18.06 Professor Strang/Ingerman Quiz 1 September 27, 2002

Your name is: ______________________________

1 (30 pts.) Start with the vectors

$$\boldsymbol{u} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} \quad \text{and} \quad \boldsymbol{v} = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}$$

(a) Find two other vectors $\boldsymbol{w}$ and $\boldsymbol{z}$ whose linear combinations fill the same plane P as the linear combinations of $\boldsymbol{u}$ and $\boldsymbol{v}$.

(b) Find a 3 by 3 matrix M whose *column space* is that same plane P.

(c) Describe all vectors $\boldsymbol{x}$ in the nullspace ($M\boldsymbol{x} = \mathbf{0}$) of your matrix M.

2 (30 pts.) (a) By elimination put A into its upper triangular form U. Which are the pivot columns and free columns?

$$A = \begin{bmatrix} 1 & 3 & 2 & 1 \\ 2 & 8 & 5 & 2 \\ 1 & 5 & 3 & 1 \end{bmatrix}$$

(b) Describe specifically the vectors in the nullspace of A. One way is to find the "special solutions" (*how many*??) to $A\boldsymbol{x} = \mathbf{0}$ by setting the free variables to 1 or 0.

(c) Does $A\boldsymbol{x} = \boldsymbol{b}$ have a solution for the right side $\boldsymbol{b} = (3, 8, 5)$? If it does, find one particular solution and then the complete solution to this system $A\boldsymbol{x} = \boldsymbol{b}$.

3 (40 pts.) (a) Apply row elimination to A and find the pivots and the upper triangular U. Factor this "Pascal matrix" into L times U.

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 1 & 4 & 10 & 20 \end{bmatrix}$$

(b) How do L and U and the pivots confirm that A is invertible?

(c) If you change the entry "20" to what number (??) then A will become singular.

(d) What permutation matrix P will multiply A so that the rows of PA are in reverse order (rows $1, 2, 3, 4$ of A become rows $4, 3, 2, 1$ of PA)? What matrix multiplication would put the *columns* in reverse order?