18.06 Professor Edelman Quiz 3 November 20, 1998

Your name is ____.

1. (a.) (10 pts) Find ALL the eigenvalues and ONE eigenvector of each of the matrices below:

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 5 & 0 \\ -2 & 0 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ -3 & 0 & -2 \end{bmatrix}$$

1. (b.) (10 pts) Find ONLY one eigenvalue of each of the matrices below: (This can be done with no arithmetic.)

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix}$$

2. (20 pts) Let A have eigenvalues $\lambda_1, \ldots, \lambda_n$ (all nonzero) and corresponding eigenvectors $x_1, \ldots, x_n$ forming a basis for R^n. Let C be its cofactor matrix. (The answers to the questions below should be in terms of the λ_i.)

(a) (5 pts) What is trace(A^{-1})? det(A^{-1})?

(b) (15 pts) What is trace(C)? What is det(C)? (Hint: $A^{-1} = \frac{C^T}{\det A}$)

3. (30 pts.) Suppose A is symmetric ($n \times n$) with rank $r = 1$ and one eigenvalue equal to 7. Let the general solution to

$$\frac{du}{dt} = -Au$$

be written as $u(t) = M(t)u(0)$. (Note the minus sign!)

(a) (5 pts.) Write down an expression for $M(t)$ in terms of A and t.

(b) (15 pts.) Is it true that for all t, $\text{trace}(M(t)) \geq \det(M(t))$? Explain your answer by finding all the eigenvalues of $M(t)$.

(c) (5 pts.) Can $u(t)$ blow up when $t \to \infty$? Explain.

(d) (5 pts.) Can $u(t)$ approach 0 when $t \to \infty$? Explain.

4. (30pts.) (a). If B is invertible prove that AB has the same eigenvalues as BA. (Hint: Find a matrix M such that $ABM = MBA$.)

(b). Find a diagonalizable matrix $A \neq 0$ that is similar to $-A$. Also find a nondiagonalizable matrix A that is similar to $-A$.