

## 18.06 Midterm Exam 1, Spring, 2001

Name \_\_\_\_\_

Optional Code \_\_\_\_\_

Recitation Instructor \_\_\_\_\_

Email Address \_\_\_\_\_

Recitation Time \_\_\_\_\_

This midterm is closed book and closed notes. No calculators, laptops, cell phones or other electronic devices may be used during the exam.

There are 3 problems. Good luck.

1. (20pts.) Find a general formula for the solutions of the following linear system of equations,

$$\begin{array}{ccccccc} -x_1 & +3x_2 & & +2x_4 & = & 1 & \\ 4x_2 & -12x_2 & +2x_3 & -4x_4 & = & -4 & \\ -7x_1 & +21x_2 & +2x_3 & +18x_4 & = & 7 & \end{array}$$

2. (40pts.) Let  $A = \begin{pmatrix} 1 & 1 & b \\ a & b & b-a \\ 1 & 1 & 0 \end{pmatrix}$ .

- (a) For  $a = 2$  and  $b = 1$ , find the inverse of  $A$ .
- (b) For which values of  $a$  and  $b$  is the matrix  $A$  not invertible, i.e. it has less than three pivots?
- (c) For what values of  $a$  and  $b$  is the rank of  $A$  equal to 3 ? For what values is it equal to 2, equal to 1 ?
- (d) For  $a = b = 2$ , describe the nullspace of  $A$ .

3. (40pts.) Let  $A = \begin{pmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}$ .

- (a) For what vectors  $\mathbf{b} = (b_1, b_2, b_3)^T$  does the linear system  $A\mathbf{x} = \mathbf{b}$  have a solution?
- (b) Prove that the column space of  $A$  is made up of those vectors  $(x, y, z)^T \in \mathbb{R}^3$  that satisfy  $x + y + z = 0$ .
- (c) Prove that the vectors  $(x, y, z)^T \in \mathbb{R}^3$  that satisfy  $x + y + z = c$  form a subspace of  $\mathbb{R}^3$  if and only if  $c = 0$ .