Simple, zero-feedback, collaborative beamforming for emergency radio

Simple, Zero-Feedback, Collaborative Beamforming for Emergency Radio

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***Abstract*—Collaborative beamforming among a set of distributed terminals is studied, assuming a) no specialized RF hardware for carrier frequency synchronization, and b) zero feedback from destination (either in the form of pilot signals or explicit messages). A solution is provided for conventional radios in relevant critical applications, such as emergency radio. The proposed scheme simply exploits *lack* of synchronization among distributed carriers, operating at the same *nominal* carrier frequency. It is shown that such beamforming is possible and its performance is analytically quantified.**

I. Introduction

Collaborative beamforming of distributed radio terminals has recently attracted attention as a means to boost connectivity in various critical scenarios. Current proposals utilize either specialized RF hardware for carrier synchronization among distributed terminals or some type of feedback from the destination, in order to adjust the collaborative transmissions; from simple pilot signals for channel state information (CSI) (e.g. [1]) or single-bit feedback (e.g. [2], [3], [4]) to several-bit messages from destination to transmitters that assist the required carrier phase adjustments at the local oscillator system of each transmitter (e.g. [5]).

This work studies collaborative beamforming, assuming a) no specialized RF hardware, and b) zero feedback from the destination. Our goal is to provide a solution for conventional radios, when the link between destination and distributed terminals is already too weak, so that no feedback signal can reliably reach the distributed terminals. This may be the case in ground sensor networks, where the destination is located at an airplane flying too high, or fire-fighters in a building unable to communicate outside, unless collaborative beamforming is utilized. We categorize such critical, zero-feedback scenarios as emergency radio.

Simple, zero-feedback, collaborative beamforming is shown possible and could be employed in emergency radio situations, even with conventional (i.e. no carrier-phase adjustment capability) distributed transceivers. Section II provides the problem formulation and the basic idea, Section III offers analysis, Section IV discusses numerical results and finally, Section V concludes this work.

II. Problem Formulation and Basic Idea

The received baseband signal $y[n]$ at the destination, when M distributed terminals collaboratively transmit common symbol $x[n]$, is given by

$$\begin{aligned} y[n] &= \widetilde{x[n]} + w[n] \\ &= \underbrace{x[n] \sum_{k=1}^{M} A_k \exp\left\{+j\left(2\pi \Delta f_k n T_s + \phi_k\right)\right\}}_{\widetilde{x[n]}} + w[n], \end{aligned} \tag{1}$$

where $w[n]$ is the complex additive noise at the destination with $\mathbb{E}\left\{|w[n]|^2\right\} = N_0$. Δf_k is the carrier frequency offset of transmitter k, between nominal carrier frequency f_c and actual carrier frequency; it depends on the frequency skew (in parts-per-million or ppm) of the oscillator crystal used at each radio and it is due to crystal manufacturing errors.[1] Parameter T_s denotes the symbol $x[n]$ duration, while A_k, ϕ_k are the amplitude and phase of the end-to-end channel between transmitter k and destination. These parameters remain constant within T_c, the channel coherence time, inversely proportional to Doppler shift (and relative mobility).

The carrier offsets $\{\Delta f_k\}$ vary slowly with temperature, and thus, remain constant for a duration of several transmitted symbols. It is assumed that $\{\Delta f_k\}$ are independent, identically distributed and zero-mean ($\mathbb{E}\{\Delta f_i\} = 0$), according to probability density function $p_{\Delta f}(\Delta f)$. The standard deviation is given by $\sigma = \sqrt{\mathbb{E}\{\Delta f^2\}} = f_c \times \mathsf{ppm}$, where ppm is the frequency skew of the clock crystal, with typical values of $1-20$ ppm. For example, clock crystals of 20 ppm offer carrier frequency offsets on the order of 2.4 GHz$\times 20\ 10^{-6} = 48$ kHz.

[1]For time/frequency metrology, the interested reader could refer to [6] and references therein.

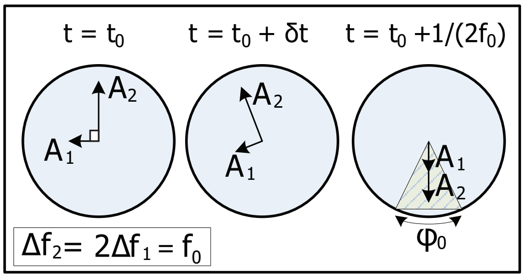

Fig. 1. The two signals align at $t = t_0 + 1/(2f_0)$, providing constructive addition at the destination (beamforming gain). This work studies the general-M signal alignment case (within angle ϕ_0) for any carrier frequency offset distribution $p_{\Delta f}(\Delta f)$.

In line with all previous distributed beamforming research, Eq. (1) implicitly assumes that the distributed transmitters employ a low-complexity packet/symbol synchronization algorithm, i.e. even though the distributed transmitters are not carrier-synchronized, the distributed transmitters do manage to transmit the same symbol at a given time. Experimental examples of low-complexity, distributed symbol synchronization can be found in [7], [8].

Furthermore, denoting that

$$\widetilde{\phi_k}[n] = 2\pi n T_s \Delta f_k + \phi_k, \tag{2}$$

and assuming equal energy signal constellation $\mathbb{E}\left\{|x[n]|^2\right\} =$ constant (e.g. PSK), with $\mathcal{P}_T = \mathbb{E}\left\{|x[n]|^2\right\}$ the transmitted power *per individual terminal*, then the signal-to-noise ratio (SNR) at the destination is given by:

$$\begin{aligned} \mathsf{SNR}[n] &\triangleq \frac{\mathbb{E}\left\{|\widetilde{x[n]}|^2\right\}}{\mathbb{E}\left\{|w[n]|^2\right\}} = \frac{\mathcal{P}_T}{N_0}\ \mathsf{L_{BF}}[n] \qquad (3)\\ &= \frac{\mathcal{P}_T}{N_0} \underbrace{\left\{\sum_{k=1}^{M} A_k^2 + 2\sum_{k\neq m} A_k A_m \cos\left(\widetilde{\phi_k}[n] - \widetilde{\phi_m}[n]\right)\right\}}_{\mathsf{L_{BF}}[n]}, \end{aligned}$$

where the second sum for $k \neq m$ above includes all possible $\binom{M}{2}$ terms.

According to the above expression, the beamforming factor $\mathsf{L_{BF}}[n]$ can become positive or negative, depending on the symbol n, the phase offsets $\{\phi_i\}$, as well as the distribution of the carrier frequency offsets $\{\Delta f_i\}$ $i \in \{1, 2, \ldots, M\}$. Fig. 1 depicts the special case of two distributed transmitters ($M = 2$) with carrier frequency offsets $\Delta f_2 = 2\Delta f_1 = f_0$; their signals arrive at the destination with phase difference $\pi/2$ at time instant $t = t_0$. The two signals align at $t = t_0 + 0.5/f_0$, providing constructive addition at the destination, i.e *beamforming gain.*

For constant $a \in (0, 1]$, if $\cos\left(\widetilde{\phi_k}[n] - \widetilde{\phi_m}[n]\right) \geq a$ for *all* pairs $\{k, m\}$, $k \neq m$ and $k, m \in \{1, 2, \ldots, M\} \equiv \mathcal{S}_M$, then the beamforming gain becomes strictly positive, since *all* M transmitted signals align constructively, without any type of feedback from the destination:

$$\mathsf{Align}[n, a, M] \triangleq \bigcap_{k\neq m} \left\{\cos\left(\widetilde{\phi_k}[n] - \widetilde{\phi_m}[n]\right) \geq a\right\}, \quad \forall\ k, m \in \mathcal{S}_M \tag{4}$$

$$\Rightarrow \mathsf{L_{BF}}[n] \geq \left\{\sum_{k=1}^{M} A_k^2 + 2a\sum_{k\neq m} A_k A_m\right\}. \tag{5}$$

The above event defines *signal alignment of M signals with parameter a*, or equivalently, alignment within angle $\phi_0 = \cos^{-1}(a)$.

Furthermore, denote the following random variable:

$$\beta_n[a, M] = \begin{cases} 1, & \text{with prob. } \Pr\{\mathsf{Align}[n, a, M]\} \\ 0, & \text{with prob. } 1 - \Pr\{\mathsf{Align}[n, a, M]\} \end{cases} \tag{6}$$

Assuming that the M distributed, carrier-unsynchronized transmitters repeatedly transmit the same information (i.e. symbol) for $N \leq T_c/T_s \triangleq \tau_c$ symbols. The random variable $\beta(M) \triangleq \beta_1[a, M] + \beta_2[a, M] + \beta_3[a, M] + \ldots + \beta_N[a, M]$ denotes the number of symbols (out of total N) where the M *signals* align with beamforming factor $\mathsf{L_{BF}}[n]$ at least equal to

$$\mathsf{L_{BF}}[n] \geq \left\{\sum_{k=1}^{M} A_k^2 + 2a\sum_{k\neq m} A_k A_m\right\} \triangleq \mathsf{L_0}(M). \tag{7}$$

It can be easily shown that $\mathsf{L_0}(M) = \mathcal{O}\left(M + 2a\binom{M}{2}\right)$, where $\mathcal{O}(\cdot)$ is the mathematical symbol for order of magnitude. For perfect alignment ($a = 1$), $\mathsf{L_0}(M) = \mathcal{O}\left(M^2\right)$, as expected.

The average number of symbols in $[1, N]$ with minimum beamforming factor $\mathsf{L_0}(M)$ becomes:

$$\mathbb{E}\{\beta(M)\} = \sum_{n=1}^{N\leq\tau_c} \Pr\{\mathsf{Align}[n, a, M]\}. \tag{8}$$

The above can be used to estimate the alignment delay i.e. the amount of symbols that must be repeatedly transmitted, in order to guarantee one symbol on average, with *minimum* beamforming gain $\mathsf{L_0}(M)$. Equivalently, the ratio $\mathbb{E}\{\beta(M)\}/N$ provides the effective communication rate with *minimum* beamforming gain $\mathsf{L_0}(M)$ per information symbol.

The following section studies $\Pr\{\mathsf{Align}[n, a, M]\}$ for any given n, $a \equiv \cos\phi_0 \in (0, 1]$, M, $\overline{\phi} = [\phi_1\ \phi_2\ \ldots\ \phi_M]^T$ and p.d.f. $p_{\Delta f}(\Delta f)$. In other words, this work exploits absence of carrier synchronization among distributed, conventional radios for collaborative beamforming, without any type of feedback from the destination.

III. Analysis of M Signal Alignment Probability

It is already denoted that $\phi_0 = \cos^{-1}(a)$. Taking into account the fact that $a \in (0, 1]$, the value of ϕ_0 is further restricted in $[0, \pi/2)$ (even though $[2k\pi, 2k\pi + \pi/2)$ or $(2k\pi - \pi/2, 2k\pi]$ for any $k \in \mathbb{Z}$ could be considered):

$$0 \leq \phi_0 = \cos^{-1}(a) < \pi/2. \tag{9}$$

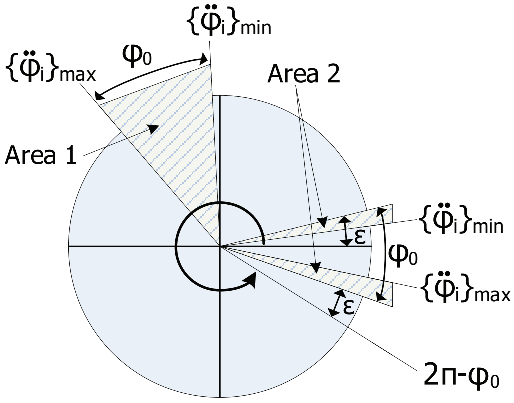

Fig. 2. Shaded areas 1 and 2 describe the M-signal alignment event with parameter $a = \cos(\phi_0)$ and $0 \leq \phi_0 < \pi/2$. Area 2 decreases with decreasing ϕ_0 in $[0, \pi/2)$.

Furthermore, the following M independent, non-identically distributed random variables in $[0, 2\pi)$ are set:

$$\ddot{\phi}_i(n) \triangleq \widetilde{\phi}_i(n) \bmod 2\pi = (2\pi n T_s \Delta f_i + \phi_i) \bmod 2\pi, \quad (10)$$
$$\ddot{\phi}_i \in [0, 2\pi),\ i \in \{1, 2, \ldots, M\},$$

where $x \bmod 2\pi$ denotes the modulo 2π operation. Assuming knowledge of $p_{\Delta f}(\Delta f)$, it becomes straightforward to find out the p.d.f. of $\ddot{\phi}_i(n)$ [9] as:

$$p_{\ddot{\phi}_i}\left(\ddot{\phi}_i\right) = \frac{1}{2\pi n T_s} \sum_{k \in \mathbb{Z}} p_{\Delta f}\left(\frac{\ddot{\phi}_i + 2k\pi - \phi_i}{2\pi n T_s}\right), \quad (11)$$
$$\ddot{\phi}_i \in [0, 2\pi), \forall i \in \mathcal{S}_M$$

Appendix Lemma 2 provides numerical calculation of the above p.d.f. for the special case of uniform or normal carrier offset distribution $p_{\Delta f}(\Delta f)$. It is emphasized that $\{\ddot{\phi}_i\}'s$ are independent but not identically distributed because of the different $\{\phi_i\}'s$. The auxiliary variables $\{\ddot{\phi}_i\}'s$ are limited in $[0, 2\pi)$, as opposed to the variables $\{\phi_i\}'s$, which span $(-\infty, +\infty)$; thus, Eq. (4) alignment event at transmitted symbol n becomes:

$$\mathsf{Align}[n, a, M] \equiv \bigcap_{k \neq m} \cos\left(\ddot{\phi}_k[n] - \ddot{\phi}_m[n]\right) \geq a, \quad (12)$$
$$\forall\ k, m \in \mathcal{S}_M,\ \ddot{\phi}_i \in [0, 2\pi).$$

The above states that *all* pairwise differences of the auxiliary angles should be less than a limit which is determined by a.

Lemma 1 (Alignment Probability): For any given symbol time n, alignment parameter $a \equiv \cos\phi_0 \in (0, 1]$ (and ϕ_0 in $[0, \pi/2)$), number of transmitters M, vector of wireless channel phases between distributed transmitters and receiver $\overline{\phi} = [\phi_1\ \phi_2\ \ldots\ \phi_M]^T$ and probability density function of carrier frequency offset $p_{\Delta f}(\Delta f)$, the following expression holds for alignment probability $\Pr\{\mathsf{Align}[n, a, M]\}$ of M signals with parameter a:

$$\Pr\{\mathsf{Align}[n, a, M]\} \geq \int_{y=0}^{2\pi} \int_{x=y}^{\min\{y+\phi_0,\ 2\pi\}} p_{y,x}(y, x)\, dx\ dy, \quad (13)$$

where
$$p_{y,x}(y, x) = \begin{cases} g_0(y, x), & y < x \\ 0, & \text{elsewhere,} \end{cases} \quad (14)$$

$$g_0(y, x) = \sum_{(k_1,\ k_2),\ k_1 \neq k_2} \left\{ \left[p_{\ddot{\phi}_{k_1}}(y)\ p_{\ddot{\phi}_{k_2}}(x) + p_{\ddot{\phi}_{k_1}}(x)\ p_{\ddot{\phi}_{k_2}}(y)\right] \times \right.$$
$$\left. \times \prod_{k_3 \neq k_1,\ k_3 \neq k_2} \left(F_{\ddot{\phi}_{k_3}}(x) - F_{\ddot{\phi}_{k_3}}(y)\right) \right\}. \quad (15)$$

The summation in Eq. (15) involves all $\binom{M}{2}$ pairs $(k_1,\ k_2)$, with $k_1,\ k_2 \in \mathcal{S}_M,\ k_1 \neq k_2$, and the product involves all $k_3 \in \mathcal{S}_M$ excluding k_1 and k_2 ($\mathcal{S}_M - \{k_1\} - \{k_2\}$). Moreover, $F_{\ddot{\phi}_{k_m}}(x) = \int_0^x p_{\ddot{\phi}_{k_m}}(t)\ dt$ and $p_{\ddot{\phi}_i}\left(\ddot{\phi}_i\right)$ already calculated in Eq. (11).

Proof: The lower bound of alignment (within angle ϕ_0) probability of M signals is:

$$\Pr\{\mathsf{Align}[n, a, M]\} \geq \Pr\left\{\max_{i \in \mathcal{S}_M}\left\{\ddot{\phi}_i\right\} \leq \min_{i \in \mathcal{S}_M}\left\{\ddot{\phi}_i\right\} + \phi_0\right\}. \quad (16)$$

The event of the RHS probability in Eq. (16) guarantees the desired event of the LHS probability. However, there are cases when $\max_{i \in \mathcal{S}_M}\left\{\ddot{\phi}_i\right\} > 2\pi - \phi_0$ and $\min_{i \in \mathcal{S}_M}\left\{\ddot{\phi}_i\right\} < \phi_0$ (shaded area 2 in Fig. 2), where alignment can still occur and such cases are not captured by the RHS probability above. Fig. 2 and shaded area 2 describes the later event, while shaded area 1 describes the RHS event above. It can be shown that area 2 decreases with decreasing ϕ_0, suggesting that the above lower bound is tight. Numerical results for moderate values of ϕ_0 ($\pi/4$ or less) further validate that observation.

The joint probability density function $p_{y,x}\left(y = \min_{i \in \mathcal{S}_M}\left\{\ddot{\phi}_i\right\},\ x = \max_{i \in \mathcal{S}_M}\left\{\ddot{\phi}_i\right\}\right)$ is calculated with the help of Appendix Theorem 1. Integration of that joint pdf, provides the calculation of the RHS in Eq. (16), concluding the proof. ■

Only knowledge of the carrier frequency offset p.d.f. $p_{\Delta f}(x)$ is required in order to calculate the above bound.[2] Thus, alignment probability (Eq. (13)) and alignment delay (Eq. (8)) can be assessed for *any* carrier frequency offset distribution $p_{\Delta f}(\Delta f)$, as well as propagation environment ($\overline{\phi}$).

IV. Numerical Results

Numerical results assume carrier frequency at 2.4 GHz, normal (or uniform) zero-mean carrier frequency offset distribution and symbol duration at $T_s = 1\ \mu$sec (corresponding to 1 Mbps for binary modulation). Moreover, channel is assumed constant during collaborative beamforming ($N << \tau_c$).

Fig. 3 provides alignment probability results for various values of M (number of transmitters), 20 ppm crystals, normal zero-mean carrier frequency offset distribution and $\phi_0 = \pi/4$ (alignment parameter $a = \cos(\phi_0) = \sqrt{(2)}/2$). It is observed

[2] We note again that calculation examples for the special case of normal or uniform distribution is given through Appendix Lemma 2.

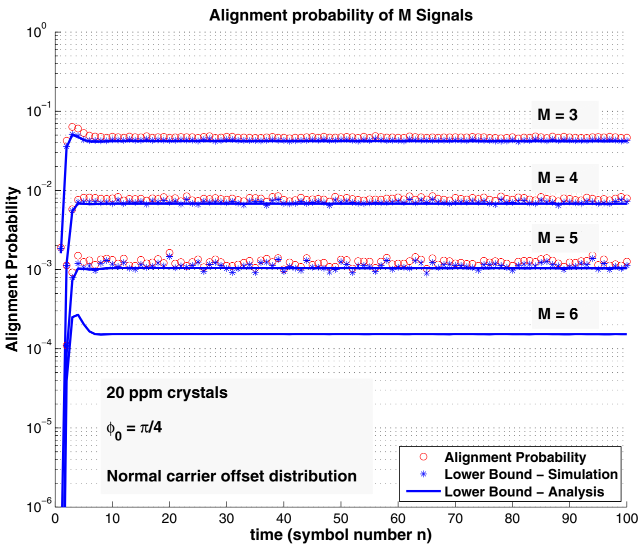

Fig. 3. Alignment probability as a function of time and number of transmitters M, with normal carrier offset distribution and $a = cos(\phi_0) = \sqrt{2}/2$.

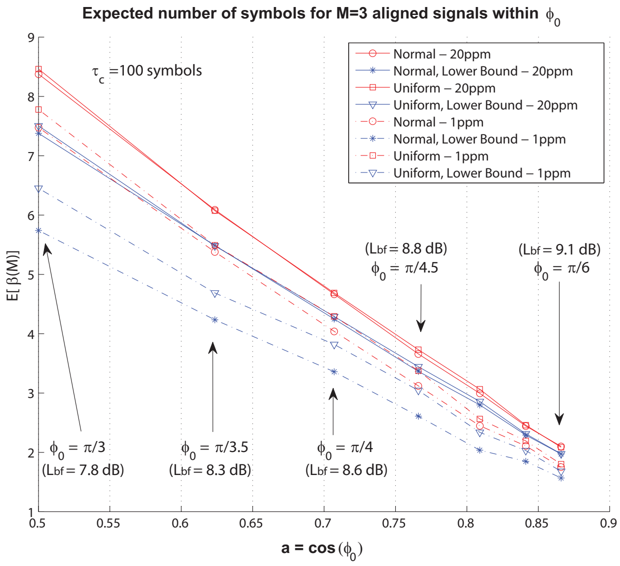

Fig. 4. Expected number of symbols (out of $\tau_c = 100$) with $M = 3$ aligned signals within at most ϕ_0 ($a = \cos(\phi_0)$), and $[\phi_1 \quad \phi_2 \quad \phi_3] = [6.19 \quad 0.24 \quad 1.77]$.

that simulation matches analysis results, while the lower bound of Eq. (13) is indeed tight.

Fig. 3 also shows that alignment probability drops linearly with M in a logarithmic scale; that means that alignment probability drops exponentially with M. This is also true for asymptotic $M \to \infty$ [10]. Thus, higher beamforming gains with the utilization of increasing number of transmitters come at the price of increased alignment delay, according to Eq. (8).

It has been also found that, even though alignment probability is a function of time, it reaches a steady-state value; the latter does not depend on $\overline{\phi}$, but only on $p_{\Delta f}(\Delta f)$ [10]. This observation can be explained intuitively; consider the signal from each transmitter i as a phasor in Figs. 1, 2 (or runner in a circular stadium), with frequency (rotating speed) equal to Δf_i. Then, after a certain period of time, the probability all phasors (or runners) "meet" (alignment event) does not depend on their initial starting points (initial phases in $\overline{\phi}$) but it is affected by their relative speeds (distribution of $\{\Delta f_i\}$'s).

Fig. 4 provides the expected number of symbols $\mathbb{E}\{\beta(M)\}$ (out of total $N = 100$ transmitted symbols) with $M = 3$ aligned signals (three distributed transmitters), for various values of parameter $a = \cos(\phi_0)$, or equivalently, angle ϕ_0. In other words, Fig. 4 offers an estimate of alignment delay. It is shown that for normal or uniform carrier offset distribution, oscillator crystals on the order of $1 - 20$ ppm, and $N = 100$ transmitted symbols of the same information, there are approximately 4 symbols with $M = 3$ aligned signals of parameter $a = \cos(\pi/4) = \sqrt{2}/2$. At those 4 symbols, there is *minimum* beamforming gain factor L_{BF} on the order of $\mathsf{L}_{\mathsf{BF}} = 3 + 2\ 3\ \sqrt{2}/2 = 3(1 + \sqrt{2}) \to 8.6$ dB (assuming identically distributed channel amplitudes $\{A_k\}$ in Eq. (5)). Compared to the case of a single transmitter with $3\mathcal{P}_T$ transmission power instead, the minimum beamforming gain of the proposed work for $M = 3$ and $a = \sqrt{2}/2$ becomes on the order of $8.6 - 10\log_{10}(3) = 3.9$ dB.

The example shows that for 100 transmitted symbols of the same information, there are approximately 4 symbols on average, with beamforming gain on the order of 8.6 dB, compared to the non-collaborative transmission with transmission power $\mathcal{P}_T$ (or 3.9 dB with non-collaborative transmission power $3\mathcal{P}_T$). Therefore, the above beamforming gains can be offered at effective communication rates of $4/100 \times 1$ Mbps $= 40$ kbps, without any type of destination feedback or carrier synchronization. Moreover, according to Fig. 4 the higher the clock frequency skew (20 ppm vs 1 ppm), the larger the expected number of symbols where alignment occurs; thus, the more unsynchronized are the carriers (higher frequency skew), the smaller the alignment delay and the higher the effective communication rate with beamforming gains.

The same plot also shows that one can reduce the alignment parameter a (increase angle ϕ_0) and thus, reduce the minimum beamforming gain factor L_{BF}. In that way, $\mathbb{E}\{\beta(M)\}$ increases and thus, alignment delay can be also reduced. In short, there is an interesting tradeoff between beamforming gains and alignment delay (or equivalently effective communication rate). Additional results can be found in [10].

V. Conclusion

There are cases where inadequate signal-to-noise ratio at the receiver prohibits reliable communication (e.g. forward error correction is not sufficient). In such scenarios, reliable feedback from the destination, either in the form of pilot signals or in the form of explicit feedback messages are not available. By requiring no carrier synchronization capability among the transmitters, an additional constraint was imposed; this way, the proposed algorithm's complexity is vastly reduced.

It was shown that lack of carrier synchronization among distributed radios can be turned to an advantage. The proposed beamforming approach utilizes no feedback from the destination or carrier synchronization and could be used in relevant critical radio applications. Its performance was quantified and its tradeoffs were highlighted. Future work includes experimental validation.

APPENDIX

Theorem 1: Assume M independent, not identically distributed (i.n.i.d.) random variables $X_1, X_2, \ldots, X_M$, with probability density function (p.d.f.) $p_{X_i}(x)$ and cumulative distribution function (c.d.f.) $F_{X_i}(x)$ per X_i, $i \in \{1, 2, \ldots, M\} \equiv \mathcal{S}_M$. Denote $Y_1 < Y_2 < \ldots < Y_M$ the ordered random variables $\{X_i\}$. The joint probability density function of the minimum and maximum of the i.n.i.d. random variables $\{X_i\}$ $p_{Y_1,Y_M}(Y_1 = y = \min_{i\in\mathcal{S}_M}\{X_i\}, Y_M = x = \max_{i\in\mathcal{S}_M}\{X_i\})$ is given by:

$$p_{y,x}(y,x) = \begin{cases} g_0(y,x), & y < x \\ 0, & \text{elsewhere,} \end{cases} \tag{17}$$

where $g_0(y,x)$ is given by Eq. (15).

Proof:

$$p_{y,x}(y,x)\ dy\ dx = \Pr\{Y_1 \in dy,\ Y_M \in dx\} \tag{18}$$

$$= \Pr\{\text{one } X_i \in dy, \text{ one } X_j \in dx \text{ (with } y < x \text{ and } i \neq j) \text{ and all the rest } \in (y,x)\} \tag{19}$$

$$= \sum_{k_1 \neq k_2} \Big\{ \big[p_{X_{k_1}}(y)\ p_{X_{k_2}}(x) + p_{X_{k_1}}(x)\ p_{X_{k_2}}(y)\big]\ dy\ dx \times \prod_{k_3 \neq k_1,\ k_3 \neq k_2} \big(F_{X_{k_3}}(x) - F_{X_{k_3}}(y)\big) \Big\}, \text{ for } y < x. \tag{20}$$

The double sum of $p_{X_{k_1}}(y)\ p_{X_{k_2}}(x)\ dy\ dx + p_{X_{k_1}}(x)\ p_{X_{k_2}}(y)\ dy\ dx$ above stems from the fact that even though there are exactly $\binom{M}{2}$ pairs among the set of M $\{X_i\}$'s, ordering among each pair matters. Simplifying the last line above concludes the proof. ■

Lemma 2: Assume zero-mean uniform or normal carrier offset distribution $p_{\Delta f}(\Delta f)$ with $\mathbb{E}\{\Delta f^2\} = \sigma^2$. The p.d.f. of $\{\ddot{\phi}_j\}$'s can be numerically calculated by:

$$p_{\ddot{\phi}_i}\left(\ddot{\phi}_i\right) = \frac{1}{2\pi n T_s} \sum_{k=-K_0}^{K_0} p_{\Delta f}\left(\frac{\ddot{\phi}_i + 2k\pi - \phi_i}{2\pi n T_s}\right), \quad \ddot{\phi}_i \in [0, 2\pi),\ \forall i \in \mathcal{S}_M, \tag{21}$$

where $K_0 = \lfloor nT_s b + 1 \rfloor$, $\lfloor x \rfloor$ is the floor function and $b = \sqrt{3}\ \sigma$ or $b = 3\ \sigma$ for uniform or normal carrier offset distribution, respectively.

Proof: For zero-mean uniform distribution $p_{\Delta f}(x)$ in $[-b,\ b]$, the standard deviation σ is expressed through b as: $\mathbb{E}\{\Delta f^2\} = \sigma^2 = 4\ b^2/12 \Rightarrow b = \sqrt{3}\ \sigma$. Given that $p_{\Delta f}(x)$ is zero outside $[-b,\ b]$, the following holds:

$$-b \leq \frac{\ddot{\phi}_i + 2k\pi - \phi_i}{2\pi n T_s} \leq b \Rightarrow \tag{22}$$

$$-1 - nT_s b \leq \frac{\phi_i - \ddot{\phi}_i}{2\pi} - nT_s b \leq k, \tag{23}$$

$$k \leq nT_s b + \frac{\phi_i - \ddot{\phi}_i}{2\pi} \leq nT_s b + 1, \tag{24}$$

where $\phi_i \in [0,\ 2\pi)$. Given that k is an integer, the above expression justifies the selected $K_0 = \lfloor nT_s b + 1 \rfloor$.

For zero-mean normal distribution $p_{\Delta f}(\Delta f)$, the justification is the same for $b = 3\ \sigma$; about 99.7% of values drawn from a normal distribution are within $3\ \sigma$ from the mean. ■

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