18.06 Professor Strang Quiz 3 May 4, 2005

Your PRINTED name is: ____________________

Grading

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1 (37 pts.) (a) (16 points) Find the three eigenvalues and *all* the real eigenvectors of A. *It is a symmetric Markov matrix with a repeated eigenvalue.*

$$A = \begin{bmatrix} \frac{2}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{2}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{2}{4} \end{bmatrix}.$$

(b) (9 points) Find the limit of A^k as $k \to \infty$. (You may work with $A = S\Lambda S^{-1}$ without computing every entry.)

(c) (6 points) Choose any positive numbers r, s, t so that

$$\begin{aligned} A - rI &\quad \text{is} \quad \text{positive definite} \\ A - sI &\quad \text{is} \quad \text{indefinite} \\ A - tI &\quad \text{is} \quad \text{negative definite} \end{aligned}$$

(d) (6 points) Suppose this A equals $B^{\mathrm{T}}B$. What are the singular values of B?

2 (41 pts.) (a) (14 points) Complete this 2 by 2 matrix A (depending on a) so that its eigenvalues are $\lambda = 1$ and $\lambda = -1$:

$$A = \begin{bmatrix} a & 1 \\ & \end{bmatrix}$$

(b) (9 points) How do you know that A has two independent eigenvectors?

(c) (9 points) Which choices of a give orthogonal eigenvectors and which don't?

(d) (9 points) Explain why any two choices of a lead to matrices A that are *similar* (with the same Jordan form).

3 (22 pts.) Suppose the 3 by 3 matrix A has independent eigenvectors in $Ax_1 = \lambda_1 x_1$, $Ax_2 = \lambda_2 x_2$, $Ax_3 = \lambda_3 x_3$. (Those λ's might not be different.)

(a) (11 points) Describe the general form of every solution $u(t)$ to the differential equation $\frac{du}{dt} = Au$. (The answer $e^{At}u(0)$ does not use the λ's and x's.)

(b) (11 points) Starting from any vector u_0 in $\mathbf{R}^3$, suppose $u_{k+1} = Au_k$. What are the conditions on the x's and λ's to guarantee that $u_k \to 0$ (as $k \to \infty$)? Why?