

Your name is: _____

Grading 1
2
3
4

- 1 (25 pts.) Suppose that row operations (elimination) reduce the matrices A and B to the same row echelon form

$$R = \begin{bmatrix} 1 & 2 & 0 & 7 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix} .$$

- (a) Which of the four subspaces are sure to be the same for A and B ? ($C(A) = C(B)$? $N(A) = N(B)$? $C(A^T) = C(B^T)$? $N(A^T) = N(B^T)$?)
- (b) Each time the subspaces in part (a) are the same for A and B , find a basis for that subspace.
- (c) True or False (A is any matrix and x, y are two vectors): If Ax and Ay are linearly independent then x and y are linearly independent.

2 (25 pts.) Suppose

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 7 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 4 & 5 \\ 0 & 1 & 2 & 2 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

- (a) Find a basis for the nullspace of A .
- (b) Find a basis for the column space of A .
- (c) Give the complete solution to

$$Ax = \begin{bmatrix} 3 \\ 3 \\ 21 \end{bmatrix} .$$

- 3 (25 pts.)** Suppose A is a 3×5 matrix and the solutions to $A^T y = 0$ are spanned by the vectors

$$y = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}.$$

- (a) What is the rank of this A ?
- (b) For all A , why does the rank of A equal the rank of the block matrix

$$B = \begin{bmatrix} A & A \\ A & A \end{bmatrix}?$$

- (c) If the rank of a matrix A equals the number of rows ($r = m$), what do we know about the equation $Ax = b$?

4 (25 pts.) Suppose A is a 4 by 3 matrix, and the complete solution to

$$Ax = \begin{bmatrix} 1 \\ 4 \\ 1 \\ 1 \end{bmatrix} \quad \text{is} \quad x = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + c_1 \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} .$$

- (a) What is the third column of A ?
- (b) What is the second column of A ?
- (c) Give all known information about the first column of A .