# 18.06 - Spring 2005 - Problem Set 2 

Solution to the Challenge Problem

Denote by $C_{i j}$ the $n \times n$ matrix having a 1 in the $(i, j)$ entry, and 0 's everywhere else. In terms of these matrices we may write $E_{i j}=I-\ell_{i j} C_{i j}$ and $E_{i j}^{-1}=I+\ell_{i j} C_{i j}$. Thus we have

$$
M E_{i j}^{-1}=M\left(I+\ell_{i j} C_{i j}\right)=M+\ell_{i j} M C_{i j}
$$

The matrix $M C_{i j}$ has all columns different from the $j$-th consisting entirely of 0 's. The $j-$ th column of $M C_{i j}$ is simply the $i-$ th column of $M$. Since the matrix $E_{i j}$ is a lower triangular matrix we have $i>j$, and since $M$ is only filled in up to column $j$, the $i-$ th column of $M$ has exactly one 1 in the $i-$ th row and 0 's everywhere else. Therefore the matrix $M C_{i j}$ has exactly one non-zero entry in position $(i, j)$ and this entry is a 1, i.e. $M C_{i j}=C_{i j}$

We conclude that $M E_{i j}^{-1}=M+\ell_{i j} C_{i j}=N$, as required.
The effect of right multiplication by $E_{i j}^{-1}$ on a matrix $M$ is to leave the columns of $M$ different from the $j$-th one unchanged, and replacing the $j-$ th column of $M$ by the sum of the $j-$ th column of $M$ with $\ell_{i j}$ times the $i-$ th column of $M$.

