18.06 Strang, Edelman, Huhtanen Quiz 2 November 5, 2001

Your name is: ______________________________

1 (36 pts.) Let A be the square matrix

$$A = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} q_1^T + \begin{bmatrix} -1 \\ a \\ -1 \end{bmatrix} q_2^T,$$

where q_1 and q_2 are orthonormal vectors in $\mathbf{R}^3$. (12p)

(a) Find x such that

$$Ax = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

(b) Choose a such that the column space of A has dimension 1. (8p)

(c) If

$$q_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \text{ and } q_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

and $a = 0$, solve

$$Ay = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

in the least squares sense. (16p)

2 (28 pts.) Let

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}.$$

(a) By Gram-Schmidt, factor A into QR where Q is orthogonal and R is upper triangular. (16p such that 10p from Q and 6p from R)

(b) Find the inverse of R and then give the inverse of A by using $A = QR$. (12p such that 4p from R^{-1} and 4p from Q^{-1} and 4p from A^{-1})

3 (36 pts.) (a) Let u, v and w be linearly independent. How is the matrix A with columns u, v, w related to the matrix B with columns $u+v$, $u-v$, $u-2v+w$? Show that those three columns are linearly independent. (12p)

(b) Using Cramer's rule, find b_3 such that $x_3 = 0$ for the solution of

$$\begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & 1 \\ 1 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ b_3 \end{bmatrix}.$$

(12p)

(c) Using rules for the determinant (so do not compute it with any of the 3 formulas), show the steps and rules that lead to

$$\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} = 0$$

(12p)