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with location-dependent sensing models*

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Distributed Coverage Control for Mobile Sensors with Location-dependent Sensing Models

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Abstract—This paper addresses the problem of coverage control of a network of mobile sensors. In the current literature, this is commonly formulated as a *locational optimization problem* under the assumption that sensing performance is independent of the locations of sensors. We extend this work to a more general framework where the sensor model is location-dependent. We propose a distributed control law and coordination algorithm. If the global sensing performance function is known a priori, we prove that the algorithm is guaranteed to converge. To validate this algorithm, we conduct experiments with indoor and outdoor deployments of Cyclops cameras and model its sensing performance. This model is used to simulate deployments on 1D pathways and study the coverage obtained. We also examine the coverage in the case when the global sensing function is not known and is estimated in an online fashion.

I. INTRODUCTION

In this paper we address the coverage-control problem of a network of mobile sensors. Coverage is of fundamental importance in sensor networks. It captures the notion of quality-of-service of the sensing task. More formally, a coverage problem involves formulating an overall coverage-metric based on sensing performance of an individual sensor and finding an *optimal placement* of sensors such that the overall metric is optimized. Recently, there has been a lot of papers (e.g. [4], [5], [11]) in which this problem is formulated as a classic *locational optimization problem* [6]. An optimal solution to the locational optimization problem is based on the generalized Voronoi partitioning of the region [6]. An algorithm for control and coordination of mobile sensors to obtain an optimal sensor placement was first proposed in [2]. This initiated a number of variations of the problem formulations and coordination algorithms [4], [3], [1], [5], [11], [9], [10]. To the best of our knowledge, with the exception of [5], in the work so far, each sensor behavior is assumed to be identical everywhere, *i.e.*, each sensor has the same performance function everywhere. Each sensor's performance is assumed to reduce monotonically with the distance from the sensor location. This leads to the classic Euclidean Voronoi partitioning of the region. However, in

reality, in several situations a sensor's performance is not identical everywhere. The performance of each sensor is impacted by the environmental conditions in which the sensor is located. In our recent work [7], we demonstrate that sensing performance is not only a function of the distance but also a function of the location of a Cyclops camera [8] used as an image sensor. In another work [12], the authors report that underwater sound propagation is susceptible to changes in water salinity and temperature and therefore the listening range, or coverage, of a sound detector is largely influenced by its location underwater. Currently, coverage algorithms do not address such location dependence. This leads to inefficient usage of resources.

In this paper, we address the case of sensors where sensing performance depends on the sensor's location also. We formulate the coverage problem as the locational optimization problem with the sensing performance function which is location-dependent, *i.e.*, it is a function of the distance between the sensor and the sampling location as well as the location of the sensor. In the same spirit of the coverage algorithms in [4], we propose a distributed control and coordination algorithm for the new formulation. We prove the convergence of our algorithm in the case where sensing performance is known globally. As a case study, we consider the problem of covering 1D loopy pathways with a network of mobile Cyclops cameras. In our recent work [7], we modeled the sensing behavior of Cyclops cameras and proposed the locational optimization problem formulation work in 1D. In our results based on indoor experiments [7], we assumed that sensing performance is known globally. Here, we compare and contrast simulation results for the case where we estimate the sensing performance in the online fashion with the case where the performance is assumed to be known globally. We also present coverage results based on simulations using data from outdoor experiments.

II. RELATED WORK

There is a rich body of literature on locational optimization which has applications for problems such as facility-location [6]. The solution to this problem is based on the generalized Voronoi partitioning. Cortes et. al. proposed a control and coordination algorithm to obtain an optimal coverage configuration for mobile sensor networks starting from the initial configuration. They proposed a distributed, adaptive and asynchronous control algorithm with guaranteed convergence. This work initiated a number of variations that involved obstacles, communication constraints, *etc.* The algorithm in [4] requires global knowledge of the *density* function

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and in that sense the algorithm is not truly distributed. Schwager, Slotine and Rus proposed a consensus based distributed algorithm that simultaneously learns the density function and obtains a coverage solution [11]. Recently, Lekein and Leonard [5] have considered the case of non-Euclidean distance metrics. Their solution is to map the non-Euclidean metric to a near-Euclidean metric using transformations known as *Cartograms*. They show that convergence of a control algorithm in the transformed Euclidean space implies convergence in the original non-Euclidean space. The location-dependent sensing model that we use can be thought of as a non-Euclidean distance metric and therefore, in terms of objectives, [5] is closest to our work. The key difference is that in the Cartograms based approach the transformation requires global knowledge of the distance metric and density function. In contrast, our approach is distributed and the sensing performance function is *learned* based on local observations. In our recent work [7], we addressed the problem of covering 1D loopy pathways with a network of mobile Cyclops cameras. We proposed an empirical model for the sensing performance of a Cyclops camera and formulated a locational optimization problem in 1D. In this paper, we generalize the formulation to higher dimensions for a general class of sensors.

III. PROBLEM FORMULATION

We build on the notation used in [4] to define our coverage problem. Let Q denote the bounded convex domain in \mathbb{R} . We seek to *cover* Q using n mobile sensors. Let $\phi : Q \rightarrow \mathbb{R}$ denote the scalar density function. It captures the importance of covering a particular location. Let p_1, p_2, \dots, p_n denote the current locations of n sensors. We assume a location-dependent sensing model for each sensor, *i.e.*, the *sensing performance* at point q for the i^{th} sensor not only depends on $\|q - p_i\|$ but also on the location, p_i itself. Here, $\|\cdot\|$ denotes the Euclidean norm. We denote the sensing performance function for sensor i at point q by $f(\|q - p_i\|, p_i)$, where $f : \mathbb{R}^+ \times Q \rightarrow \mathbb{R}$. Note that we explicitly indicate the dependence of the performance function on the sensor location. The performance function naturally induces a *generalized Voronoi partition* of Q . Each sensor has its own *dominance region*, *i.e.*, its Voronoi cell, where the sensing performance is better than that of any other sensor. More formally, if W_i denotes the dominance region for the i^{th} sensor, then

$$W_i = \{q \in Q \mid f(\|q - p_i\|, p_i) \geq f(\|q - p_j\|, p_j), \forall j \neq i\}. \quad (1)$$

We define our *coverage* metric or *net utility function* as follows:

$$\mathcal{H}(p_1, p_2, \dots, p_n) = \sum_{i=1}^n \int_{W_i} \phi(q) f(\|q - p_i\|, p_i) dq. \quad (2)$$

The utility function is the sum of the sensing performance function of each sensor over its dominance region including the weighing density function. We refer to $\int_{W_i} \phi(q) f(\|q - p_i\|, p_i) dq$ as the *utility* of the i^{th} sensor. We formulate the

coverage problem as finding the optimal set of locations of the sensors that maximizes the net utility.

$$(p_1^*, p_2^*, \dots, p_n^*) = \arg \max_{(p_1, p_2, \dots, p_n)} \mathcal{H}(P). \quad (3)$$

Note that it is possible to formulate other metrics such as maximizing the minimum over the utility functions of all the sensors [4]. We would also like to note that in the common literature the coverage objective function is known as the *locational optimization function* and the coverage problem is formulated as the minimization problem, *e.g.*, [4]. In the spirit of the application we consider in this paper, we formulate the problem as the maximization problem.

A. Partial derivative of the overall utility function

We note that the result about the partial derivative of the net utility function based on the divergence theorem [4] generalizes easily to our case. We use the divergence theorem [2] in the proof of our result.

Theorem 3.1: [2] Let $\mathcal{V} = \mathcal{V}(x) \subset Q$ be a region that depends smoothly on real parameter $x \in \mathbb{R}$. \mathcal{V} has a well-defined closed boundary $\partial\mathcal{V}(x)$ for all x . Let $\phi(q)$ be the density function over Q . Then

$$\begin{aligned} \frac{d}{dx} \int_{\mathcal{V}(x)} \phi(q) f(\|q - y\|, y) dq &= \\ \int_{\partial\mathcal{V}(x)} \left\langle \frac{dq}{dx}, n(q) \right\rangle \phi(q) f(\|q - y\|, y) dq, \end{aligned}$$

where $\langle \cdot, \cdot \rangle$ denotes the dot-product and $n(q)$ denotes the normal vector along $\partial\mathcal{V}(x)$.

We modify the above result for a special case as follows:

Lemma 3.2: Let $\mathcal{V} = \mathcal{V}(x) \subset Q$ be a region that depends smoothly on real parameter $x \in \mathbb{R}$. \mathcal{V} has a well-defined closed boundary $\partial\mathcal{V}(x)$ for all x . Let $\phi(q)$ be the density function over Q . Then

$$\begin{aligned} \frac{d}{dx} \int_{\mathcal{V}(x)} \phi(q) f(\|q - x\|, x) dq &= \\ \int_{\mathcal{V}(x)} \phi(q) \frac{\partial f(\|q - x\|, x)}{\partial x} dq &+ \\ \int_{\partial\mathcal{V}(x)} \left\langle \frac{dq}{dx}, n(q) \right\rangle \phi(q) f(\|q - x\|, x) dq. \end{aligned}$$

Theorem 3.3: The partial derivative of the net utility function with respect to the position of the i -th sensor is given by,

$$\frac{\partial \mathcal{H}}{\partial p_i} = \int_{W_i} \phi(q) \frac{\partial f(\|q - p_i\|, p_i)}{\partial p_i} dq \quad (4)$$

Proof:

Below we sketch the steps of the proof. We refer readers to [2] for details. Let $p_{j_1}, p_{j_2}, \dots, p_{j_k}$ be p_i 's generalized Voronoi neighbors. Let Δ_{ij_l} be the Voronoi cell boundary between i and neighbor j_l . Note that the following two normal directions are in opposite directions. $n_{ij_l}(q) = -n_{j_l i}(q)$. Then

$$\frac{\partial \mathcal{H}}{\partial p_i} = \frac{\partial}{\partial p_i} \int_{W_i} \phi(q) f(\|q - p_i\|, p_i) dq + \sum_{l=1}^k \int_{\Delta_{j_l i}} \phi(q) f(\|q - p_{j_l}\|, p_{j_l}) dq.$$

Based on Theorem 3.1 and Lemma 3.2, and the definition of the generalized Voronoi diagrams, we obtain the desired result. ■

IV. COVERAGE CONTROL ALGORITHM

We propose a distributed coverage control algorithm that is based on iterative local sub-gradient method.

A. Control Law for each sensor

Let $p_i(n)$ be the position of the i -th sensor after n updates. We state the control law for the position update of the i -th sensor to $p_i(n+1)$. The local control law involves searching for the locally optimal solution. During the time the i -th sensor updates its position, our coverage algorithm does not allow the simultaneous update at the Voronoi neighbors of the i -th sensor. Given that the positions of the Voronoi neighbors are fixed, the net utility \mathcal{H} and the local gradient, $\frac{\partial \mathcal{H}}{\partial p_i}$ are just the functions of p_i . We state the control law as follows:

$$p_i(n+1) = p_i^*, \text{ where, } p_i^* = \arg \max_{p_i \in W_i} \mathcal{H}(\dots, p_i, \dots) \quad (5)$$

Essentially, the control law involves local search over the Voronoi partition W_i for the optimal solution p_i^* by solving the constrained optimization problem. If p_i^* does not lie on the boundary of W_i , then it is the root of equation $g(p_i) = 0$ for some function g . Since the search involves finding the optimal location just for one sensor, it is computationally tractable. We state the following lemma about the increase in the net utility after every pose update.

Lemma 4.1: The control law for an individual sensor in Equation 5 guarantees that the net utility increases after the position update of the sensor.

Implementation of the above control law requires the exact knowledge of the sensing performance function $f(\|q - p_i\|, p_i)$ for each sensor i and the density function $\phi(q)$. Later we show in our case study of camera sensor network that $f(\|q - p_i\|, p_i)$ can be estimated on the fly using measurements. We also note that [11] shows that the global knowledge of $\phi(q)$ is not required.

B. Distributed Coverage Algorithm

Below we state the coverage algorithm that runs at each node. We assume discrete time steps, $1, 2, \dots$. The algorithm involves the implementation of the local control law that we described in the previous section. A node updates its position only if its Voronoi neighbors are not updating their positions.

Algorithm 1 Coverage algorithm at the i -th sensor: Let p_i denote its position. Let V_i denote the set of its Voronoi neighbors.

t : Update V_i
t+1: Check if any $j \in V_i$ is updating its position
t+2: if yes, wait for random duration of time and repeat the above steps
t+2: if no, schedule position update at t+3 and broadcast over V_i
t+3: start moving to optimal p_i^* as obtained using Equation 5
t+k: After the move, wait for random duration before the next update

C. Convergence

Theorem 4.2: The distributed coverage algorithm at each sensor as described in Algorithm IV-B converges, as the number of position updates tends to infinity.

The proof follows from the fact that the net coverage utility is bounded and by lemma 4.1, it increases monotonically with each iteration of the algorithm. The algorithm yields a locally optimal solution to the coverage problem.

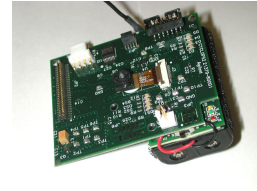


Fig. 1. Cyclops camera with an attached Mica2 Mote [8]

V. CASE STUDY: CAMERA NETWORKS

In this section, we present results of our algorithm for coverage optimization of Cyclops camera sensors. The Cyclops camera sensor consists of a CMOS imager with CIF image capability and an internal image processor unit for image interpretation and analysis [8]. It couples with a Mote and periodically captures images (figure 1). Our objective is to *cover* loopy environments such as indoor corridors or outdoor pathways such that an intruder object can be detected.

A. Camera Sensing Performance

To detect moving objects, the Cyclops uses standard background-subtraction algorithm because of its low memory and computation requirements. It is well known that the performance of the algorithm varies with environmental factors such as lighting, luminance, background contrast, etc. We use the number of pixels detected on the object as a measure of the quality of coverage. In our earlier work [7], we developed an empirical model for the number of pixels as a function of the location of Cyclops and the distance to the object. We assume that each sensor is a bi-directional camera obtained by combining two cyclops camera modules aligned

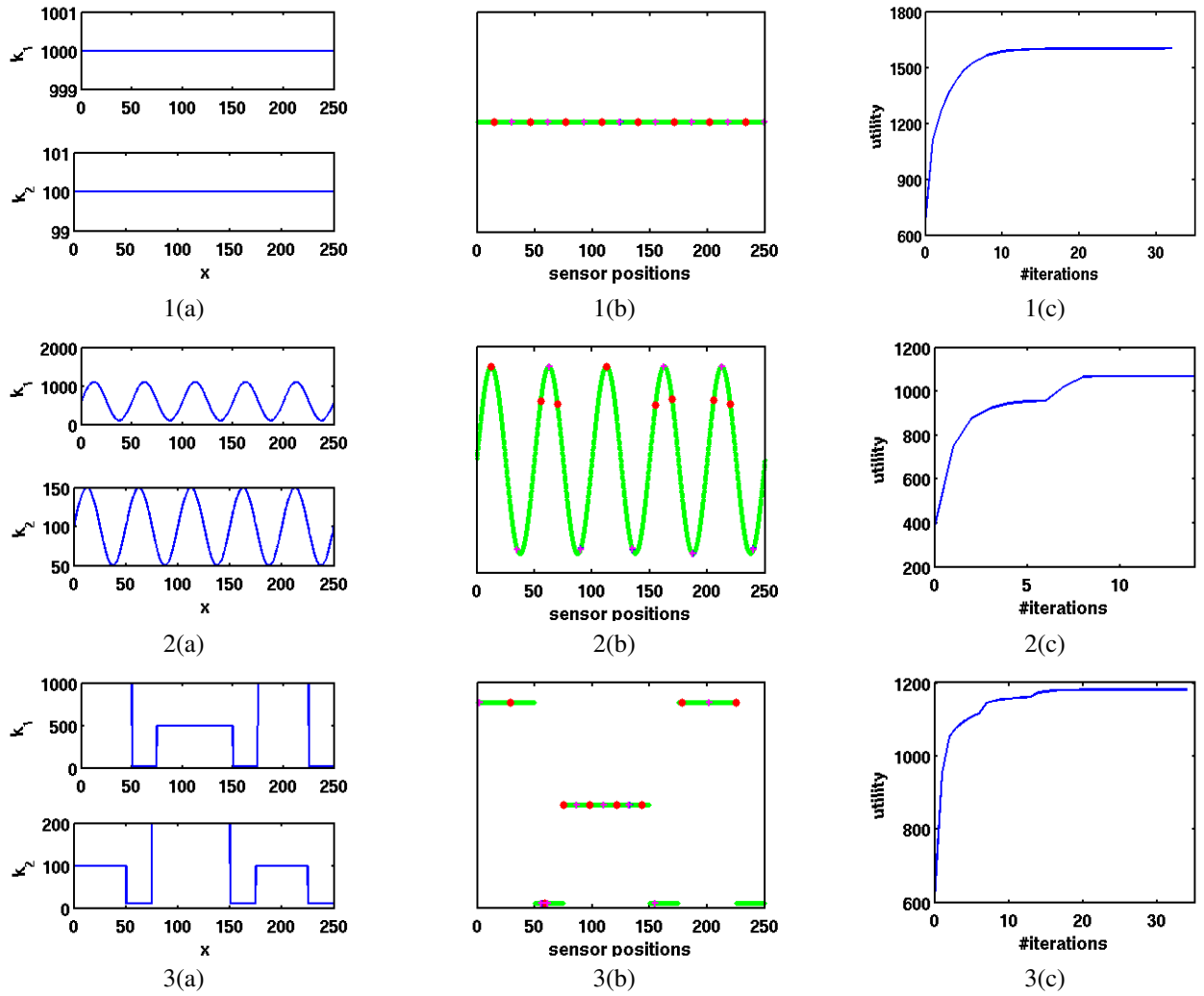


Fig. 2. Performance of the distributed control law for three types of parameter functions on a 1D loop which is a 250 feet long smooth curve. 1(a), 2(a) and 3(a) show the parameter values constant, sine and step. 1(b), 2(b) and 3(b) show the corresponding final positions of sensors (red circles) superimposed on the k_1 function. The crosses show the dominance region boundaries between sensors. 1(c), 2(c) and 3(c) show the variation in the net coverage utility. The utility increases monotonically with each iteration.

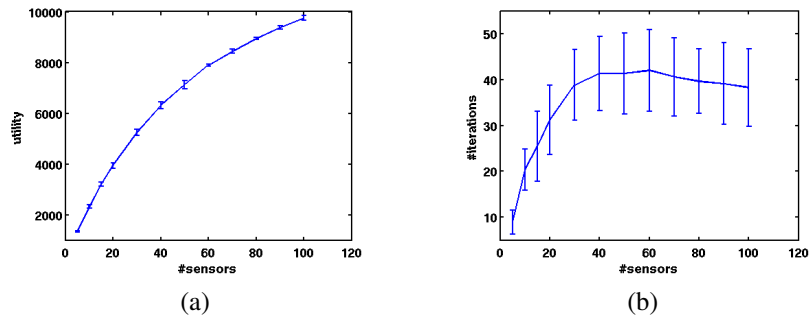


Fig. 3. Variation in coverage utility and convergence time for different network sizes averaged over 50 iterations.

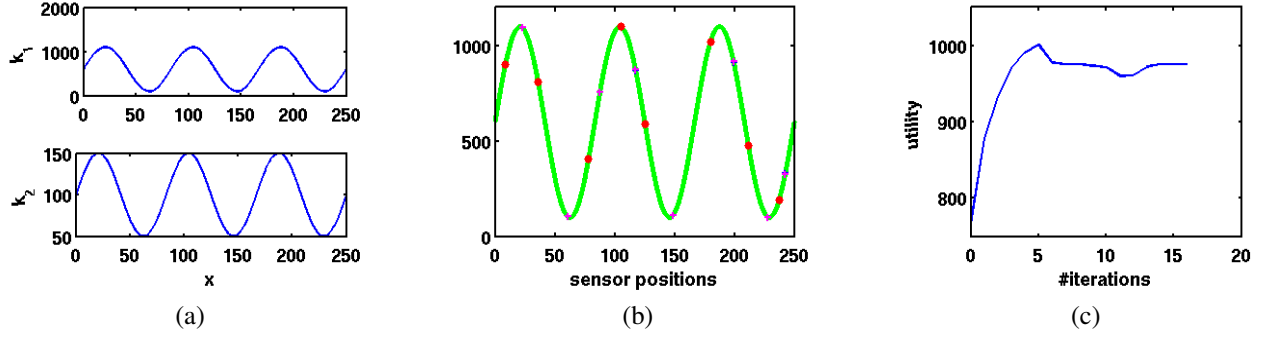


Fig. 4. Performance of the distributed control law with partial knowledge of $k_1(x)$ and $k_2(x)$.

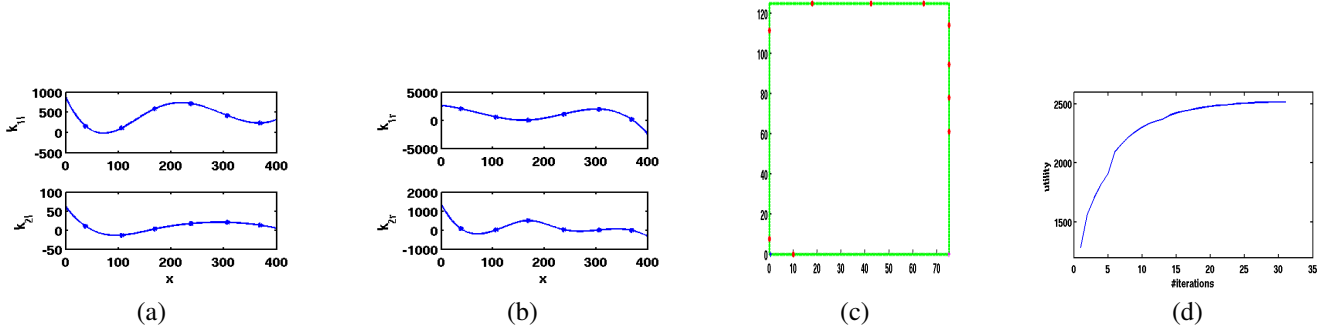


Fig. 5. Performance of the distributed control law in an outdoor environment.

along the same axis. Thus each sensor can *see* on its *left* and *right* directions on the loop. The sensing performance function along each direction can be expressed as follows

$$f_l(\|p_i - q\|, p_i) = \frac{k_{1l}(p_i)}{k_{2l}(p_i) + \|p_i - q\|^2} \quad (6)$$

$$f_r(\|p_i - q\|, p_i) = \frac{k_{1r}(p_i)}{k_{2r}(p_i) + \|p_i - q\|^2} \quad (7)$$

where k_1 and k_2 are parameters that vary with the location of the Cyclops.

B. Simulation Environment

We ran simulations in MATLAB to test the convergence and resulting coverage of our distributed control law. The path is a 1D loop that can have sharp corners and obstacles that cause the sensing performance function to vary abruptly. The sensors start at randomly chosen initial positions and in each iteration of the algorithm, they update their position according to Equation 5 using a constrained-optimization routine in MATLAB. We assume that sensors are localized and adjacent sensors on the path can communicate with each other. The simulation converges when the no sensor changes its position. We test for different models of $k_1(x)$ and $k_2(x)$, different shapes of the path, and different number of sensors.

C. Global knowledge of sensor performance function

We first consider the ideal scenario where sensors have perfect knowledge of the sensing performance function. In

practice, the sensors will estimate the performance function based on observed data the performance of the coverage algorithm will depend on the accuracy of this estimation. Figure 2 shows the performance of the algorithm for three different sensing performance functions. In each case, the sensors start at random initial locations. As expected, for constant parameter values (Figure 2.1(a)), the sensors converge to a uniform distribution(Figure 2.1(b)). When the parameters have significant variations (Figure 2.2(a), 2.3(a)), the distribution is non-uniform (Figure 2.2(b), 2.3(b)). Sensors try to move to positions where the value of k_1 is large and k_2 is small so that their local utility is maximized. Note that in each case the global coverage utility increases monotonically even though the position updates at each sensor are local. This is in agreement with Theorem 4.2. To study the impact of the network size on the algorithm performance, varied the network size from 5 to 100. For each network size, we repeated the simulation 50 times starting with a different random initial position each time. The coverage utility increases exponentially and eventually saturates (Figure 3(a)) since as the number of sensors increases, the marginal utility of each additional sensor decreases and tends to zero. The variation in the utility is very small indicating that the initial locations do not impact the final coverage. The convergence time initially increases with the number of sensors and then stops increasing (Figure 3(b)) proving that the algorithm is scalable. The large deviations indicate that unlike coverage, the convergence time varies significantly with the initial sensor locations.

D. Online estimation of sensor performance function

In the coverage algorithm in the previous section we assumed that each sensor has global knowledge of the sensing performance function $f(\|p_i - q\|, p_i)$. This permits a sensor to take the best step according to Equation 5. However in practice this is not possible. Specifically, in the case of camera sensors, functions $k_1(x)$ and $k_2(x)$ are not globally known. This necessitates modification of the current coverage algorithm. We assume that at every step of the algorithm, a camera *learns* its sensing performance function, *i.e.*, the empirical values of k_1 and k_2 .

In the simulations shown in Figure 4, each sensor builds piecewise linear models of k_1 and k_2 based on data from its neighboring sensors and its current data. To improve the estimation in the first iteration, we assume that sensors share data over multiple hops. We observe that the utility sometimes decreases because of erroneous estimation (Figure 4(c)). But even with this very simple estimation, the coverage utility is close to the case when global knowledge was assumed.



Fig. 6. Cyclops outdoor setup

1) *Outdoor experiments:* We conducted experiments in an outdoor rectangular pathway surrounded by trees and varying lighting conditions. Figure 6 shows the outdoor setup of the camera. The pathway was of size $150\text{ft} \times 75\text{ft}$. We captured background images and foreground images at 8 locations around the pathway in both directions. We observed that the lighting conditions in the outdoor setting change rapidly and because of this effect we collected data at relatively a few spots. Based on these images, we estimated the values of k_1 and k_2 at the sampling points. We interpolate spline functions through these values to obtain the sensing performance function over the domain and Figure 5(a) shows this model. We use this model to test our coverage algorithm. In our simulations, we placed 10 sensors initially uniformly randomly. In our earlier work [7], we presented results based on similar experiments at finer resolution of sampling locations in indoor environments. Lighting conditions in outdoor environments change rapidly relative to the time scale of collecting all the measurements. However we believe that with some degree of automation, our coverage algorithm can easily implemented in dynamic environments that demand frequent reconfiguration of the sensor positions.

VI. CONCLUSIONS

In this paper, we addressed the coverage problem where the performance of each sensor depends on its location. We formulated the coverage problem as a locational optimization problem and proposed a distributed algorithm with guaranteed convergence. We presented a set of results for the problem of covering 1D pathways with a network of mobile Cyclops cameras. As a part of future work we plan to implement generalized Voronoi diagrams and apply our algorithm in 2D situations.

VII. ACKNOWLEDGEMENTS

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