8 October 1997 **Profs. S. Lee and A. Kirillov**

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Hour Exam I for Course 18.06: Linear Algebra

Recitation Instructor: Your Name:

Recitation Time: Lecturer:

Grading

1.

2.

3.

4.

TOTAL:

Do all your work on these pages.
No calculators or notes.
Please work carefully, and check your intermediate results whenever possible.
Point values (total of 100) are marked on the left margin.

1. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 10 \\ 3 & 6 & 10 \end{bmatrix}$.

[16] **1a.** Give an LU-factorization of A.

[8] **1b.** Give a basis for the column space of A.

[8] **1c.** Give a basis for the nullspace of A.

[8] **1d.** Give the complete solution to $Ax = \begin{bmatrix} 3 \\ 4 \\ 7 \\ 7 \end{bmatrix}$. Recall that $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 10 \\ 3 & 6 & 10 \end{bmatrix}$.

[8] **2a.** If possible, give a matrix A which has $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ as a basis for the column space, and $\begin{bmatrix} 0 \\ 3 \\ 2 \\ -1 \end{bmatrix}$ as a basis for its row space. If not possible, give your reason.

[8] **2b.** Are the vectors $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$ a basis for the vector space $\mathbf{R}^3$?
(More than 'yes' or 'no' is needed for full credit.)
Show your work, then briefly explain your answer.

[16] **3.** Given that $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix} A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$, find A^{-1}.

4. Suppose $x = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$ is the only solution to $Ax = \begin{bmatrix} 1 \\ 3 \\ 5 \\ 7 \\ 9 \end{bmatrix}$.

[12] **4a.** Fill in each (blank) with a number.
The columns of A span a ____(blank)____-dimensional subspace of the vector space $\mathbf{R}^{(\text{blank})}$.

[16] **4b.** After applying elementary row operations to A, the <u>reduced</u> row echelon form will be $R =$ _____(give the matrix)_____.