18.06 Professor Edelman Quiz 2 October 23, 1998

Your name is ____.

This exam is harder than quiz 1, but in many cases there is an easy solution to the problem that does not require much work.

1. (a.) (20 pts) Find the dimensions and bases for the four fundamental subspaces of

$$M = \begin{bmatrix} 0 & 1 & 2 & 0 & 4 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

1. (b.) (10 pts) Find orthonormal bases for the same four subspaces.

2. **(a.)** **(10 pts)** Find the QR factorization of

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}.$$

(b.) **(10 pts)** What is A^{-1}?

3. (20 pts) Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$ and $P = A(A^T A)^{-1} A^T = \frac{1}{6} \begin{bmatrix} 5 & 2 & -1 \\ 2 & 2 & 2 \\ -1 & 2 & 5 \end{bmatrix}$.

What is P^3? (Hint: This problem requires no arithmetic.)

4. (30 pts) Let A be an $n \times n$ matrix

(a.) If the row space of A is R^n then the column space of A is ____________________ ?

(b.) If the nullspace of A is R^n then the column space of A is ____________________ ?

(c.) If the left nullspace of A is R^n then the column space of A is ____________________ ?

The next three questions consider a square matrix A whose column space is orthogonal to the row space.

(d.) Give an example of a square matrix A such that the column space is orthogonal to the row space.

(e.) If the column space of an $n \times n$ matrix A is orthogonal to the row space there is an inequality relating the rank r to n. What is the strongest possible inequality? (Hint: $r \leq n$ is a true inequality, but is not the strongest and hence will be considered an incorrect answer. Only the right answer will be given credit.)

(f.) If the column space is orthogonal to the row space, then $\det(A) =$ ____________________ ?