

18.06

Professor Strang

Quiz 2

April 1, 2005

Grading

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Your **PRINTED** name is: _____

1 (17 pts.) If the output vectors from Gram-Schmidt are

$$q_1 = \begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix} \quad \text{and} \quad q_2 = \begin{bmatrix} -\sin \theta \\ \cos \theta \end{bmatrix}$$

describe all possible input vectors a_1 and a_2 .

2 (15 pts.) If a and b are nonzero vectors in \mathbf{R}^n , what number x minimizes the squared length $\|b - xa\|^2$?

- 3 (17 pts.)** Find the projection p of the vector $b = (1, 2, 6)$ onto the plane $x + y + z = 0$ in \mathbf{R}^3 . (You may want to find a basis for this 2-dimensional subspace, even an orthogonal basis.)

4 (17 pts.) Find the determinants of A and A^{-1} and the $(1, 2)$ entry of A^{-1} if

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 3 \\ 1 & 3 & 1 & 7 \end{bmatrix}.$$

- 5 (17 pts.) By recursion or cofactors or otherwise(!) compute the determinant of this 5 by 5 circulant matrix C :

$$C = \begin{bmatrix} 2 & -1 & 0 & 0 & -1 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ -1 & 0 & 0 & -1 & 2 \end{bmatrix}$$

- 6 (17 pts.)** Suppose P_1 is the projection matrix onto the 1-dimensional subspace spanned by the first column of A . Suppose P_2 is the projection matrix onto the 2-dimensional column space of A . After thinking a little, compute the product P_2P_1 .

$$A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 0 & 1 \\ 1 & 2 \end{bmatrix}.$$