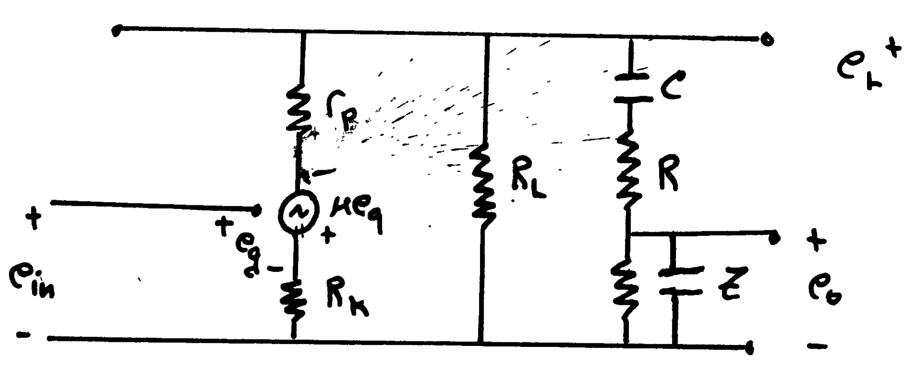


BLOCK DIAGRAM OF CIRCUIT TO TRANSFER THE MAGNETIC TAPE SIGNAL TO THE PAPER TAPE.

APPENDIX II

Low Frequency Amplifier

The basic design of the amplifier employs three RC coupled stages to provide high loop gain plus a cathode follower to provide a low output impedance. The choice of tube (12AY7) is based on the fact that it is the same tube which is used at the input stage to the Krohn-Hite filter which has a grid resistance of 22 Megs. hung across it. Most amplifiers specify 1 Meg. as an upper limit to the load impedance but this special purpose tube was designed to have low noise and be capable of operating with a large load impedance, which is what the amplifier faces at the output of filter "M". The 12AY7 is a medium-mu twin triode with $r_p = 23K$ and $u = 40$. The first two stages are designed to have a quiescent operating point around 100 plate volts and 2 ma. plate current. The third stage operates at 125 plate volts and 1 ma. plate current. The signal is fed directly into the grid of the first stage. To find the gain per stage of the amplifier, we look at the incremental model of the first stage.



Since the impedance of the coupling circuit $\frac{1}{Cs} + R + Z$ is very large compared with R_L, the parallel

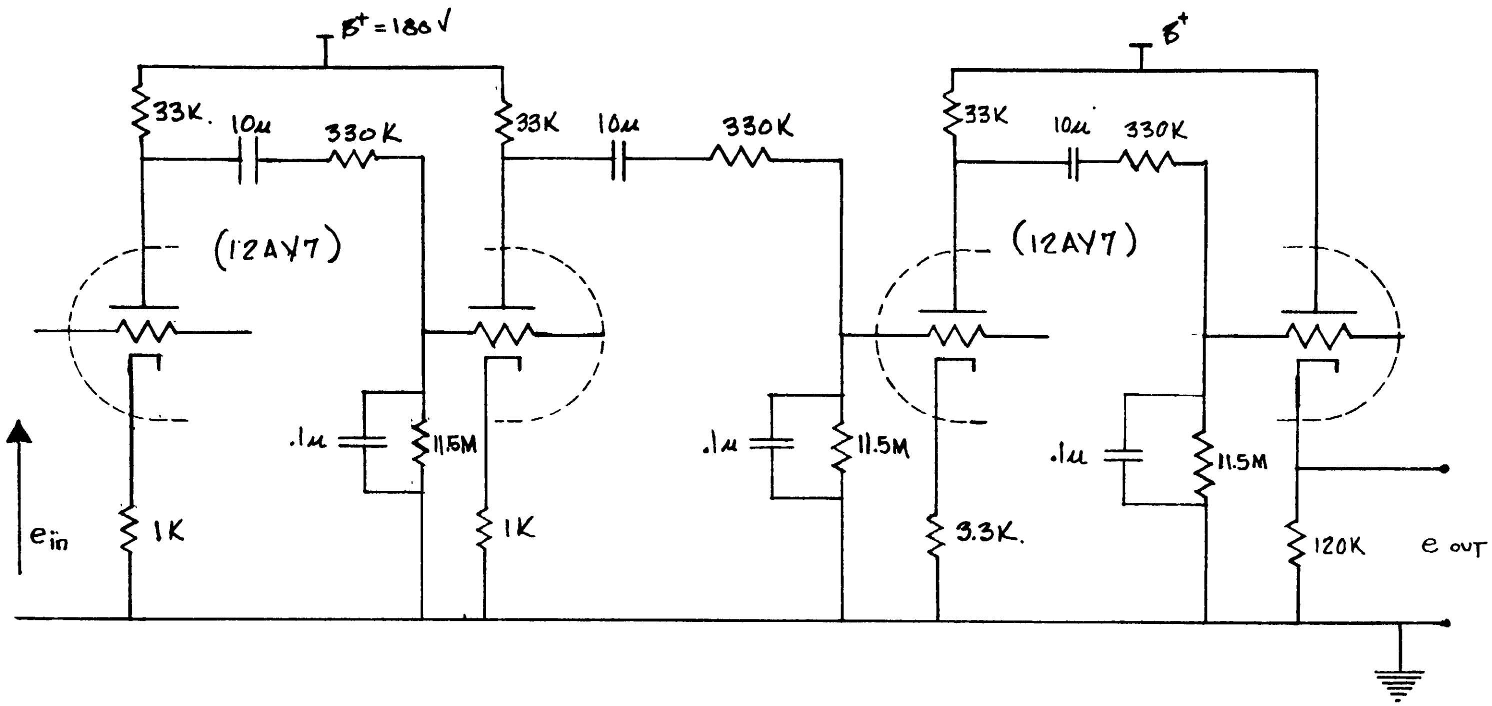


FIGURE "A"

LOW FREQUENCY AMPLIFIER

combination of these two is approximately equal to R_1.

$$Z_c = \frac{1}{cs} + R + Z = 1/10^{-2} \times 2\pi f + 330k + 1.15 \times 10^{7}/1.15 \times 2\pi f + 1$$

since the highest frequency of interest is 2 c.p.s.,

$$Z_c = 10^5/12 + 330k + 11.5 \times 10^6/14.3 \simeq 10k + 330k + 800k$$

$$Z_c = 1.14 \text{ Meg} \quad (\text{minimum})$$

Writing the loop equations:

$$e_m = e_g + i_p R_K$$

$$\mu e_g = i_p (r_p + R_K + R_L)$$

$$\mu e_{in} = i_p [r_p + R_L + R_K(\mu+1)]$$

$$\text{gain} \quad \frac{e_L}{e_{in}} = \frac{i_p R_L}{e_{in}} = \frac{\mu R_L}{r_p + R_L + (\mu+1) R_K}$$

$$\frac{e_L}{e_{in}} = \frac{(40)(33k)}{23k + 33k + 41k} \simeq 13.6$$

This gain is RC coupled to the next stage. A capacitor of 10 μf is used to block the passage of d.c. and still pass the low frequency components of interest. A larger capacitor would be better, but there is a practical limit to the physical size of the capacitor used capable of withstanding the plate voltage present in the circuit. The 330K resistor acts as the a.c. load line resistance. The output to the next stage is taken off the parallel combination of 11.5 Megs. and .1uf. This is designed to be a high impedance relative to the 10 uf capacitor and load resistor so that most of the output voltage of the preceding stage will be transferred to the next grid input. The capacitor is used as a high frequency pass for 60 cycle noise.

At .02 c.p.s. the percent signal passed to the next stage is given by:

$$100 \times \frac{11.5 \times 10^6}{1.15(2\pi f)+1} \Big/ \frac{11.5 \times 10^6}{1.15(2\pi f)+1} + 330\,K + 10^5/2\pi f$$

which is very close to 90%. Only about 7/10 of the signal is passed at 2 c.p.s.

The ratio of the grid input impedance $Z_i = R/RCS + 1$ at 1 c.p.s. to that at 60 c.p.s. is given by

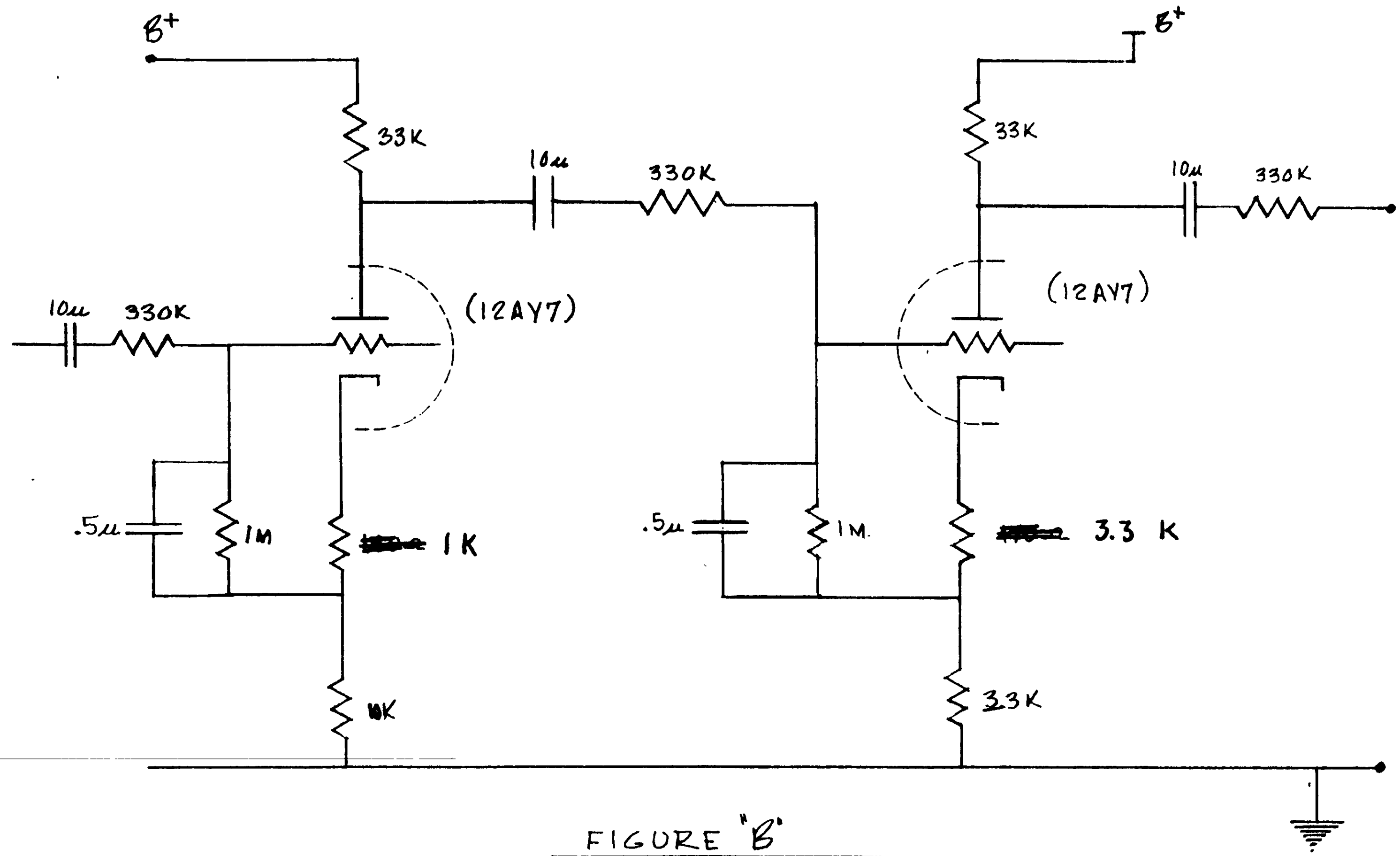
$$\frac{Z_1}{Z_{60}} = RCS_{60} + 1 \Big/ RCS_1 + 1 = \frac{1.15 \times 6 \times 60 + 1}{1.15 \times 6 \times 1 + 1} \doteq 53$$

In other words, 60 cycle noise is attenuated by a factor of 50 relative to 1 cycle signals at each grid input. Thus, using an order of magnitude estimate, if the original input was .1 mv. signal and 1 volt 60 cycle noise, which was actually measured in the field, the amplifier output, for a total gain of 1000, would be about .1 volt signal and 2×10^{-5} volts noise since, in addition to the attenuation due to the by-pass condenser, high frequency components appear across the 330 K resistor and are not passed to the next stage.

Two separate plate voltage supplies are used to prevent the possibility of a feed-back loop existing between stages one and three or two and four.

The cathode follower stage at the output has a gain of approximately one and gives the amplifier a low impedance output.

For the circuit model of Fig. B, the aim was to adjust the two cathode resistors so that the grid resistance

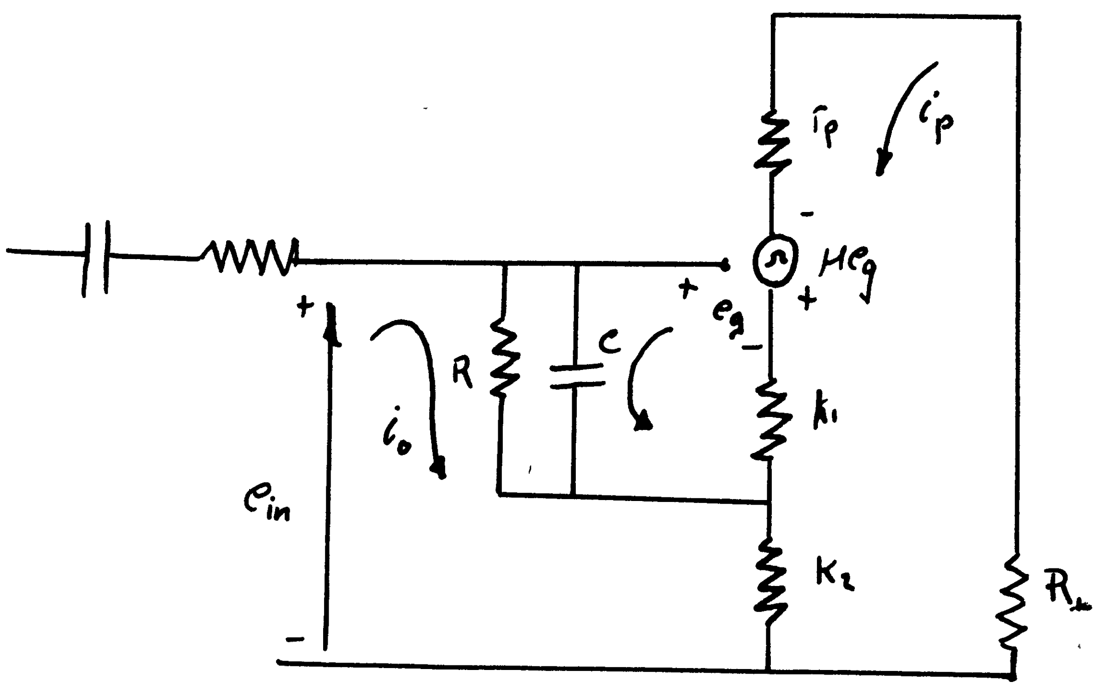


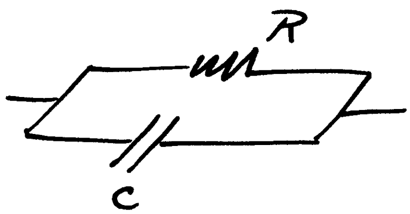
B+
33K
10µ
330K
(12AY7)
10µ
330K
.5µ
1M
1K
10K
B+
33K
10µ
330K
(12AY7)
.5µ
1M.
3.3 K
33K

FIGURE "B"

could be changed from 10 Meg to 1 Meg, but it would still look like 10 Meg as far as the a.c. load line of the previous stage was concerned. In this manner most of the signal would be passed by the coupling circuit, and any noise generated by grid current flowing across the grid resistance would be diminished by a factor of 10 relative to the circuit of Fig. A.

Looking at the incremental model for this circuit and writing the loop equations:



let 

$= Z = \dfrac{R}{RCs+1} \simeq \dfrac{10^6}{3f+1}$

$i_g << i_o, i_p$

$\mu e_g = i_p(R_L + r_p + k_1 + k_2) + i_o k_2$

$e_g = Z i_o - k_1 i_p$

$e_{in} = Z i_o + k_2(i_o + i_p)$

$Z_{in} = e_{in} / i_o$

$$i_p = (\mu z i_o - i_o k_2) / R_L + r_p + k_2 + (\mu+1) k_1$$

$$e_{in} = z i_o + k_2 (i_o + i_p)$$

$$= i_o (z + k_2) + \frac{k_2 [\mu z i_o - i_o k_2]}{[R_L + r_p + k_2 + (\mu+1) k_1]}$$

$$Z_{in} = e_{in} / i_o$$

$$= z + k_2 + \frac{k_2 [\mu z - k_2]}{R_L + r_p + k_2 + (\mu+1) k_1}$$

$$Z_{in} \simeq z + \frac{k_2 (1 + \mu z)}{R_L + r_p + k_2 + (\mu+1) k_1}$$

To find the incremental gain;

$$i_p (R_L + r_p + k_1 + k_2) = \mu e_g - i_o k_2$$

$$= \mu [z i_o - k_1 i_p] - i_o k_2$$

$$e_{in} = i_o (z + k_2) + k_2 i_p$$

$$i_o = \frac{e_{in} - k_2 i_p}{z + k_2}$$

$$i_p (R_L + r_p + k_1 + k_2) = \mu \left[z \frac{(e_{in} - k_2 i_p)}{z + k_2} - k_1 i_p \right] - k_2 \left[\frac{e_{in} - k_2 i_p}{z + k_2} \right]$$

$$i_p\left[R_L + r_p + (\mu+1)k_1 + k_2\left(1 + \frac{\mu z - k_2}{z+k_2}\right)\right] = \frac{\mu z e_{in}}{z+k_2} - \frac{k_2 e_{in}}{z+k_2}$$

$$G = i_p R_L / e_{in}$$

$$= \frac{R_L\left[\mu z/_{z+k_2} - \frac{k_2}{z+k_2}\right]}{R_L + r_p + (\mu+1)k_1 + k_2\left(1 + \frac{\mu z - k_2}{z+k_2}\right)}$$

using the approximations, $z \gg k_1$, $z \gg k_2$

$$G = \frac{R_L\left[\mu - k_2/z\right]}{R_L + r_p + (\mu+1)(k_1+k_2)} + \left(\frac{i_g}{i_o}\right) \text{ terms}$$

Since incremental $i_g \simeq 0$, $G \simeq \mu R_L / R_L + r_p + (\mu+1)(k_1+k_2)$

From these equations it can be seen that increasing K_2 will increase the effective input impedance Z_{in}, which is what we set out to do. However, this has also decreased the gain, since the term $(u+1)K_2$ appears in the denominator of G. Therefore the factor by which the grid resistor may actually be reduced and still comply with the demands of the rest of the circuit is limited by the loss in gain which the system can undergo and still be of use.

APPENDIX III

Field Operating Techniques and Comments

The electric and magnetic recording systems are connected as shown in the block diagrams. The plane of the inductance coils should be aligned parallel to the direction in which the pot wire is laid so that the B field lines associated with the horizontal E lines in the wire direction cut the coil at right angles to contribute their maximum effect towards inducing a voltage. The power supply should be kept as far as possible from all recording equipment, since the field that it generates is a non-magneto-telluric source which will obscure the main MT field.

Recording Procedure for Electric Field

Set the pots into the ground at some measured interval, e.g. 500 meters. If the ground is exceptionally dry, it may be saturated with water to insure better electrical contact. Record wire length, as this will be needed for calibration. The nearest pot is placed close to the recording apparatus and acts as the ground for the whole system. Adjust the bias so that all the voltage variations which appear on the VTVM remain above zero. If any of the fluctuations go through zero, the original signal will not be regained on detection due to the effect of the amplifier in the VTVM. The VTVM should be adjusted during recording to keep the meter reading between about 3 and 7.

Recording Procedure for Magnetic Field

Set the inverter so that it amplifies the signal by 100. Then using the a.c. amplifier as a voltmeter, adjust bias "A" so as to keep all signal variations slightly above zero. After this is done, switch back to the "amplify" position. The Krohn-Hite filter settings are then adjusted to the particular band of interest. The input voltage setting should be on "high". Using a VTVM, observe the signal at the chopper output and adjust bias "B" to keep all variations above zero. Finally, observe the signal after the voltage divider and adjust the divider setting so that the output to the tape is about 4 mv. RMS. Record all amplifier and voltage divider positions so that they may be used for calibration.

The best way of keeping records is to monitor the magnetic tape with a microphone so that all pertinent information will be permanently preserved with the data.

Field Data

Three magnetic tapes containing field data are available in the Geophysics Lab. They were all made in Lincoln, Massachusetts. The pot wire was laid southeast and the Permalloy rod pointed southwest. The electric field was recorded unfiltered, while the filter settings for the magnetic signal were .02-2, .6-2, .2-.6, .06-.2, and .02-.06 c.p.s. On the record of May 1 the electric signal was not monitored at the tape input, and there are indications on observing the Sanborn recording that the magnetic tape was saturated. A 1200 foot pot separation was used for this record. For the May 12 tape the electric signal is completely invalid due to an instrumentation error which was later corrected. The magnetic signal was saturating the tape in several sections of this record. The record of May 18 has no defects that are known. The pot separation here was about 830 feet.

Suggestions for Further Work

Recordings should be made using a band-pass filter in the electric as well as the magnetic system.

The range of frequency investigation could be extended.

A valid interpretation scheme should be formulated and applied to field records or, for more meaningful results, to a modeling problem.

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