# Solution to 18.06 Challenge Problem Set 5, Spring 2005

1. Minimize $$\int_0^1 (c+dt-t^2)^2 dt \;=\; \int_0^1 (c^2+2cd+d^2t-2ct^2-2dt^3+t^4)dt$$
$$= c^2+cd+\frac{1}{3}d^2-\frac{2}{3}c-\frac{2}{4}d+\frac{1}{5}$$

   $$\begin{array}{lrcrcl} c\text{-derivative:} & 2c & + & d & = & \frac{2}{3} \\ d\text{-derivative:} & c & + & \frac{2}{3}d & = & \frac{2}{4} \end{array}$$

   Solution: $c=-\frac{1}{6}$ and $d=1$: Best line $y=t-\frac{1}{6}$.

   Note: Dividing by 2 shows the 2 by 2 Hilbert matrix with $h_{ij}=1/(i+j-1)$:

   $$\texttt{hilb}(2)=\begin{bmatrix}1 & 1/2\\ 1/2 & 1/3\end{bmatrix}$$

2. The 10 by 2 matrix is $A=\begin{bmatrix}\texttt{ones}(10,1) & (1:10)'/10\end{bmatrix}$ and the column vector is $b=(1:10)'.*(1:10)'/100$.

   $$A^{\mathrm{T}}A\begin{bmatrix}C\\D\end{bmatrix}=A^{\mathrm{T}}b \quad\text{is}\quad \begin{bmatrix}10 & 5.5\\ 5.5 & 3.85\end{bmatrix}\begin{bmatrix}C\\D\end{bmatrix}=\begin{bmatrix}3.85\\3.02\end{bmatrix}$$
   $$\text{giving}\quad \begin{bmatrix}C\\D\end{bmatrix}=\begin{bmatrix}-.22\\1.1\end{bmatrix}.$$

3. The same calculation with 10 changed to 20 (and 100 to 400) comes closer to $c=-\frac{1}{6}, d=1$:

   $$A^{\mathrm{T}}A\begin{bmatrix}C\\D\end{bmatrix}=A^{\mathrm{T}}b \quad\text{is}\quad \begin{bmatrix}20 & 10.5\\ 10.5 & 7.175\end{bmatrix}\begin{bmatrix}C\\D\end{bmatrix}=\begin{bmatrix}7.175\\5.5125\end{bmatrix}$$
   $$\text{giving}\quad \begin{bmatrix}C\\D\end{bmatrix}=\begin{bmatrix}-.1925\\1.0500\end{bmatrix}.$$

   The error in comparing $D$ to $d=1$ dropped from .1 to .05 (exactly in half). The error in comparing $C$ to $c=-\frac{1}{6}$ dropped from $c-C=.0533$ to $c-C=.0258$ (almost exactly half).