18.06 Strang, Edelman, Huhtanen Quiz 1 October 1, 2001

Your name is: ______________________________

1 (32 pts.) Suppose A is the tridiagonal matrix

$$A = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 3 & -1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

(a) Carry out the row elimination to find the upper triangular factor U. (10)

(b) What matrix L yields $A = LU$? (6)

(c) Solve $Ax = b$ with

$$b = \begin{bmatrix} -1 \\ 2 \\ -2 \\ 0 \end{bmatrix}.$$

All components of the solution x happen to be 0's or 1's. What linear combination of the columns of A produces b? (10)

(d) If you change the entry $A_{4,4} = 0$ in the right lower-corner of A to $A_{4,4} =$ ____ the matrix becomes singular. (Hint: look at pivots) (6)

2 (36 pts.) (a) Suppose $A^n = 0$. Show that $(I - A)^{-1} = I + A + A^2 + \cdots + A^{n-1}$. (10)

(b) Assume A and B are commuting matrices (that is, $AB = BA$). If they both are also nonsingular, show that A^{-1} and B^{-1} commute. (10)

(c) Which are true and which false. (Give a good reason!!!)

Let A be an m-by-n matrix. Then $Ax = 0$ has always a non-zero solution if

(i) $\text{rank}(A) < m$ (5)

(ii) $\text{rank}(A) < n$ (5)

(iii) $m = n$ and $A^2 = 0$ (6)

3 (32 pts.) Suppose after elimination on a matrix A we reach its row reduced echelon form

$$R = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

(a) Find the null space matrix of A. (10)

(b) What is the null space of A^T? (6)

(c) What is the rank of 2-by-9 block matrix $[\ A\ A\ A\ ]$? (6)

(d) Find a complete solution to $Rx = d$ with $d = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. (10)