

18.06 Professor A.J. de Jong Exam 1 March 3, 2003

Your name is: \_\_\_\_\_

Please circle your recitation:

**1 (30 pts.)**

(a) Compute the following matrix product

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ -5 & -4 & -3 & -2 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{pmatrix}$$

No explanation is necessary.

(b) Let  $U$  be the matrix below. Reduce  $U$  to a reduced row echelon matrix by row operations (upward elimination). Find the “special solutions” to  $Ux = 0$ . Also give an expression for the general solution to  $Ux = 0$ .

$$U = \begin{pmatrix} 1 & 1 & 1 & -2 & 0 \\ 0 & 0 & 1 & 7 & 5 \\ 0 & 0 & 0 & 0 & 7 \end{pmatrix}$$

**2 (35 pts.)**

- (a) Let  $A$  and  $b$  be as below. For any real number  $t$ , and any real number  $s$ : Find the complete solution to the equation  $Ax = b$  using the algorithm described in class and in the book. (It depends on  $t$  and  $s$ .)

$$A = \begin{pmatrix} 1 & 0 & 0 & 4 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 3 & t \end{pmatrix} \text{ and } b = \begin{pmatrix} 2 \\ 0 \\ 0 \\ s \end{pmatrix}$$

- (b) First part: For which  $t$  are the columns of the matrix  $A$  linearly dependent? Second part: Consider  $b$  and the first three columns of  $A$ . For which  $s$  are these linearly dependent?

**3 (35 pts.)** The elimination algorithm explained in the course (with “row swapping after Gaussian elimination”) was applied to the matrix  $A$ . Suppose it yields the following equality:

$$\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 11 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 10 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$

- (a) Which row operations do the four elimination matrices in the product correspond to? Please write them down in words in the order in which they were performed on  $A$ . Why is the upper left hand corner of  $A$  zero? (This is the  $(1, 1)$  entry of  $A$ .)
- (b) The equation implies that  $A$  factors as  $A = LPUR$ . Here  $R$  is the matrix on the right hand side of the  $=$  sign. The matrices  $U$ ,  $P$ , and  $L$  are invertible  $4 \times 4$  matrices. The matrix  $U$  is upper triangular. The matrix  $P$  is a permutation matrix. And  $L$  is lower triangular. Find  $U$ ,  $P$ , and  $L$ , and explain how you got them.