5 November 1997 **Profs. S. Lee and A. Kirillov**

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Hour Exam II for Course 18.06: Linear Algebra

Recitation Instructor: Your Name:

Recitation Time: Lecturer:

Grading

1.

2.

3.

4.

TOTAL:

Do all your work on these pages.
No calculators or notes.
Please work carefully, and check your intermediate results whenever possible.
Point values (total of 100) are marked on the left margin.

[10] **1a.** Give a vector $\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$ that makes $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 5 \\ 11 \\ -8 \end{bmatrix}, \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$ an orthogonal basis for the vector space $\mathbf{R}^3$.

[10] **1b.** Given that $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix} A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 3 \\ 0 & 3 & 0 \end{bmatrix}$, find $\det(A)$.

[12] **1c.** Can you find a matrix A such that $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is a basis for the <u>left</u> nullspace of A and $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is a basis for the nullspace of A?

If 'yes', give a matrix A.

If 'no', briefly explain why the matrix A cannot exist.

[16] **2.** Find an orthogonal basis for the column space of $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 4 & 0 \\ 1 & 4 & 6 \\ 1 & 4 & 6 \end{bmatrix}$.

3. Let $A = \begin{bmatrix} 1 & -2 & -5 & 1 \\ 2 & -4 & -10 & 3 \end{bmatrix}$.

[16] **3a.** Find the solution to $Ax = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ that is closest to $\begin{bmatrix} 5 \\ 5 \\ 11 \\ 11 \end{bmatrix}$.

[12] **3b.** Give an ortho<u>normal</u> basis for the nullspace of $A = \begin{bmatrix} 1 & -2 & -5 & 1 \\ 2 & -4 & -10 & 3 \end{bmatrix}$.

4. Let $A = \begin{bmatrix} 5 & -12 \\ 2 & -5 \end{bmatrix}$.

[8] **4a.** Find the eigenvalues of A.

[8] **4b.** Find an eigenvector for each eigenvalue of A.

[8] **4c.** Find A^{99}. Recall that $A = \begin{bmatrix} 5 & -12 \\ 2 & -5 \end{bmatrix}$.