## 18.06 - Spring 2005 - Problem Set 2

Solution to the Challenge Problem

Denote by  $C_{ij}$  the  $n \times n$  matrix having a 1 in the (i, j) entry, and 0's everywhere else. In terms of these matrices we may write  $E_{ij} = I - \ell_{ij}C_{ij}$  and  $E_{ij}^{-1} = I + \ell_{ij}C_{ij}$ . Thus we have

$$ME_{ij}^{-1} = M(I + \ell_{ij}C_{ij}) = M + \ell_{ij}MC_{ij}$$

The matrix  $MC_{ij}$  has all columns different from the j-th consisting entirely of 0's. The *j*-th column of  $MC_{ij}$  is simply the *i*-th column of M. Since the matrix  $E_{ij}$  is a lower triangular matrix we have i > j, and since M is only filled in up to column j, the i-th column of M has exactly one 1 in the i-th row and 0's everywhere else. Therefore the matrix  $MC_{ij}$  has exactly one non-zero entry in position (i, j) and this entry is a 1, i.e.  $MC_{ij} = C_{ij}$ . We conclude that  $ME_{ij}^{-1} = M + \ell_{ij}C_{ij} = N$ , as required.

The effect of *right* multiplication by  $E_{ij}^{-1}$  on a matrix M is to leave the columns of M different from the j-th one unchanged, and replacing the j-th column of M by the sum of the j-th column of M with  $\ell_{ij}$  times the i-th column of M.