Elastic Properties of Sedimentary Anisotropic Rocks
(Measurements and Applications)

by

Franklin J. Ruiz Peña

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Signature of Author

Department of Earth, Atmospheric, and Planetary Sciences
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Certified by

Professor M. Nafi Toksöz
Thesis Advisor

Accepted by

Professor Ronald G. Prinn
Chairman, Department of Earth, Atmospheric, and Planetary Sciences
ABSTRACT

In multidisciplinary studies carried out in the Budare Oil Field of the Great Oficina Oil Field, there was difficulty matching well log synthetic seismograms with 2D and 3D seismic data. In addition, the seismically determined depths of reservoir horizons are greater than the well sonic log depths. To examine this discrepancy we conducted an experimental study of dynamic elastic parameters of the rocks in the oil field. We chose core representative samples of the lower Oficina Formation, the main reservoir of the field. The rocks selected were sandstones, sandy shales and dolomitized shales.

For the velocity measurements, we used the ultrasonic transmission method to measure P-, Sh- and Sν-wave travel times as a function of orientation, and pore and confining pressures to 60 and 65 MPa, respectively. We found that, in room dry condition, most of the rocks studied are transversely isotropic. The stiffnesses constants, Young’s moduli, Poisson’s ratios, and bulk moduli of these rocks, were also calculated.

The velocity anisotropies, together with the behavior of the elastic constants for dry rocks, indicate that: (1) the elastic anisotropy of the sandstones and sandy shales is due to the combined effects of pores, cracks, mineral grain orientation, lamination and foliation.
The velocity anisotropies caused by the preferred oriented cracks decrease with increasing confining pressure. (2) For the dolomitized shales, the elastic anisotropy is due to mineral orientation and microlamination. In these cases the very high intrinsic anisotropy does not decrease with increasing confining pressure. (3) The velocities of compressional waves are greater in sandstones saturated with water than in the dry specimens, but the opposite behavior was found for shear waves. (4) The P-wave velocity anisotropy decreases after saturation; the magnitude of the decrease depends on the crack density and on the abundance and distribution of clay. (5) The $V_{sh}$-anisotropy does not show a pronounced change after saturation, and it is only slightly affected by confining pressure.

Visual description, petrography and mineralogical analyses from thin sections and x-ray diffraction revealed the vertical and lateral heterogeneous nature of sandstones and sandy shales, whereas the dolomitized shale specimens looked homogeneous.

The results of the laboratory measurements are consistent with an elastic model, using the equivalent medium theory for fine-layered isotropic and anisotropic media. However, in order to do reliable seismic migration and solve the problem of thickness calculations and time-to-depth conversion of surface seismic data, the ultrasonic data need to be extrapolated to low frequencies.

Determining rock mechanical properties in situ is important in many applications in the oil industry such as reservoir production, hydraulic fracturing, estimation of recoverable reserves, and subsidence. Direct measurement of mechanical properties in situ is difficult. Nevertheless, experimental methods exist to obtain these properties, such as measurements of the stress-strain relationships (static) and elastic wave velocities (dynamic).

We investigate the static and dynamic elastic behavior of sedimentary, anisotropic rock specimens over a range of confining and pore pressures up to 70 MPa, the original reservoir conditions. The static and dynamic properties are simultaneously measured for
room dry shales, room dry sandstones, and brine saturated sandstones. We found that (1) All the ratios of dynamic to static velocities and of dynamic to static elastic parameters in all directions, $R_m(\theta)$, decrease with increasing confining pressure. However, the rate of decrease is greater in the vertical direction than in the horizontal direction. (2) After saturation, all the ratios of dynamic to static moduli and dynamic to static velocities, $R_m(\theta)$, decrease, except the bulk compressibility ratio, $R_{kb}$, which increases. (3) All the ratios of dynamic to static moduli, $R_m(\theta)$, decreases when the pore pressure is raised, except $R_{kb}$ which increases. (4) The magnitude of the ratio of dynamic to static velocities or moduli, $R_m(\theta)$, depends on the direction of the measurements. Not all the ratios $R_m(\theta)$ are equally affected. The ratio of dynamic to static P-wave velocity, $R_P(\theta)$, is greater in the vertical direction than in the horizontal direction. On the other hand, the ratio of dynamic to static Sh-wave velocity, $R_{sh}(\theta)$, does not depend on the direction of propagation. (5) The modulus determined from: uniaxial stresses, hydrostatic compression or any other stress system yields different values. This is because of the rock porosity. (6) All the static and dynamic velocities and elastic parameters decrease with increasing confining pressure. (7) The static velocity anisotropies and static modulus anisotropies are always greater than the corresponding dynamic anisotropies, over the entire range of confining pressure and directions. (8) After saturation, the dynamic Vp-anisotropy, $\varepsilon_d$, decrease, while the dynamic Vsh-anisotropy, $\gamma_d$, is affected much less. The static anisotropy also decreases after saturation. (9) Both $V_p^{(dyn)}$ and $V_p^{(stat)}$ increase after saturation and with increasing pore pressure. However, the increase is more pronounced in the $V_p^{(stat)}$. (10) $V_s^{(dyn)}$ decreases after saturation and with increasing pore pressure. On the other hand, $V_s^{(stat)}$ increases both after saturation and with increasing pore pressure. (11) The increase of elastic moduli with confining pressure is much larger than the increase in the corresponding dynamic ones.

Thesis supervisor: M. Nafi Toksöz

Title: Professor of Geophysics
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Static versus dynamic bulk modulus in dry and saturation conditions

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4 Summary and Conclusions

References

A Dynamic elastic parameters in isotropic and transverse isotropic materials

(Dynamic, Theory)

Linearized theory of elasticity

Bounds on the stiffnesses and compliances. The strain energy

Relation between phase velocities and elastic parameters in isotropic media

Phase velocities in anisotropic media

Velocity anisotropies

B Isotropic and transverse isotropic materials under some simple stress systems

(Static, Theory)

B.1 Introduction

B.2 Isotropic materials

B.3 Transverse isotropic materials

Principal axes

Confining pressure, $P_c$

Uniaxial extension or compresion

Shear stress system

Interconversion of the stiffnesses and compliances

Measures of anisotropies

Representation of the elastic anisotropies by surfaces

C Non-linear, transverse isotropic materials

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Chapter 1

Introduction

Exploration seismology methods are often used to delineate rock interfaces in sedimentary basins. The use of seismic wave information for the determination of rock properties, or the direct detection of hydrocarbon has not been used extensively. It is necessary to establish relationships between the seismic properties of the sedimentary column and porosity, permeability, lithology, saturation, fluid properties, and pore pressure. If these relationships are established, they can be used to evaluate stratigraphic oil traps, fracture detection, spatial distribution of porosity and permeability, and the mechanical properties of the rock mass.

Direct measurement of mechanical properties \textit{in situ} is difficult. Nevertheless, experimental methods exist to obtain these properties, such as measurements of the stress-strain relationships (static) and elastic wave velocities (dynamic). In an ideal elastic medium these two techniques yield identical results. However, rocks are not ideal elastic materials, their stress-strain curves show nonlinearity, hysteresis, and sometimes permanent deformation, and the curves’ magnitudes and characteristics depend on the rock type. Different rock types are differentiated by mineralogy, grain size-shape spectra, pore-cracks density and size distribution, and fabric. The presence of porosity causes strain amplitude and frequency dependencies on the elastic coefficients. Consequently,
differences are noted between static and dynamic measurements. There are different strain amplitude and frequency-dependent effects that take place during a deformation process or during the perturbation produced by a traveling wave in a medium. The magnitude of the effects varies with the rock type, the physical state and scale, so it is not easy to extrapolate the behavior of the rock mass from the results of laboratory tests on small samples. In static measurements, the rocks are stressed at very low frequency (~$10^{-4}$ Hz) and large strain amplitude (commonly greater than $10^4$). Wave propagation methods cover a wide range of frequencies from laboratory measurements ($10^5$-$10^6$ Hz), well logging ($~10^4$ Hz), exploration seismology ($10$-$10^2$ Hz), and small strain amplitude ($10^{-6}$-$10^{-3}$).

Adams and Williamson (1923) attributed the nonlinearity of the strain-stress curves of crystalline rocks to the closure of thin, crack-like voids. The dependencies on the strain amplitudes observed in static measurements are attributed to frictional losses (Gordon and Davis, 1968; Mavko, 1979; Winkler, 1979). All rocks, even low porosity crystalline varieties, contain small cracks. A considerable reduction in magnitude of the elastic moduli is associated with the presence of these cracks. In the laboratory, Zisman (1933), Ide (1936), King (1969), Simmons and Brace (1965), Cheng and Johnston (1981), Jizba (1991), and Tutuncu et al. (1998), as well as, Don Leet and Ewing (1932) in the field, found that at, low confining pressures, the static moduli are generally smaller than the corresponding dynamic ones. The elastic properties approach those of the uncracked material as the cavities are closed by higher pressures (Walsh, 1965a). The presence of cracks affects the static and dynamic mechanical properties differently (Walsh and Brace, 1966). Walsh (1965a,b) and Cook and Hodgson (1965) showed, theoretically, that for cracked materials these differences are predictable.

Frequency dependent mechanisms are often ascribed to inertial and viscous losses in saturated rocks (Biot, 1956a,b; Usher, 1962; Mavko and Jizba, 1991; Wulff and Burkhardt, 1997; Tutuncu et al., 1998) although viscoelasticity of the rock frame may also be important in shales (Johnston, 1987). Global and local fluid flow have been considered the two main mechanisms explaining the influence of fluids on wave
velocities and attenuation. Biot's model (1956a,b) described the global fluid flow for fully saturated, porous material. The Biot theory is based on the viscous coupling of the fluid and the solid frame. At low frequencies, fluid moves with the frame and the Biot theory reduces to the theory of Gassmann (1951). At high frequencies the inertia of the fluid causes relative motion between fluid and frame. The consequent viscous flow causes velocity dispersion. The model of Murphy et al. (1986) describes the effect that local flow has on velocity and attenuation. At low frequencies the pressure will equilibrate, whereas at high frequency equilibrium is not possible and the rock is in an unrelaxed or undrained state. At high frequencies the rock looks stiffer than at low frequencies.

The anisotropy of sedimentary rocks has been recognized for about 70 years (McCollum and Snell, 1932). The anisotropy observed in surface seismic data may be produced by the combined effect of mineral grain orientation (Simmons and Wang, 1971), microstructure fabric (Jones, 1983), layering (Backus, 1962), and preferred oriented cracks (Nur and Simmons, 1969) at different scales, ranging from $\sim 10^{-8}$ m (mineral foliation) to $\sim 10$ m (thick layering).

Interpretation of seismic data requires an understanding of how elastic anisotropy affects the kinematics and dynamics of wave propagation. Two key steps in the analysis of surface seismic data that can be affected by anisotropy are (1) time-to-depth conversion using velocities derived from seismic data, and, (2) correlation of sonic log synthetic seismograms with surface seismic data. Lucas et al. (1980) demonstrated that elastic anisotropy causes large errors in computing layer thickness when velocities are determined from measurements made at the Earth's surface.

The presence of thick shale layers is associated with high elastic anisotropy (Banik, 1984). However, when the clays are disseminated, the observed anisotropy is not large, $< 10\%$ (Banik, 1984). Shales comprise about 70 percent of sedimentary basins. However, due to the friable nature of shales, there are very few laboratory measurements of velocity anisotropy (Kaarsberg, 1959; Podio et al. 1968; Jones and Wang, 1981; Lo et al. 1986).
In spite of the fact that shale formations exhibit high anisotropies, the study of fracture-related anisotropy has been more intensive, both theoretically (Brown and Korringan, 1975; Hudson, 1980, 1981, 1990, 1991, 1996a; Hudson et al, 1996b; Mukerji and Mavko, 1994; Thomsen, 1995) and experimentally (Jones, 1983; Lucet and Tarif, 1988; Zamora and Porier, 1990). This is most likely because if anisotropy is observed, it can be associated with fractures, which have a strong impact on permeability (Walls, 1983). Highly permeable rocks can be good oil reservoirs. However, in order to associate anisotropy with a fracture zone, a correction for the anisotropic effect of the upper layers of the sedimentary column that is traversed by the wave field is necessary. This correction is not an easy task; knowledge of the anisotropy of the whole column is fundamental.

Despite the numerous studies that have confirmed the anisotropic nature of sedimentary rocks, for the sake of simplicity rock is usually treated as an ideally elastic, isotropic material. It is necessary to investigate the effects of elastic and anelastic anisotropy and nonlinearity on the mechanical behavior of porous rocks at different scales. Recently, many relations already established between rock properties of isotropic rocks and seismic wave information have been extended for anisotropic material. At low frequencies, Brown and Korringa (1975) extended the Gassmann’s relations for anisotropic media. At high frequencies, Mukerji and Mavko (1994) presented a methodology for predicting the amount of local flow or “squirt” dispersion in anisotropic media. They pointed out that velocities and velocity anisotropy in sedimentary rocks, at low and high frequencies, can be significantly different. Tutuncu (1998) concluded that the anisotropic, anelastic behavior of sedimentary rocks is a strong function of frequency, strain amplitude, and the properties of the saturating fluid. Tutuncu (1998) also showed that Young’s moduli and Poisson’s ratio obtained from ultrasonic laboratory measurements, low-frequency measurements, and static measurements exhibit significant differences under identical stress conditions.
The purpose of this work is to study the dynamic and static anisotropic behavior of different sedimentary rock types as a function of confining pressure, pore pressure and saturation.

In Chapter II, we study: (1) the strong anisotropy exhibited by some nonfriable, microlaminated rocks (dolomitized calcareous shales); (2) the effect of saturation on the velocity anisotropies exhibited in sandstones; and (3) the anisotropic behavior of a fractured rock with an intrinsic anisotropic matrix (sandy shales). We used the ultrasonic transmission method to measure $V_{p}$-, $V_{sh}$-, and $V_{sv}$-wave velocities as functions of confining and pore pressure to 60 and 65 MPa, respectively.

The rock specimens tested were cored from 10 wells in the Budare Oil Field. This oil field, in the western extreme of the Great Oficina Oil Field, Budare, Venezuela, is a traditional field with shallow reservoirs between 4500 and 5000 feet, crudes with API grades greater than 30, and remnant reserves greater than 40 Mmbls (Mahmoudi and Rodriguez, 1995). In this field, several multidisciplinary studies are being conducted to test new tools and techniques to increase oil production. New seismic data, drilling and well logs, detailed sedimentologic studies, and a review of the production history support these studies. However, the discrepancies found in the correlation of synthetic seismograms from sonic logs with seismic sections (2D and 3D) make it difficult to interpret the sections and apply geological models that facilitate increased production with less risk.

In Chapter III, we study the effect of saturation on statically and dynamically-determined elastic properties and velocities. It can be ambiguous to interpret in situ seismic anisotropy without a complete understanding of the effects of pore fluid on the static and dynamic elastic properties. It is necessary to make this connection for the interpretation of seismic data and when trying to determine the static moduli of a rock mass from ultrasonic measurements. In this chapter III, we also study the static and dynamic velocity and modulus anisotropies as functions of confining pressures. We worked with rocks from the Toruno Oil Field, Barinas, Venezuela. The tests were performed on room
dry shale and dry and saturated sandstones cored from two different wells in the field. Pressures are varied from atmospheric conditions to in situ confining and pore pressure conditions. Simultaneously, velocities and strains are measured over a range of hydrostatic pressures and uniaxial stress exerted on specimens cored in three different directions. The elastic parameters and anisotropies obtained from uniaxial experiments, hydrostatic pressures and dynamic experiments are compared.
Chapter 2

Elastic anisotropy in sedimentary rocks (Dynamic measurements)

2.1 Introduction

The anisotropy of sedimentary rocks has been recognized for about 70 years (McCollum and Snell, 1932). The anisotropy observed in surface seismic data may be produced by the combined effect of mineral grain orientation (Simmons and Wang, 1971), microstructure fabric (Jones, 1983), layering (Backus, 1962), and preferred oriented cracks (Nur and Simmons, 1969) at different scales, ranging from $\sim 10^{-8}$ m (mineral foliation) to $\sim 10$ m (thick layering).

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The presence of thick shale layers is associated with high elastic anisotropy (Banik, 1984). However, when the clays are disseminated, the observed anisotropy is not large, < 10% (Banik, 1984). Shales comprise about 70 percent of sedimentary basins. However, due to the friable nature of shales, there are very few laboratory measurements of velocity anisotropy (Kaarsberg, 1959; Podio et al. 1968; Jones and Wang, 1981; Lo et al. 1986). In spite of the fact that shale formations exhibit high anisotropies, the study of fracture-related anisotropy has been more intensive, both theoretically (Brown and Korringan, 1975; Hudson, 1980, 1981, 1990, 1991, 1996a; Hudson et al, 1996b; Mukerji and Mavko, 1994; Thomsen, 1995) and experimentally (Jones, 1983; Lucet and Tarif, 1988; Zamora and Porier, 1990). This is most likely explained because when anisotropy is observed, it can be associated with fractures, which have a strong impact on permeability (Walls, 1983). Highly permeable rocks can be good oil reservoirs. However, in order to associate anisotropy with a fracture zone, a correction for the anisotropic effect of the upper layers of the sedimentary column that is traversed by the wave field is necessary. This correction is not an easy task; knowledge of the anisotropy of the whole column is fundamental.

The purpose of this work is to: (1) study the strong anisotropy exhibited by some nonfriable, microlaminated rocks (dolomitized calcareous shales); (2) study the effect of saturation on the velocity anisotropies exhibited in sandstones; and (3) study the anisotropic behavior of a fractured rock with an intrinsic anisotropic matrix (sandy shales). We used the ultrasonic transmission method to measure $V_p$, $V_{sh}$, and $V_{sv}$-wave velocities, as functions of confining and pore pressure to 60 and 65 MPa, respectively. We worked with rocks from the Budare Oil Field. This oil field, in the western extreme of the Great Oficina Oil Field, Budare, Venezuela, is a traditional field with shallow reservoirs between 4500 and 5000 feet, crudes with API grades greater than 30, and remnant reserves bigger than 40 Mmbls (Mahmoudi and Rodriguez, 1995). In this field, several multidisciplinary studies are being conducted to test new tools and techniques to increase oil production. New seismic data, drilling and well logs, detailed sedimentologic studies, and a review of the production history support these studies. However, the discrepancies found in the correlation of synthetic seismograms from sonic logs with
seismic sections (2D and 3D) make it difficult to interpret the sections and apply geological models that facilitate increased production with less risk.

2.2 Lithological description and stratigraphic sequences in the Budare Oil Field

Paleozoic sediments were deposited at the base of the Budare Oil Field stratigraphic column. During the Late Cretaceous, sandstones and shallow marine to continental shelf carbonates accumulated to about 2000 feet. In discordant contact with the Cretaceous sediments, Terciary and Quaternary carbonate shales accumulated to 5400 feet. The sedimentary environment varies from continental deltaic at the botton, to shallow marine in the middle, to continental deltaic at the top of the column (Mahmoudi and Rodriguez, 1995).

The main hydrocarbon reservoirs are found in the sandstones of the Oligocene Merecure Formation and Early and Middle Miocene Oficina Formation. The present study was carried out with cores from the bottom of Oficina Formation, which is at the top of the Merecure Formation’s sandstones. These sandstones belong to a fluvial to deltaic continental sedimentary environment, with local variations, which caused the characteristic geometry of the sandstones and shales.

We studied specimens plugged from cores of 10 wells in an area that contains several oil fields. X-ray diffraction and thin section analyses were performed on cores at two depth intervals of one of the wells. Figure 2.1 shows the variation of the specimens mineral content versus depth. An optical microscopic analysis of thin sections showed the presence of organic material, which was not differentiated by X-ray diffraction study of the rock specimens. The sedimentary column studied is characterized by three rock types: sandstones with varying clay content (shaley sandstone), sandy shale, and dolomitized shale (dolostone). The mineralogical and lithologic description of the specimens in the two selected intervals are summarized in Table 2.1. The distribution and percentage of
organic material were estimated from the thin section analyses. Clays were determined to be kaolinite and illite; however, they both were included as clay in Table 1.

2.3 Experimental procedure

Sample preparation and experimental methodology

The test specimens were prepared from larger cores (figure 2.3). Each test specimen had a diameter of $25.4 \pm 9$ mm. The specimens were cored using a diamond drill. The ends of the specimens were ground flat and parallel to $\pm 0.001$ mm/mm. Next, the samples were checked for flaws and defects that might produce undesirable effects in subsequent testing. Then, the specimens were dried in a vacuum oven at a $65^\circ$ C for 24 hours. Next, the dry mass of the specimens was measured with a digital balance. The dry bulk density was computed by dividing the volume by the mass of the sample. After completing the measurements in the dry state, the sandstone specimens were vacuum water saturated and the bulk density was computed.

One compressional and two orthogonal shear wave velocities were measured on each specimen as a function of pore and confining pressure. The measurements were carried out in an Autolab/1000 system designed and fabricated by New England Research, Inc. Each specimen was jacketed and secured between a matched set of ultrasonic transducers. The resonant frequency of the transducer is 1.0 MHz. Each polarization is sequentially propagated through the rock and each waveform is recorded. The first arrival of each waveform is determined from the data using the appropriate correction for the travel time through the transducer assembly. The experiments, on dry and saturated specimens, were conducted at effective confining pressures from 1 to 65 MPa.

To measure transverse isotropy completely, it is necessary to core specimens in three directions: vertical (X$_3$), horizontal (X$_1$ or X$_2$), and at an intermediate angle between X$_3$ and the plane of isotropy (X$_1$X$_2$) (figure 2.2). Nine velocities are measured, three in each direction. The particle polarizations and the direction of propagation of the three modes of propagation, P, $S_h$ and $S_v$, with respect to the layers, are shown in figure 2.3.
Shale and dolomitized shale specimens were tested only in room dry state because of the difficulty in saturating these rocks. Sandstone specimens were tested in room dry and full water saturation conditions.

The effective hydrostatic pressure (EP) is assumed to be the difference between confining pressure (CP) and pore pressure (PP). The pore pressure was kept constant while the confining pressure varied (drained regime).

For dry specimens, the confining pressure was varied from atmospheric pressure up to the reservoir effective pressure conditions. In saturated specimens, the effective pressure never exceeded the reservoir pressure. However, the pore pressure was varied from atmospheric pressure up to a pressure approximately 30% higher than the reservoir pore pressure conditions.

**Experimental Measurement of Phase Velocities**

The quasi-compressional wave phase velocity \( V_{qp} \), vertically polarized shear wave velocity \( V_{qsv} \), and horizontally polarized shear wave velocity \( V_{sh} \), in a transversely isotropic medium are given by (Musgrave, 1970)

\[
\rho V_{qp}^2 = C_{44} + \frac{1}{2}(h \cos^2 \theta + a \sin^2 \theta) + \frac{1}{2}(h \cos^2 \theta + a \sin^2 \theta)^2 - 4(a - d^2) \cos^2 \theta \sin^2 \theta, \tag{2.3.1}
\]

\[
\rho V_{qsv}^2 = C_{44} + \frac{1}{2}(h \cos^2 \theta + a \sin^2 \theta) - \frac{1}{2}(h \cos^2 \theta + a \sin^2 \theta)^2 - 4(a - d^2) \cos^2 \theta \sin^2 \theta, \tag{2.3.2}
\]

and

\[
\rho V_{sh}^2 = C_{44} \cos^2 \theta + C_{66} \sin^2 \theta \tag{2.3.3}
\]

where

\[ a = C_{11} - C_{44}, \quad h = C_{33} - C_{44}, \quad and \quad d = C_{13} + C_{44} \]
and $\theta$ is the angle measured from the symmetry axis, in this case $X_3$.

Using equations 2.3.1 to 2.3.3 we find that for hexagonal symmetry the relation of the phase velocities, in the vertical direction, horizontal direction, and at $45^0$ with respect to the plane of isotropy, and stiffness are given by

\[
C_{33} = \rho V_{qp}^2 (90^0),
\]

\[
C_{13} = -C_{44} + \sqrt{4 \rho^2 V_{qp}^4 (45^0) - 2 \rho V_{qp}^2 (45^0)(C_{11} + C_{33} + 2C_{44}) + (C_{11} + C_{44})(C_{33} + C_{44})},
\]

\[
C_{11} = \rho V_p^2 (90^0),
\]

\[
C_{44} = \rho V_{sv}^2 (0^0) = \rho V_{sh}^2 (0^0) = \rho V_{sv}^2 (90^0) \quad \text{and}
\]

\[
C_{12} = C_{11} - 2\rho V_{sh}^2 (90^0).
\]

In the dynamic experiments $C_{66}$ is obtained directly from $V_{sh}(90^0)$ as

\[
C_{66} = \rho V_{sh}^2 (90^0)
\]

These stiffnesses must satisfy the following restrictions (Nye, 1957):

\[
C_{44} > 0, \quad C_{11} > |C_{12}|, \quad \text{and} \quad (C_{11} + C_{12})C_{33} > 2C_{13}^2.
\]

As shown in Appendix B, using the five elastic stiffnesses $C_{ij}$, a bulk modulus, one vertical Young’s moduli, $E_3$, and one horizontal Young’s modulus $E_1$, and three dynamic Poisson’s ratios can be determined for a hexagonal material as follows:

\[
K = \frac{C_{33}(C_{11} + C_{12}) - 2C_{13}^2}{2C_{33} + C_{11} + C_{12} - 4C_{13}}.
\]
\[ E_1 = \frac{[C_{33}(C_{11} + C_{12}) - 2C_{13}^2](C_{11} - C_{12})}{C_{11}C_{33} - C_{13}}, \]  
\[ E_3 = \frac{C_{33}(C_{11} + C_{12}) - 2C_{13}^2}{C_{11} + C_{12}}, \]  
\[ \nu_{31} = \frac{C_{13}}{C_{11} + C_{12}}, \]  
\[ \nu_{12} = \frac{C_{33}C_{12} - C_{13}^2}{C_{11}C_{33} - C_{13}^2}, \]  
\[ \nu_{13} = \frac{C_{13}(C_{11} - C_{12})}{C_{11}C_{33} - C_{13}^2}. \]  

These dynamic Poisson’s ratios, \( \nu \), are indirect measure of the ratio of the lateral (\( \theta \)) to axial strains (\( \theta \)) when the uniaxial stress is applied in the direction \( X_i \).

Figure 2.4 shows the direction of propagation and polarization of the particles in the three oriented samples. We measured three velocities in each of the three samples cored at different orientations. A total of 9 velocities were obtained for each core at a given depth.

**Velocity Anisotropies**

As a measure of velocity anisotropy we introduce the notation suggested by Thomsen (1986):

\[ \varepsilon = \frac{C_{11} - C_{33}}{2C_{33}} = \frac{V_p^2(90^\circ) - V_p^2(0^\circ)}{2V_p^2(0^\circ)}, \]  

\[ 22 \]
\[
\gamma = \frac{C_{66} - C_{44}}{2C_{44}} = \frac{V_{S}^2(90^\circ) - V_{S}^2(0^\circ)}{2V_{S}^2(0^\circ)},
\]

(2.3.14)

and

\[
\delta = \frac{(C_{13} + C_{44})^2 - (C_{33} - C_{44})^2}{2C_{33}(C_{33} - C_{44})}
\]

(2.3.15)

where \(\varepsilon\) and \(\gamma\) represent measurements of anisotropy of P-wave velocity and S\(_h\)-wave velocity, respectively. \(\delta\) is a parameter that is useful in reflection velocity analyses because it describes weak anisotropy in transversely isotropic media, and it is almost totally independent of the horizontal velocities. As concluded by Banik (1987), variations in \(\delta\) describes, both variations in the moveout velocity and variations in the offset-dependent P-P reflection amplitude at short offsets, which are very important parameters in seismic exploration (Banik, 1987).

Thomsen (1986) pointed out that in cases of weak anisotropy, as those observed in sandstone specimens, an error in \(V_p(45^\circ)/V_p(0^\circ)\) is propagated into \(\delta\) magnified by a factor of 4. To reduce these errors, we precisely identified the plane of anisotropy in the specimens, and the polarizations \(S_v\) and \(S_h\) were also precisely oriented with respect to this plane, as shown in figure 2.4, as well as the very precise orientation of the sample at 45° with respect to the vertical axis.

Using equations 2.3.4, we compute 5 stiffnesses with only 5 of the 9 measured velocities. Subsequently, these stiffnesses are substituted in the same phase velocity equations 2.3.1 to 2.3.3. Making use of these equations, we compare the calculated versus measured velocities in the remaining directions.

The measured and computed velocities must satisfy the following relations:
\[ V_{sh}(0^\circ) = V_{sv}(0^\circ) \]

\[ V_{sv}(0^\circ) = V_{sv}(90^\circ) \]

\[ V_{sv}(45^\circ) = \frac{1}{2} \left[ \frac{(C_{33} + C_{11} + 2C_{44}) - \left( C_{33} - C_{11} \right)^2 + 4(C_{13} + C_{44})^2}{\rho} \right]^{1/2} \]

\[ V_{sh}(45^\circ) = \left[ \frac{C_{44} + C_{66}}{2\rho} \right]^{1/2} \]

(2.3.13)

We found that the largest differences between calculated and measured velocities is found for the Sv-wave velocities and less pronounced for the Sh-wave velocities when the sample is oriented at 45° with respect to the bedding.

2.4 Experimental Results

We selected three rock types that represent elastic and anisotropic behavior in the sedimentary column: sandy shales, immature sandstones, and dolomitized shales. Each of these rock types exhibit varying sensitivities to confining pressure, and as a consequence the anisotropies are affected differently as the confining pressure varies. All the rock specimens in this section corresponds to well-10 in table 2.3.

Figures 2.5 to 2.7 show the Vp-, Vsh- and Vsv-wave velocities in different directions for the three rock types. Using these velocities we compute 5 independent stiffnesses and plots as functions of pore and confining pressures (figures 8 and 9). Substituting these stiffnesses in equations 2.3.6 to 2.3.11, we compute the bulk moduli, the Young’s moduli and the Poisson’s ratios. The variation of these elastic parameters versus pressure is shown in figures 2.10 and 2.11.

Figure 2.12 shows the behavior of the elastic anisotropy for the different rock types as the confining pressure is increased.
The thin sections and X-ray diffraction analyses, together with the visually observed sedimentary structures, suggest that the anisotropy exhibited by the tested sedimentary rock specimens is due to preferred orientation of mineral grains constituents, sedimentary structure, pores and fractures.

**Dolomitized Shales**

The dolomitized shale, dolomitized calcareous shale, is made up of very well-cemented layers. Even though the process of dolomitization causes a loss of 6% to 13% volume, we find that these dolomitized shales have a porosity of approximately 1%. It appears that the dolomitization process did not disrupt the laminated depositional fabric of the original calcareous shale. These rock specimens, comprised of approximately 80% dolomite, show lineation of organic material parallel to the layering plane. Very few randomly oriented microcracks were observed in some specimens. This suggests that the strong velocity anisotropies observed in figure 2.12 is caused mainly by the micro-fine layering.

There is a linear increase in all velocities as the confining pressure is raised. The rate of increase is approximately the same for all velocities. Velocity hysteresis was not observed during the stepwise hydrostatic loading and unloading path. After finishing the test in the dry specimen, the sample was vacuum-saturated with water. We did not observe any significant variation in density, or physical changes. We infer that the slight monotonic increase in the elastic properties (figures 2.8 and 2.10) may be caused by the low-density organic material observed between the layers and by the randomly oriented and disconnected microfractures observed optically.

Figure 2.11 shows the variation of the Poisson's ratios ($v_{12}$, $v_{13}$ and $v_{31}$) as the confining pressure is increased. While $v_{12}$ remains approximately constant as the confining pressure is increased, $v_{13}$ and $v_{31}$ show a slight monotonic increase. The difference between $v_{13}$ and $v_{31}$ remains approximately constant over the whole range of frequency.
\( v_{12} \) is an indirect measure of the ratio of lateral strain \( (\varepsilon_2) \) to axial strain \( (\varepsilon_1) \) when the specimen is under uniaxial stress in the direction \( X_1 \). The high and almost constant value of \( v_{12} \) indicates that the rock is very stiff and linearly elastic in the plane \( X_1X_2 \), as consequence of the absence of voids and discontinuities in this plane.

\( v_{31} \) is an indirect measurement of the ratio of lateral strain \( (\varepsilon_1) \) to axial strain \( (\varepsilon_3) \) when the sample is subjected to uniaxial stress in the direction normal to the plane of isotropy \( (X_3) \). \( v_{31} \) is smaller than \( v_{12} \) because the specimen is softer in the vertical direction than in the horizontal direction. The vertical direction is perpendicular to the foliation plane. Between layers, there are discontinuities or void space filled with gas, liquid, or viscous organic material that increases the compliance of the rock in this direction. Thus the axial strain is larger than the lateral strain, resulting in \( v_{31} < v_{12} \). Consequently, the axial strain rate is also larger than the lateral strain rate, resulting in an increase in \( v_{31} \) as the pressure is increased.

\( v_{13} \) shows behavior similar to \( v_{31} \); however, the coefficient is larger because the strain in the \( X_1 \) direction is smaller than the strain in the \( X_3 \) direction when the rock is compressed in the \( X_3 \) direction.

Figure 2.12 shows the variation of velocity anisotropies as a function of confining pressure. While the velocity anisotropies, \( \varepsilon \) and \( \gamma \), remain approximately constant as pressure is increased, \( \delta \) increases. The P-wave anisotropy is about 18% and the Sh anisotropy is about 13%. The parameter \( \delta \) is negative and exhibits a consistent increase as confining pressure is raised.

**Sandy Shale**

This specimen is comprised mainly of quartz (39%) and clays (41%). The anisotropy is caused by the combined effect of cracks with a preferred orientation parallel to the isotropy plane and the fine layering or foliation. As the confining pressure increases, fractures close and there is an increase in contact area between foliation planes;
consequently the velocities and stiffnesses \((C_{ij})\) increase (figure 2.6). At high pressures the anisotropic behavior is determined mainly by the foliation in the shale specimen. The rate of decrease of velocity anisotropy is greater at low pressures, where the rock specimen is more sensitive to the stress amplitude and where the nonlinearity of the stress and strain relationship is stronger.

At low confining pressure, the \(V_p\)-anisotropy, \(\varepsilon\), is bigger than the \(S_h\)-anisotropy. Oriented fractures affect the \(V_p\)-anisotropy more than the \(S_h\)-anisotropy. When an \(S_h\)-wave propagates in the vertical and horizontal directions, the particle is polarized parallel to the plane of isotropy, and it always encounters more rigid material. However, when a \(P\)-wave propagates in the vertical and horizontal directions, the polarization of the particle and the direction of propagation are perpendicular and parallel to the plane of isotropy, respectively. The \(P\)-waves polarized in the vertical direction encounter softer material than the \(P\)-waves polarized in the horizontal direction. At higher confining pressures, when fractures close, \(V_p\) and \(V_s\) anisotropies approach the same value (figure 2.12). However, the anisotropies caused by preferred orientation of minerals, by fractures, and by layering are always due to the fact that the material is softer in the direction perpendicular to the direction of preferential orientation than in the direction parallel to it.

**Dry and saturated sandstone**

The sandstone is comprised of 70% quartz and 20% clay. At low confining pressures, the anisotropy is determined mainly by the preferred orientation of fractures parallel to the bedding plane. Figures 2.7, 2.9 and 2.10 show the pronounced increase in the velocities, stiffnesses, and bulk and Young’s moduli, for the room dry sandstone as the confining pressure is raised. At low pressures, the \(V_p\) anisotropy, \(\varepsilon\), is a little higher than the \(V_s\) anisotropy, \(\gamma\), and they tend to approach the same value at higher confining pressures. However, \(\delta\) shows a pronounced increase and then a decrease as the confining pressure is increased. At high pressures, the anisotropy is still controlled by the incomplete closure of the cracks.
After saturation, $V_p(0^\circ)$ and $V_p(45^\circ)$ increase, while $V_{sh}(0^\circ)$ ($= V_{sv}(0^\circ)$) decreases. $V_p(0^\circ)$ and $V_p(45^\circ)$ increase because the material is less compressible. A pore filled with fluid resists compression in a similar way when it is filled with a solid material. The difference between $V_p(0^\circ)$ for dry and for saturated sandstones decreases as the confining pressure is increased. However, the difference between $V_s(0^\circ)$ for dry and for saturated sandstones remains approximately constant. This decrease in $V_s(0^\circ)$ is caused by a significant increase in the bulk density, while $C_{44}$ remains approximately constant. This explains why the difference between $V_s(0^\circ)$-dry and $V_s(0^\circ)$-saturated does not change as the confining pressure is increased. $V_p(90^\circ)$ does not show a significant change after saturation. It does not change because the polarization of the particle and the direction of propagation are parallel to the plane of isotropy, and the fluid does not have a strong effect in this direction. Both $V_{sh}(90^\circ)$ and $V_{sv}(90^\circ)$ show the same decrease in value after saturation. In dry and saturated states, the difference between $V_{sh}(90^\circ)$ and $V_{sv}(90^\circ)$ remains approximately constant as the confining pressure is increased, caused by the greater increase in bulk density than the increase in $C_{66}$.

After saturation, $C_{33}$ shows a pronounced increase reaching the same value as $C_{11}$. $C_{44}$ remains approximately constant and $C_{66}$ shows a slight decrease.

After saturation, the P-wave anisotropy disappears ($\varepsilon = 0$). However, $\gamma$ shows only a slight increase and remains approximately constant as the effective pressure is increased.

The parameter $\delta$ shows a pronounced decrease after saturation. This parameter can be interpreted as the difference between P- and Sv- anisotropies. Because the P-wave anisotropy disappears after saturation, it may be interpreted as the $V_{sv}$-anisotropy in the saturated sandstone.
Vertical and lateral variation of the elastic properties

Figure 2.1 shows the vertical variation of the minerals that comprise the rock specimens in the selected depth intervals. Note that there is a strong variation of mineral composition with depth over a very short vertical interval. At the same time, in this sedimentary environment, fluvial to deltaic, there is a strong vertical variation caused by staking of deltas, and a strong lateral variation caused by change in clastic facies. Consequently, the elastic properties are expected to exhibit strong variations locally both vertically and horizontally. Table 2.3 shows the stiffnesses computed from velocity measurements in different rock types cored from different depths in 10 wells. The wells are randomly distributed in an area that includes several oil fields. For shales and dolomitized shales, we studied the anisotropic behavior as a function of confining pressure, and for the sandstone specimens as a function of confining pressure and pore pressures as well. The three rock type specimens formerly studied describe the typical behavior exhibited by all rocks in the area. Table 2.3 shows the stiffnesses at reservoir pressure conditions. For the sandstones, in most cases, the specimens were studied in dry and saturated conditions. However, shale and dolomitized shale specimens were only studied in dry conditions.

The data in Table 2.3 highlights some general features which are dependent on the rock types and which can be used for building an elastic model and for calibration of anisotropic velocity analyses, geological interpretation of seismic data, modeling purposes, and other possible uses in oil production.

1. The elastic stiffnesses of shales and sandy shales increase only slightly as the confining pressure is increased. Thick formations of massive shale are commonly in found in sedimentary basins. As the shale formation or layer deepens, the elastic properties do not show strong variations. Thus the elastic properties of a few specimens can be extrapolated to the whole formation. For stiffer anisotropic rocks, like the dolomitized shale specimens we discussed earlier, this extrapolation can be made.
2. Sandy shales are also commonly found in sedimentary basins. The anisotropy of these rocks is produced by the combined effect of fractures and fine layering or foliation. At low pressure, in the upper part of the sandy shale formation, the preferred orientation of not completely closed fractures contributes strongly the velocity anisotropies. In the lower part of the formation, where fractures are closer because of the higher pressures, the fine layering typical of shales mainly causes the anisotropy.

3. The Vp- and Vsh-velocity anisotropy, $\epsilon$ and $\gamma$, decrease as the confining pressure increases for all rock types, but the rate of decrease is more pronounced in rocks with higher fracture density. However, the behavior of the parameter $\delta$ as pressure increases is more complicated because its behavior is highly dependent on the behavior of the difference between P and Sv-wave anisotropies. At low confining pressure $\epsilon$ is higher than $\gamma$ and they approach approximately the same value in the lower part of the formation, where the confining pressure is higher. The behavior of $\epsilon$ and $\gamma$ was not studied at pressures higher than the reservoir confining pressures.

4. The anisotropy in P-waves generally decreases after saturation. For some sandstone specimens, the rock specimens do not show P-wave anisotropy. The fluid distribution in sedimentary sequences depends on porosity, permeability and pore pressure gradients. A rock that shows similar sedimentary structure and mineral composition may be found in gas saturated conditions, fluid full or partial saturated conditions, and under different pore and confining pressures. Thus a spatial variation of anisotropy is expected.

If the sedimentary column is well sampled, each rock specimen can be considered as an anisotropic layer and an equivalent media of this finely layered anisotropic media can be built for the purpose of modeling and seismic imaging. Care needs to be taken in these measurements in order to do reliable seismic migrations.

As pointed out by Mukerji and Mavko (1994), the velocity anisotropies obtained at low frequencies, as in static or seismic exploration, may be different than those obtained from laboratory experiments at ultrasonic frequencies. Winkler (1986) found that dispersion
between zero frequency and ultrasonic frequencies is on the order of 10 percent at low effective stress, and it decreases to only a few percent at higher stresses. Extending the work done by Winkler (1986) to anisotropic media an extrapolation of the laboratory data to seismic frequencies can be made.

2.5 Conclusions

(1) The large velocity anisotropies \((\varepsilon, \gamma, \delta)\) exhibited by the dolomitized calcareous shales may be caused by lamination not visible at an optical microscopic scale. An electron microscope should be used to find the orientation of the assumed platy dolomite grains and to verify our assumption about its sedimentary origin.

(2) After saturation, the P-wave anisotropy in fractured sandstones shows a pronounced decrease; however, the \(S_h\)-wave anisotropy is only slightly affected.

(3) The velocity anisotropies measured in uncracked rock specimens might be extrapolated to the whole formation, based on the assumption that their intrinsic anisotropies do not exhibit a significant change with confining pressure. However, before any extrapolation to low seismic frequencies a correction due to velocity dispersion should be made.

(4) In the depth intervals studied, the data suggest that cracks in sandy shales and sandstones are not always completely close at the oil reservoir effective pressure conditions.
Table 2.1 The physical descriptions of the rock specimens are shown in the upper table, the X-ray diffraction mineralogical description is shown in the middle table, and the thin section lithological descriptions are in the lower table. Dol.Shale = Dolomitized shale.
Table 2.2 Experimental P-, Sh-, Sv-wave velocities for: dry and saturated sandstone, sandy shale, and dolomitized shale. Vs_d and Vs_s are the S-wave velocities in the vertical direction from two perpendicular polarized S-wave transducers. Notice that Vs_d ≈ Vs_s.
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<th>Density (sat.)</th>
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<th>C_{12}</th>
<th>C_{33}</th>
<th>C_{44}</th>
<th>C_{13}</th>
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<th>γ (%)</th>
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<td></td>
<td>Dol.Shale</td>
</tr>
<tr>
<td>Well-10</td>
<td>4494</td>
<td>2.39</td>
<td>3.51</td>
<td>0.69</td>
<td>2.33</td>
<td>0.99</td>
<td>1.41</td>
<td>25</td>
<td>21</td>
<td></td>
<td></td>
<td>Shale</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.3 Stiffnesses in dry and saturated specimens at reservoir pressure conditions for rock specimens from 10 oil wells in the area. (*) Sandstone specimens tested in dry and saturated conditions. ϕ is the porosity and ε and γ are the Vp-anisotropy and the Vsh-anisotropy, respectively.
Figure 2.1 Mineralogy of the sandstones, on the left, and mineralogy of the shales and the dolomitized shales, on the right.

Figure 2.2 Three specimens in three different directions are necessary to study the elastic anisotropy in a hexagonal material. A vertical sample along $X_3$, a horizontal sample along $X_1$ or $X_2$, and an inclined sample oriented at $45^\circ$ with respect $X_3$. 
Figure 2.3 Nine velocities are measured in the experiment. The solid lines indicate the directions of propagation and the dashed lines the polarization of the particle.

Figure 2.4 Generic wave front showing the phase (\( V \)) angle and the ray angle (\( \mu \)).
Figure 2.5 Measured Vp-, Vsh-, and Vsv-wave velocities in different directions in the dolomitized shale.

Figure 2.6 Measured Vp-, Vsh-, and Vsv-wave velocities in different directions in the shale specimens.

Figure 2.7 Measured Vp-, Vsh, and Vsv-wave velocities in different directions in the room dry sandstone specimen.
Figure 2.8 Stiffnesses for the shale specimens on the left and for the dolomitized shale specimens on the right.

Figure 2.9 Stiffnesses for room dry and saturated sandstone specimens. The dashed curves show the stiffnesses in saturation conditions.

Figure 2.10 Young's and bulk moduli for dry and saturated sandstones on the left and for the dolomitized shale on the right. The dashed lines show the moduli in saturation conditions.
Figure 2.11 Poisson's ratios for dry and saturated sandstone specimens on the left and for room dry dolomitized shale specimens on the right. The dashed lines show the Poisson's ratios in saturated conditions.

Figure 2.12 Velocity anisotropies for dry and saturated sandstones on the left, for the shale in the middle and for the dolomitized shale on the right. The dashed lines show the anisotropies after saturation.

Figure 2.13 $V_p$, $V_{sh}$-, and $V_{sv}$-wave velocities in dry and saturated sandstones, versus effective pressure. For saturated sandstones. The pore pressure was kept constant at 52 Mpa, while confining pressure was varied.
Chapter 3

Static and dynamic elastic parameters in sedimentary anisotropic rocks

3.1 Introduction

Determining rock mechanical properties \textit{in situ} is important in many applications in the oil industry such as reservoir production, hydraulic fracturing, estimation of recoverable reserves, and subsidence. Direct measurement of mechanical properties \textit{in situ} is difficult. Nevertheless, experimental methods exist to obtain these properties, such as measurements of the stress-strain relationships (static) and elastic wave velocities (dynamic). In an ideal elastic medium these two techniques yield identical results. However, rocks are not ideal elastic materials; their stress-strain curves show nonlinearity, hysteresis, and sometimes permanent deformation, and the curves’ magnitudes and characteristics depend on the rock type. Different rock types are differentiated by mineralogy, grain size-shape spectra, pore-cracks density and size distribution, and fabric. The presence of porosity causes strain amplitude and frequency dependencies on the elastic coefficients. Consequently, differences are noted between static and dynamic measurements. There are different strain amplitude and frequency-dependent effects that take place during a deformation process or during the perturbation produced by a traveling wave in a medium. The magnitude of the effects varies with the
rock type, the physical state and scale, so it is not easy to extrapolate the behavior of the rock mass from the results of laboratory tests on small samples. In static measurements, the rocks are stressed at very low frequency (~$10^{-4}$ Hz) and large strain amplitude (commonly greater than $10^{-4}$). Wave propagation methods cover a wide range of frequencies from laboratory measurements ($10^5$-$10^6$ Hz), well logging ($~10^4$ Hz), exploration seismology ($10^2$ Hz), and small strain amplitude ($10^{-6}$-$10^{-8}$).

Adams and Williamson (1923) attributed the nonlinearity of the strain-stress curves of crystalline rocks to the closure of thin, crack-like voids. The dependencies on the strain amplitudes observed in static measurements are attributed to frictional losses (Gordon and Davis, 1968; Mavko, 1979; Winkler, 1979). All rocks, even low porosity crystalline varieties, contain small cracks. A considerable reduction in magnitude of the elastic moduli is associated with the presence of these cracks. In the laboratory, Zisman (1933), Ide (1936), King (1969), Simmons and Brace (1965), Cheng and Johnston (1981), Jizba (1991), and Tutuncu et al. (1998), as well as, Don Leet and Ewing (1932) in the field, found that at, low confining pressures, the static moduli are generally smaller than the corresponding dynamic ones. The elastic properties approach those of the uncracked material as the cavities are closed by higher pressures (Walsh, 1965a). The presence of cracks affects the static and dynamic mechanical properties differently (Walsh and Brace, 1966). Walsh (1965a,b) and Cook and Hodgson (1965) showed, theoretically, that for cracked materials these differences are predictable.

Frequency dependent mechanisms are frequently ascribed to inertial and viscous losses in saturated rocks (Biot, 1956a,b; Usher, 1962; Mavko and Jizba, 1991; Wulff and Burkhardt, 1997; Tutuncu et al., 1998) although viscoelasticity of the rock frame may also be important in shales (Johnston, 1987). Global and local fluid flow have been considered the two main mechanisms explaining the influence of fluids on wave velocities and attenuation. Biot's model (1956a,b) described the global fluid flow for fully saturated, porous material. The Biot theory is based on the viscous coupling of the fluid and the solid frame. At low frequencies, fluid moves with the frame and the Biot theory reduces to the theory of Gassmann (1951). At high frequencies the inertia of the
fluid causes relative motion between fluid and frame. The consequent viscous flow causes velocity dispersion. The model of Murphy et al. (1986) describes the effect that local flow has on velocity and attenuation. At low frequencies the pressure will equilibrate, whereas at high frequency equilibrium is not possible and the rock is in an unrelaxed or undrained state. At high frequencies the rock looks stiffer than at low frequencies.

Most rocks found in sedimentary basins are found to exhibit elastic anisotropy. At low frequencies Brown and Korringa (1975) extended the Gassmann’s relations for anisotropic media. At high frequencies, Mukerji and Mavko (1994) presented a methodology for predicting the amount of local flow or “squirt” dispersion in anisotropic media. Mukerji and Mavko (1994) pointed out that velocities and velocity anisotropy in sedimentary rocks, at low and high frequencies can be significantly different.

The distribution of porosity, clay content mineral and quartz cement affects the velocities and mechanical properties (Klimentos and McCann, 1990; Jizba, 1991; Tutuncu 1994).

The purpose of this work is to: (1) study the effect of saturation on the statically and dynamically determined elastic properties and velocities. It can be misleading to interpret in situ seismic anisotropy without a complete understanding of pore fluid effect in the static and dynamic elastic properties. It is fundamental to make this connection for the interpretation of seismic data and when trying to determine the static moduli of a rock mass from ultrasonic measurements. (2) study the static and dynamic velocity and modulus anisotropies, as functions of confining pressures. The tests were performed on room dry shale and dry and saturated sandstones cored from two Venezuelan oil wells. Pressures are varied from atmospheric conditions to in situ confining and pore pressure conditions. Simultaneously, velocities and strains are measured over a range of hydrostatic pressures and uniaxial stress exerted on specimens cored in three different directions. The elastic parameters and anisotropies obtained from uniaxial experiments, hydrostatic pressures and dynamic experiments are compared.
3.2 Elastic Anisotropy

In an isotropic material only two elastic constants are needed to specify the stress-strain relation completely (Timoshenko and Goodier, 1934). If the two parameters are the Young’s modulus \( E \) and Poisson’s ratio \( v \), Hooke’s law can be expressed as

\[
\varepsilon_{ij} = C_{ijkl} \sigma_{kl} = \frac{1}{E} \left[ (1 + v) \varepsilon_{ij} - v \delta_{ij} \sigma_{kk} \right].
\]  

(3.2.1)

The two elastic constants can be obtained from a stress-strain experiment (static test) or from a wave propagation experiment (dynamic test). From a single, uniaxial stress test, \( E_{\text{(stat)}} \) and \( v_{\text{(stat)}} \) are defined and obtained as follows:

\[ E_{\text{(stat)}} \] is the rate of change of axial stress with axial strain at any particular stress,

\[
E_{\text{(stat)}} = \frac{d\sigma_{\parallel}}{d\varepsilon_{\parallel}}.
\]  

(3.2.2)

\[ v_{\text{(stat)}} \] is the rate of change of lateral strain with axial strain at any particular stress (Walsh, 1965b),

\[
v_{\text{(stat)}} = \frac{d\varepsilon_{\perp}}{d\varepsilon_{\parallel}}.
\]  

(3.2.3)

The definitions 3.2.2 and 3.3.3 are advantageous because nonlinear stress-strain relationships are considered.

From dynamic experiments, instead of measuring stress-strain relationships, we measure the velocity of compressional and shear waves, \( V_p \) and \( V_s \), respectively. The relations between \( V_p \) and \( V_s \)-velocities and the Poisson’s ratio and the Young’s modulus are given by,
\[
\nu = \frac{1}{2} \frac{\left( \frac{V_p}{V_s} \right)^2 - 2}{\left( \frac{V_p}{V_s} \right)^2 - 1}
\]
and

\[
E = \rho V_t^2 \left( 3 \frac{\left( \frac{V_p}{V_s} \right)^2 - 4}{\left( \frac{V_p}{V_s} \right)^2 - 1} \right). \tag{3.2.5}
\]

Other useful elastic parameters that can be obtained from the dynamic or static tests are the bulk and shear moduli, \( K_b \) and \( G \), respectively. \( K_b \) and \( G \) can be determined as follows:

\[
G = \frac{E}{2(1 + \nu)} = \rho V_t^2 \quad \text{and} \quad \tag{3.2.6}
\]

\[
K = \frac{E}{3(1 - 2\nu)} = \rho \left( V_p^2 \frac{1}{3} \frac{4}{3} V_t^2 \right). \tag{3.2.7}
\]

The bulk modulus \( (K_b) \) can be also determined from a hydrostatic loading. \( K_b \) can be defined and obtained as follows,

\[
K_{b(\text{stat})} = \frac{O_h}{\varepsilon_b} \tag{3.2.8}
\]

where the volumetric strain is given by \( \varepsilon_b = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 3\varepsilon_1 \).
For an ideal elastic material, the elastic parameters obtained from hydrostatic compression, uniaxial stress, or wave propagation experiments are equivalent. However, in rocks the presence of porosity causes the elastic parameters obtained from the different tests to be different. Strain amplitude and frequency dependent mechanisms will appear. The anelastic part of the deformation process becomes as important as the elastic part. The resulting elastic parameters from different tests will depend on (1) the mode a material is deformed, for both dynamic and static tests; (2) the strain or stress amplitudes; and (3) the velocity of the deformation.

In a transverse isotropic material 5 elastic constants are needed to specify the stress-strain relationship completely (Appendix B). These elastic constants can be expressed as

\[
C_{33} = \frac{(1 - \nu_{12})E_3}{(1 - \nu_{12} - 2\nu_{13}\nu_{31})},
\]

\[
C_{13} = \frac{\nu_{31}E_1}{(1 - \nu_{12} - 2\nu_{13}\nu_{31})},
\]

\[
C_{11} = \frac{E_1(1 + \nu_{12}) - \nu_{13}^2E_3}{(1 - \nu_{12} - 2\nu_{13}\nu_{31})(1 + \nu_{12})},
\]

\[
C_{44} = \frac{1}{4} \frac{1}{E(45°)} \frac{1}{E_1} \frac{1}{E_3},
\]

and

\[
C_{12} = \frac{(\nu_{13}\nu_{31} + \nu_{12})E_1}{(1 - \nu_{12} - 2\nu_{13}\nu_{31})(1 + \nu_{12})}.
\]

where $E_1$ and $E_3$ are the Young's moduli in the horizontal ($X_1$ or $X_2$) and vertical directions ($X_3$).

$\nu_{12}$ is the ratio of $\epsilon_1$ to $\epsilon_2$, when a uniaxial stress is applied in the direction $X_1$.  

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\[ \nu_{13} \] is the ratio of \( \sigma_1 \) to \( \varepsilon_3 \), when a uniaxial stress is applied in the direction \( X_1 \).

\[ \nu_{31} \] is the ratio of \( \varepsilon_3 \) to \( \varepsilon_1 \), when a uniaxial stress is applied in the direction \( X_3 \).

To facilitate subsequent discussions, the direction \( X_3 \) in a Cartesian system will be called “vertical direction or axis of symmetry,” and the plane \( X_1X_2 \), “horizontal direction, bedding plane or plane of isotropy.”

If the moduli \((C_{ij})\) or any other elastic parameters are determined from stress-strain tests, we call them “static stiffnesses or static elastic parameters.” If the velocities are determined from the static moduli, we call them “static velocities.”

In a transversely anisotropic material there are, in general, three modes of propagation (quasi longitudinal, quasi shear and pure shear) with mutually orthogonal polarizations. For an arbitrary direction of propagation there are three independent modes of propagation, corresponding to three directions of particle displacement which are mutually orthogonal, but none are orthogonal to the direction of propagation nor coincident with it. The problem is simplified considerably when the direction of wave propagation is in particular material directions in which “pure modes” (longitudinal or shear) are obtained. Then, a transducer vibrating either longitudinally or transversely generates a wave in only one of the three modes of propagation. However, in general, it is not possible to obtain all of the elastic constants with the use of pure modes alone. In hexagonal systems \( C_{33}, C_{44}, C_{11}+C_{12}, \) and \( C_{11}-C_{12} \) may all be obtained by pure wave propagation perpendicular and parallel to the plane of isotropy (the bedding plane). On the other hand, \( C_{13} \) can only be obtained by using wave propagation in a direction for which coupling occurs. In loading cyclic experiments this coupling between modes also exists if the stress is introduced in a direction different than the principal axis.

In a transverse isotropic material the wave slowness surface is always symmetric about the axis of symmetry \((X_3)\). The relation between the the quasi-compressional wave phase velocity, \( V_{qp} \), vertically polarized shear wave velocity, \( V_{qsv} \), horizontally polarized shear
wave velocity, $V_{sh}$, and the stiffness constants in a transversely isotropic material are given by (Appendix A)

$$C_{33} = \rho V_s^2 (90^\circ),$$

$$C_{13} = -C_{44} + \sqrt{4\rho^2 V_s^4 (45^\circ) - 2\rho V_s^2 (45^\circ)(C_{11} + C_{33} + 2C_{44}) + (C_{11} + C_{44})(C_{33} + C_{44})},$$

$$C_{11} = \rho V_p^2 (90^\circ),$$

$$C_{44} = \rho V_s^2 (0^\circ) = \rho V_{sh}^2 (0^\circ) = \rho V_{sv}^2 (90^\circ),$$

and

$$C_{12} = C_{11} - 2\rho V_{sh}^2 (90^\circ). \tag{3.2.10}$$

Other elastic parameters that are useful in the interpretation of elastic anisotropy are the bulk modulus, $K_b$, and the inverse of linear compressibilities in the vertical and horizontal direction, $K_{l1}$ and $K_{l3}$, defined as follows

$$K_b = \frac{1}{2\beta l_1 + \beta l_3},$$

$$K_{l1} = \frac{E_1}{1 - \nu_{12} - \nu_{13}} = \frac{C_{33}(C_{11} + C_{12}) - 2C_{13}^2}{C_{33} - C_{13}}, \tag{3.2.11}$$

$$K_{l3} = \frac{E_3}{1 - 2\nu_{31}} = \frac{C_{33}(C_{11} + C_{12}) - 2C_{13}^2}{C_{11} + C_{12} - 2C_{13}}.$$

The static bulk modulus and the inverse of linear compressibilities may be obtained from hydrostatic compression or from uniaxial stress experiments.
3.3 Experimental procedure

3.3.1 Description of the rock specimens

In this study we worked with 2 sets of rocks from two Venezuelan oil wells (Table 3.1). Set I is comprised of transverse isotropic shale specimens and set II of isotropic sandstone specimens. We cored shale specimens in three different orientations. The characteristics and description of the rock specimens tested are summarized in Tables 3.1 and 3.2.

3.3.2 Sample preparation

For transverse isotropic material, it is necessary to core cylindrical specimens in three different orientations: vertical (X₃), horizontal (X₁ or X₂) and at an intermediate angle between X₃ and the plane of isotropy, X₁X₂, (figure D.1).

The specimens to be tested were prepared from larger pieces of cores (figure D.1). Each test specimen had a diameter of about 38.10 mm. The specimens were cored using a diamond drill. The ends of the specimens were ground flat and parallel to ± 0.025 mm/mm. Next, the samples were checked for flaws and defects that might produce undesirable effects in subsequent testing. Then, the specimens were dried in a vacuum oven at a temperature of 65°C for 24 hours. Next, the dry mass of the specimens was measured with a digital balance. The dry bulk density was computed by dividing the volume by the mass of the sample. After completing the measurements in the dry state, the sandstone specimens were vacuum saturated with 20,000 ppm sodium chloride brine solution, and the bulk densities of the saturated specimens were computed.
3.3.3 Sample instrumentation

Measurements of the principal strains, \( \varepsilon_1 \), \( \varepsilon_2 \) and \( \varepsilon_3 \), from static experiments and velocities of waves with particle polarizations parallel to the main axes of the material are needed to characterize the material elastically (Appendix B).

The sandstone and shale specimens obtained at different orientations were analysed with strain gauges to measure strain as a function of differential axial stress and confining pressure (figures C.3 to C.5). Simultaneously, P-, Sv-, and Sh-wave velocities were measured (figure D.2).

Each cylindrical specimen was jacketed with a 0.013 mm thick soft copper jacket. The copper jacket was seated to the sample by pressurizing the sealed core and jacket assembly to 21 MPa. Subsequently, the copper was then cleaned with acetone. For the vertically oriented specimens a minimum of two strain gauges were epoxied to the jacket: one parallel (axial) and one normal (tangential) to the axis of symmetry (figure 3.2). For horizontally and 45°-oriented specimens a minimum of three gauges were applied: one parallel to the axis of the specimen, one perpendicular to the axis of symmetry, and one parallel to the plane of isotropy (figures C.4 and C.5). The specimens were jacketed and secured between a matched set of one P- and two perpendicular polarized S-wave ultrasonic velocity transducers, which are housed in two titanium hollow end pieces, as described in Chapter 2. The assembly was placed in a pressure vessel where a confining pressure cycle of 20 MPa was applied to seat the sample in the vessel to calibrate and set the necessary electronic filtering and to check the strain-stress response. In the vertically and horizontally oriented samples, the polarizations and direction of propagation of the waves and the direction of uniaxial deformation correspond to the main axes of the hexagonal system (figures C.3 and C.5). The strains are measured parallel to the particle polarizations P, Sv, and Sh for the three oriented specimens. In the dynamic experiment we need only five velocity measurements to obtain the five stiffnesses of the hexagonal system. In the statics experiments, under uniaxial stresses, we only need to measure 6
strains: 2 from the vertical sample ($\varepsilon_1 = \varepsilon_2$ and $\varepsilon_3$), three from the horizontal sample ($\varepsilon_1$, $\varepsilon_2$, and $\varepsilon_3$), and one at $45^\circ$ ($\varepsilon(45^\circ)$).

### 3.3.4 Test Procedure

Static and dynamic moduli were computed as a function of pore and confining pressures to reservoir conditions. A diagram of the loading sequence is shown in figure 3.1. The experiment is subdivided into three parts. In the first part, the specimen was stepwise loaded to 70 MPa and unloaded. At each step or pressure level, $P_-$, $S_v$, and $S_h$ wave velocities were measured. The object of this cycle is to measure wave velocities and compressibilities simultaneously. In the second part, the specimen was monotonically hydrostatically loaded to a pressure of 70 MPa and unloaded at a constant rate without interruption. The object of this cycle is to measure compressibilities as a continuous function of pressure. The specimen is then hydrostatically reloaded to a pressure of 10 MPa. Subsequently, a differential axial stress cycle was applied on the specimen keeping the effective confining pressure constant at three different levels, 10, 35, and 60 MPa. $P_-$, $S_v$, and $S_h$-wave velocities were measured at the peak of the axial stress-strain curve and at the completion of the axial cycle. The same stress path is applied on the specimens cored in the three directions. The experiment lasted approximately 4000 sec. The characteristics of the shale and sandstone specimens tested in the experiments are shown in Table 3.1.

Shale specimens were tested only in a room dry state because of the difficulty of saturating these rocks. Sandstone specimens were tested in room dry and fully brine saturated conditions.

The effective hydrostatic pressure (EP) is assumed to be the difference between confining pressure (CP) and pore pressure (PP). The pore pressure was kept constant while the confining or axial stress was varied (drained regime). The velocities ($V_p$ and $V_s$) and density of the rock specimens were corrected for changes in sample length and
volumetric strain as pressure was varied. The changes in the dimension of the specimens were determined from the strain measurements in different directions.

In order to compare the static and the dynamic elastic moduli, it is necessary to consider small axial differential stress excursions. The differential stresses for these measurements was less than 41 MPa.

3.3.5 Determination of the elastic and dynamic velocities and elastic parameters

The static moduli \( C_{ij(\text{stat})} \) were determined by substituting the strain-stress data, obtained from uniaxial stress tests, into equations 3.2.2 and 3.2.3 for the isotropic sandstone specimens and into equations 3.2.9 for the transverse isotropic shale specimens (Appendices A, B, and C). Two different sets of static linear and volumetric compressibilities were computed: one from the uniaxial stress tests and one from a hydrostatic compression test (Appendix B). The static velocities were determined by substituting the static moduli \( C_{ij(\text{stat})} \) into equations 3.2.11 (Appendix A).

The dynamic stiffnesses \( C_{ij(\text{dyn})} \) were determined by substituting 5 of the 9 experimental velocities measured into equations 3.2.10 for transverse isotropic rocks. For the isotropic sandstones, the \( V_p \) and \( V_s \)-dynamic velocities were substituted into equations 3.2.4 and 3.2.5 to obtain the dynamic isotropic elastic parameters.

The static moduli and the compressibilities are computed from the stress-strain curve (Appendix C) by using the slopes obtained from piece wise linear least square fits. We used linear least square fits for 10 to 20 samples to find the following: volumetric strain versus confining pressure for the bulk compressibility \( K_b \); vertical \( (e_3) \) and horizontal \( (e_1) \) linear strains versus confining pressures for the vertical and horizontal linear compressibilities \( K_{13} \) and \( K_{31} \), axial stress versus axial strain for the Young’s moduli \( \left( E_1, E_3 \text{ and } E(45^{\circ}) \right) \), and lateral and axial strains versus axial strains for Poisson’s ratios \( (\nu_{12}, \nu_{13}, \text{ and } \nu_{31}) \).
3.3.6 Time dependence

Strain-stress analysis must take into account time and space scale effects. If the same rock is driven with different stress functions they may show different elastic responses. The same rock may show an elastic response at a given range of frequencies and a viscoelastic response at another range of frequencies. In general, it is not possible to describe all the deformations at different scales with the same physical laws.

As shown in figure 3.1, we used linear functions for both hydrostatic and uniaxial stress. For linear stress functions (uniaxial or hydrostatic), the strain rate ($\dot{\varepsilon}$) and linear or volume compressibilities ($\beta$) are related as follows:

$$\sigma = t\sigma_0 = K\varepsilon = \frac{\varepsilon}{\beta}. \quad (3.3.6.1)$$

If we derive equation 3.3.6.1 with respect to time, we obtain

$$\dot{\varepsilon} = \dot{\sigma}_0 (\beta + t \frac{d\beta}{dt}) \quad (3.3.6.2)$$

where $\sigma_0$ is a constant stress rate and $\dot{\varepsilon}$ may be the horizontal, vertical or volumetric strain rate, $\varepsilon_1$, $\varepsilon_3$, and $\varepsilon$, respectively. $\beta$ may be the horizontal or vertical linear compressibilities or volumetric compressibility, $\beta_1$, $\beta_2$, $\beta_3$, respectively. Note that if $\dot{\beta} = 0$, the strain rate $\dot{\varepsilon}$ is linearly proportional to $\beta$. However, if frequency dependent mechanisms take place during the deformation process $\dot{\beta} \neq 0$. 

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3.4 Experimental results

3.4.1 Dynamic and static elastic behavior of dry and saturated, isotropic, homogeneous sandstones

In this section we work with homogeneous, isotropic sandstone specimens in dry and 20,000 ppm brine saturated conditions at two different pore pressures (2 MPa and 40 MPa) and versus effective pressure up to \textit{in situ} conditions. The characteristics and descriptions of the sample are shown in Tables 3.1 and 3.2.

In order to facilitate subsequent discussions we introduce the following notation:

(1) Ratios of dynamic to static elastic parameters,

\[
R_P(\theta) = \frac{V_p(\theta) - \text{static}}{V_p(\theta) - \text{dynamic}}. \tag{3.4.1}
\]

\(R_P(\theta)\) is the ratio of dynamic to static P-wave velocity as a function of direction.

\[
R_C(\theta) = \frac{C_C(\theta) - \text{static}}{C_C(\theta) - \text{dynamic}}. \tag{3.4.2}
\]

\(R_C(\theta)\) is the ratio of dynamic to static \(C_{11}\) as a function of direction.

\[
R_E(\theta) = \frac{E(\theta) - \text{static}}{E(\theta) - \text{dynamic}}. \tag{3.4.3}
\]

\(R_E(\theta)\) is the ratio of dynamic to static Young's modulus, \(E\), as a function of direction.

In general the ratio, \(R_M(\theta)\), may be represented by surfaces and be useful for studying the anisotropic behavior of rocks.
As a measure of static and dynamic velocity anisotropies we use Thomsen’s notation:

\[ \varepsilon , \gamma , \text{ and } \delta \text{ are the static velocity anisotropies.} \]

\[ \varepsilon_d , \gamma_d , \text{ and } \delta_d \text{ are the dynamic velocity anisotropies.} \]

As a measure of Young’s and shear moduli (E and G) and inverse of linear compressibilities (KI) anisotropies, we use the following relations and notations (Appendix B):

\[ A_{Es} = \left( \frac{E_1}{E_3} \right)_{stat}, \quad A_{Gs} = \left( \frac{G_{12}}{G_{23}} \right)_{stat}, \quad \text{and} \quad A_{kls} = \left( \frac{K_1}{K_3} \right)_{stat}; \]

which are the static anisotropies and

\[ A_{Ed} = \left( \frac{E_1}{E_3} \right)_{dyn}, \quad A_{Gd} = \left( \frac{G_{12}}{G_{23}} \right)_{dyn}, \quad \text{and} \quad A_{kld} = \left( \frac{K_1}{K_3} \right)_{dyn}; \]

which are the dynamic anisotropies.

Figures 3.2 and 3.3 show the \( V_p \) and \( V_s \) velocities obtained from the dynamic experiment (\( V_{s(dyn)} \) and \( V_{p(dyn)} \)) and from the static experiment (\( V_{s(stat)} \) and \( V_{p(stat)} \)).

In dry and saturated conditions, the static and dynamic P-wave velocities increase with increasing confining pressure (figures 3.2 and 3.3). After saturation, both \( V_{p(dyn)} \) and \( V_{p(stat)} \) increase, and they also increase with rising pore pressure. However, the increase is much more pronounced for \( V_{p(stat)} \) than \( V_{p(dyn)} \). In dry and saturated conditions, \( V_{p(stat)} \) is always smaller than \( V_{p(dyn)} \) over the whole range of confining pressures, but at high pore and hydrostatic pressures, the difference between \( V_{p(dyn)} \) and \( V_{p(stat)} \) is reduced pronouncedly, reaching almost the same value.

In dry and saturated conditions, the static and dynamic S-wave velocities increase with increasing the confining pressure over the whole range of pressure. In dry conditions, \( V_{s(stat)} \) is smaller than \( V_{s(dyn)} \), but the difference decreases with increasing confining pressure. After saturation, \( V_{s(stat)} \) increases while \( V_{s(dyn)} \) decreases. At high pore pressure,
Vs(stat) is bigger than Vs(dyn) over the whole range of confining pressures. However, at low pore pressure, Vs(stat) is smaller than Vs(dyn) and the behavior is reversed at high confining pressure.

In dry and saturated conditions, the ratios of dynamic to static Rs and Rp decrease with increasing confining pressures. In dry conditions, Rs and Rp, are approximately equal in magnitude as the confining pressure is varied. After saturation, Rs shows a pronounced decrease, reaching a value of one at high confining pressures. Rp also decreases after saturation, but not as strongly as Rs does. In saturated conditions, Rs and Rp decrease when the pore pressure is raised from 2 MPa to 40 MPa; however, the decrease is greater in the shear velocity ratio.

The ratios of dynamic to static shear and Young's moduli, RG and RE, respectively, decrease as the effective hydrostatic pressure increases (figures 3.5). After saturation, RG and RE decrease, and they also decrease when the pore pressure is raised (appendix C). At high pressure, RG and RE remain approximately constant (figures 3.4 to 3.5).

Static versus dynamic bulk modulus in dry and saturation conditions

Figure 3.6 shows the behavior of the dynamic and static bulk moduli, Kdyn and Kstat, respectively, as the effective hydrostatic pressure is varied from 5 to 70 MPa. The static modulus in figure (3.6) corresponds to the loading path of a continuous hydrostatic compression cycle, as shown in the second part of figure (3.1). Kdyn is always bigger than Kstat, but RK becomes smaller as the hydrostatic stress increases (figure 3.7). Kstat exhibits more linear behavior than Kdyn in both dry and saturated conditions. Kdyn shows a pronounced increase after saturation, while Kstat decreases slightly. For the static experiment, the rock seems to get softer after saturation, while for the dynamic one, the rock seems to get stiffer. At hydrostatic pressures below 21 MPa, Kstat is smaller in dry than saturated conditions, but after reaching a pressure of about 21 MPa, Kstat in dry conditions gets bigger than those obtained in saturated conditions, and the behavior remains similar up to 70 MPa, having an increasing difference.
Notice that the nonlinearity in the stress-strain relationship causes a pronounced increase in $K_{\text{dyn}}$ and $K_{\text{stat}}$ as the pressure is increased. However, the presence of fissures or cracks affects the dynamic and static bulk modulus differently (figure 3.6). While the rate of increase of $K_{\text{dyn}}$ becomes less as the stress is increased, the rate of increase of $K_{\text{stat}}$ remains approximately constant in both dry and saturated conditions. $K_{\text{stat}}$ shows a linear relationship with pressure and the slope decreases as the confining pressure is increased.

In dry conditions, $R_K$ decreases with confining pressure, and it reaches a value of approximately 1.15 at 10 MPa. After saturation, $R_K$ has a pronounced increase (figure 3.7). In saturation conditions, $R_K$ decreases with increasing confining pressure at approximately the same rate as it does in dry conditions. The difference between $R_K$ in dry and saturated conditions is slightly bigger at low pressure than at high pressure.

The dynamic bulk modulus, $K_{\text{dyn}}$, and two static bulk moduli, $K_{\text{stat1}}$ and $K_{\text{stat2}}$, are presented in Table D.4. $K_{\text{stat1}}$ is obtained from a uniaxial stress cycle, and $K_{\text{stat2}}$ is obtained from a hydrostatic compression cycle. $K_{\text{stat2}}$ is always less than $K_{\text{stat1}}$ in dry and saturated conditions. The differences between $K_{\text{stat1}}$ and $K_{\text{stat2}}$ is caused by the fact that under hydrostatic pressures, crack faces may slide less relative to one another than in uniaxial experiments. Under uniaxial stresses, cracks and pores change shapes more than in hydrostatic pressure experiments. Both cracks and friction forces between sliding crack faces introduce non-linearity in the stress-strain relationship, but the amount of nonlinearity depends on how the material is strained. In dry sandstones, the difference between $K_{\text{dyn}}$ and $K_{\text{stat}}$ is smaller than the difference in saturated conditions.

### 3.4.2 Static and dynamic elastic behavior of room dry anisotropic shales

In this section we work with a set of three shale specimens cored in three different directions as described in section 3.3.2. The characteristics of the specimens are presented in Tables 3.1 and 3.2.
Figures 3.8 and 3.9 show the inverse of linear compressibilities, $K_{11}$ and $K_{13}$, and the bulk modulus, $K_b$, as functions of confining pressure, obtained from three different experiments: a step-wise hydrostatic cycle ($K_{b\text{ (stat3)}}$), a continuous hydrostatic cycle ($K_{b\text{ (stat2)}}$) and a dynamic experiment ($K_{b\text{ (dyn)}}$) as described in section 3.4 (Table D.5). Notice that there are differences among the compressibility values obtained from the different experimental procedures. $K_{b\text{ (stat3)}}$ is always smaller than $K_{b\text{ (stat2)}}$, and $K_{b\text{ (stat2)}}$ is always smaller than $K_{b\text{ (dyn)}}$. They also behave differently as the confining pressure is increased. $K_{b\text{ (stat2)}}$ is a linear function of pressure and increases monotonically over the range of pressures; however, the rate of increase of $K_{b\text{ (dyn)}}$ is greater at low pressures than at high pressures where it tends to be constant. $K_{11\text{ (dyn)}}$ and $K_{13\text{ (dyn)}}$ are always greater than the corresponding static values over the whole range of confining pressures.

$K_{11\text{ (stat2)}}$ and $K_{13\text{ (stat2)}}$, obtained from the continuous hydrostatic cycle, increase monotonically as the stress is increased, and they are linearly related with hydrostatic pressure. On the other hand, $K_{11\text{ (dyn)}}$ and $K_{13\text{ (dyn)}}$ show greater increases at low pressures than at higher pressures.

Figure 3.9 also shows the bulk modulus obtained from three uniaxial cycles, one in the vertical direction ($X_3$), one in the horizontal direction ($X_1$), and one oriented at 45° with respect to $X_3$ (Table D.8). Notice that $K_{b\text{ (stat2)}}$ is smaller than $K_{b\text{ (stat1)}}$ at low pressures and tends to be equal at high pressures. The static bulk modulus obtained from uniaxial experiments ($K_{b\text{ (stat1)}}$) is always smaller than the dynamics bulk modulus ($K_{b\text{ (dyn)}}$); however, they both behave similarly as the confining pressure is increased (figure 3.9). The ratios of dynamic to static bulk modulus and inverse of linear compressibilities, $R_K$, $R_{KL1}$ and $R_{KL3}$, decrease as the confining pressure is increased. $R_{KL3}$ reaches a value of one when the hydrostatic pressure is at its maximum (figure 3.10).

The dynamic inverse of linear compressibility anisotropy, $A_{KLd}$, was found to be always smaller than the static anisotropy, $A_{KLs}$ (Appendix D). $A_{KLs}$, obtained from continuous hydrostatic cycles, was found to be smaller than that obtained from stepwise cycle. The
static and dynamic anisotropies, $A_{KLd}$ and $A_{KLs}$, decrease as confining pressure increases (Appendix D).

Figures 3.11 and 3.12 show the P- and Sh-wave velocities in different directions as functions of confining pressures. All the velocities, static and dynamic, increase monotonically as the confining pressure is increased, but the rate of increase becomes smaller at high pressure. The static velocities, $V_p(\theta)_{(\text{stat})}$, $V_{sh}(\theta)_{(\text{stat})}$, and $V_{sv}(\theta)_{(\text{stat})}$, are smaller than the dynamic velocities $V_p(\theta)_{(\text{dyn})}$, $V_{sh}(\theta)_{(\text{dyn})}$, and $V_{sv}(\theta)_{(\text{dyn})}$, over the whole range of confining pressures and in all directions; however, the difference tends to decrease as the confining pressure is increased (figures 3.11 to 3.13).

The ratios of dynamic to static velocities, $R_p(\theta)$ and $R_{sh}(\theta)$, decrease with increasing confining pressure (figure 3.14). At high confining pressure these ratios approach a value of approximately 1.1, except $R_p(90^\circ)$ which reaches a value of approximately 1.0 (figure 3.16).

The ratio $R_v(0^\circ)$ is greater than $R_v(90^\circ)$ at low confining pressures (10 MPa); however, at high pressures (70 MPa), the ratios of dynamic to static P-wave velocities in the vertical and horizontal direction approach almost the same value (figure 3.17). $R_{sh}(\theta)$ is independent of the direction ($\theta$) over the whole range of pressures (figure 3.18).

The static and dynamic P-wave and Sh-wave velocity anisotropies ($\varepsilon_s, \gamma_s$, and $\varepsilon_d, \gamma_d$) all decrease with increasing confining pressure (figure 3.19). The differences between $\varepsilon_s$ and $\varepsilon_d$, and between $\gamma_s$ and $\gamma_d$ are more pronounced at low pressures than at high confining pressures, where they tend to be equal (figure 3.19). At low pressures, there is a pronounced difference between the static and the dynamic P-wave anisotropy, $\varepsilon_s$ and $\varepsilon_d$. However, this difference decreases sharply as the pressure is raised, reaching a value of almost zero at high pressures. The difference between $\gamma_s$ and $\gamma_d$ is small over the whole range of pressures.
The difference between static and dynamic moduli arises from the strain amplitude, mode of deformation of the material, frequency dependent mechanisms, pore pressure, and anisotropy, and it is strongly related to both crack density and distribution and confining pressure.

In uncracked, dry shales the ratio of dynamic to static is less sensitive to the confining pressure than it is in room dry cracked sandstone. This behavior may be attributed to the differing sensitivity of static and dynamic moduli to cracks. In saturated conditions, these ratios are less sensitive to confining pressure than they are in dry conditions.

### 3.5 Conclusions

1. The ratios of dynamic to static velocities, and the moduli, $R_M(\theta)$, are affected by:

   1.1 *Confining pressure.* All the ratios of dynamic to static velocities and of dynamic to static elastic parameters in all directions, $R_M(\theta)$, decrease with increasing confining pressure. However, the rate of decrease is greater in the vertical direction than in the horizontal direction (figures 3.4 to 3.7, 3.10, and 3.14 to 3.16).

   1.2 *Saturation.* After saturation, all the ratios of dynamic to static moduli and velocities, $R_M(\theta)$, decrease, except the bulk compressibility ratio, $R_{kB}$, which increases. $R_{kB}$ increases because while $K_{b(\text{stat})}$ decreases after saturation, $K_{b(\text{dyn})}$ decreases (figures 3.4, 3.5, and 3.7).

   1.3 *Pore pressure.* All the ratios of dynamic to static moduli, $R_M(\theta)$, decrease when the pore pressure is raised, except $R_{kB}$ which increases (figures 3.4, 3.5 and D.6).

   1.4 *Anisotropy.* The magnitudes of the ratios of dynamic to static velocities or moduli, $R_M(\theta)$, depend on the direction the measurements are made in. Not all the ratios, $R_M(\theta)$, are equally affected. The ratio of dynamic to static P-wave velocity, $R_P(\theta)$, is greater in the vertical direction than in the
horizontal direction. On the other hand, the ratio of dynamic to static Sh-wave velocity, \( R_{sm}(\theta) \), does not depend on the direction of propagation (figures 3.15 to 3.18).

(1.5) **Mode of deformation.** The presence of porosity in the rock, results in different moduli determined from: uniaxial stresses, hydrostatic compression or any other stress system (figures 3.8, 3.9 and D.9). It also may occurs if the modulus is obtained from different kinds of wave propagation experiments. Cracks open and close differently depending on the state of stress. Crack faces slide relative to the other, but the slip across crack faces may be small or large depending on the direction of the stress with respect to the plane of preferred orientation of cracks. The closure of cracks and frictional forces between sliding crack faces introduce hysteresis and nonlinearity into the problem; consequently, nonlinearity anisotropy may be observed. The rock may exhibit different degrees of nonlinearity depending on the direction in which linear strain are measured (figure D.10, E.1, and E.5), and at the same time different amounts of elastic energy may be dissipated depending on the direction in which the measurements are being made (anelastic anisotropy).

(1.6) **Crack density and pore structure.** Cracks affect static and dynamics moduli differently (Simmons and Brace, 1965). The ratio of dynamic to static velocities and moduli depends on the crack density and the stress level. In uncracked rocks, like some shales, \( R_m(\theta) \) is less sensitive to pressure than it is in cracked rocks, like some sandstones (figures 3.4 and 3.5). In a saturated state, \( R_m(\theta) \) becomes less sensitive to confining pressure (figures 3.4, 3.5, D.14, E.13, and E.14).

(2) The elastic anisotropy is affected by:

(2.1) **Confining pressure.** All the static and dynamic velocities and elastic parameters decrease with increasing confining pressure (figures 3.19, D.21 and D.16).
(2.2) **Static and dynamic.** The static velocity anisotropies and static modulus anisotropies are always greater than the corresponding dynamic anisotropies over the entire range of confining pressures and in all directions (figures 3.13, 3.19, D.21, and D.16).

(2.3) **Saturation.** After saturation, the dynamic Vp-anisotropy, \( \varepsilon_p \), decreases while the dynamic Vsh-anisotropy, \( \varepsilon_s \), is much less affected (Chapter II). The static anisotropy also decreases after saturation (figures E.1 and E.3); however, this decrease was only observed in plots of vertical and horizontal linear strain.

(2.4) **Mode of deformation.** As a consequence of obtaining different elastic moduli when they are determined from, for instance, uniaxial or hydrostatic tests, the resulting computed anisotropies are also different (figure D.16).

(2.5) **Sedimentary structure, foliation or fine lamination, and preferred orientation of cracks** (Chapter II).

(2.6) **Frequency.** Heterogeneities in the rock much smaller than the wavelength associated to the deformation method may increase the observed anisotropy. The apparent anisotropy caused by scattering is superimposed on the intrinsic anisotropy associated to foliation, for instance.

(3) Both V\(_p\)\(_{\text{dyn}}\) and V\(_p\)\(_{\text{stat}}\) increase after saturation and with increasing pore pressure. However, the increase is more pronounced in the V\(_p\)\(_{\text{stat}}\) (figure 3.2).

(4) V\(_s\)\(_{\text{dyn}}\) decreases after saturation and also with increasing pore pressure. On the other hand, V\(_s\)\(_{\text{stat}}\) increases after saturation and also increases with pore pressure (figure 3.3)

(5) The increase of static elastic moduli with confining pressure is much larger than the increase in the corresponding dynamic ones (figures 3.8, 3.9, D.7, D.9, D.17, and D.19).
<table>
<thead>
<tr>
<th>Sample Name</th>
<th>Sets</th>
<th>P.P. (MPa)</th>
<th>Dry Density (g/c.c)</th>
<th>Saturated Density (g/c.c)</th>
<th>Por. (%)</th>
<th>Depth (feet)</th>
<th>Sample Length (m.m)</th>
<th>Sample Diameter (m.m)</th>
<th>Fluid</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1v44d</td>
<td>Vertical</td>
<td>0</td>
<td>2.592</td>
<td></td>
<td></td>
<td>10854</td>
<td>50.550</td>
<td>38.202</td>
<td>Shales</td>
</tr>
<tr>
<td>T1h44d</td>
<td>Horizontal</td>
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<td>2.543</td>
<td></td>
<td></td>
<td>10854</td>
<td>50.850</td>
<td>38.202</td>
<td>Dry</td>
</tr>
<tr>
<td>T14445d</td>
<td>At 45 deg. (I)</td>
<td>0</td>
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<td></td>
<td></td>
<td>10854</td>
<td>29.540</td>
<td>25.400</td>
<td>Dry</td>
</tr>
<tr>
<td>T259</td>
<td>Vertical (II)</td>
<td>40</td>
<td>2.13</td>
<td>2.30</td>
<td>17.2</td>
<td>12 mD</td>
<td>10342</td>
<td>38.125</td>
<td>Sandstone</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Brine Sat.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Dry</td>
</tr>
</tbody>
</table>

Table 3.1 Characteristic of the shale and sandstone specimens tested. P or. = Porosity, \( \phi \) = permeability in millidarcies.

<table>
<thead>
<tr>
<th>Shale (I)</th>
<th>Layer friable shale with inclusions of preferred oriented fine micromic grains and cracks parallel to the foliation plane with intercalation of fine sheets of organic clay.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sandstone (II)</td>
<td>Medium quartz grains, no lamination observed, approx. less than 5% of disseminated clay.</td>
</tr>
</tbody>
</table>

Table 3.2 Description of the rock specimens from thin sections analyses.
Figure 3.1 Stress Path for the confining pressure and axial stress. The experiment is subdivided into three parts. During the first part, we have no axial stress and the specimen is stepwise hydrostatically loaded and unloaded. At each confining pressure level, $P_-$, $S_v-$, and $S_h-$ wave velocities are measured. The second part consists of a hydrostatic constant rate loading and unloading without interruption and without axial stress. After finishing the second part, the specimen is hydrostatically reloaded to a pressure of 10 MPa. Then, the confining pressure is held constant, and a differential constant rate axial load cycle is applied to the specimen. $P_-$, $S_v-$, and $S_h-$ wave velocities are measured at the peak of the axial stress-strain curve and at the completion of the cycle. The confining pressure is then sequentially increased to 35 MPa and 60 MPa. A similar axial load/unload cycle is performed at each level of the confining pressures. At the conclusion of the final axial stress-strain loading cycle, the specimen is unloaded. The total duration of the experiment is approximately 4000 sec.
**Figure 3.2** Static and dynamic Vp-velocities for dry and saturated sandstone versus pore and effective hydrostatic pressure.

**Figure 3.3** Static and dynamic Vs velocities for dry and saturated sandstones versus pore and effective hydrostatic pressure.

**Figure 3.4** Ratio of dynamic to static Vs and Vp velocities for dry and saturated sandstones versus pore and effective hydrostatic pressure.
Figure 3.5 Ratio of dynamic to static Young's modulus ($E$) and shear modulus ($G$) for dry and saturated sandstone versus pore and effective hydrostatic pressure.

Figure 3.6 Static and dynamic bulk moduli for dry and brine saturated sandstone at two pore pressures, $PP = 290$ psi and $PP = 5800$ psi.

Figure 3.7 Ratio of dynamic to static bulk modulus ($K_{b(dyn)}/K_{b(stat)}$) versus effective pressure for dry and brine saturated sandstones at two pore pressures.
Figure 3.8 Static and dynamic inverse of linear compressibilities versus confining pressure obtained from a step wise hydrostatic compression, a continuous hydrostatic compression and from a dynamic experiment.

Figure 3.9 Bulk moduli obtained from a step wise and continuous hydrostatic compression, from a three uniaxial experiments, and from a dynamic experiment.

Figure 3.10 Ratio of dynamic to static inverse of linear compressibilities and bulk modulus.

Figure 3.11 Static and dynamic Vp-velocities in the vertical, horizontal and 45° to the vertical directions versus confining pressure.
Figure 3.12 Static and dynamic Vsh-velocities in the vertical and horizontal directions versus confining pressure.

Figure 3.13 Static and dynamic Vp-, Vsh-, and Vsv-phase velocities (m/sec) versus orientation at low confining pressure (10 MPa).
Figure 3.14 Ratio of dynamic to static $V_p$ and $V_{sh}$ velocities at three different directions versus confining pressure.

Figure 3.15 Ratio of dynamic to static $P$-, $Sh$-, and $Sv$-velocities versus direction.

Figure 3.16 Ratio of dynamic to static $P$-, $Sh$-, and $Sv$-wave velocities versus direction.

Figure 3.17 Ratio of dynamic to static $V_p$-velocities as a function of orientation and confining pressure.

Figure 3.18 Ratio of dynamic to static $V_{sh}$-velocities as a function of orientation and confining pressure.
Figure 3.19 Static and dynamic Thomsem's parameters, $\epsilon_{\text{stat}}$, $\gamma_{\text{stat}}$ and $\epsilon_{\text{dyn}}$, $\gamma_{\text{dyn}}$, versus confining pressure.
Chapter 4

Summary and Conclusions

In multidisciplinary studies carried out in the Budare Oil Field of the Great Oficina Oil Field, there was difficulty matching well log synthetic seismograms with 2D and 3D seismic data. In addition, the seismically determined depths of reservoir horizons are greater than the well sonic log depths. To examine this discrepancy we conducted an experimental study of dynamic elastic parameters of the rocks in the oil field. We chose core representative samples of the lower Oficina Formation, the main reservoir of the field. The rocks selected were sandstones, sandy shales and dolomitized shales.

For the velocity measurements, we used the ultrasonic transmission method to measure P-, Sh- and Sv-wave travel times as a function of orientation, and pore and confining pressures to 60 and 65 MPa, respectively. We found that, in room dry condition, most of the rocks studied are transversely isotropic. The stiffnesses constants, Young’s moduli, Poisson’s ratios, and bulk moduli of these rocks, were also calculated.

The dynamic velocity anisotropies, together with the behavior of the elastic constants for dry rocks, indicate that:
(1) The large velocity anisotropies \((\epsilon, \gamma, \delta)\) exhibited by the dolomitized calcareaous shales may be caused by lamination not visible at an optical microscopic scale. An electron microscope should be used to find the orientation of the assumed platy dolomite grains and to verify our assumption about its sedimentary origin.

(2) After saturation, the P-wave anisotropy in fractured sandstones shows a pronounced decrease; however, the \(S_h\)-wave anisotropy is only slightly affected.

(3) The velocity anisotropies measured in uncracked rock specimens might be extrapolated to the whole formation, based on the assumption that their intrinsic anisotropies do not exhibit a significant change with confining pressure. However, before any extrapolation to low seismic frequencies a correction due to velocity dispersion should be made.

(4) In the depth intervals studied, the data suggest that cracks in sandy shales and sandstones are not always completely close at the oil reservoir effective pressure conditions.

Determining rock mechanical properties \(\text{in situ}\) is important in many applications in the oil industry such as reservoir production, hydraulic fracturing, estimation of recoverable reserves, and subsidence. Direct measurement of mechanical properties \(\text{in situ}\) is difficult. Nevertheless, experimental methods exist to obtain these properties, such as measurements of the stress-strain relationships (static) and elastic wave velocities (dynamic).

We investigate the static and dynamic elastic behavior of sedimentary, anisotropic rock specimens over a range of confining and pore pressures up to 70 MPa, the original reservoir conditions. The static and dynamic properties are simultaneously measured for room dry shales, room dry sandstones, and brine saturated sandstones. We found that:

(1) The ratios of dynamic to static velocities, and the moduli, \(R_M(\theta)\), are affected by:
(1.1) **Confining pressure.** All the ratios of dynamic to static velocities and of dynamic to static elastic parameters in all directions, $R_m(\theta)$, decrease with increasing confining pressure. However, the rate of decrease is greater in the vertical direction than in the horizontal direction (figures 3.4 to 3.7, 3.10, and 3.14 to 3.16).

(1.2) **Saturation.** After saturation, all the ratios of dynamic to static moduli and velocities, $R_m(\theta)$, decrease, except the bulk compressibility ratio, $R_b$, which increases. $R_b$ increases because while $K_{b(stat)}$ decreases after saturation, $K_{b(dyn)}$ decreases (figures 3.4, 3.5, and 3.7).

(1.3) **Pore pressure.** All the ratios of dynamic to static moduli, $R_m(\theta)$, decrease when the pore pressure is raised, except $R_b$ which increases (figures 3.4, 3.5 and D.6).

(1.4) **Anisotropy.** The magnitudes of the ratios of dynamic to static velocities or moduli, $R_m(\theta)$, depend on the direction the measurements are made in. Not all the ratios, $R_m(\theta)$, are equally affected. The ratio of dynamic to static P-wave velocity, $R_p(\theta)$, is greater in the vertical direction than in the horizontal direction. On the other hand, the ratio of dynamic to static S-wave velocity, $R_s(\theta)$, does not depend on the direction of propagation (figures 3.15 to 3.18).

(1.5) **Mode of deformation.** The presence of porosity in the rock, results in different moduli determined from: uniaxial stresses, hydrostatic compression or any other stress system (figures 3.8, 3.9 and D.9). It also may occurs if the modulus is obtained from different kinds of wave propagation experiments. Cracks open and close differently depending on the state of stress. Crack faces slide relative to the other, but the slip across crack faces may be small or large depending on the direction of the stress with respect to the plane of preferred orientation of cracks. The closure of cracks and frictional forces between sliding crack faces introduce hysteresis and nonlinearity into the problem; consequently, nonlinearity anisotropy may be
observed. The rock may exhibit different degrees of nonlinearity depending on the direction in which linear strain are measured (figure D.10, E.1, and E.5), and at the same time different amounts of elastic energy may be dissipated depending on the direction in which the measurements are being made (anelastic anisotropy).

(1.6) Crack density and pore structure. Cracks affect static and dynamics moduli differently (Simmons and Brace, 1965). The ratio of dynamic to static velocities and moduli depends on the crack density and the stress level. In uncracked rocks, like some shales, $R_M(\theta)$ is less sensitive to pressure than it is in cracked rocks, like some sandstones (figures 3.4 and 3.5). In a saturated state, $R_M(\theta)$ becomes less sensitive to confining pressure (figures 3.4, 3.5, D.14, E.13, and E.14).

(2) The elastic anisotropy is affected by:

(2.1) Confining pressure. All the static and dynamic velocities and elastic parameters decrease with increasing confining pressure (figures 3.19, D.21 and D.16).

(2.2) Static and dynamic. The static velocity anisotropies and static modulus anisotropies are always greater than the corresponding dynamic anisotropies over the entire range of confining pressures and in all directions (figures 3.13, 3.19, D.21, and D.16).

(2.3) Saturation. After saturation, the dynamic $V_p$-anisotropy, $\varepsilon_d$, decreases while the dynamic $V_{sh}$-anisotropy, $\gamma_d$, is much less affected (Chapter II). The static anisotropy also decreases after saturation (figures E.1 and E.3); however, this decrease was only observed in plots of vertical and horizontal linear strain.

(2.4) Mode of deformation. As a consequence of obtaining different elastic moduli when they are determined from, for instance, uniaxial or hydrostatic tests, the resulting computed anisotropies are also different (figure D.16).
(2.5) Sedimentary structure, foliation or fine lamination, and preferred orientation of cracks (Chapter II).

(2.6) Frequency. Heterogeneities in the rock much smaller than the wavelength associated to the deformation method may increase the observed anisotropy. The apparent anisotropy caused by scattering is superimposed on the intrinsic anisotropy associated to foliation, for instance.

(3) Both \( \nu_p^{(dy)} \) and \( \nu_p^{(stat)} \) increase after saturation and with increasing pore pressure. However, the increase is more pronounced in the \( \nu_p^{(stat)} \) (figure 3.2).

(4) \( \nu_s^{(dy)} \) decreases after saturation and also with increasing pore pressure. On the other hand, \( \nu_s^{(stat)} \) increases after saturation and also increases with pore pressure (figure 3.3).

(5) The increase of static elastic moduli with confining pressure is much larger than the increase in the corresponding dynamic ones (figures 3.8, 3.9, D.7, D.9, D.17, and D.19).
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Appendix A

Dynamic elastic parameters in isotropic and transverse isotropic materials (theory)

Linearized theory of elasticity

In the linearized theory of isothermal elasticity (in materials coordinates), the field equations are given by (Malvern, 1969),

\[
\frac{\partial \sigma_{ij}}{\partial x_i} + \rho b_i = \rho \frac{\partial^2 w}{\partial t^2}, \tag{A.1}
\]

\[
\sigma_{ij} = C_{ijkl} \epsilon_{kl} \quad \text{(anisotropic elasticity)} \quad \text{or} \quad \sigma_{ij} = \lambda \delta_{ij} + 2\mu \epsilon_{ij} \quad \text{(isotropic elasticity)}, \tag{A.2}
\]

and

\[
\epsilon_{ij} = \frac{1}{2} \left( \frac{\partial w}{\partial x_i} + \frac{\partial w}{\partial x_j} \right). \tag{A.3}
\]
There are 3 equations of motion, 6 Hooke’s laws and 6 geometric equations. A total of 21 equations for 9 stresses, 9 strains and 3 displacements are reduced to 15 equations for 6 stresses, 6 strains and 3 displacements due to the symmetry of the tensors $\sigma_{ij}, \varepsilon_{ij}$, and $C_{ijkl}$.

In order to facilitate subsequent discussions, we rewrite the stress and strain relations in their expanded matrix form. To obtain this expansion, it is necessary to use the symmetry of the strain tensor and the properties of the stiffness tensor,

$$\sigma_{ij} = \sigma_{ji}, \quad \varepsilon_{ij} = \varepsilon_{ji}, \quad C_{ijkl} = C_{jikl} = C_{ijlk}, \quad and \quad C_{ijkl} = C_{klij},$$

and the adoption of the conventional contracted index notation,

$$11 \rightarrow 1, \quad 22 \rightarrow 2, \quad 33 \rightarrow 3, \quad 23 \rightarrow 4, \quad 13 \rightarrow 5, \quad and \quad 12 \rightarrow 6.$$

The stress-strain relation for a linear elastic isotropic media is described by,

$$\begin{pmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\sigma_4 \\
\sigma_5 \\
\sigma_6 
\end{pmatrix} =
\begin{pmatrix}
C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\
C_{12} & C_{11} & C_{12} & 0 & 0 & 0 \\
C_{12} & C_{12} & C_{11} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{C_{11} - C_{12}}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{C_{11} - C_{12}}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{C_{11} - C_{12}}{2}
\end{pmatrix}
\begin{pmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3 \\
\varepsilon_4 \\
\varepsilon_5 \\
\varepsilon_6
\end{pmatrix} \tag{A.4}$$

The stress-strain relation for a linear elastic transversely isotropic media, in the plane $X_1X_2$, is described by,
where

\[ \varepsilon_q = \begin{vmatrix} \varepsilon_{11} & \varepsilon_{22} & \varepsilon_{33} & 2\varepsilon_{23} & 2\varepsilon_{13} & 2\varepsilon_{12} \end{vmatrix} \quad \text{with} \quad \sigma_q = C_{qp} \varepsilon_p \quad \text{or} \]

\[ \sigma_q = \begin{vmatrix} \sigma_{11} & \sigma_{22} & \sigma_{33} & 2\sigma_{23} & 2\sigma_{13} & 2\sigma_{12} \end{vmatrix} \quad \text{with} \quad \varepsilon_q = S_{qp} \sigma_p \]

and \( C_{qp} \) and \( S_{qp} \) are the stiffness and the compliance matrix, respectively.

**Bounds on the stiffnesses and compliances. The strain energy.**

The work necessary to produce a strain \( \varepsilon \) in a unit cube of a body is,

\[ W = \frac{1}{2} C_{ijkl} \varepsilon_i \varepsilon_j \]

(A.6)

\[ W = \frac{1}{2} S_{ijkl} \sigma_i \sigma_j. \]

This strain energy must be positive, otherwise the material would be unstable. Positive energy means that the quadratic form (2.2.4) must be positive definite (Nye, 1957). This implies some restrictions on the stiffnesses and compliances. In the hexagonal system, these restrictions are given by (Nye, 1957),
\( C_{44} > 0, \quad C_{11} > |C_{12}|, \quad (C_{11} + C_{12})C_{33} > 2C_{13}^2. \) \( \text{(A.7)} \)

**Relations between phase velocities and elastic parameters in isotropic media**

In an isotropic medium there are only two independent elastic stiffnesses, \( C_{11} \) and \( C_{12}. \) From the hexagonal system we can obtain the isotropic system by making

\[ C_{33} \rightarrow C_{11} = \lambda + 2G \]

and

\[ C_{66} \rightarrow C_{44} = G, \]

and consequently,

\[ C_{13} \rightarrow C_{12} \rightarrow (C_{33} - 2C_{44}) \]

where \( \lambda \) and \( G \) are the Lame’s parameters.

The velocities of the two independent modes of body wave propagation, shear and compressional, are given by

\[ V_p = \sqrt{\frac{C_{33}}{\rho}} \quad \text{and} \quad V_s = \sqrt{\frac{C_{44}}{\rho}}. \]  \( \text{(A.8)} \)

The dynamic Poisson’s ratio, Young’s modulus, and the bulk modulus for these media can be expressed as follows:
The relation between shear modulus and shear velocity ($V_s$) is given as

$$G = \rho V_s^2.$$  \hspace{1cm} (A.12)

### Phase velocities in anisotropic media

For a particular direction of propagation in a material of a given symmetry, one of the three directions of particle displacement will be parallel, and the other two directions perpendicular, to the direction of propagation.

There are three different modes of propagation, one corresponding to a longitudinal wave, and the other two to transverse (shear) waves. In general these three velocities are different.

For an arbitrary direction of propagation, one does not obtain separately one longitudinal and two shear waves. Instead, there are three independent modes of propagation,
corresponding to three directions of particle displacement which are mutually orthogonal, but none are orthogonal to the direction of propagation nor coincident with it.

The three velocities are the roots of a third-order determinant in which each term depends on the elastic constant and the direction cosines of the normal to the wavefront (Musgrave, 1970).

The problem is simplified considerably when the direction of wave propagation is in particular simple material directions in which "pure modes" (longitudinal or shear) are obtained. Then a transducer vibrating either longitudinally or transversely generates a wave in only one of the three modes of propagation. However, in general, it is not possible to obtain all of the elastic constants with the use of pure modes alone.

In hexagonal systems $C_{33}$, $C_{44}$, $C_{11}+C_{12}$, and $C_{11}-C_{12}$ may all be obtained by pure wave propagation perpendicular and parallel to the plane of isotropy (the bedding plane). On the other hand, $C_{13}$ can only be obtained by using wave propagation in a direction for which coupling occurs. In loading cyclic experiments this coupling between modes also exists if the stress is introduced in a direction different from the principal axis.

The quasi-compressional wave phase velocity $v_q^p$, vertically polarized shear wave velocity $v_{qs}$, and horizontally polarized shear wave velocity $v_{sh}$, in a transversely isotropic medium, are given by (Musgrave, 1970)

\[
\rho V_{qp}^2 = C_{44} + \frac{1}{2}(h \cos^2 \theta + a \sin^2 \theta) + \frac{1}{2}\left[(h \cos^2 \theta + a \sin^2 \theta)^2 - 4(ah - d^2)\cos^2 \theta \sin^2 \theta \right]^{1/2}, \quad (A.13)
\]

\[
\rho V_{sv}^2 = C_{44} + \frac{1}{2}(h \cos^2 \theta + a \sin^2 \theta) - \frac{1}{2}\left[(h \cos^2 \theta + a \sin^2 \theta)^2 - 4(ah - d^2)\cos^2 \theta \sin^2 \theta \right]^{1/2}, \quad (A.14)
\]

and

\[
\rho V_{sh}^2 = C_{44}\cos^2 \theta + C_{66}\sin^2 \theta \quad (A.15)
\]
where

\[ a = C_{11} - C_{44}, \quad h = C_{33} - C_{44}, \quad \text{and} \quad d = C_{13} + C_{44} \]

and \( \theta \) is the angle measured from the symmetry axis, in this case \( X_3 \).

Using equations A.13 to A.15 we find that for hexagonal symmetry the relationships of the phase velocities, in the vertical direction, horizontal direction, and at \( 45^\circ \) with respect to the plane of isotropy, and stiffness are given by

\[ C_{33} = \rho V_{qp}^2(90^\circ), \]

\[ C_{13} = -C_{44} + \sqrt{4\rho^2 V_{qp}^2(45^\circ) - 2\rho^2 V_{qp}^2(45^\circ)(C_{11} + C_{33} + 2C_{44}) + (C_{11} + C_{44})(C_{33} + C_{44})}, \]

\[ C_{11} = \rho V_p^2(90^\circ), \quad (A.16) \]

\[ C_{44} = \rho V_{sv}^2(0^\circ) = \rho V_{sh}^2(0^\circ) = \rho V_{sv}^2(90^\circ) \quad \text{and} \]

\[ C_{66} = \rho V_{sh}^2(90^\circ). \]

**Velocity anisotropies**

As a measure of velocity anisotropy, we introduce the notation suggested by Thomsen (1986):

\[ \varepsilon \equiv \frac{C_{11} - C_{33}}{2C_{33}} = \frac{V_p^2(90^\circ) - V_p^2(0^\circ)}{2V_p^2(0^\circ)}, \quad (A.17) \]

\[ \gamma \equiv \frac{C_{66} - C_{44}}{2C_{44}} = \frac{V_{sh}^2(90^\circ) - V_{sh}^2(0^\circ)}{2V_{sh}^2(0^\circ)}, \quad (A.18) \]

and
\[ \delta = \frac{(C_{13} + C_{44})^2 - (C_{33} - C_{44})^2}{2C_{33}(C_{33} - C_{44})} \]  \hspace{1cm} (A.19)

where \( \varepsilon \) and \( \gamma \) represent a measure of anisotropy of P-wave velocity, and S\(_h\)-wave velocity, respectively. \( \delta \) is a parameter that is useful in reflection velocity analyses because it describes weak anisotropy in transversely isotropic media, and it is almost totally independent of the horizontal velocities. As concluded by Banik (1987), variations in \( \delta \) describe both variations in the moveout velocity and variations in the offset-dependent P-P reflection amplitude at short offsets, which are very important parameters in seismic exploration (Banik, 1987).

Thomsen (1986) pointed out that in cases of weak anisotropy, as those observed in sandstone specimens, an error in \( V_p(45^\circ)/V_p(0^\circ) \) is propagated into \( \delta \) magnified by a factor of 4. To reduce these errors, we precisely identified the plane of anisotropy in the rock specimens. The polarization \( S_v \) and \( S_h \) were also precisely oriented with respect to this plane as shown in figure D.2, as well as the very precise orientation of the sample at 45\(^\circ\) with respect to the vertical axis.
Appendix B

Isotropic and transverse isotropic materials under some simple stress systems (Static, Theory)

B.1 Introduction

In this appendix the elastic parameters for isotropic and transverse isotropic materials are defined. We derive the stiffnesses and compliances for a hexagonal system, their interconversion, bounds, measures of anisotropy, and representation by surfaces. We analyze the deformation under two simple stress systems: hydrostatic and uniaxial stresses.

The direction $X_3$ in a Cartesian system will be called “vertical direction” or “axis of symmetry,” and the plane $X_1X_2$, “horizontal direction”, “layering plane,” or “plane of isotropy.”

In a general, linear, elastic material the stress and strain are related by Hooke’s law as follow, (Timoshenko and Goodier, 1934):

$$\sigma_{ij} = C_{ijkl} \varepsilon_{kl}$$  \hspace{1cm} (B.1.1)

where $\sigma_{ij}$ and $\varepsilon_{ij}$ are the elements of the stress and strain tensors, respectively, and $C_{ijkl}$ are the 81 elastic stiffness coefficients.
As an alternative relationship between stress and strain, we could write

\[ \varepsilon_{ij} = S_{ijkl} \sigma_{kl} \]  

(B.1.2)

where \( S_{ijkl} \) are the 81 elastic compliance constants.

Due to the symmetry of the tensors \( C_{ijkl} \) and \( S_{ijkl} \) and thermodynamic considerations the independent compliances and stiffneses, for a general anisotropic material, are reduced to 21 (Nye, 1957).

**B.2 Isotropic materials**

In an isotropic, linear elastic material equation, B.1.2 takes the following form:

\[ \varepsilon_{ij} = \frac{1}{E} \left[ (1 + \nu)\sigma_{ij} - \nu \delta_{ij} \sigma_{kk} \right] \]  

(B.2.1)

where  

\[ \delta_{ij} = 0 \quad \text{if} \quad i \neq j, \quad 1 \quad \text{if} \quad i = j \]

\( \sigma_{kk} = \sigma_{11} + \sigma_{22} + \sigma_{33} \)

\( \varepsilon_{kk} \) is the volumetric strain,

In an isotropic material, only two constants are needed to specify the stress-strain relation completely, the Poisson’s ratio \( \nu \), and the Young’s modulus, \( E \).

There are other useful elastic parameters that can be obtained from different experiments, for example:

**Poisson’s Ratio**, \( \nu \), is the ratio of lateral strain (\( \varepsilon_{11} \)) to axial strain (\( \varepsilon_{33} \)) in a uniaxial stress (\( \sigma_{11} \)),

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\[ \nu = -\frac{\varepsilon_{11}}{\varepsilon_{33}}. \]  

(B.2.2)

**Shear modulus**, \( G \), is the ratio of shear stress to shear strain, \( \sigma_{ij} = 2G\varepsilon_{ij}, \quad i \neq j \).  

(B.2.3)

**Young's Modulus**, \( E \), is the ratio of extensional stress to extensional strain in a uniaxial stress, \( \sigma_{33} = E\varepsilon_3 \) and \( \sigma_{11} = \sigma_{22} = \sigma_{13} = \sigma_{12} = \sigma_{23} = 0 \).

(B.2.4)

**Volumetric compressibility**, \( \beta \), is the ratio of volumetric strain, \( \varepsilon_0 \), to hydrostatic stress \( \sigma_0 \), \[ \beta = \frac{\varepsilon_0}{\sigma_0}. \]  

(B.2.5)

**Linear compressibility**, \( \beta l \), is the decrease in length of a line when the material is subjected to hydrostatic pressure. For isotropic material it does not vary with direction, \[ \beta l = \frac{\varepsilon_1}{\sigma_0}. \]  

(B.2.6)

**P-wave modulus**, \( M = \rho V_p^2 \), is the ratio of axial stress (\( \sigma_{33} \)) to axial strain (\( \varepsilon_{33} \)) in a uniaxial strain, \( \sigma_{33} = M\varepsilon_{33} \) and \( \varepsilon_{11} = \varepsilon_{22} = \varepsilon_{13} = \varepsilon_{12} = \varepsilon_{23} = 0 \).

(B.2.7)
S-wave Modulus, \( G = \rho V_p^2 \), where \( G \) is the shear modulus (Eq. A.2.3).

Some useful relationships between these elastic parameters are

\[
K = \frac{E}{3(1-2v)} \quad \text{and} \quad G = \frac{E}{2(1+v)}.
\]

The elastic constants in isotropic material must satisfy the following restriction:

\[-1 < v < \frac{1}{2}; \quad E \geq 0 \quad \text{or} \]

\[
K = \frac{1}{\beta} = \frac{E}{3(1-2v)} \geq 0; \quad G \geq 0.
\]

**B.3 Transverse isotropic materials**

**Principal axes**

An important property of a second order tensor, \( \varepsilon_{ij} \) and \( \sigma_{ij} \), is the possession of principal axes. There are three directions at right angle so that when

\[
\begin{vmatrix}
\sigma_{11} & \sigma_{12} & \sigma_{13} \\
\sigma_{21} & \sigma_{22} & \sigma_{23} \\
\sigma_{31} & \sigma_{32} & \sigma_{33}
\end{vmatrix}
\]

is transformed to its principal axes, it becomes

\[
\begin{vmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3
\end{vmatrix}
\]
where $\sigma_1$, $\sigma_2$, and $\sigma_3$ are the principal components of $\sigma_i$. The same applies for $\varepsilon_i$.

**Confining pressure, $Pc$**

A hydrostatic confining pressure is defined as

$$\sigma_1 = \sigma_2 = \sigma_3 = -P_c \quad \text{and} \quad \sigma_4 = \sigma_5 = \sigma_6 = 0. \quad (B.3.1)$$

The strains in a general material subjected to hydrostatic pressure are given by

$$\varepsilon_1 = -(S_{11} + S_{12} + S_{13})P_c,$$
$$\varepsilon_2 = -(S_{12} + S_{22} + S_{23})P_c,$$
$$\varepsilon_3 = -(S_{13} + S_{23} + S_{33})P_c,$$
$$\varepsilon_4 = -(S_{14} + S_{24} + S_{34})P_c,$$
$$\varepsilon_5 = -(S_{15} + S_{25} + S_{35})P_c, \quad \text{and}$$
$$\varepsilon_6 = -(S_{16} + S_{26} + S_{36})P_c. \quad (B.3.2)$$

Under confining pressures there is, therefore, a change in the angles as well as a volume change in the body (Hearmon, 1961).

The volumetric strain, $\varepsilon_o$, is given by

$$\varepsilon_o = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = -[S_{11} + S_{22} + S_{33} + 2(S_{12} + S_{23} + S_{13})]P_c. \quad (B.3.3)$$

The volume compressibility, $\beta_o$, and the linear compressibilities in the main axes: $\beta_{l1}$ in the direction $X_1$, $\beta_{l2}$ in the direction $X_2$ and $\beta_{l3}$ in the direction $X_3$ are given by

$$\beta_o = -\frac{\varepsilon_o}{P_c} = \frac{1}{K_o} = \frac{1}{[S_{11} + S_{22} + S_{33} + 2(S_{12} + S_{23} + S_{13})]}, \quad (B.3.4)$$
\[ \beta_1 = -\frac{\varepsilon_1}{P_c} = \frac{1}{K_{11}} = S_{11} + S_{12} + S_{13}, \]  

(B.3.5)

\[ \beta_2 = -\frac{\varepsilon_2}{P_c} = \frac{1}{K_{12}} = S_{12} + S_{22} + S_{23}, \]  

(B.3.6)

and

\[ \beta_3 = -\frac{\varepsilon_3}{P_c} = \frac{1}{K_{13}} = S_{13} + S_{23} + S_{33} \]  

(B.3.7)

where \( K_{13} \) and \( K_{13} \) are the bulk modulus and the inverse of linear compressibilities.

These equations will be simplified in systems of higher symmetry. For instance, in the hexagonal system, the compliance matrix has the form

\[
\begin{pmatrix}
S_{11} & S_{12} & S_{13} \\
S_{12} & S_{11} & S_{13} \\
S_{13} & S_{13} & S_{13}
\end{pmatrix}
\]

(B.3.8)

\[
\begin{array}{c}
S_{44} \\
S_{44} \\
2(S_{11} - S_{12})
\end{array}
\]

The stiffness matrix is similar to equation B.3.8, but \( S_{66} \) is substituted by

\[ C_{66} = \frac{1}{2} (C_{11} - C_{12}). \]

From equation (B.3.2) and (B.3.8) we find that the volumetric and the linear strains in a hexagonal material under hydrostatic stress are given by

\[ \varepsilon_v = 2\varepsilon_1 + \varepsilon_3 \]  

(volumetric strain),
\[ \varepsilon_1 = -(S_{11} + S_{12} + S_{13})P_c = \varepsilon_2, \quad \text{and} \]
\[ \varepsilon_3 = -(2S_{13} + S_{33})P_c \]

where \( \varepsilon_4 = \varepsilon_5 = \varepsilon_6 = 0 \), and \( \varepsilon_1 = \varepsilon_2 \).

From equations B.2.5-B.2.8 we understand that for the hexagonal system, \( \beta l_1 = \beta l_2 \), there are only two independent linear compressibilities. One in the direction \( X_1 \), and another in the direction \( X_3 \),

\[ \beta l_1 = \beta l_2 = S_{11} + S_{12} + S_{13} = \frac{1}{Kl_1} = \frac{1}{Kl_2} \]  
and

\[ \beta l_3 = 2S_{13} + S_{33} = \frac{1}{Kl_3}. \]

From equations B.2.4 and B.2.8 we understand that, for a hexagonal material that the volume compressibility is reduced to

\[ \beta_0 = 2S_{11} + S_{33} + 2S_{12} + 4S_{13}. \]  

(B.3.11)

**Uniaxial extension or compression**

If a cylindrical, general anisotropic material is cut with its length parallel to some arbitrary direction, \( X_i \), and loaded in simple tension or compression, the tension produces, in general, not only longitudinal and lateral strains but also shear strains (Hearmon, 1961).

If the cylindrical specimen, with its long axis in the \( X_3 \) direction, is extended by the stress \( \sigma_3 \), we have
\[ \varepsilon_1 = S_{13}\sigma_3, \]
\[ \varepsilon_2 = S_{23}\sigma_3, \]
\[ \varepsilon_3 = S_{33}\sigma_3, \]
\[ \varepsilon_4 = S_{43}\sigma_3, \]
\[ \varepsilon_5 = S_{53}\sigma_3, \]
\[ \varepsilon_6 = S_{63}\sigma_3, \]

and the specimen will be sheared in all three coordinate planes as well as extended in all three coordinate directions.

For a cylindrical specimen of hexagonal symmetry, \( S_{43} = S_{53} = S_{63} = 0 \), and consequently, \( \varepsilon_4 = \varepsilon_5 = \varepsilon_6 = 0 \). Therefore, equations B.2.12 are simplified as

\[ \varepsilon_1 = \varepsilon_2 = \varepsilon_3 = S_{13}\sigma_3 \quad \text{and} \quad \varepsilon_4 = \varepsilon_5 = \varepsilon_6 = 0. \] (B.3.13)

The Young's moduli for a general anisotropic media may be defined as follows:

\[ E_j = \frac{1}{S_{ji}} = \frac{\sigma_j}{\varepsilon_j}, \quad (j = 1 \text{ or } 2 \text{ or } 3, \text{ it does not follow the summation convention}), \] (B.3.14a)

and the Poisson's ratios for a general anisotropic media may be defined as follows:

\[ \nu_{ij} = \frac{S_{ji}}{S_{ii}} = -\frac{\varepsilon_j}{\varepsilon_i} \quad (i, j \text{ are the directions of the uniaxial stress and strain, respectively}). \] (B.3.14b)

From equations B.2.14a and B.2.14b the elements of the compliance are given by

\[ S_{ij} = -\frac{\nu_{ij}}{E_i} = -\frac{\nu_{ji}}{E_j}. \]
For a hexagonal material with a stress in the direction $X_3$, we can compute a Young’s modulus and a Poisson’s ratio, using equations (B.3.12) and (B.3.14), as follows:

$$E_3 = \frac{\sigma_3}{\varepsilon_3} = \frac{1}{S_{33}} \quad \text{ (B.3.16)}$$

and

$$v_{31} = \frac{-\varepsilon_1}{\varepsilon_3} = \frac{-S_{13}}{S_{33}}.$$

Note that, for hexagonal materials, $v_{31} = v_{32}$ and $S_{13}$ can be obtained as

$$S_{13} = -\frac{v_{31}}{E_3} \quad \text{ (B.3.17)}$$

If the cylindrical specimen, with its long axis in the $X_1$ direction is extended by the stress $\sigma_1$, for hexagonal symmetry we obtain

$$\varepsilon_1 = S_{11}\sigma_1,$$

$$\varepsilon_2 = S_{12}\sigma_1,$$

and

$$\varepsilon_3 = S_{13}\sigma_1 \quad \text{ (B.3.18)}$$

We can define a Young’s Modulus in this direction $X_1$ and two Poisson’s ratios as

$$E_1 = \frac{1}{S_{11}}, \quad \nu_{12} = -\frac{\varepsilon_2}{\varepsilon_1} = -\frac{S_{12}}{S_{11}}, \quad \text{and} \quad \nu_{13} = -\frac{\varepsilon_3}{\varepsilon_1} = -\frac{S_{13}}{S_{11}} \quad \text{ (B.3.19)}$$

then we can compute $S_{12}$ and $S_{13}$ as

$$S_{12} = -\frac{\nu_{12}}{E_1} \quad \text{ and }$$

$$S_{13} = -\frac{\nu_{13}}{E_1}$$
Finally, if the cylindrical specimen, with its long axis in the $X_2$ direction is extended by the stress $\sigma_2$, for hexagonal symmetry we obtain,

\[ \varepsilon_1 = S_{12} \sigma_2, \]
\[ \varepsilon_2 = S_{11} \sigma_2, \quad \text{and} \]
\[ \varepsilon_3 = S_{13} \sigma_2, \]

and we can define a Young’s Modulus in the direction $X_2$ and two Poisson’s ratios as

\[ E_2 = \frac{1}{S_{22}}, \quad \nu_{21} = -\frac{\varepsilon_1}{\varepsilon_2} = -\frac{S_{12}}{S_{11}}, \quad \text{and} \quad \nu_{13} = -\frac{\varepsilon_3}{\varepsilon_1} = -\frac{S_{13}}{S_{11}}. \]  

(B.3.22)

Under uniaxial stress, the linear compressibilities for a hexagonal material can be expressed as follows:

\[ \beta_1 = \frac{\varepsilon_1 + \varepsilon_2 + \varepsilon_3}{\sigma_1} \quad \text{and} \]

\[ \beta_3 = \frac{\varepsilon_3 + 2\varepsilon_1}{\sigma_3} \]

(B.3.23)

where $\sigma_1$ is a uniaxial stress in the direction $X_1$ and $\sigma_3$ is a uniaxial stress in the direction $X_3$.

**Shear stress system**

Under shear stress, $S_{ij} \quad (\equiv 4S_{ij})$ with $j = 4,5,6$ relates shear strain to shear stress in the same plane (Hearmon, 1961).
If we apply a stress $\sigma = \sigma_4$, in a general anisotropic material, only one element of the strain tensor is different than zero,

$$\varepsilon_4 = S_{44}\sigma_4 \quad (S_{44} = S_{2323}), \quad (B.3.24)$$

and the shear modulus in the plane $X_2X_3$ is given by

$$S_{44} = \frac{1}{G_{23}}. \quad (B.3.25)$$

If $\sigma = \sigma_5$ we have only one strain element,

$$\varepsilon_5 = S_{55}\sigma_5 \quad (S_{55} = S_{1313}), \quad (B.3.26)$$

and the shear modulus in the plane $X_1X_3$ is given by

$$S_{55} = \frac{1}{G_{13}}. \quad (B.3.27)$$

If $\sigma = \sigma_6$ we have again only one strain element different than zero,

$$\varepsilon_6 = S_{66}\sigma_6 \quad (S_{66} = S_{1212}), \quad (B.3.28)$$

and the shear modulus in the plane $X_1X_2$ is given by

$$S_{66} = \frac{1}{G_{12}}. \quad (B.3.29)$$

For hexagonal symmetry $S_{55} = S_{44}$, the shear modulus ($G_{13}$) in the plane $X_1X_3$ is equal to the shear modulus ($G_{12}$) in the plane $X_1X_2$. Consequently, we have only two different
shear moduli, \( G_{13} \) and \( G_{23} \). However, one of them can be obtained from \( S_{11} \) and \( S_{12} \) as

\[
G_{13} = \frac{1}{S_{66}} \left( \frac{1}{2(S_{11} - S_{12})} \right) = \frac{E_1}{2(1 + \nu_{12})};
\]  

(B.3.30)

therefore, for a hexagonal material, we have only one independent shear modulus, \( G_{23} \).

As we will show in next section, if the Young’s modulus is measured at 45°, \( G_{23} \) can be obtained from the knowledge of the young’s moduli in three direction and a Poison’s ratio.

Thus it has been shown that a hexagonal material can be characterized, for instance, by two Young’s moduli, two Poisson’s ratios, and a shear modulus,

\[
E_1 = E_2, \quad E_3, \quad \nu_{13} = \nu_{23}, \quad \nu_{31}, \quad G_{23}
\]

**Interconversion of the stiffness \( C_{ij} \) and compliance \( S_{ij} \)**

The sets of the equations \( C_{ij} \) and \( S_{ij} \) can be written in determinant forms (Hearmon, 1961)

\[
\Delta C = \begin{vmatrix}
C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\
C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\
C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36} \\
C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46} \\
C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56} \\
C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66}
\end{vmatrix}
\]

(B.3.31)

and
\[ \Delta S = \begin{vmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ S_{12} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ S_{13} & S_{23} & S_{33} & S_{34} & S_{35} & S_{36} \\ S_{14} & S_{24} & S_{34} & S_{44} & S_{45} & S_{46} \\ S_{15} & S_{25} & S_{35} & S_{45} & S_{55} & S_{56} \\ S_{16} & S_{26} & S_{36} & S_{46} & S_{56} & S_{66} \end{vmatrix} . \]  
(B.3.32)

It follows from the definition of \( C_{ij} \) and \( S_{ij} \) that

\[ e_{ij} = S_{ijkl} \delta_{kl} = S_{ijk} C_{ij} e_i = \delta_{ij} e_i \]  
(B.3.33)

where \( \Delta S \Delta C = 1 \),

\[ \Delta S = \frac{\Delta C_{ij}}{\Delta C}, \quad \text{and} \quad C_{ij} = \frac{\Delta S_{ij}}{\Delta S} . \]  
(B.3.35)

\( \Delta C_{ij} \) and \( \Delta S_{ij} \) are the minor determinants associated with \( C_{ij} \) and \( S_{ij} \).

Equations (4.2.31) to (4.2.32) can be simplified by the reduction in the number of elastic constants due to symmetry. In the hexagonal system the determinant can be expanded to give relatively simple expressions as follows:

\[ C_{11} = \frac{1}{2} \left[ \frac{S_{33}}{S_{33}(S_{11} + S_{12}) - 2 S_{13}^2} + \frac{1}{S_{11} - S_{12}} \right], \]

\[ C_{12} = \frac{1}{2} \left[ \frac{S_{33}}{S_{33}(S_{11} + S_{12}) - 2 S_{13}^2} - \frac{1}{S_{11} - S_{12}} \right], \]  
(B.3.36)

\[ C_{13} = \frac{-S_{13}}{S_{33}(S_{11} + S_{12}) - 2 S_{13}^2}, \]

\[ C_{33} = \frac{(S_{11} + S_{12})}{S_{33}(S_{11} + S_{12}) - 2 S_{13}^2}, \]
\[ C_{44} = \frac{1}{S_{44}}. \]

These equations can also be used to convert the stiffnesses into compliances simply by substituting \( C_{ij} \) for \( S_{ij} \) and \( S_{ij} \) for \( C_{ij} \).

We can write the compressibilities in term of Poisson’s ratios and Young’s moduli as

\[
K_{11} = \frac{E_1}{1 - \nu_{12} - \nu_{13}} = \frac{C_{33}(C_{11} + C_{12}) - 2C_{213}}{C_{33} - C_{13}} = \frac{1}{\beta_{11}},
\]

\[
K_{13} = \frac{E_3}{1 - 2\nu_{31}} = \frac{C_{33}(C_{11} + C_{12}) - 2C_{213}}{C_{11} + C_{12} - 2C_{13}} = \frac{1}{\beta_{13}},
\]

\[
K_b = \frac{1}{2\beta_1 + \beta_3} = \frac{E_1}{[2 - \nu_{12} - 3\nu_{13} + \frac{\nu_{13}}{\nu_{31}}]} = \frac{E_1E_3}{2E_3(1 - \nu_{12} - \nu_{13}) + E_1(1 - 2\nu_{31})} = \frac{1}{\beta_b},
\]

and

\[
K_b = \frac{C_{33}(C_{11} + C_{12}) - 2C_{213}}{2C_{33} + C_{11} + C_{12} - 4C_{13}}.
\]

Finally, using equations (4.2.16) to (4.2.22) and (4.2.36) we can write the Young’s Moduli, Poison’s ratios and compressibilities in terms of the stiffnesses as follows:

\[
E_1 = \frac{[C_{33}(C_{11} + C_{12}) - 2C_{213}](C_{11} - C_{12})}{C_{11}C_{33} - C_{13}}, \quad \text{(B.3.38)}
\]

\[
E_3 = \frac{C_{33}(C_{11} + C_{12}) - 2C_{213}}{C_{11} + C_{12}}, \quad \text{(B.3.39)}
\]
\[ \nu_{31} = \frac{C_{13}}{C_{11} + C_{12}}, \quad \text{(B.3.40)} \]

\[ \nu_{12} = \frac{C_{33}C_{12} - C_{213}^2}{C_{11}C_{33} - C_{213}^2}, \quad \text{(B.3.41)} \]

and

\[ \nu_{13} = \frac{C_{13}(C_{11} - C_{12})}{C_{11}C_{33} - C_{213}^2}. \quad \text{(B.3.42)} \]

Using equations 4.2.36 we can obtain stiffnesses from static measurements as follows:

\[ C_{33} = \frac{(1 - \nu_{12})E_3}{(1 - \nu_{12} - 2\nu_{31}\nu_{13})}, \]

\[ C_{13} = \frac{\nu_{31}E_1}{(1 - \nu_{12} - 2\nu_{31}\nu_{13})}, \]

\[ C_{11} = \frac{E_1(1 + \nu_{12}) - \nu_{213}^2E_3}{(1 - \nu_{12} - 2\nu_{31}\nu_{13})(1 + \nu_{12})}, \quad \text{(B.3.43)} \]

\[ C_{44} = G_{23}, \]

\[ C_{12} = \frac{(\nu_{31}\nu_{13} + \nu_{12})E_1}{(1 - \nu_{12} - 2\nu_{31}\nu_{13})(1 + \nu_{12})}, \]

and

\[ C_{66} = G_{12} = \frac{E_1}{2(1 + \nu_{12})}. \]
Measures of anisotropies

Anisotropic stiffnesses

The Poisson’s ratio and Young’s modulus anisotropies for a general anisotropic material are given by

\[
\frac{\nu_{31}}{\nu_{13}} = \frac{S_{11}}{S_{33}} = \frac{E_3}{E_1},
\]

\[
\frac{\nu_{12}}{\nu_{21}} = \frac{S_{22}}{S_{11}} = \frac{E_1}{E_2},
\]

and

\[
\frac{\nu_{23}}{\nu_{32}} = \frac{S_{33}}{S_{22}} = \frac{E_2}{E_3}
\]

for a hexagonal symmetry \( \frac{\nu_{12}}{\nu_{21}} = 1 \) and \( \frac{\nu_{13}}{\nu_{31}} = \frac{\nu_{23}}{\nu_{32}} \) because

\( \nu_{13} = \nu_{23}, \quad \nu_{32} = \nu_{31}, \quad \text{and} \quad \nu_{12} = \nu_{21}. \)

The only measurement of Young’s modulus anisotropy is given by

\[
\frac{E_1}{E_3} = \frac{C_{11}^2 - C_{12}^2}{C_{11}C_{33} - C_{13}^2}.
\]

The shear modulus anisotropies for a general anisotropic material are given by

\[
\frac{G_{12}}{G_{13}} = \frac{S_{44}}{S_{66}} \quad \text{and} \quad \frac{G_{12}}{G_{23}} = \frac{S_{55}}{S_{66}}
\]
where \( G_{12} = G_{21} \), \( G_{13} = G_{31} \) and \( G_{23} = G_{32} \).

For hexagonal symmetry \( S_{44} = S_{55} \), there is only one measurement of anisotropy given by

\[
\frac{G_{12}}{G_{23}} = \frac{S_{44}}{S_{66}} = \frac{C_{66}}{C_{44}} = \frac{C_{11} - C_{12}}{2C_{44}}.
\]  
(B.3.47)

The linear compressibilities anisotropy is given by

\[
\frac{\beta_{li}}{\beta_{lj}} = \frac{S_{lj}}{S_{lj}}
\]  
(B.3.48)

where \( \beta_{li} = S_{lj} \) and \( i,j = 1,2,3 \), and Einstein’s convention follows.

For hexagonal symmetry \( \beta_{l1} = \beta_{l2} \), and the only measured of anisotropy is given by

\[
\frac{\beta_{l1}}{\beta_{l3}} = \frac{C_{33} - C_{13}}{C_{11} + C_{12} - 2C_{13}}.
\]  
(B.3.49)

**Representation of the anisotropic elastic behavior by surfaces**

It is possible to generate surfaces of stiffness or compliance to represent the elastic behavior of an anisotropic material. More than one surface is needed for a full description of anisotropic behavior. Hexagonal systems can be described by a young’s modulus surface, a shear modulus surface, and a linear compressibility surface.
The Young's Modulus surface

The Young's modulus for an arbitrary direction of tension, $X_i'$, is defined as the ratio of the longitudinal stress to the longitudinal strain; that is

$$ E' = \frac{1}{S'_{i111}}. $$

For hexagonal symmetry, $S'_{i111}$ is given by

$$ S'_{i111} = \frac{1}{E'} = a_{i1} a_{1j} a_{1k} S_{ijkl}. \quad (B.3.50) $$

Using the compliances for this symmetry, the Young's moduli is given by (Nye, 1957)

$$ E(\theta) = S'_{i111} = S_{11} \sin^4 \theta + S_{33} \cos^4 \theta + (S_{44} + 2S_{13}) \sin^2 \theta \cos^2 \theta \quad (B.3.51) $$

where $\theta$ is the angle between the arbitrary direction $X_i'$ and the material axes, $X_3$.

For instance, for $\theta = 45^\circ$ we obtain

$$ S_{11}(45) = \frac{1}{4} (S_{11} + S_{33} + S_{44} + 2S_{13}) = \frac{1}{E(45^\circ)}, \quad (B.3.52) $$

and using

$$ S_{13} = \frac{\nu_{31}}{E_3}, \quad S_{11} = \frac{1}{E_1}, \quad \text{and} \quad S_{33} = \frac{1}{E_3} \quad (B.3.53) $$

we can compute $S_{44}$ by measuring the Young's modulus in three different directions and a Poisson's ratio as follows:

$$ S_{44} = \frac{4}{E(45^\circ)} - \frac{1}{E_1} - \frac{1-2\nu_{31}}{E_3} = \frac{1}{G_{23}} \quad (B.3.54) $$
where \( G_{23} \) is the only independent rigidity modulus for the hexagonal system.

**The rigidity surface**

The rigidity modulus \( S'_{44} = G(\theta)^{-1} \) in an arbitrary plane also depends on the direction in which shear is applied. For a hexagonal system it is given by

\[
G(\theta)^{-1} = S_{44} + (S_{11} - S_{12} - \frac{S_{44}}{2}) \sin^2(\theta) + 2(S_{11} + S_{33} - 2S_{13} - S_{44}) \cos^2(\theta) \sin^2(\theta).
\] (B.3.55)

**The linear compressibility surface**

In general the linear compressibility varies with direction. For a material of any symmetry it is given by (Nye, 1956)

\[
\beta_l = S_{ijkl} l^i
\] (B.3.56)

where \( l^i \) is a unit vector.

For hexagonal symmetry the linear compressibility surface is given by Nye (1956) as

\[
\beta_l(\theta) = (S_{11} + S_{12} + S_{13}) - (S_{11} + S_{12} - S_{13} - S_{33}) \cos^2 \theta.
\] (B.3.57)

For the following angles the equation B.3.57 becomes

\[
\begin{align*}
\beta_l(0^\circ) &= 2S_{13} + S_{33}, \quad \text{and} \\
\beta_l(90^\circ) &= S_{11} + S_{12} + S_{13}, \\
\beta_l(45^\circ) &= \frac{1}{2} (S_{11} + S_{12} + 3S_{13} + S_{33}).
\end{align*}
\] (B.3.58) (B.3.59) (B.3.60)
Appendix C

Nonlinear, transverse isotropic materials

As discussed in the introduction, rocks are not ideal elastic materials. In contrast to common engineering materials, such as metals or ceramics, the elastic properties vary with the state of the stress. Differences of as much as a factor of ten between the high pressure and low pressure values of, for instance, bulk modulus, are not uncommon (Zemmerman, 1991). The elastic properties of rocks should be calculated at the appropriate stress levels. Different rock types can be characterized by the behavior of the elastic properties as a function of pressures.

Young’s Moduli

Young’s moduli are obtained from three uniaxial stress experiments parallel, perpendicular and at 45° with respect to the plane of isotropy (X1X2) (figures 4.2 to 4.4).

\[ E_3 = \frac{\partial \sigma_3}{\partial \varepsilon_3} \quad \text{(from a vertical sample)}, \quad (C.1.1) \]

\[ E_1 = E_2 = \frac{\partial \sigma_1}{\partial \varepsilon_1} = \frac{\partial \sigma_2}{\partial \varepsilon_2} \quad \text{(from a horizontal sample)} \quad (C.1.2) \]

and

\[ E(45) = \frac{\partial \sigma}{\partial \varepsilon} \quad \text{(from an inclined sample)} \quad (C.1.3) \]
The third part of figure 3.1 shows the uniaxial stresses exerted on the sample specimens at three confining pressure levels.

**Poisson’s Ratio**

Poisson’s ratios are obtained as follows:

\[
\nu_{31} = \nu_{32} = -\frac{\partial \varepsilon_1}{\partial \varepsilon_3} = -\frac{\partial \varepsilon_2}{\partial \varepsilon_3} \quad (\text{obtained from a vertical sample})
\]

(C.1.4)

and

\[
\begin{cases}
\nu_{12} = -\frac{\partial \varepsilon_2}{\partial \varepsilon_1} \\
\nu_{13} = -\frac{\partial \varepsilon_3}{\partial \varepsilon_1}
\end{cases} \quad (\text{obtained from a horizontal sample}).
\]

(C.1.5)

**Bulk Modulus and Linear Compressibilities**

Bulk moduli, linear and volume compressibilities can be obtained from a hydrostatic load cycle as follows:

\[
\beta_b = \frac{d \varepsilon_b}{d P_c} = \frac{1}{K_b},
\]

(C.1.6)

\[
\beta_{l1} = \frac{d \varepsilon_1}{d P_c} = \frac{1}{K_{l1}}
\]

(C.1.7)

and

\[
\beta_{l3} = \frac{d \varepsilon_3}{d P_c} = \frac{1}{K_{l3}}
\]

(C.1.8)

where the volumetric strain is given by \( \varepsilon_b = 2\varepsilon_1 + \varepsilon_3 \).
The linear compressibilities, $\beta_i$ or their inverse, $K_i$, can also be obtained from uniaxial stresses using the obtained Poisson’s ratios and Young’s moduli as follows:

$$\beta_{i1} = \frac{\partial \varepsilon_1}{\partial \sigma_1} + \frac{\partial \varepsilon_2}{\partial \sigma_1} + \frac{\partial \varepsilon_3}{\partial \sigma_1} = \frac{1 - \mu_{12} - \mu_{13}}{E_1}$$  \hspace{1cm} (C.1.9)

and

$$\beta_{i3} = \frac{\partial \varepsilon_3}{\partial \sigma_3} + 2 \frac{\partial \varepsilon_1}{\partial \sigma_3} = \frac{1 - 2\mu_{31}}{E_3}.$$  \hspace{1cm} (C.1.10)
Appendix D

Static and dynamic elastic behavior of isotropic and transversely isotropic shales and sandstones.

D.1 Introduction

In this appendix we present the laboratory results and data of this study, as well as an extension of the discussion on the static versus dynamic elastic parameters in anisotropic shales and in dry and brine fully saturated sandstones. Instead of analyzing the velocities’ behavior, as done in Chapter 3, we base our discussion on the study of the behavior of elastic parameters. The elastic parameters chosen are the dynamic and static Young’s, shear, and bulk moduli, Poisson’s ratios, linear compressibilities, strain-stress and strain rate-stress relationships. We consider that the behavior of the chosen parameters is easier to be interpreted than the behavior of stiffnesses (C_{ij}) or compliances (S_{ij}). However, the information obtained from all possible elastic parameters may be useful tools to fully understand possible mechanisms of energy loss (anelasticity) and the anisotropic behavior of shales and sandstones.

D.2 Sample description and instrumentation

The descriptions and characteristics of the shales and sandstone specimens tested are shown in Tables 3.1 and 3.2 (set I and II). Figure D.1 show how smaller oriented rock specimens were cut from larger cores. Figure D.2 shows the directions of wave
propagation and polarizations of the particle in the dynamic experiments. Figures D.3 and D.5 show how the rock specimens were instrumented with strain gauges and the orientation of the velocity transducers with respect to the main axes of the material.

D.3 Static versus dynamic elastic parameters in dry and brine saturated isotropic sandstones

Tables D.1, D.2 and D.3 show the $V_p$ and $V_s$ velocities, the ratio of $V_p/V_s$, the dynamic Young's ($E_{dyn}$), shear ($G_{dyn}$) and bulk ($K_{dyn}$) moduli, and the dynamic Poisson's ratio ($\nu_{dyn}$), for a sandstone specimen in dry and brine saturated conditions, as functions of confining pressures (5 MPa to 70 Mpa). In the saturated state, the pore pressure was kept constant at 2 MPa and 40 MPa during the hydrostatic compression and uniaxial stress cycles (figure 3.1). We computed the dynamic elastic parameters for the isotropic sandstone specimen substituting the velocities listed in Tables D.1 and D.3 into equations A.9 to A.12.

Table D.4 shows the dynamic and static Poisson's ratios ($\nu_{dyn}$ and $\nu_{stat}$), Young's ($E_{dyn}$ and $E_{stat}$), shear ($G_{dyn}$ and $G_{stat}$), and bulk ($K_s$ and $K_b$) moduli versus pore and hydrostatic pressures. We computed the static parameters for the isotropic sandstone, using equations B.2.2 to B.2.10 and the measured stress-strain data. The static and dynamic elastic parameters computed satisfy the restrictions given by equations A.7.

Figures D.6 shows the behavior of the static and dynamic Poisson's ratios ($\nu_{stat}$ and $\nu_{dyn}$) as a function of hydrostatic pressure for a sandstone specimen in room dry and saturated conditions. In the dry state, $\nu_{dyn}$ remains approximately constant as the confining pressure is increased, while $\nu_{stat}$ decreases slightly. After saturation, $\nu_{dyn}$ shows a pronounced increase, while $\nu_{stat}$ increases only slightly, and both $\nu_{stat}$ and $\nu_{dyn}$ decrease as the confining pressure is increased. When the pore pressure is raised from 2 MPa to 40 MPa, both $\nu_{stat}$ and $\nu_{dyn}$ increase; however, the increase in $\nu_{dyn}$ is so much larger than $\nu_{stat}$, which at high confining pressure is only slightly affected by the saturation and pore
pressure. In both dry and saturated conditions, \( v_{\text{stat}} \) is always smaller than \( v_{\text{dyn}} \), over the whole range of pressures.

Figure D.7 shows the behavior of the static and dynamic Young’s moduli (\( E_{\text{stat}} \) and \( E_{\text{dyn}} \)) as functions of the confining pressure. Both \( E_{\text{dyn}} \) and \( E_{\text{stat}} \) increase as confining pressure is increased. The rate of increase is smaller at high high pressures. \( E_{\text{dyn}} \) is always smaller than \( E_{\text{stat}} \) over the whole range of confining pressures. However, the difference between \( E_{\text{stat}} \) and \( E_{\text{dyn}} \) is much smaller in the saturated state.

After saturation, \( E_{\text{dyn}} \) is affected only slightly, but \( E_{\text{stat}} \) shows a pronounced increase. When the pore pressure is raised from 2 MPa to 40 Mpa, \( E_{\text{dyn}} \) shows a slight decrease, while \( E_{\text{stat}} \) increases dramatically, and both approach the same value.

The behavior of the dynamic shear modulus (\( G_{\text{dyn}} \)) is similar to that observed in the dynamic shear velocities (\( V_s \)). In dry conditions, \( G_{\text{stat}} \) is smaller than \( G_{\text{dyn}} \) over the whole range of pressure. After saturation, \( G_{\text{stat}} \) and \( G_{\text{dyn}} \) show opposite behavior, while \( G_{\text{stat}} \) increases, and \( G_{\text{dyn}} \) decreases. The difference between \( G_{\text{dyn}} \) and \( G_{\text{stat}} \) decreases with increasing confining pressure and is much smaller in the saturated state.

The value of the static modulus depends on the loading path (figure D.9). This dependence is due to the nonlinear relationship between stress and strain. If the bulk modulus is measured from a stepwise hydrostatic cycle, as in the first part of figure 3.1, we obtain smaller values than if we measure it using continuous cycles, as illustrated in the second part of the loading path shown in figure 3.1.

### D.4 Static versus dynamic elastic parameters of anisotropic shales

We compute the static elastic parameters for the transversely isotropic shales, substituting the stress-strain measurements into the equations derived in section B.2. The dynamic elastic parameters were computed substituting 5 of the 9 experimental velocities measured into equations A.13 to A.16.
Figures D.10 and D.11 show the resulting vertical ($\varepsilon_3$), horizontal ($\varepsilon_1 = \varepsilon_2$) and volumetric ($\varepsilon_8 = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$) strains when a constant rate hydrostatic cycle, as illustrated in the second part of figure 3.1, is exerted on shale specimens. Under confining pressure in a transverse isotropic material, $\varepsilon_1$ is equal to $\varepsilon_2$. Notice, however, in figure D.10 that $\varepsilon_1$ and $\varepsilon_2$ differ slightly, which is due to the fact that the sample is not an ideal transverse isotropic material due to the errors introduced by the experimental set-up.

Notice that the area enclosed by the hysteresis loop is bigger in the vertical direction ($X_3$), normal to the plane of isotropy, than in the horizontal direction ($X_1$) (figure D.10). Even though the stress applied is the same in all directions (hydrostatic), we observe elastic and anelastic anisotropy. As a consequence, we may observe linear compressibility anisotropy, $\beta_{11} / \beta_{13} \neq 1$ (elastic anisotropy), as well as anisotropy in the attenuation factor $Q\beta$ associated with linear compressibilities, $Q\beta(0^\circ) / Q\beta(90^\circ) \neq 1$ (anelastic anisotropy). The elastic anisotropy is caused, in part, by the presence of cracks oriented parallel to the plane of isotropy (foliation plane), which makes the rock softer in the vertical direction than in the horizontal, and by fine lamination characteristic of shales. The hysteresis observed in plots of the volumetric strain versus confining pressure (figure D.11) indicates that there is an important amount of energy loss under hydrostatic compression. The presence of preferred oriented cracks causes nonlinearity anisotropy. The stress-strain relationship is more linear in the direction perpendicular to the plane of preferred fractures orientation.

Figures D.12 shows that there is a significant difference between the static linear compressibility in the vertical direction ($K_{13}$) and in the horizontal direction ($K_{11}$). The difference remains approximately constant with increasing the confining pressure. Both $K_{11}$ and $K_{13}$ increase and decrease linearly during the loading and unloading paths of the hydrostatic cycle, respectively. However, the slope of the loading path is smaller than the slope of the unloading. Figure D.13 shows that $K_6$ exhibits similar behavior to $K_{11}$ and $K_{13}$.
The ratio of horizontal strain to vertical strain, $\varepsilon_1/\varepsilon_3$, decreases from 1.0 to 0.4 when the confining pressure is increased from 7 MPa to 17 MPa (figure D.14). However, it shows a monotonic increase above 17 MPa. For confining pressure higher than 34 MPa, when cracks are closed, $\varepsilon_1/\varepsilon_3$ becomes a single value function of the pressure. At low confining pressures, preferential orientation of cracks in the layering plane of the shale specimen induces an elastic and anelastic anisotropy that are superimposed on the intrinsic elastic and anelastic anisotropy of shales. These fractures close after reaching a pressure of about 28 MPa. Above this pressure the elastic behavior is governed by the fabric of the shale frame, and the intrinsic elastic and anelastic anisotropic behavior of shale becomes dominant (figure D.14). In uncracked shales the ratio $\varepsilon_1/\varepsilon_3$ increases with increasing confining pressure (figure E.1).

Figure D.15 shows the vertical and horizontal strain rate, $\dot{\varepsilon}_3$ and $\dot{\varepsilon}_1$, versus confining pressure. $\dot{\varepsilon}_1$ and $\dot{\varepsilon}_3$ do not behave exactly in the same way as the linear compressibilities $\beta_{11}$ and $\beta_{13}$ because the relation between $\dot{\varepsilon}$ and $\beta$ is not linear (equations 3.3.4.1 and 3.3.4.2). The existence of a nonlinear relationship between $\dot{\varepsilon}$ and $\beta$ indicates that there are frequency dependent mechanisms taking place during the deformation process. Notice that the difference between the strain rate during the loading and unloading part of the cycle is more pronounced in the vertical than in the horizontal directions (figure D.15). The slope of the horizontal strain rate, $\dot{\varepsilon}_1$, versus confining pressure remains approximately constant over the whole range of confining pressure. However, in the vertical direction there is a change of slope in the strain rate $\dot{\varepsilon}_3$ after reaching a pressure of approximately 28 MPa to 35 MPa that may be related with the closure of the cracks.

Figures D.16 shows the behavior of the inverse of linear compressibility anisotropy, $K_{11}/K_{13}$, as a function of confining pressure. $K_{11}/K_{13}$ decreases as the pressure is increased; however, the dynamic anisotropy is always smaller than the static anisotropy over the whole range of pressure (figure D.16). The static linear compressibility anisotropy is larger for the stepwise hydrostatic cycle than for the continuous cycle. It is an indication of an amplitude dependent mechanisms caused by the nonlinear relationship between stress and strain.
Table C.6 shows the 5 experimental dynamic velocities needed to describe elastically the transverse isotropic shale specimens. Substituting these velocities in equations A.16, we computed the 5 dynamic stiffnesses: $C_{11}$, $C_{33}$, $C_{44}$, $C_{66}$, and $C_{13}$. From these five stiffnesses and equations B.3.37 to B.3.42, we determined the dynamic bulk modulus and the inverse of linear compressibilities, the dynamic Young’s and shear moduli, and the dynamic Poisson’s ratios (Table C.7). The static elastic parameters are obtained using the equations derived in section B.3 and the measured stress-strain data. The computed dynamic and static stiffnesses are listed in Tables D.9 and D.10, respectively.

Figure D.17 shows the variation of the static Young’s moduli ($E_{3}\text{(stat)}$, $E(45^\circ)\text{(stat)}$ and $E_{1}\text{(stat)}$) and the dynamic Young’s moduli ($E_{3}\text{(dyn)}$, $E(45^\circ)\text{(dyn)}$ and $E_{1}\text{(dyn)}$) with confining pressure. $E_{1}\text{(stat)}$, $E_{3}\text{(stat)}$ and $E(45)\text{(stat)}$ increase with confining pressure and the rate of increase is more pronounced at low confining pressure. On the other hand, the dynamic Young’s moduli $E_{3}\text{(dyn)}$ and $E(45)\text{(dyn)}$ increase with increasing confining pressure while $E_{1}$ decreases. For hydrostatic stresses higher than 41 MPa, the dynamic and static Young’s moduli approach a constant value. The rate of increase of the dynamic Young’s moduli is not as pronounced as that of the static Young’s moduli. At high confining pressures the ratio of dynamic to static Young’s moduli tends to be one, for both vertical ($E_{3}$) and horizontal ($E_{1}$) moduli (figure D.20). The static and dynamic Young’s modulus anisotropies, $E_{1}\text{(stat)}/E_{3}\text{(stat)}$ and $E_{1}\text{(dyn)}/E_{3}\text{(dyn)}$, decrease with confining pressure; however, the rate of decrease is stronger for the static moduli. For stresses higher than 35 MPa the static and dynamic anisotropies tend to be equal (figure D.21). Even though the static and dynamic anisotropy in Young’s moduli decrease due to the closing of the cracks parallel to the plane of isotropy, at high pressures the intrinsic Young’s modulus anisotropic behavior of shales remains.

The static Poisson’s ratio $\nu_{31}\text{(stat)}$ increases sharply and linearly, as the confining pressure is increased (figure D.18). However, the opposite behavior is observed for the static Poisson’s ratios. $\nu_{13}\text{(stat)}$ and $\nu_{12}\text{(dyn)}$ approach a constant value for confining pressure higher than 41 MPa, but $\nu_{31}\text{(stat)}$ shows a monotonic increase. The dynamic Poisson’s ratios $\nu_{31}\text{(dyn)}$ and $\nu_{13}\text{(dyn)}$ show a constant rate increase as the pressure is raised, while
\(v_{12}(\text{dyn})\) decreases. For confining pressures higher than 21 MPa, \(v_{31}(\text{dyn})\) and \(v_{13}(\text{dyn})\) are always higher in value than \(v_{31}(\text{stat})\) and \(v_{13}(\text{stat})\), respectively; however, at low confining pressure the opposite behavior is observed. \(v_{12}(\text{stat})\) is always bigger than \(v_{12}(\text{dyn})\), and both decrease as the confining pressure is raised (figure D.18).

The static shear moduli, \(G_{13}(\text{stat})\) and \(G_{23}(\text{stat})\), increase with increasing pressure, but the rate of increase becomes less as the confining pressure is raised (figure D.19). The difference between \(G_{13}(\text{stat})\) and \(G_{23}(\text{stat})\) remains approximately constant as pressure is increased. \(G_{23}(\text{dyn})\) is always bigger than \(G_{23}(\text{stat})\), and they vary similarly as the stress is raised. \(G_{13}(\text{dyn})\) is always bigger than \(G_{13}(\text{stat})\), and it decreases as the pressure is raised, which is the opposite behavior to that observed in \(G_{13}(\text{stat})\). The difference between \(G_{13}(\text{dyn})\) and \(G_{13}(\text{stat})\) becomes less as the pressure is increased (figure D.19).

The static and dynamic anisotropies in shear moduli do not approach the same value as pressure is increased as observed for Young’s moduli (figure D.21). The dynamic shear anisotropy remains higher than the static anisotropy over the whole range of confining pressures.
Table D. 1 $V_p$ and $V_s$-velocities, ratio $V_p/V_s$, dynamic bulk modulus $K_{dyn}$, dynamic Young's modulus $E_{dyn}$, dynamic Poisson's ratio, and dynamic shear modulus $G_{dyn}$ versus confining pressure for a room dry sandstone.

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<th>Effective Pressure (MPa)</th>
<th>$V_p$ (m/sec)</th>
<th>$V_s$ (m/sec)</th>
<th>$V_p/V_s$</th>
<th>$K_{dyn}$ (GPa)</th>
<th>$E_{dyn}$ (GPa)</th>
<th>$v_{dyn}$</th>
<th>$G_{dyn}$ (GPa)</th>
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<td>10.10</td>
<td>22.20</td>
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</table>

Table D. 2 $V_p$ and $V_s$-velocities, ratio $V_p/V_s$, dynamic bulk modulus $K_{dyn}$, dynamic Young’s modulus $E_{dyn}$, dynamic Poisson’s ratio $\nu_{dyn}$, and dynamic shear modulus $G_{dyn}$ versus effective pressure for a saturated sandstone. The pore pressure is kept constant at 290 psi.

<table>
<thead>
<tr>
<th>Effective Pressure (MPa)</th>
<th>$V_p$ (m/sec)</th>
<th>$V_s$ (m/sec)</th>
<th>$V_p/V_s$</th>
<th>$K_{dyn}$ (GPa)</th>
<th>$E_{dyn}$ (GPa)</th>
<th>$\nu_{dyn}$</th>
<th>$G_{dyn}$ (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>3598</td>
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<td>25.09</td>
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<td>29.05</td>
<td>0.225</td>
<td>11.86</td>
</tr>
<tr>
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<td>2374</td>
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<td>0.193</td>
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<td>26.60</td>
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</table>

Table D. 3 $V_p$ and $V_s$-velocities, ratio $V_p/V_s$, dynamic bulk modulus $K_{dyn}$, dynamic Young’s modulus $E_{dyn}$, dynamic Poisson’s ratio $\nu_{dyn}$, and dynamic shear modulus $G_{dyn}$ versus effective pressure for a saturated sandstone. The pore pressure is kept constant at 5850 psi.

<table>
<thead>
<tr>
<th>Effective Pressure (MPa)</th>
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<th>$V_s$ (m/sec)</th>
<th>$V_p/V_s$</th>
<th>$K_{dyn}$ (GPa)</th>
<th>$E_{dyn}$ (GPa)</th>
<th>$\nu_{dyn}$</th>
<th>$G_{dyn}$ (GPa)</th>
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<td>Saturated</td>
<td>Saturated</td>
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<td>-----------</td>
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<td>Saturated</td>
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<td>(m/sec)</td>
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<td>32.30</td>
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</tr>
</tbody>
</table>

**Table D.4** Static (stat) and dynamic (dyn) elastic parameters of dry and saturated sandstone versus effective pressure. 

*K_{stat1}^2* is the bulk modulus obtained from a uniaxial stress cycle, and *K_{stat2}^2* is the bulk modulus obtained from a hydrostatic compression cycle. This is a sandstone specimen.

<table>
<thead>
<tr>
<th>Shale Specimen</th>
<th>Hydrostatic Step Wise</th>
<th>Hydrostatic Continous</th>
<th>Dynamic</th>
<th>Ratio d/s</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>C.P. (psi)</td>
<td>C.P. (MPa)</td>
<td>K_{b(stat)}</td>
<td>K_{l3}</td>
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</tr>
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<td>9.36</td>
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<tr>
<td></td>
<td>10143</td>
<td>69.9</td>
<td>17.11</td>
<td>35.17</td>
</tr>
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</table>

**Table D.5** Dynamic and static bulk modulus and inverses of the linear compressibilities in the vertical and horizontal directions. The static moduli were obtained from a stepwise hydrostatic compression cycle and a continuous hydrostatic cycle. The dynamic experiment was performed during the stepwise cycle. This is a Shale specimen.

<table>
<thead>
<tr>
<th>C.P. (psi)</th>
<th>C.P. (MPa)</th>
<th>V_p(0°) (m/sec)</th>
<th>V_p(45°) (m/sec)</th>
<th>V_p(90°) (m/sec)</th>
<th>V_d(0°) (m/sec)</th>
<th>V_m(90°) (m/sec)</th>
</tr>
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<tbody>
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<td>4056</td>
<td>4644</td>
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</tr>
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<td>4224</td>
<td>4702</td>
<td>2529</td>
<td>2961</td>
</tr>
<tr>
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<td>3928</td>
<td>4360</td>
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</tr>
<tr>
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</tr>
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<td>4257</td>
<td>4628</td>
<td>4853</td>
<td>2706</td>
<td>3014</td>
</tr>
</tbody>
</table>

**Table D.6** Dynamic velocities at different directions, versus confining pressure.
<table>
<thead>
<tr>
<th>C.P. (psi)</th>
<th>C.P. (MPa)</th>
<th>( E_3 ) (GPa)</th>
<th>( E_1 ) (GPa)</th>
<th>( E(45^\circ) )</th>
<th>( v_{31} )</th>
<th>( v_{12} )</th>
<th>( v_{13} )</th>
<th>( G_{13} ) (GPa)</th>
<th>( G_{23} ) (GPa)</th>
<th>( K_b ) (GPa)</th>
</tr>
</thead>
<tbody>
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<td>52.78</td>
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<td>50.72</td>
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<td>29.8</td>
<td>32.97</td>
<td>49.98</td>
<td>37.19</td>
<td>0.152</td>
<td>0.163</td>
<td>0.243</td>
<td>21.50</td>
<td>16.49</td>
<td>22.87</td>
</tr>
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<td>0.149</td>
<td>0.282</td>
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<td>18.08</td>
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<td>37.74</td>
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<td>0.126</td>
<td>0.342</td>
<td>20.78</td>
<td>18.88</td>
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</tr>
</tbody>
</table>

**Table D. 7.** Dynamic elastic parameters versus confining pressure for transverse isotropic shale specimen. 

\( v_{ij} \) are the Poisson’s ratios. \( K_b(\text{stat1}) \) and \( K_b(\text{stat2}) \) are the bulk moduli obtained from uniaxial stress cycles and a hydrostatic compression cycle, respectively. \( G_{13} \) and \( G_{23} \) are the shear moduli for a T.I. medium.

<table>
<thead>
<tr>
<th>C.P. (psi)</th>
<th>C.P. (MPa)</th>
<th>( E_3 ) (GPa)</th>
<th>( E_1 ) (GPa)</th>
<th>( E(45^\circ) )</th>
<th>( v_{31} )</th>
<th>( v_{12} )</th>
<th>( v_{13} )</th>
<th>( v(45^\circ) )</th>
<th>( G_{13} ) (GPa)</th>
<th>( G_{23} ) (GPa)</th>
<th>( K_b(\text{stat1}) ) (GPa)</th>
<th>( K_b(\text{stat2}) ) (GPa)</th>
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**Table D. 8.** Static elastic parameters versus confining pressure for transverse isotropic shale specimen.

<table>
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<th>( C_{12} ) (GPa)</th>
<th>( C_{13} ) (GPa)</th>
<th>( C_{33} ) (GPa)</th>
<th>( C_{44} ) (GPa)</th>
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</tr>
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<tr>
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</tbody>
</table>

**Table D. 9** Dynamic stiffnesses versus confining pressure for a transverse isotropic shale specimen.

<table>
<thead>
<tr>
<th>C.P. (psi)</th>
<th>C.P. (MPa)</th>
<th>( C_{11} ) (GPa)</th>
<th>( C_{12} ) (GPa)</th>
<th>( C_{13} ) (GPa)</th>
<th>( C_{33} ) (GPa)</th>
<th>( C_{44} ) (GPa)</th>
<th>( C_{66} ) (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1400</td>
<td>9.7</td>
<td>48.87</td>
<td>9.87</td>
<td>3.02</td>
<td>16.07</td>
<td>8.97</td>
<td>15.06</td>
</tr>
<tr>
<td>5045</td>
<td>34.8</td>
<td>56.48</td>
<td>9.99</td>
<td>6.41</td>
<td>30.91</td>
<td>13.54</td>
<td>19.01</td>
</tr>
<tr>
<td>8700</td>
<td>60.0</td>
<td>57.83</td>
<td>10.44</td>
<td>8.83</td>
<td>39.25</td>
<td>15.69</td>
<td>19.37</td>
</tr>
</tbody>
</table>

**Table D. 10** Dynamic stiffnesses versus confining pressure for a transverse isotropic shale specimen.
Figure D.1  Three specimens at three different directions are necessary to study the elastic anisotropy in a hexagonal system. A vertical sample along X3, a horizontal sample along X1 or X2, and an inclined sample oriented at 45° with respect to X3.

\[
\frac{V_p(0^\circ)}{V_s(0^\circ)} = \frac{V_s(90^\circ)}{V_p(90^\circ)}
\]

Figure D.2. Nine velocities are measured in the experiment. The solid line indicates the directions of propagation and the dashed line the polarization of the particle. \(V_{s1}\) and \(V_{s2}\) are perpendicular polarized S-modes. In the vertical direction \(V_{s1} = V_{s2}\).
Figure D. 3  Vertical Sample. For uniaxial stress in the direction $X_3$, it is necessary to instrument the specimen with a minimum of two strain gauges, one axial to measure the vertical strain, $\varepsilon_3$, and one tangential to measure the horizontal strain, $\varepsilon_1$ or $\varepsilon_2$.

Figure D. 4  Horizontal sample. Under Uniaxial load in the direction $X_3$, a minimum of three strain gauges at three different directions are necessary, one axial to measure $\varepsilon_1$, one parallel to $X_3$, and one parallel to $X_1$ to compute $\varepsilon_1$ and $\varepsilon_2$. 
Figure D. 5 Inclined Sample. To compute the rigidity modulus $G_{23}$ and the Young’s modulus $E$ at $45^\circ$, it is necessary to measure the strain with a gauge oriented at $45^\circ$ with respect to the vertical axis $X_3$. The sample is instrumented with three strain gauges; however, only the axial strain is necessary to compute the elastic parameters.

Figure D. 6 Static and dynamic Poisson’s ratios ($\nu_{dyn}$ and $\nu_{stat}$) for dry and saturated sandstone, versus pore and effective hydrostatic pressure.
Figure D. 7 Static and dynamic Young's moduli for dry and saturated sandstone versus pore and effective hydrostatic pressure.

Figure D. 8 Static and dynamic shear moduli for dry and saturated sandstone versus pore and effective hydrostatic pressure.

Figure D. 9 Bulk moduli obtained from a step wise hydrostatic compression and from a continuous cycle.
Figure D.10. Vertical ($e_3$) and horizontal ($e_1$ and $e_2$) strains versus confining pressure.

Figure D.11. Volumetric strain ($e_b$) versus confining pressure.

Figure D.12 Static bulk modulus versus confining pressure.

Figure D.13 Static vertical and horizontal inverse of linear compressibilities, $K_{1i}$ and $K_{13}$, versus confining pressure.

Figure D.14 Ratio vertical to horizontal linear strain versus confining pressure.

Figure D.15 Vertical and horizontal strain rate versus confining pressure.
Figure D. 16 Static and dynamic inverse of linear compressibility anisotropies from a step wise and a continuous hydrostatic compression and from a dynamic experiment.

Figure D. 17 Static and dynamic Young's Moduli in the vertical, horizontal and at $45^\circ$ versus confining pressure.

Figure D. 18 Static and dynamic Poisson's ratios versus confining pressure.
Figure D. 19  Static and dynamic shear moduli versus confining pressure.

Figure D. 20  Ratio of dynamic to static Young's moduli and shear moduli versus confining pressure.

Figure D. 21  Static and dynamic Young's and shear moduli anisotropies versus confining pressure.
Appendix E

Comparison between the static elastic behavior of shales and sandstones under hydrostatic compression

E.1 Introduction

In this appendix we compare the static elastic behavior of shale and sandstone under a continuous hydrostatic compression cycles. We present the results obtained from testing a dry and a brine saturated, weak, tranverse, anisotropic sandstone and an anisotropic shale specimen.

E.2 Sample description

Tables E.1 and E.2 show the descriptions and characteristics of the tested rock specimens. The shale specimen does not contain visible cracks. The thin sections analysis revealed the presence of cracks with preferred horizontal orientations, parallel to a no well defined bedding plane. In hydrostatic compression tests, we need only one rock specimen in an arbitrary direction to determine the bulk and linear compressibilities. We chose cylindrical rock specimens in the vertical direction (Figure D.1 and D.3). The stress path exerted on the rock specimens corresponds to the second cycle of figure 3.1.
<table>
<thead>
<tr>
<th>Sample Name</th>
<th>Sets</th>
<th>P.P. (MPa)</th>
<th>Dry Density (g/c.c)</th>
<th>Saturated Density (g'/c.c)</th>
<th>Por. (%)</th>
<th>Depth (feet)</th>
<th>Sample Length (mm)</th>
<th>Sample Diameter (mm)</th>
<th>Fluid</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1x54Vd</td>
<td>Vertical (I)</td>
<td>0</td>
<td>2.597</td>
<td></td>
<td></td>
<td>10875</td>
<td>25.380</td>
<td>38.180</td>
<td>Dry</td>
</tr>
<tr>
<td>T2E59VS</td>
<td>Vertical (II)</td>
<td>2</td>
<td>2.146</td>
<td>2.318</td>
<td>17.4</td>
<td>10341</td>
<td>49.900</td>
<td>25.400</td>
<td>Brine Sat</td>
</tr>
</tbody>
</table>

Table E.1 Description and characteristics of the rock specimens tested. Por. = Porosity. P.P. = Pore pressure.

**E.3 Experimental results and discussions**

For the shale and sandstone specimens under confining pressure (C.P.), the strains parallel to the bedding plane ($X_1X_2$) are consistently less than those measured normal to it (figures E.1 and E.2). The difference is more pronounced for the anisotropic shale than for the weak transverse isotropic sandstone. After saturating the sandstone, the difference between horizontal and vertical strains gets smaller, which indicates a reduction in the linear compressibility anisotropy, $K_1/K_3$ (figure E.3). Under confining pressure both rock types, shale and sandstone (dry and saturated), exhibit hysteresis in the radial, axial and volumetric strains (figure E.1 to E.6). The area between the loading and unloading curves, for the linear and volumetric strains, is bigger for the shale specimen than for the sandstone specimen (figure E.2 and E.6). This behavior was observed for all the sandstone and shale specimens tested in this work. In sandstones, the exhibited hysteresis is approximately the same in both vertical and horizontal strains (figure E.1). In shales, the exhibited hysteresis is bigger for the strain normal to the plane of isotropy (figure E.5).
The sandstone specimen exhibits a stronger non-linear strain-pressure relationship than the shale (figure E.2 and E.6). This effect is more pronounced at lower confining pressures, where fractures are open. The differing in the nonlinearity may be attributed to the different crack density encountered in these rock types. Saturated sandstone exhibits a more linear behavior than dry sandstone (figure E.4 and E.6). Shales exhibit more hysteresis in the linear and volumetric strains than sandstones.

The difference between the vertical and horizontal inverse of linear compressibilities (K\textsubscript{11} and K\textsubscript{13}) is much larger in shales than in sandstones (figure E.7 and E.11), indicating a stronger anisotropy in the linear compressibilities of shales.

After saturation, sandstones show a decrease in the bulk modulus and inverse of linear compressibilities over the whole range of effective hydrostatic pressure (figure E.7 and E.10).

In shales the slope of the loading arm, in the bulk modulus and inverse of linear compressibilities, is bigger than the slope of the unloading arm (figure E.11 and E.12). This behavior is not so pronounced in the dry sandstone (figure E.7 and E.8), but it becomes more important after saturation.

Shales, dry sandstones, and saturated sandstones, exhibit different behavior in the ratio of horizontal (\(\varepsilon_1 = \varepsilon_2\)) to vertical (\(\varepsilon_3\)) strains as the confining pressure is increased. Figures E.13 and E.14 show the behavior of the ratio \(\varepsilon_1 / \varepsilon_3\) for the uncracked shale specimen and for the sandstone specimen in dry and saturated conditions. For the shale specimen, the ratio \(\varepsilon_1 / \varepsilon_3\) increases monotonically as the hydrostatic pressure is raised. For the dry sandstone \(\varepsilon_1 / \varepsilon_3\) decreases greatly when the confining pressure goes from 7 MPa to 21 MPa and decreases monotonically and at a low rate as the pressure is increased (figure E.14). At low pressures \(\varepsilon_1 / \varepsilon_3\) is different during the loading and unloading paths of the cycle; nevertheless, it tends to be equal at high pressures. After saturation, the ratio \(\varepsilon_1 / \varepsilon_3\) decreases, but the decrease is much smaller at low confining pressures. In saturated
sandstone the hysteresis observed in $\varepsilon_1/\varepsilon_3$ is small, and it disappears completely when the pore pressure is increased (PP = 40 MPa).

In general, for dry and saturated sandstones, the ratio $\varepsilon_1/\varepsilon_3$ decreases with increasing hydrostatic pressure, but the rate of increase is almost zero at high confining pressure.

It has been observed that for both rock types the relationship between stress and strain is not linear. This is more pronounced in the dry sandstone specimen. Consequently, one expects the elastic parameters to vary greatly between hydrostatic pressure levels and they should be computed at the appropriate stress levels. The elastic parameters of shales, dry sandstones and saturated sandstones behave differently when increasing the stress level.
Figure E.1 Vertical ($e_3$) and horizontal ($e_1$) linear strain versus confining pressure on dry sandstone.

Figure E.2 Volumetric strain ($e_b$) versus confining pressure.

Figure E.3 Vertical ($e_3$) and horizontal ($e_1$) linear strains versus confining pressure at pore pressure = 290 psi.

Figure E.4 Volumetric strain ($e_b$) versus confining pressure at pore pressure = 290 psi.

Figure E.5 Vertical ($e_3$) and horizontal ($e_1$) linear strain for the shale specimen versus confining pressure.

Figure E.6 Volumetric strain ($e_b$) versus confining pressure.
Figure E. 7 Vertical ($K_{l3}$) and horizontal ($K_{l1}$) inverse of linear compressibilities versus confining pressure.

Figure E. 8 Bulk modulus ($K_b$) versus confining pressure.

Figure E. 9 Vertical and horizontal inverse of linear compressibilities ($K_{l1}$ and $K_{l3}$) versus confining pressure, at pore pressure = 2 MPa.

Figure E. 10 Bulk modulus ($K_b$) versus confining pressure, at pore pressure = 2 MPa.

Figure E. 11 Vertical and horizontal inverse of linear compressibilities ($K_{l1}$ and $K_{l3}$) versus confining pressure.

Figure E. 12 Bulk modulus ($K_b$) versus confining pressure.
Figure E. 13 Ratio horizontal ($e_1$) to vertical ($e_3$) linear strain for room dry shale versus confining pressure.

Figure E. 14 Ratio horizontal ($e_1$) to vertical ($e_3$) strains for dry and brine saturated sandstone at two pore pressure, $PP = 2$ MPa and $PP = 40$ MPa.