18.06   Professor Strang   Quiz 1   March 1, 2004

Your name is: _____

Please circle your recitation:

1 (30 pts.)   For the system $Ax = b$ given by

$$\begin{bmatrix} 1 & 0 & 4 \\ 2 & 1 & 10 \\ 3 & 1 & c \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 14 \\ 20 \end{bmatrix}$$

(a) Find the value of c that makes the matrix A not invertible. *Use this value of c in parts (b) and (c).*

(b) Find the complete solution to $Ax = b$.

(c) Describe **EITHER** the column picture for the three columns of A and the column vector b, **OR** the row picture for the three equations in $Ax = b$.

2 (30 pts.) Suppose A has reduced echelon form R,

$$A = \begin{bmatrix} 1 & 2 & 1 & b \\ 2 & a & 1 & 8 \\ (\text{row 3 of } A) \end{bmatrix}, \qquad R = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

(a) What can you say about row 3 of A?

(b) What are the numbers a and b?

(c) Describe the nullspace of A.

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3 (20 pts.) Suppose you have 4 column vectors u, v, w, z in 3-dimensional space $\mathbf{R}^3$.

(a) Give an example where the column space of a matrix A contains u, v, w but not z. [Tell us u, v, w, z, and A.]

(b) What are the dimensions of the column space and the nullspace of your A?

4 (20 pts.) (a) Factor this matrix A into LU (lower triangular times upper triangular)

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 4 & 5 & 2 \\ 4 & 6 & 0 \end{bmatrix}$$

(b) Is A invertible?   YES   NO