

LINEAR ALGEBRA IN A NUTSHELL

((*A is n by n*))

Nonsingular

A is invertible

The columns are independent

The rows are independent

The determinant is not zero

$A\mathbf{x}=\mathbf{0}$ has one solution $\mathbf{x}=\mathbf{0}$

$A\mathbf{x}=\mathbf{b}$ has one solution $\mathbf{x}=A^{-1}\mathbf{b}$

A has n (nonzero) pivots

A has full rank $r=n$

The reduced row echelon form is $R=I$

The column space is all of \mathbf{R}^n

The row space is all of \mathbf{R}^n

All eigenvalues are nonzero

$A^T A$ is symmetric positive definite

A has n (positive) singular values

Singular

A is not invertible

The columns are dependent

The rows are dependent

The determinant is zero

$A\mathbf{x}=\mathbf{0}$ has infinitely many solutions

$A\mathbf{x}=\mathbf{b}$ has no solution or infinitely many

A has $r < n$ pivots

A has rank $r < n$

R has at least one zero row

The column space has dimension $r < n$

The row space has dimension $r < n$

Zero is an eigenvalue of A

$A^T A$ is only semidefinite

A has $r < n$ singular values

Each line of the singular column can be made quantitative using r .