18.06 Professor Strang Quiz 1 March 6, 1998

Your name is: ____________________

1. Suppose the complete solution to the equation

$$Ax = \begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix} \quad \text{is} \quad x = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} + c \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + d \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

(6) (a) What is the dimension of the row space of A?

(12) (b) What is the matrix A?

(6) (c) Describe exactly all the vectors b for which $Ax = b$ can be solved. (Don't just say that b must be in the column space.)

ANSWER BELOW AND ON THE NEXT PAGE

2. Suppose the matrix A is this product BC (not L times U!):

$$A = BC = \begin{bmatrix} 2 & 0 & 0 \\ 3 & 4 & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 2 & 4 & 4 \\ 0 & 0 & 0 & 0 \\ 2 & 2 & 6 & 6 \end{bmatrix}$$

(16) (a) Find bases for the row space and the column space of A.

(8) (b) Find a basis for the space of all solutions to $Ax = 0$.

(8) (c) All these answers will be different if you correctly change one entry in the first factor B. Tell me the new matrix B.

3. **(12) (a)** Find the row-reduced echelon form R of A and also the inverse matrix E^{-1} that produces $A = E^{-1}R$.

$$A = \begin{bmatrix} 1 & 0 & 3 & 3 \\ 2 & 0 & 6 & 6 \\ 1 & 1 & 3 & 3 \end{bmatrix}. \qquad \text{Find } R \text{ and } E^{-1}.$$

(9) (b) Separate that multiplication $E^{-1}R$ into columns of E^{-1} times rows of R. This allows you to write A as the sum of *two rank-one matrices.* What are those two matrices?

4. **(16) (a)** Suppose A is an m by n matrix of rank r. Describe exactly the matrix Z (its shape and all its entries) that comes from transposing the row echelon form of R' (prime means transpose):

$$Z = \text{rref}(\text{rref}(A)')'\,.$$

(7) (b) Compare Z in Problem 4a with the matrix ZZ that comes from starting with the transpose of A (and not transposing at the end):

$$ZZ = \text{rref}(\text{rref}(A')')\,.$$

Explain in one sentence why ZZ is or is not equal to Z.