Distributed event-triggered control strategies for multi-agent systems

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***Abstract*— Event-driven strategies for distributed multi-agent systems are motivated by the future use of embedded microprocessors with limited resources that will gather information and actuate the individual agent controller updates. The event-driven control actuation updates considered in this paper are distributed, in the sense that agents require knowledge only of the states of their neighbors for the controller implementation. The proposed distributed strategy is compared with an earlier approach.**

I. Introduction

Decentralized control of networked multi-agent systems is an important research field due to its role in a number of applications, including multi-agent robotics [1]–[4], distributed estimation [5], [6] and formation control [7]–[9] just to name a few.

Recent advances in communication technologies have facilitated multi-agent control over communication networks. On the other hand, the need to increase the number of agents leads to a demand for reduced computational and bandwidth requirements per agent. In that respect, a future control design may equip each agent with a small embedded microprocessor, which will collect information from neighboring nodes and trigger controller updates according to some rules. The control update scheduling can be done in a time-driven or an event-driven fashion. The first case involves the traditional approach of sampling at pre-specified time instances, usually separated by a specific period. Since our goal is allowing more agents into the system without increasing the computational cost, an event-driven approach seems more suitable. Stochastic event-driven strategies have appeared in [10], [11]. Similar results on deterministic event-triggered feedback control have appeared in [12]–[14]. A comparison of time-driven and event-driven control for stochastic systems favoring the latter can be found in [15].

Motivated by the above discussion, in previous work [16], [17] a deterministic event-triggered strategy was provided for a large class of cooperative control algorithms, namely those that can be reduced to a first order agreement problem [18]. The distributed control design in [16], [17] enforced each agent to update its control law whenever a certain error measurement threshold was violated, as well as when the control law of its neighbors was updated. In this paper we review the previous control designs and compare it with a distributed event-triggered strategy where an agent does not have to update its control law when when the control law of its neighbors is updated.

The rest of this paper is organized as follows: Section II presents some necessary background and discusses the problem treated in the paper. In Section III where we first review the distributed event-triggered formulation of [16], and then the corresponding formulation of [17]. Section IV presents the novel distributed event-triggered control approach. Some examples comparing the three different designs are given in Section V while Section VI includes a summary of the results of this paper and indicates further research directions.

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II. Preliminaries

A. System Model

We consider N agents, with $x_i \in \mathbb{R}$ denoting the state of agent i. Note that the results of the paper are extendable to arbitrary dimensions. We assume that the agents' dynamics obeys a single integrator model:

$$\dot{x}_i = u_i, \qquad i \in \mathcal{N} = \{1, \ldots, N\}, \tag{1}$$

where u_i denotes the control input for each agent.

Each agent is assigned a subset $N_i \subset \mathcal{N}$ of the rest of the team, called agent i's *communication set*, that includes the agents with which it can communicate. The undirected *communication graph* $G = \{V, E\}$ of the multi-agent team consists of a set of vertices $V = \{1, ..., N\}$ indexed by the team members, and a set of edges, $E = \{(i, j) \in V \times V | i \in N_j\}$ containing pairs of vertices that correspond to communicating agents.

B. Background and Problem Statement

The agreement control laws in [19], [18] were given by

$$u_i = -\sum_{j \in N_i} (x_i - x_j), \tag{2}$$

and the closed-loop equations of the nominal system (without quantization) were $\dot{x}_i = -\sum_{j \in N_i} (x_i - x_j,)$, $i \in \mathcal{N}$, so that

$$\dot{x} = -Lx,$$

where $x = [x_1, \ldots, x_N]^T$ is the stack vector of agents' states and L is the Laplacian matrix of the communication graph. For a review of the Laplacian matrix and its properties, see the above references and [20]. For a connected graph, all agents' states converge to a common agreement point which coincides with the average $\frac{1}{N}\sum_i x_i(0)$ of the initial states.

We redefine the above control formulation to take into account event-triggered strategies for the system (1). The

formulation of the distributed event-triggered strategies is provided next.

1) Distributed Event-triggered Multi-agent Control: We assume that there is a separate sequence of events, occurring at times $t_0^k, t_1^k, \ldots$, defined for each agent k. A separate distributed condition triggers the events for agent $k \in \mathcal{N}$. The decentralized control law for k is updated both at its own event times $t_0^k, t_1^k, \ldots$, as well as at the last event times of its neighbors $t_0^j, t_1^j, \ldots, j \in N_k$. Thus it is of the form

$$u_k(t) = u_k\left(t_i^k, \bigcup_{j\in N_k} t_{i'(t)}^j\right), \tag{3}$$

where $i'(t) \triangleq \arg\min_{l\in\mathbb{N}: t\geq t_l^j} \left\{t - t_l^j\right\}$.

A different formulation of the distributed event-triggered control law relaxes the need for the agents to update their control laws at the event updates of their neighbors. Such a control has the general form

$$u_k(t) = u_k(t_i^k) \tag{4}$$

where each agent uses the values of its own state and the states of its neighbors to update its control law at its own event times. This formulation will be used in the strategy of Section IV.

The decentralized cooperative control problem can be stated as follows: "derive control laws of either the form (3) or (4, and event times $t_0^k, t_1^k, \ldots$, for each agent $k \in \mathcal{N}$ that drive system (1) to an agreement point."

III. Review of previous approaches

The distributed event-triggered formulations of [16], [17] is reviewed in this section.

A. Review of Event-Triggered Control Design in [17]

The measurement error for agent i is defined as

$$e_i(t) = x_i(t_k^i) - x_i(t), \quad t \in [t_k^i, t_{k+1}^i). \tag{5}$$

The decentralized control strategy for agent i is now given by:

$$u_i(t) = -\sum_{j\in N_i}\left(x_i(t_k^i) - x_j(t_{k'(t)}^j)\right), \tag{6}$$

where

$$k'(t) \triangleq \arg\min_{l\in\mathbb{N}: t\geq t_l^j}\left\{t - t_l^j\right\}.$$

Hence, each agent takes into account the last update value of each of its neighbors in its control law. The control law for i is updated both at its own event times $t_0^i, t_1^i, \ldots$, as well as at the event times of its neighbors $t_0^j, t_1^j, \ldots, j \in N_i$.

We then have

$$\begin{aligned}\dot{x}_i(t) &= -\sum_{j\in N_i}\left(x_i(t_k^i) - x_j(t_{k'}^j)\right) = \\ &= -\sum_{j\in N_i}(x_i(t) - x_j(t)) - \sum_{j\in N_i}(e_i(t) - e_j(t)).\end{aligned}$$

Denote by $\bar{x}(t) = \frac{1}{N}\sum_i x_i(t)$ the average of the agents' states.

$$\begin{aligned}\dot{\bar{x}} = \frac{1}{N}\sum_i \dot{x}_i = &-\frac{1}{N}\sum_i\sum_{j\in N_i}(x_i(t) - x_j(t)) \\ &-\frac{1}{N}\sum_i\sum_{j\in N_i}(e_i(t) - e_j(t)) = 0,\end{aligned}$$

so that $\bar{x}(t) = \bar{x}(0) = \frac{1}{N}\sum_i x_i(0) \equiv \bar{x}$, i.e., the average of the agents' states remains constant and equal to its initial value.

Denote now $Lx \triangleq z = [z_1, \ldots, z_N]^T$ and consider

$$V = \frac{1}{2}x^T L x.$$

Then

$$\dot{V} = x^T L\dot{x} = -x^T L(Lx + Le) = -z^T z - z^T Le.$$

From the definition of the Laplacian matrix we get

$$\begin{aligned}\dot{V} &= -\sum_i z_i^2 - \sum_i\sum_{j\in N_i} z_i(e_i - e_j) \\ &= -\sum_i z_i^2 - \sum_i |N_i| z_i e_i + \sum_i\sum_{j\in N_i} z_i e_j.\end{aligned}$$

Using now the inequality $|xy| \leq \frac{a}{2}x^2 + \frac{1}{2a}y^2$, for $a > 0$, we can bound $\dot{V}$ as

$$\begin{aligned}\dot{V} \leq &-\sum_i z_i^2 + \sum_i a|N_i| z_i^2 \\ &+\sum_i \frac{1}{2a}|N_i| e_i^2 + \sum_i\sum_{j\in N_i}\frac{1}{2a}e_j^2,\end{aligned}$$

where $a > 0$.

Since the graph is symmetric, by interchanging the indices of the last term we get

$$\sum_i\sum_{j\in N_i}\frac{1}{2a}e_j^2 = \sum_i\sum_{j\in N_i}\frac{1}{2a}e_i^2 = \sum_i\frac{1}{2a}|N_i|e_i^2,$$

so that

$$\dot{V} \leq -\sum_i(1 - a|N_i|)z_i^2 + \sum_i\frac{1}{a}|N_i|e_i^2.$$

Assume that a satisfies

$$0 < a < \frac{1}{|N_i|}, \tag{7}$$

for all $i \in \mathcal{N}$. Then, enforcing the condition

$$e_i^2 \leq \frac{\sigma_i a(1 - a|N_i|)}{|N_i|}z_i^2, \tag{8}$$

we get

$$\dot{V} \leq \sum_i(\sigma_i - 1)(1 - a|N_i|)z_i^2$$

which is negative definite for $0 < \sigma_i < 1$.

Thus for each i, an event is triggered when

$$e_i^2 = \frac{\sigma_i a(1 - a|N_i|)}{|N_i|}z_i^2, \tag{9}$$

where $z_i = \sum_{j \in N_i} (x_i - x_j)$. The main result of [17] is summarized in the following:

Theorem 1: Consider the system $\dot{x} = u$ with the control law (6), (9) and assume that the communication graph G is connected. Suppose that $0 < a < \frac{1}{|N_i|}$ and $0 < \sigma_i < 1$ for all $i \in \mathcal{N}$. Then the states of all agents converge to their initial average, i.e., $\lim_{t\to\infty} x_i(t) = \bar{x} = \frac{1}{N}\sum_i x_i(0)$ for all $i \in \mathcal{N}$.

B. Review of Event-Triggered Control Design in [16]

The same control design but a different event-triggered formulation was proposed in [16] and is reviewed in the following paragraphs.

We use the decomposition $x(t) = \bar{x}(t)\mathbf{1} + \delta(t)$, where, as shown previously, we have $\dot{\bar{x}}(t) = 0$ and where δ is called the disagreement vector in [18] and $\mathbf{1}$ is the vector of ones. We now have

$$\dot{x} = \dot{\delta} = -L(x + e) = -L(\bar{x}\mathbf{1} + \delta + e)$$

so that

$$\dot{\delta} = -L(\delta + e) \tag{10}$$

For an undirected graph, an important property of δ proven in [18] is $\delta^T L\delta \geq \lambda_2(G)\|\delta\|^2$ for all δ satisfying $x = \bar{x}\mathbf{1} + \delta$.

The difference with respect to the design in [17] is the use of

$$V = \frac{1}{2}\|\delta\|^2 = \frac{1}{2}\sum_i \delta_i^2$$

as a candidate Lyapunov function. Then

$$\dot{V} = \delta^T\dot{\delta} = -\delta^T L(\delta + e) = -\delta^T L\delta - \delta^T Le$$

so that

$$\begin{aligned}\dot{V} \leq &-\lambda_2(G)\|\delta\|^2 - \delta^T Le = \\ &-\lambda_2(G)\sum_i \delta_i^2 - \sum_i\sum_{j\in N_i}\delta_i(e_i - e_j)\end{aligned}$$

and thus,

$$\begin{aligned}\dot{V} &\leq -\lambda_2(G)\sum_i \delta_i^2 + \sum_i\sum_{j\in N_i}|\delta_i|\,|e_i - e_j| \\ &\leq -\lambda_2(G)\sum_i\left(\delta_i^2 - \left|\frac{\delta_i}{\lambda_2(G)}\right|\sum_{j\in N_i}(|e_i| + |e_j|)\right)\end{aligned}$$

Enforcing the condition

$$\sum_{j\in N_i}(|e_i| + |e_j|) \leq \lambda_2(G)\,\sigma_i\,|\delta_i| \tag{11}$$

we get

$$\sigma_i\delta_i^2 \geq \left|\frac{\delta_i}{\lambda_2(G)}\right|\sum_{j\in N_i}(|e_i| + |e_j|)$$

so that

$$\dot{V} \leq -\lambda_2(G)\sum_i\left(\delta_i^2 - \sigma_i\delta_i^2\right) = -\lambda_2(G)\sum_i(1 - \sigma_i)\,\delta_i^2$$

which is negative semidefinite for $0 < \sigma_i < 1$.

Thus for each i, an event in this formulation is triggered when

$$\sum_{j\in N_i}(|e_i| + |e_j|) = \lambda_2(G)\,\sigma_i\,|\delta_i|, \tag{12}$$

At an event time t_k^i, we have $e_i(t_k^i) = x_i(t_k^i) - x_i(t_k^i) = 0$, and since $\sum_{j\in N_i}(|e_i(t)| + |e_j(t)|) \geq \sum_{j\in N_i}|e_j(t)|$ for all $t \geq 0$, the condition (11) is enforced.

The main result of [16] is summarized in the following:

Theorem 2: Consider the system $\dot{x} = u$ with the control law (6), (12) and assume that the communication graph G is connected. Suppose that $0 < \sigma_i < 1$ for all $i \in \mathcal{N}$. Then the states of all agents converge to their initial average, i.e., $\lim_{t\to\infty} x_i(t) = \bar{x} = \frac{1}{N}\sum_i x_i(0)$ for all $i \in \mathcal{N}$.

The main drawback of this approach is that knowledge of the initial average of the states is required by the agents in order to implement the control strategy. In contract, the formulation of [17] present previously relaxes this assumption. In particular, no knowledge of the initial average is required.

IV. Novel Distributed Event-triggered Strategy

In this section, we propose a control law of the form (4) for each agent. In particular, the decentralized control strategy for agent i is now given by:

$$u_i(t) = -\sum_{j\in N_i}\left(x_i(t_k^i) - x_j(t_k^i)\right), \tag{13}$$

and thus each agent updates its control law only at its own error update times. We then have

$$\begin{aligned}\dot{x}_i(t) &= -\sum_{j\in N_i}\left(x_i(t_k^i) - x_j(t_k^i)\right) = \\ &= -\sum_{j\in N_i}(x_i(t) - x_j(t)) - \sum_{j\in N_i}e_i(t) + \sum_{j\in N_i}e_{ij}(t)\end{aligned}$$

where we use the notation

$$e_{ij}(t) = x_j(t_k^i) - x_j(t), t \in [t_k^i, t_{k+1}^i).$$

Note that initial average is not invariant in this case, and thus agents may reach a different agreement point.

Using now

$$V = \frac{1}{2}x^T Lx$$

as a candidate Lyapunov function we get

$$\begin{aligned}\dot{V} &= -\sum_i z_i^2 - \sum_i\sum_{j\in N_i}z_i(e_i - e_{ij}) \\ &= -\sum_i z_i^2 - \sum_i |N_i|z_ie_i + \sum_i\sum_{j\in N_i}z_ie_{ij}.\end{aligned}$$

The derivative of V is now bounded as follows:

$$\begin{aligned}\dot{V} \leq &-\sum_i z_i^2 + \sum_i |N_i||z_i||e_i| \\ &+\sum_i\sum_{j\in N_i}|z_i||e_{ij}|,\end{aligned}$$

Enforcing the condition

$$\sum_{j\in N_i} (|e_i| + |e_{ij}|) \leq \sigma_i |z_i| \tag{14}$$

we get

$$\dot{V} \leq \sum_i (1-\sigma_i) z_i^2$$

which is negative semidefinite for $0 < \sigma_i < 1$.

Thus for each i, an event in this formulation is triggered when

$$\sum_{j\in N_i} (|e_i| + |e_{ij}|) = \sigma_i |z_i|, \tag{15}$$

At an event time t_k^i, we have $e_i(t_k^i) = x_i(t_k^i) - x_i(t_k^i) = 0$, and since $\sum_{j\in N_i} (|e_i(t)| + |e_{ij}(t)|) \geq \sum_{j\in N_i} |e_{ij}(t)|$ for all $t \geq 0$, the condition (14) is enforced.

Following the proofs of Theorems 1 and 2, the following is easily derived:

Theorem 3: Consider the system $\dot{x} = u$ with the control law (13), (15) and assume that the communication graph G is connected. Suppose that $0 < \sigma_i < 1$ for all $i \in \mathcal{N}$. Then the states of all agents converge to an agreement point.

Note that the agreement point is not guaranteed to be the initial average in this case.

V. Examples

The results of the previous sections are illustrated through computer simulations. In the following paragraphs, we consider all three distributed event-triggered algorithms presented previously and compare the derived results.

Consider a network of four agents whose Laplacian matrix is given by

$$L = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 3 & -1 & -1 \\ 0 & -1 & 2 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

The four agents start from the same initial conditions and evolve under the control laws (6),(12),(13) respectively. We have set $\sigma_1 = \sigma_2 = 0.55$, $\sigma_3 = \sigma_4 = 0.75$ and $a = 0.2$ for the examples of the paper.

The next simulations depicts how the framework is realized in each of the three cases for agent 4. In particular, the solid line in the top plot of Figure 1 shows the evolution of $|e_4(t)|$ in the case of the first control strategy (6),(9). This stays below the specified state-dependent threshold given by (9) $|e_4|_{\max} = \sqrt{\frac{\sigma_4 a(1-a|N_4|)}{|N_4|}}|z_4|$, which is represented by the dotted line in the plot. In the middle plot, the solid line shows the evolution of $|e_3(t)| + |e_4(t)|$ in the case of the second control strategy (6),(12). This also stays below the specified state-dependent threshold given by (12) $M_4 = \lambda_2(G)\sigma_4|\delta_4|$, represented by the dotted line in the Figure. Finally, the solid line in the bottom plot of Figure 1 shows the evolution of $|e_4(t)| + |e_{43}(t)|$ in the case of the third control strategy (13),(15), which also stays below the specified threshold given by (15) $M_4 = \sigma_4|z_4|$, represented by the dotted line in the plot.

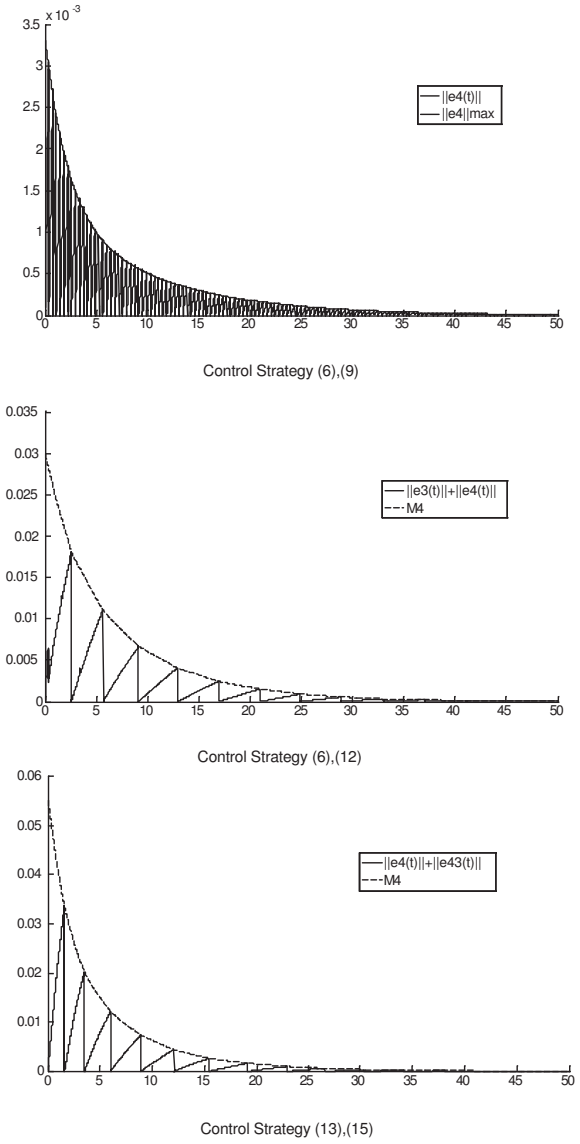

Fig. 1. Four agents evolve under the three distributed event-triggered strategies.

As can be seen in the figure, the first approach that uses less information has a slightly slower convergence rate. The third approach seems to have less updates and a faster convergence rate, however, the property of converging to the initial average is lost in this case.

VI. Conclusions

Distributed event-triggered control strategies for a multi-agent system with single integrator agents were reviewed and proposed. Future work will involve extending the proposed approach to more general dynamic models, as well as finding sufficient conditions for a strict lower bound on the inter-execution times of all agents in the decentralized case.

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